Environment and mass dependencies of galactic $\lambda$ spin parameter: cosmological simulations and observed galaxies compared

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ABSTRACT

We use a sample of galaxies from the Sloan Digital Sky Survey (SDSS) to search for correlations between the $\lambda$ spin parameter and the environment and mass of galaxies. In order to calculate the total value of $\lambda$ for each observed galaxy, we employed a simple model of the dynamical structure of the galaxies, which allows a rough estimate of the value of $\lambda$, using only readily obtainable observables from the luminous galaxies. Use of a large volume-limited sample (upwards of 11 000) allows reliable inferences of mean values and dispersions of $\lambda$ distributions. We find, in agreement with some N-body cosmological simulations, no significant dependence of $\lambda$ on the environmental density of the galaxies. For the case of mass, our results show a marked correlation with $\lambda$, in the sense that low-mass galaxies present both higher mean values of $\lambda$ and associated dispersions, than high-mass galaxies. These results provide interesting constrain on the mechanisms of galaxy formation and acquisition of angular momentum, a valuable test for cosmological models.

Key words: galaxies: formation – galaxies: fundamental parameters – galaxies: statistics – galaxies: structure – cosmology: observations.

1 INTRODUCTION

One of the most studied parameters in numerical simulations of formation and evolution of galaxies is the $\lambda$ spin parameter, first introduced by Peebles in 1969, where it was used to test the gravitational instability picture as a theory for the origin of galaxies. Since then, there have been several studies using the $\lambda$ parameter as an indispensable tool of analysis, or characterizing its distribution in cosmological N-body simulations. As the first numerical simulations were done, some estimates of mean values of $\lambda$ were obtained (Peebles 1971; Efstathiou & Jones 1979; Blumenthal et al. 1984; Barnes & Efstathiou 1987), as well as probability distributions. In general, the distributions of this parameter are well fitted by a lognormal function, characterized by two parameters; $\lambda_0$, the most probable value, and $\sigma_\lambda$, which accounts for the spread of the distribution. In a recent work, Shaw et al. (2006) showed a compilation of results for estimates of these parameters, arising from numerical studies performed by several authors, lying in the ranges: 0.03 < $\lambda_0$ < 0.05 and 0.48 < $\sigma_\lambda$ < 0.66. Besides numerical simulations, there have been several analytical attempts (Peebles 1969; Doroshkevich 1970; White 1984; Heavens & Peacock 1988) at predicting $\lambda_0$ and $\sigma_\lambda$ with similar results. More recently, Syer, Mao & Mo (1999) confirmed the form of the distribution for $\lambda$ using observational data of 2500 objects, and some of us in a previous work (Hernandez et al. 2007), using the same sample of 11 597 galaxies from the Sloan Digital Sky Survey (SDSS) we treat here, obtained the distribution of this parameter, with results well fitted by a lognormal function with parameters $\lambda_0 = 0.04 \pm 0.005$ and $\sigma_\lambda = 0.51 \pm 0.05$, for the first time derived from a statistical sample of real galaxies. Using the same model, Puech et al. (2007) obtained a similar result using a sample of intermediate-redshift galaxies.

The origin of galactic angular momentum is commonly explained as a result of tidal torques of neighbouring protogalaxies on the forming galactic halo. As the protogalaxy breaks away from the general expansion of the Universe, and since in the general case, it is unlikely to be spherically symmetric, the forming galaxy is torqued up through coupling with the ambient tidal field. This picture appeared with Hoyle in 1949 and was developed at first order analytically where the growth of the angular momentum is proportional to time, and later explored using numerical N-body simulations where the complicated processes of gravitational interactions can be tracked (e.g. Sugerman, Summers & Kamionkowski 2000). Other explanations for the origin of the angular momentum have been proposed, like the model of Vitvitska et al. (2002), in which the haloes obtain their spin through the cumulative acquisition of angular momentum from satellite accretion, obtaining distributions well modelled by lognormal functions with parameters similar to the ones obtained with the tidal torque theory. Variations to the general

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theory have been tested by e.g. Maller, Dekel & Somerville (2002), proposing mainly two scenarios; in the first, the halo spin is generated by the transfer of orbital angular momentum from satellites that merge with the main halo, and a second one where linear tidal-torque theory is applied to shells of infalling matter. The evolution for $\lambda$ at different redshift is completely different for both scenarios, as well as the dependence with mass, where a trend in the tidal-torque scenario is clear, in the sense that more massive galaxies tend to present low $\lambda$ values and low dispersions. The study by Barnes & Efstathiou (1987) revealed a weak trend towards decreasing $\lambda_0$ with increasing mass, confirmed in some N-body simulations (Cole & Lacey 1996; Bett et al. 2007) and not found in others (Maccio et al. 2007).

A direct measure of the real distribution of galactic $\lambda$ distributions, as a function of environment density and mass, would therefore constrain and inform theories of angular momentum acquisition and galaxy formation. It is precisely this that we attempt, therefore constrain and inform theories of angular momentum acquisitions, as a function of environment density and mass, would be established by the transfer of orbital angular momentum from satellites that merge with the main halo. The evolution for $\lambda$ at different redshift is completely different for both scenarios, as well as the dependence with mass, where a trend in the tidal-torque scenario is clear, in the sense that more massive galaxies tend to present low $\lambda$ values and low dispersions.

2 ESTIMATION OF $\lambda$ FROM OBSERVABLE PARAMETERS

The currently accepted picture for galaxy formation is the Lambda cold dark matter ($\Lambda$CDM) model where dark matter overdensities in the expanding Universe at high redshift, accrete baryonic material through their gravitational potential, and via gravitational evolution grow to become galaxies. Their principal integral characteristics, according to theoretical studies, are their mass and angular momentum. In this section we briefly summarize the hypothesis behind the estimates derived in Papers I and II, used to estimate $\lambda$ for the haloes of observed galaxies, from observable parameters. A comparison of our estimate with results coming from numerical simulations is also included. The following are intended only as first-order estimates. The errors for individual galaxies are of the order of 30 per cent (see below), it is only through the use of extensive samples that meaningful inferences on the distribution of $\lambda$ can be derived.

2.1 Estimates of halo $\lambda$ parameters from observed galactic properties

The angular momentum is commonly characterized by the dimensionless angular momentum parameter

$$\lambda = \frac{L}{GM^2},$$

(1)

where $E$, $M$ and $L$ are the total energy, mass and angular momentum of the configuration, respectively. In Paper I we derived a simple estimate of $\lambda$ for disc galaxies in terms of observational parameters, and showed some clear correlations between this parameter and structural parameters, as the disc to bulge ratio, the scale height and the colour, after estimating $\lambda$ for a sample of real galaxies. Here we recall briefly the main ingredients of the simple model. The model consider only two components, a disc for the baryonic component with an exponential surface mass density $\Sigma(r)$:

$$\Sigma(r) = \Sigma_0 e^{-r/R_d},$$

(2)

where $r$ is a radial coordinate and $\Sigma_0$ and $R_d$ are two constants which are allowed to vary from galaxy to galaxy, and a dark matter halo having an isothermal density profile $\rho(r)$, responsible for establishing a rigorously flat rotation curve $V_d$ throughout the entire galaxy:

$$\rho(r) = \frac{1}{4\pi G} \left( \frac{V_d}{r} \right)^2.$$

(3)

We assume that (1) the specific angular momentum of the disc and halo are equal, e.g. Fall & Efstathiou (1980), Mo, Mao & White (1998); (2) the total energy is dominated by that of the halo which is a virialized gravitational structure; (3) the disc mass is a constant fraction of the halo mass $F = M_d/M_H$. These assumptions allow us to express $\lambda$ as

$$\lambda = \frac{2^{1/2} V_d^2 R_d}{GM_H}.$$

(4)

Finally, we introduce a disc Tully–Fisher (TF) relation: $M_d = A_{TF} V_d^3$, and taking the Milky Way as a representative example, we evaluate $F$ and $A_{TF}$ to obtain

$$\lambda = 21.8 \left( \frac{R_d}{\text{kpc}} \right) \left( \frac{V_d}{\text{km s}^{-1}} \right)^{3/2}.$$

(5)

In Paper II, we presented a derivation for an equivalent expression to equation (5) for elliptical galaxies, again using a model of two components: a baryonic matter distribution with a Hernquist density profile and a dark matter halo, in principle, showing no difference from that of disc galaxies (White & Rees 1978; Kahlström 1982). Following this hypothesis, the energy can be obtained from the density profile described by equation (3) assuming again a virialized structure, which allow us to calculate the mass at the half-light radius, $R_0$:

$$M_{\text{dyn}} = \frac{(1.65\sigma)^2 R_0}{G},$$

(6)

from Padmanabhan et al. (2004), where $\sigma$ is the velocity dispersion. For the angular momentum, again we suppose that the specific angular momentum of dark matter and baryons are equal, and using dimensional analysis we expect that it will be proportional to the eccentricity of the system, $e$, and the factor $(GM_0\alpha)^{1/2}$, where $M_0$ is...
the baryonic mass and $a$ is the mayor axis. Introducing a numerical factor to account for the dissipation of the baryonic component, its dynamical pressure support and the projection effects, we finally obtain

$$\lambda = \frac{0.1173(a/kpc)^{2/7}}{(\sigma/km\ s^{-1})^{3/7}}.$$  

(7)

With equations (5) and (7), we can obtain an estimate of $\lambda$ for the halo of any galaxy using just the most basic information available.

2.2 Testing the model against detailed galactic simulations

To check our estimate of $\lambda$, we made use of numerical simulations where this parameter acts as an input parameter, or its value can be estimated with high accuracy.

The model by Hernandez, Avila-Reese & Firmani (2001) includes a very wide range of physics, initial conditions are supplied by an statistical sampling of a cosmological primordial fluctuation spectrum, giving a mass aggregation history of gas and dark matter. No TF type of relation is assumed a priori, indeed, this codes attempt to recover such a relation as a final result of the physics included. A fixed value of $\lambda$ is assumed for the infalling material which settles into a disc in centrifugal equilibrium and differential rotation, viscous friction results in radial flows of matter and angular momentum. The redistribution of mass affects the rotation curve in a self-consistent way, through a Poisson equation including the disc self gravity and a dark halo which responds to mass redistributions through an adiabatic contraction. The star formation is followed in detail, with an energy balance cycle.

A simple population synthesis code then traces the luminosities and colours of various stellar populations at all radii and times. The van den Bosch & Dalcanton (2000) models are qualitatively similar to the previous ones, but vary in numerical approaches, resolution, time-step issues and the level of approximation and inclusion of many different physical aspects of the complicated problem.

Kaufmann et al. (2007) explore the angular momentum transport, disc morphology and how radial profiles depend sensitively on force and mass resolution. They study systematically this effect with controlled N-body/smoothed hydrodynamics simulations of disc galaxy formation by cooling a rotating gaseous mass distribution inside equilibrium cuspy spherical and triaxial dark matter haloes employing up to $10^6$ gas and dark matter particles. Their models are calibrated using the Milky Way and M33 models for the comparisons. Varied and detailed physical effects are introduced self-consistently in their simulations, from the end results of which we use resulting galactic properties to estimate halo $\lambda$ using equation (5), and compare to their input values.

Avila-Reese et al. (2005) performed a $\Lambda$ CDM N-body simulation to study the properties of galaxy-size dark matter haloes as a function of global environment, where $\lambda$ was one of the studied properties of the halo, in particular its value in different environments such as clusters, voids or field. We then used reported values of the final resulting galactic properties, disc scale lengths and rotation curves to estimate the $\lambda$ parameters through equation (5), and compare against the input values used in the simulations.

With the aim of studying the origin of the TF relation, Koda, Sofue & Wada (2000), used a N-body/smoothed particle hydrodynamic method, including cooling, star formation and stellar feedback of energy, mass and metals, to trace the evolution of the galaxies from $z = 25$ to 0 in a CDM cosmology. The resulting spiral galaxies, with exponential disc profiles and generically flat rotation curves, including disc self-gravity and which reproduce the slope and scatter of the TF relation, all these as results of the evolved physics, not input suppositions, were reported.

The work of Okamoto et al. (2005), was performed using hydrodynamic simulations of galaxy formation in a $\Lambda$ CDM universe with different models of star formation and feedback, to see their effect on the morphology of disc galaxies. Also giving as output present day galactic properties from which we estimated halo $\lambda$ through equation (5), and compared against their input model value.

The comparison between the input $\lambda$ values of the galaxies modelled by the different groups ($\lambda_{\text{model}}$), and our estimate using equation (5) is presented in Fig. 1, where the $x$-axis value correspond to the exact value coming from the detailed study of the system, and the $y$-axis value is our estimate using ‘observable’ parameters through equation (5). We can appreciate a very good agreement between the calculated value with our simple model, and the actual value of $\lambda$ for simulated galaxies, regardless of the complexity of the models used. While all the above models include fairly detailed physics, it is remarkable that our simple dimensional estimate agrees so well, always better than 30 per cent, in most cases much better. As no systematics are apparent, we can now use equation (5) with a sample of real galaxies, to obtain statistical properties of the distributions of $\lambda$ in the real universe.

3 COMPARISONS OF $\lambda$ DISTRIBUTIONS FROM THE SDSS AND COSMOLOGICAL N-BODY SIMULATIONS

3.1 Observational sample

The sample of real galaxies employed for the study comes from the SDSS Data Release 5 (Adelman-McCarthy et al. 2007). It is a volume-limited sample having galaxies in the redshifts interval $0.025 < z < 0.055$ and absolute magnitudes $M_r - 5 \log h \leq -18.5$. Since most of the studies concerning spin distributions from simulations presents their results at $z = 0$, we limited the sample to low redshifts. This sample contains 32 550 galaxies for which Choi, Park & Vogeley (2007) have determined the exponential disc scales, absolute magnitudes, velocity dispersions, de Vacouleurs radii and seeing corrected isophotal ellipticities for each galaxy, assuming a
\( \Lambda \)CDM universe with \( \Omega_\Lambda = 0.27, \Omega_m = 0.73 \) and \( h = 0.71 \). To obtain \( \lambda \) for each galaxy, we need to discriminate between elliptical and disc galaxies; in order to do that we used the prescription of Park & Choi (2005), in which early (ellipticals and lenticulars) and late (spirals and irregulars) types are segregated in a \( u - r \) colour versus \( g - i \) colour gradient space and in the concentration index space. In order to apply equation (5) to the disc galaxies of the sample, we need the rotation velocity, which is inferred from the absolute magnitude using a TF relation (Barton et al. 2001), then, to avoid the problem of internal absorption in edge-on galaxies, we employed only spiral galaxies having axis ratios \( 0.6 < R_e < 0.8 \) and inferred rotation velocities in the range of \( 80 < V_e < 430 \), well within the range of applicability of the TF relation we are using. After applying these two cuts, and removing a randomly selected fraction of ellipticals, so as to maintain the original early- to late-type fraction, we are left with a total of 11,597 galaxies.

The measurement of \( R_e \) for our SDSS galaxies comes from the total distribution of light, without any decomposition of the light coming from the bulge or the disc, in this way, systems with prominent bulges, where an important fraction of mass is present in the bulge with low angular momentum, present short \( R_e \) values, which imply a low \( \lambda \) value assigned trough equation (5), accounting implicitly for the low angular momentum of the bulge fraction. We note that of the simulated galaxies in Fig. 1, Avila-Reese et al. (2005) and Koda et al. (2000) include prominent bulge components. It is reassuring of the implicit bulge accounting described above that no systematics are apparent in Fig. 1, regarding either the early-type simulated galaxies, or any other ones. For the elliptical galaxies we apply equation (7), as fully described in Paper II, using the corrected isophotal ellipticities to calculate the eccentricity.

3.2 Simulation and subhalo finding

To compare our observational sample with a numerical simulation, we have made a cosmological N-body simulation of a \( \Lambda \)CDM model in a cubic box with a side length of \( 614 h^{-1} \) Mpc. The model parameters are \( h = 0.7, \Omega_m = 0.27, \Omega_\Lambda = 0.046 \) and \( \Omega_\Lambda = 0.73 \), given by the Wilkinson Microwave Anisotropy Probe 1-yr data (Spergel et al. 2003). The simulation gravitationally evolved \( 10^{14} \) CDM particles from redshift \( z = 48 \) to 0 taking 1880 global time-steps. The linear density fluctuations of matter at the present epoch is normalized by \( \sigma_8 = 0.9 \), the RMS density fluctuation at the \( 8 h^{-1} \) Mpc top-hat smoothing scale. The mass of each simulation particle is \( M_p = 1.4 \times 10^{10} h^{-1} M_\odot \) and the force resolution is about \( 60 h^{-1} \) kpc.

To identify subhalos in the simulation, we first extracted the dark matter particles located in virialized regions by applying the standard friend-of-friend (FOF) method with a linking length equal to one-fifth of the mean particle separation. This length is a characteristic scale for the identification of virialized structures. To each FOF particle group, we applied the PSB method (Kim & Park 2006; Kim, Park & Choi 2008) to find subhalos. The method identifies subhalos that are gravitationally self-bound in terms of the total energy and stable against the external tidal force. The resulting subhalo population is complete down to \( M_h = 4.3 \times 10^{11} h^{-1} M_\odot \). This has been found from a comparison of the subhalo mass function with that of the base mass functions of subhaloes obtained from other higher resolution simulations (Kim & Park, in preparation). The minimum halo mass amounts to a collective mass of 30 particles and corresponds to an early-type galaxy with \( M_e = 19 \) in the subhalo–galaxy correspondence model, which assumes that each subhalo contains one and only one optical galaxy (Kim et al. 2008).

Henceforth, the term halo, when applied to our simulations, will refer to subhalo, as defined above.

For comparisons with our observational results we randomly selected 100,000 haloes from the simulation, for which the spin parameter \( \lambda \) was calculated numerically. The normalized local density parameter is obtained following the same method as used for our SDSS sample (see Section 3.4 for more details, see also Park et al. 2007; Kim et al. 2008).

3.3 Integral properties of the \( \lambda \) distributions

In Fig. 2, we present the distribution of the spin parameter for the SDSS and the simulated samples; the broken curves show histograms after dividing the data into 150 bins, and the dotted curves

![Figure 2](https://example.com/figure2.png)
give the best direct fit to a lognormal distribution of the form
\[
P(\lambda, \sigma_\lambda; \lambda) \, d\lambda = \frac{1}{\sigma_\lambda \sqrt{2\pi}} \exp \left[ -\frac{\ln^2(\lambda / \lambda_0)}{2\sigma_\lambda^2} \right] \, d\lambda. \tag{8}
\]

We can appreciate that the distributions are well fitted by this function, where the corresponding values from the SDSS and the simulation are: \(\lambda_0 = 0.046, \sigma_\lambda = 0.535\) and \(\lambda_0 = 0.038, \sigma_\lambda = 0.525\), both results in agreement with previous predictions from N-body simulations (from a recent compilation for \(\lambda_0\) and \(\sigma_\lambda\) values, see Shaw et al. 2006).

The empirical scaling relations used for our estimation of \(\lambda\) show a measure of dispersion. In the case that these dispersions were intrinsic to the data, they would propagate as errors in our \(\lambda\) estimate. In order to estimate what is the best underlying distribution, assuming that the distribution of the data is a degraded version of the intrinsic lognormal distribution, in Paper II we employed a full maximum likelihood analysis, which gave us a best fit with parameters \(\lambda_0 = 0.0394\) and \(\sigma_\lambda = 0.509\), showed in Fig. 2, top panel, as a solid line. The maximum likelihood fit and the direct fit over the data from the sample are extreme cases, one assuming that the dispersions in the empirical scalings used are fully intrinsic to the data, and the other that they are strictly measurement errors, the actual distribution should lie between these two cases, in very good agreement with the distribution from the simulation, we can hence see that the integral distribution of galactic halo \(\lambda\) spin parameters in real galaxies is consistent with what current structure formation theories predict.

That both the \(\lambda\) distributions inferred from the SDSS sample, and those coming from all published cosmological simulations are well fitted by lognormal distributions, is interesting. We note that lognormal distributions often arise as a consequence of the central limit theorem. Whenever the particular outcome of a variable is the sum of a large number of independent distributions, we are guaranteed that the final distribution will be a Gaussian. If the particular outcome of a variable is the product of a large number of independent distributions, it follows that the logarithm of the variable will be normally distributed. What the many independent distributions might in this case be, we can only speculate, the repeated merger events leading to the formation of a galaxy, or perhaps the tidal fields due to surrounding density fields as they sequentially detach from the overall expansion of the universe and virialize.

### 3.4 Correlations between \(\lambda\) and environment density

To study if there is any correlation between spin and the environment, we use an estimation of the local background density defined directly from the observations that is continuous and able to characterize the full range of galaxy environments. It measures the local number density of galaxies at a given smoothing scale.

Being a continuous measure, it does not make any difference on field, voids, filaments or clusters but their estimation can be done based on the present density measure. The measurement of the environmental density was done by Park et al. (2007), where they adopt a Spline kernel with adaptive smoothing scale to include a fixed number of galaxies within the centrally weighted kernel. The details of the method using the spline kernel are summarized in Park et al. (2007). The local density at a given location for each galaxy is measured by
\[
\rho(x)/\langle \rho \rangle = \sum_{i=1}^{20} W_i(|x_i - x|)/\langle \rho \rangle, \tag{9}
\]

using the spline kernel weight \(W(r)\) for the background density estimation as in Park, Gott & Choi (2008). The mean mass density of the sample is obtained by
\[
\langle \rho \rangle = \sum_{all} 1/V, \tag{10}
\]

where the summation is over all galaxies in the sample contained in the total volume \(V\).

With this measure for the local density of the environment, we are able to search for any correlation among environment and spin. In several studies, the samples of simulated galaxies are divided according to their location in field, voids, filaments or clusters, by cuts in the overdensity of the environment or simply by cuts of the normalized local density, and then mean values of \(\lambda\) for each subsample are obtained or \(\lambda_0\) values from fits of lognormal functions to the distributions at each cut. We divided our sample into six cuts according to the normalized environmental density, and for each cut we obtained the mean \(\lambda\) value and its dispersion. The result is presented in the upper left-hand panel of Fig. 3, where the value of total inferred \(\lambda\) versus \(\rho/\langle \rho \rangle\) for each galaxy is plotted and we show the mean \(\lambda\) value for each cut as a function of the normalized density, the solid error bars corresponds to the dispersion and the broken error bars to the standard error in the calculation of the mean value, this convention will be kept in the following figures. As can be seen, there is no clear trend, taking into account the dispersion in each bin.

As discussed in Paper II, our \(\lambda\) estimation for early-type galaxies is not as accurate as the estimation for disc galaxies due to the uncertainties in the way dissipation, pressure support and projection effects are taken into account to obtain equation (7); with this in mind, we also checked if being more rigorous in the estimation of \(\lambda\), we could find any trend with the environment. Using only the 7753 disc galaxies, and dividing the sample again in six bins, we obtain the result presented in the upper right-hand panel of Fig. 3, again showing little trend if any, between \(\lambda\) and the normalized local density of the environment.

Using the full sample of haloes from our N-body simulation, divided into six bins and calculating the mean \(\lambda\) value and dispersion for each bin, we obtain the result shown in Fig. 3, bottom left-hand panel. The small change in the mean value and large dispersion are consistent with there being no correlation between the two properties, in agreement with what was obtained using the SDSS sample. To compare the results arising from the numerical simulation with our sample of disc galaxies, where we are more confident of the \(\lambda\) estimation, we can take as a first-order criterion for haloes harbouring ‘late-type galaxies’ only the haloes with \(\lambda > \lambda_0\), where \(\lambda_0\) comes from the fit of the whole sample to a lognormal function, in this case, \(\lambda_0 = 0.0354\). The behaviour of the spin for the high spin haloes of the N-body simulation, as a function of density, is shown in Fig. 3 bottom right-hand panel, with the same qualitative result as obtained using the whole sample.

Recent works studying the variation of \(\lambda\) in different environments, sometimes find a weak dependence for high-mass galaxies, in the sense that higher spin galaxies are more strongly clustered than lower spin galaxies (Bett et al. 2007; Hahn et al. 2007). We inspect if keeping only the high-mass galaxies, we can find such a significant correlation. For both samples, the SDSS and the simulation, we took into account disc galaxies with total halo masses \(M > 1 \times 10^{13} M_\odot\), and simulates haloes in the same mass range. To obtain the mass of the SDSS galactic haloes we used a mass–luminosity relation which relates the baryonic mass of the galaxy with its absolute magnitude (e.g. Kassin, de Jong & Weiner 2006),
Figure 3. Relations between spin and environmental density, showing the mean $\lambda$ value for each bin as a function of the normalized density, the solid error bars correspond to the dispersion and the broken error bars to the standard error in the calculation of the mean value. Top panels correspond to the SDSS sample: on the left-hand side the entire sample using the 11,597 galaxies and on the right-hand side using only the 7,754 disc galaxies. Bottom panels correspond to the simulated dark matter haloes: on the left-hand side the whole 100,000 sample, on the right-hand side 52,950 haloes with $\lambda > 0.04$

and we hold the same baryonic to dark matter fraction introduced in Section 2. The behaviour found is presented in Fig. 4, top panel corresponding to the SDSS sample and bottom panel corresponding to the N-body simulation, where we can see that the trend, almost imperceptible in Fig. 3 is now a little more clear but still not significant, taking into account the large dispersion in each bin. In the case of the SDSS sample, we find the same behaviour as in Fig. 3.

To summarize the results of this subsection, we can conclude that we could not find any clear correlation between the $\lambda$ spin parameter and the environment where the galaxies are located, as pointed out by several theoretical studies for dark matter haloes in simulations (Lemson & Kauffmann 1999; Bett et al. 2007; Maccio et al. 2007). Neither at low nor at high masses, not for real galaxies or simulated haloes, do we find any significant trend in the parameters describing the distribution of $\lambda$ with environment density. Up to this point, current cosmological modelling appears fully consistent with real galaxies.

3.5 Correlations between $\lambda$ and halo mass

After having found no correlation between $\lambda$ and the local environment, we now search for a correlation with internal properties of the galaxies. In many analytic and numerical studies, $\lambda$ has been established as an important, if not the most important, parameter determining the galactic type (Fall & Efstathiou 1980; Flores et al. 1993; Firmani, Hernandez & Gallagher 1996; Silk 2001). In that sense, we expect it to correlate with the most important structural parameters such as thickness of the disc, bulge-to-disc ratio, colours, metallicity among others. van den Bosch (1998) emphasizes the important role of this parameter in explaining the origin of the Hubble sequence, and shows correlations of $\lambda$ with the bulge-to-disc ratio, in the sense that systems with high $\lambda$ presents systematically lower bulge-to-disc ratios. Kregel et al. (2005) explain the role of $\lambda$ in the stellar velocity dispersion and predict extended thin disc in galaxies immersed in high spin angular momentum dark matter haloes. Trends with star formation efficiencies, gas fractions, metallicities, abundance gradients and colours have also been predicted in several studies (e.g. Dalcanton et al. 1997; Boissier et al. 2001; Churches, Nelson & Edmunds 2001). In Paper I and in Cervantes-Sodi & Hernandez (2008), we showed this type of scaling but using samples of real galaxies, confirming the general results of theoretical studies.

But still, more important than all these properties is the mass of the galaxy. The results arising from early works are rich and varied, some of them pointing in the sense that no correlation is found between $\lambda$ and mass (e.g. Warren et al. 1992; Lemson &
Kauffmann 1999; Shaw et al. 2006; Maccio et al. 2007) and others showing from weakly to marked trends between these two parameters, again, always for cosmological simulations (e.g. Cole & Lacey 1996; Jang-Condell & Hernquist 2001 – although the low resolution of this particular study might make this results questionable, Bett et al. 2007). We showed in Papers I and II a strong correlation between halo $\lambda$ as estimated through equation (5), and visually assigned morphological type, in the expected sense, with later type galaxies corresponding to high values of inferred $\lambda$. Given the tight correlation that exists between Hubble type and mass, where high-mass systems tend to be of earlier types, it is natural to expect a trend to appear when cutting the sample of $\lambda$ as a function of mass. We expect high-mass galaxies to be found preferentially amongst the low-$\lambda$ haloes.

We investigate whether our SDSS sample presented this correlation or not. Once total $\lambda$ values were obtained for all the galaxies using the corresponding equation, and using the TF relation presented in the preceding subsection, we obtained the upper left-hand panel of Fig. 5, plotting $\lambda$ as a function of mass. In this case, the trend is very clear, low-mass galaxies present typically higher $\lambda$ values and larger dispersion, exactly what the association of $\lambda$ and Hubble type mentioned previously would have suggested. We note that precisely such a trend is reported in the theoretical study of Maller et al. (2002) for the case of a tidal-torque scenario for the acquisition of angular momentum, as opposed to the scenario where angular momentum grows through the merger of satellites. We take our results as evidence of a mechanism of acquisition of angular momentum for galaxies, where it is the ambient tidal field what torques up a halo, with little participation of repeated mergers.

The lower left-hand panel in Fig. 5 shows halo $\lambda$ as a function of mass for the simulation, although a certain trend is present in the same sense as in the SDSS sample, its magnitude is small and the dispersion large.

Our estimates of $\lambda$ are more precise for disc galaxies, in both parameters, the spin and the mass. Taking only these galaxies, the corresponding figures for the SDSS sample and the above average $\lambda$ simulated haloes are presented in Fig. 5, right-hand panels. In the case of the SDSS sample, the trend found using the whole sample, disc galaxies plus ellipticals, appears even stronger. The response in the case of the simulation is similar, the trend is kept but the dispersion decreases for larger galaxies, as in the SDSS sample but at a much lower level.

### 4 DISCUSSION

We have presented our results in the study of a large sample of galaxies taken from the SDSS and compared to the situation found in recent cosmological N-body simulations. We looked for correlations of the total galactic $\lambda$ spin parameter distribution with both halo mass and local environmental density. With the local environment density we find no correlation, as reported in several theoretical studies and reproduced here using a cosmological N-body simulation. The large dispersion and little trend seen when dividing the samples into bins with different normalized environment density, are consistent with no correlation at all for a $\lambda$–($\rho$) relation, for both; inferred haloes of observed galaxies and modelled haloes.

The case of the $\lambda$–mass relation is quite different. Analysing our SDSS sample, we notice that the mean value of $\lambda$ tends to decrease as the mass increases. But not only the mean value, also the dispersion of $\lambda$ tends to decrease as the value of the mass increases. The effect is stronger using only the disc galaxies, where our estimate is more precise. Large galaxies are seen to form a more coherent low-$\lambda$ sample with small dispersion, while small galaxies show mean values of $\lambda$ of about a factor of 3 larger than what is found for our most massive galaxies. The dispersion about mean values shows the same trend, with small galaxies in our SDSS sample having a much larger dispersion than the large galaxies. Analysing the N-body simulation we found a very weak trend in the same sense using the high spin galaxies, which would correspond to disc galaxies.

Burkert (2003), found a correlation between the spin parameter and the baryon fraction of the form $\lambda \propto F^{2/3}$. In our simple model used to calculate $\lambda$ from observable parameters, the dependence is even stronger; $\lambda \propto F^2$, this could be invoked to explain the relation found between mass and $\lambda$, strong star formation can drive galactic winds which clear a fraction of the baryonic material, preferentially in small systems. This would lead to a lower effective baryon
fraction $F$ in smaller systems, and hence, the assumption of a constant baryon fraction would lead us to overestimate $\lambda$ in small systems. However, as has been shown recently by several authors studying the accretion and expulsion of baryons in cosmological scenarios (e.g. Crain et al. 2007; Brooks et al. 2007), the galactic baryon fraction is almost constant in the range of masses we are dealing with in this work. A fractional variation in $F$ of over a factor of 3, over the 1.5 orders of magnitude in galactic mass we are dealing with here, is inconsistent with the physics of galaxy formation. We excluded galaxies of smaller masses, partly to guarantee completeness of the sample, and partly to stay well above the limit where galactic winds can be expected the change the baryon fraction differentially.

If the baryonic component is susceptible to angular momentum dissipation or baryonic to dark halo angular momentum transfer, and this effect is sharper in more massive systems, the trend observed with the mass could be due to a subestimation of the halo spin. Again, we can discard this hypothesis based on the study of Kaufmann et al. (2007), in which the loss of angular momentum of disc particles on a smoothed particle hydrodynamic simulation is traced. They found that almost half of the initial angular momentum is lost via different mechanisms in low-resolution cosmological simulations, but when a high-resolution simulation is used only 10-20 per cent of the original angular momenta is lost, not enough to explain the strong correlation presented here between mass and $\lambda$. Further, no trend with mass is seen.

van den Bosch et al. (2002), using a numerical simulation of structure formation in a $\Lambda$CDM cosmology, concluded that the detailed angular momentum distributions of the gas and dark matter components in individual haloes are remarkably similar, and the differences between them present no significant dependence on the halo virial mass, result that supports the model used to calculate $\lambda$. A similar conclusion was obtained by Zavala, Okamoto & Frenk (2008). Tracing the evolution of the specific angular momenta of the dark matter and baryonic components on a cosmologically forming disc galaxy, they show that the baryonic component tracks the specific angular momentum of the halo, and their actual values are very similar throughout the entire history of the galaxy. The lack of a strong trend with mass in the effect of baryonic angular momentum transfer or dissipation found in detailed hydrodynamical simulations, shows that the correlation we find in mean values of $\lambda$ which decrease with increasing galactic halo mass, is real. In addition, the

Figure 5. Relations between spin and mass. Top panels correspond to the SDSS sample: on the left-hand side the entire sample using the 11 597 galaxies and on the right-hand side using only the 7754 disc galaxies. Bottom panels correspond to the simulated dark matter haloes: on the left-hand side the whole 100 000 galaxies, on the right-hand side 52 950 galaxies with $\lambda > 0.04$. 

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relation between $R_h$, $V_h$ and $\lambda$ of equation (5) is practically the same as what is found in the results of the high-resolution simulation of Okamoto et al. (2005), as seen in Fig. 1.

To summarize, we find a good accordance between the integral distributions of $\lambda$ inferred from the large SDSS sample we studied, and results from current cosmological simulations, both in terms of the functional form of the distributions, and the parameters describing these distributions. The same is the case when comparing the samples as a function of environment density, with no clear trend apparent. However, when looking at the samples as a function of mass, no clear trend is seen in the simulated haloes, but in the sample of real galaxies we find a strong tendency for both mean values and dispersions in $\lambda$ to decrease with increasing galactic mass; Tonini et al. (2006) reached similar conclusions through a more indirect study of galactic populations, and in order to explain the inferred baryonic scaling relations for a sample of around 80 isolated disc galaxies, using models of galactic evolution, Avila-Reese et al. (in preparation) required an anticorrelation between the mass, and both, the spin and the baryon fraction. This provides constrains on the form of acquisition of the angular momenta in galactic systems and important clues to discriminate cosmological models, given the dependency of this relations on the cosmology assumed, e.g. Eke, Efstathiou & Wright (2000). Comparing with existing studies of galaxy formation, we see that the trend we find cannot be ascribed to astrophysical effects associated with baryonic loss or dissipation. It must rather be understood as evidence in favour of an angular momentum acquisition mechanism where ambient tidal fields torque up galactic haloes as a whole at high redshift, rather than the progressive spin up caused by a repeated and prolonged satellite accretion process. To first order, the trend for a lower mean value of $\lambda$ at larger halo masses can be understood by thinking of a constant ambient tidal field, which then results in higher $\lambda$ for smaller systems.

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