On the structure of tidal tails

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Accepted 2008 April 10. Received 2008 March 27; in original form 2008 February 1

ABSTRACT

We examine the longitudinal distribution of the stars escaping from a cluster along tidal tails. Using both theory and simulations, we show that, even in the case of a star cluster in a circular galactic orbit, when the tide is steady, the distribution exhibits maxima at a distance of many tidal radii from the cluster.

Key words: methods: analytical – methods: N-body simulations – galaxies: kinematics and dynamics – galaxies: star clusters.

1 INTRODUCTION

Stars escaping from a star cluster in the gravitational field of a galaxy form extended tidal tails. These have been observed in the Milky Way (Grillmair et al. 1995; Kharchenko, Scholz & Lehm 1997; Lehmann & Scholz 1997; Leon, Meylan & Combes 2000; Odenkirchen et al. 2001; Rockosi et al. 2002; Lee et al. 2003; Belokurov et al. 2006) and have often been modelled (Combes, Leon & Meylan 1999; Johnston, Sigurdsson & Hernquist 1999; Yim & Lee 2002; Odenkirchen et al. 2003; Dehnen et al. 2004; Koch et al. 2004; Lee et al. 2004; Capuzzo Dolcetta, Di Matteo & Miocchi 2005; di Matteo, Capuzzo Dolcetta & Miocchi 2005; Chumak & Rastorguev 2006; Lee, Lee & Sung 2006; Choi, Weinberg & Katz 2007; Fellhauer et al. 2007; Montuori et al. 2007). They exhibit significant longitudinal structure, that is clumps, or over- and underdensities, which are usually attributed to the influence of gravitational shocks, for example encounters with spiral arms or giant molecular clouds, or passages through a galactic disc or past a galactic bulge, or simply motion in a triaxial potential. Here, we show that clumps also arise when the tidal field is static, in the simplest case of a star cluster in a circular orbit about an axisymmetric galactic potential. In the next section, we describe a theoretical approach to the problem, which we verify in N-body calculations in Section 3. The final section is a slightly extended summary with discussion.

2 THEORY

As in standard theory (Chandrasekhar 1942), we describe the position of a star in a rotating, accelerated coordinate system with origin at the centre of the cluster, and axes aligned towards the galactic anticentre, in the direction of galactic motion of the cluster, and orthogonal to the plane of motion of the cluster, respectively. Then

the equations of motion are

\[ \ddot{x} - 2\Omega \dot{y} + (\kappa^2 - 4\Omega^2)x = -U_x, \]  

\[ \ddot{y} + 2\Omega \dot{x} = -U_y, \]  

and

\[ \ddot{z} + \omega^2 z = -U_z, \]  

where \( \Omega \) is the angular velocity of the cluster around the galaxy, \( \kappa \) is the epicyclic frequency, \( \omega \) is the frequency of motions orthogonal to the plane of motion of the cluster, and \( U \) is the gravitational potential of the cluster stars.

We are interested in escaping stars, and so we shall adopt a point-mass approximation for \( U \). We let \( U = -GM/r \), where \( G \) is the gravitational constant, \( M \) is the total mass of the cluster and \( r = \sqrt{x^2 + y^2 + z^2} \). There are equilibria at the Lagrangian points \((\pm \alpha_1, 0, 0)\), where

\[ x_1 = \left( \frac{GM}{4\Omega^2 - \kappa^2} \right)^{1/3}. \]

The total effective energy of a star (per unit mass) is

\[ E = \frac{1}{2} v^2 + \Phi, \]

where \( v \) is the star’s speed and the effective potential energy is

\[ \Phi = \frac{1}{2} \left[ (\kappa^2 - 4\Omega^2)x^2 + \omega^2 z^2 \right] - \frac{GM}{r}. \]

The Lagrangian points are saddle points of \( \Phi \), where its value is \( \Phi_\perp = -\frac{3}{2} \frac{GM}{r_\perp} \). This is also the escape energy from the potential well of the cluster in the galactic field.

Except for high-speed escapers created in few-body encounters, stars escape as a result of two-body encounters and then an escaper usually has an energy only slightly above the escape energy \( \Phi_\perp \) (Hayli 1970). Therefore, such escapers pass close to one of the Lagrange points at slow speed. Thereafter, the acceleration due to the cluster rapidly diminishes, and the motion of the escaping star...
can be well approximated by equations (1)–(3) with the right-hand side set equal to zero. The solution starting at time \( t = 0 \) at position \((x_L, 0, 0)\) with zero velocity is then easily found to be

\[
x = \frac{4\Omega^2}{\kappa} x_L + \left(1 - \frac{4\Omega^2}{\kappa^2}\right) x_L \cos \kappa t \tag{4}
\]

\[
y = -\frac{2\Omega}{\kappa} \left(1 - \frac{4\Omega^2}{\kappa^2}\right) x_L (\sin \kappa t - \kappa t) \tag{5}
\]

\[z = 0. \tag{6}\]

The solution for the stars starting at \((-x_L, 0, 0)\) can be found as easily.

After leaving the Lagrange point, the solution traces an oscillatory path along the tidal tail and comes to rest again at the cusp of the orbit, which is reached when

\[t = t_c = 2\pi/\kappa. \tag{7}\]

At this time the location of the star is \(y = -y_c\), where

\[y_c = -\frac{4\pi \Omega}{\kappa} \left(1 - \frac{4\Omega^2}{\kappa^2}\right) x_L.\]

Now let us suppose that there is a stream of stars following this motion. Because the \(y\)-component of velocity is zero at \(y = -y_c\), there is a peak (actually, an infinite peak) in the distribution of the values of \(y\) at this location. In the evolution of a star cluster, however, it will not be visible until the first escapers reach this location, at around time \(t_c\). Furthermore, as the star cluster loses mass by escape, \(x_L\) decreases, and the location of this peak moves closer to the cluster as the cluster dissolves. The existence of this peak and the time when it begins to become apparent are the two main predictions of this theory.

Now we discuss the extent to which the solution we have used is a satisfactory approximation to the motion of slow escapers from a star cluster. There are two factors to consider.

(i) The effect of neglecting the field of the cluster is shown in Fig. 1. What will be important in what follows is the first cusp after the star leaves the Lagrange point, and it is clear that this is well approximated.

(ii) Our other main approximation is the assumption that the star leaves the Lagrange point with a very small velocity. This cannot be tested thoroughly without recourse to an \(N\)-body simulation, which we postpone to Section 3. But we can develop a feel for the result by changing the initial conditions, for example by adding a small initial value to \(y\), of order \(x_L\). Then the cusp is displaced in \(y\) by the same amount, so that the relative displacement of the cusp is

\[
\frac{x_L}{y_c} = \frac{1}{4\pi \Omega (4\Omega^2 - \kappa^2)},
\]

which is \(1/(12\pi)\) in the case of a point-mass potential in which \(\Omega = \kappa\). While this is not large, it shows that the density enhancement at \(y \simeq -y_c\) will be smoothed out. Similar results are obtained if we vary other initial conditions appropriately, though usually the cusp disappears, and may be replaced by a small epicyclic loop. This calculation also shows that the distance of the density enhancement from the star cluster is of order \(12\pi\) tidal radii, again in the case of a point-mass galactic potential.

Now we consider the number of stars between the cluster and the first density enhancement (near \(y = -y_c\)). This is occupied by stars which escape in a time interval of order \(t_c\). If we assume that a constant fraction, \(\mu\), of stars escapes in each half-mass relaxation time (see, for example, Spitzer 1987), the relevant number of escaping stars is

\[N_{\text{esc}} = \mu N \frac{t_c}{t_{\text{fh}}} \tag{8}\]

where

\[t_{\text{fh}} = \frac{0.138 N^{1/2} r_h^{3/2}}{G^{1/2} m^{1/2} \log(y N)}, \tag{9}\]

in which \(r_h\) is the half-mass radius, \(N\) is the number of stars, \(m\) is the (mean) stellar mass, and \(y\) is a numerical constant of order unity. It follows that

\[N_{\text{esc}} = \frac{2\pi \mu}{0.138} \log(y N) \left(\frac{x_L}{r_h}\right)^{3/2} \left(\frac{4\Omega^2}{\kappa^2} - 1\right)^{1/2}. \tag{10}\]

The point of this result is not so much to furnish a numerical estimate, but to point out that it is only weakly dependent on \(N\). In this sense, a single numerical simulation with large \(N\) is not more useful for investigating the distribution along a tidal tail than a single small simulation. However, relaxation proceeds by the cumulative effect of many small encounters, and the mean change (in the energy of a star) per encounter decreases as \(N\) increases. Therefore, the assumption of zero velocity at the moment when the star crosses the tidal radius becomes increasingly accurate as \(N\) increases, and we may expect that the density enhancements become increasingly pronounced in this limit.

### 3 Numerical Verification

The numerical simulations used to verify the existence of the predicted substructure were carried out with the collisional \(N\)-body code \textsc{Nbody6} by Sverre Aarseth (Aarseth 1999) on the 52-processor Sun Fire E15k at the Edinburgh Parallel Computing Centre. We used the supercomputer as a work farm to compute a set of 64 renditions of our test model, which were then co-added to gain statistical accuracy.
Figure 2. Time evolution of the number density of stars along the tidal tails. Since the leading and trailing tail are symmetric they are folded together. Time is given in Myr. As expected the overdensities need more than $t_C$ to build up; they are most pronounced at about $2 - 3t_C$. The predicted location of $y_C$ agrees fairly well with the results of the simulations. At $4t_C$ ($\approx 1000$ Myr) the cluster has lost about 70 per cent of its mass, and hence $y_C$ moves closer to the cluster centre. Also the flux of escaping stars diminishes. At 2000 Myr the cluster has dissolved almost completely.

The test model is a cluster of 1000 stars of each $1\,M_\odot$, following a Plummer density profile with a half-mass radius of 0.8 pc. The cluster orbits about a point-mass galaxy of $M_G = 9.565 \times 10^{10}\,M_\odot$ at a galactic distance of $R_G = 8.5$ kpc, which corresponds to a rotational velocity of $v_{\text{rot}} = 220$ km s$^{-1}$.

In this particular case of a point-mass potential, the tidal radius, $x_L$, is given by (Spitzer 1987)

$$x_L = R_G \left(\frac{M}{3M_G}\right)^{1/3},$$

which yields an initial value of 12.9 pc for the above given numbers. Therefore, the cusps of the orbits of escaping stars are initially located at $\pm y_C$, where

$$y_C = 12\pi x_L \approx 490 \text{ pc},$$

but this decreases with the ongoing loss of cluster mass. The epicyclic frequency of the cluster’s orbit about the galactic centre is given by

$$\kappa = \Omega = v_{\text{rad}}/R_G = 0.026 \text{ Myr}^{-1}.$$  

Hence the time the first stars need to reach this first cusp, $t_C$, is about 240 Myr. Before this time, we do not expect any overdensity within the growing tidal tails.

The simulations were conducted until total dissolution, which occurred at about 2000 Myr. For several times the positions of the stars were first centred on the density centre of the remaining cluster stars within the tidal radius. This was done for each rendition before they were combined. Then the number density of stars along the direction of the tidal tails, that is the $y$-axis, was counted in bins of 25 pc.

In Fig. 2, the time evolution of the number density of stars along the tidal tails is shown. Because of the symmetry of the leading and trailing tail they have been folded together to increase the statistical accuracy. As predicted in Section 2, the first stars need at least the time $t_C$ to reach the first cusp of their oscillatory orbit along the tail at $y_C$. At $2t_C$, the overdensity has built up completely and has become very pronounced. As time goes by, the loss of mass causes a decrease in $x_L$, and by equation (2) the location of the overdensity moves closer to the cluster. At $4t_C$ ($\approx 1000$ Myr), the...
Figure 3. The same as Fig. 2, but here the mean speed of stars outside the cluster (given in km s\(^{-1}\)) is shown. In the beginning, a few high-velocity escapers dominate the outer parts. As time goes by the speed profile clearly shows the signature of the epicyclic motion described in Section 2 (i.e. the dip at about 300 pc at \(t = 1250\)). Furthermore, the low speed close to the cluster verifies the approximation that stars leave the cluster with low velocities. A similar plot is given in Capuzzo Dolcetta et al. (2005).

In Fig. 3, the evolution of the mean speed outside the cluster is shown. The velocity change as described in Section 2 is fairly well reproduced. In the beginning the mean speed is dominated by a few high-velocity escapers. But after building up, the overdensities correspond to a minimum in the mean speed while maxima correspond to underdensities. Here again the second peak is visible. Furthermore, the initial rise of speed verifies the assumption that most stars leave the cluster with small velocities.

Figure 4 shows a snapshot of the surface density along the tidal tails after 1250 Myr. In both \(x - y\) and \(y - z\) plane the over- and underdensities are fairly pronounced including (especially in the upper diagram) the additional overdensities at \(\pm 2y_C\).

4 CONCLUSIONS

We have considered the longitudinal structure of the tidal tails generated by stars escaping from a star cluster in a circular galactic orbit. Even though the tidal field is steady (in a rotating frame in which the cluster centre is at rest), the tails exhibit density enhancements. These correspond to places where escaping stars slow down in their epicyclic motion away from the star cluster. Because stars escape with a somewhat limited range of initial positions and velocities (when they cross the tidal boundary) the location at which this happens is similar for most escapers, and the resulting density enhancement is visible in simulations provided that sufficient numbers of simulations are co-added to improve the sampling statistics. We have conducted such simulations, using stars of equal mass. The distance of the main clump is around 38 tidal radii, in the case of a galaxy modelled as a central point mass.

Similar clumps have been observed by Capuzzo Dolcetta et al. (2005), but those examples were a little ambiguous because (i) the galactic potential was triaxial, (ii) the orbit was not quite circular,
and (iii) the code was collisionless. For these reasons, the mechanism of escape in their model was not clear, and could include an unquantified episodic contribution from the time-dependence of the tidal field. Nevertheless, they showed that the clumps coincide with places where the escaping stars slow down. The main additional contribution of the present paper is that we have identified the mechanism causing those overdensities, by placing the problem in a more idealized setting and providing a theoretical explanation. Whether such density enhancements would be visible in the tidal tails of real star clusters is complicated by their mass spectrum and, especially for globular clusters, by the non-circularity of their orbits. Nevertheless, our results show that the density enhancements in tidal tails, which are apparently observed, should not necessarily be taken to indicate time-dependent tides, or encounters with perturbers such as giant-molecular clouds, dark-matter clumps or spiral arms.

ACKNOWLEDGMENTS

AM thanks the School of Mathematics at the University of Edinburgh for provision of a summer vacation scholarship, funded by the William Manson Prize, when this research was begun in 2006. The simulations reported here were carried out by AHWK while a visitor to Edinburgh under the HPC-EUROPA project (RII3-CT-2003-506079), with the support of the European Community - Research Infrastructure Action under the FP6 “Structuring the European Research Area” Programme.

The authors would like to thank the referee for fruitful suggestions.

REFERENCES

di Matteo P., Capuzzo Dolcetta R., Miocchi P., 2005, CeMDA, 91, 59
Sohn Y.-J. et al., 2003, AJ, 126, 803
Yim K.-J., Lee H. M., 2002, JKAS, 35, 75

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