On the structure of the Sun and α Centauri A and B in the light of seismic and non-seismic constraints

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ABSTRACT

The small separation (Δν₀₁, Δν₀₂ and Δν₁₃) between oscillations with low degree ℓ is dependent primarily on the sound speed profile within the stellar core, where nuclear evolution occurs. The detection of such oscillations in a star offers a very good opportunity to determine the stage of its nuclear evolution, and hence its age. In this context, we investigate the Sun and α Cen A and B. For α Cen A and B, each of the small separations Δν₀₁, Δν₀₂ and Δν₁₃ gives a different age. Therefore, in our fitting process, we also employ the second difference, defined as vᵣ_{n,2} − 2vᵣ_{n,1} + vᵣ_{n,0}, which is 2Δν₀₁ − Δν₀₂. In addition to this, we use the frequency ratio (vᵣ_{n,0}/vᵣ_{n,2}). For the Sun, these expressions are equivalent and give an age of about 4.9–5.0 Gyr. For α Cen A and B, however, the small separation and the second difference give very different ages. This conflict may be resolved by the detection of oscillation frequencies that can be measured much more precisely than the current frequencies. When we fit the models to the observations, we find that (i) Z₀ = 0.020, t = 3.50 Gyr and M₀ = 1.006 M⊙ from the small separations Δν₀₁, Δν₀₂ and Δν₁₃ of α Cen B; and (ii) a variety of solutions from the non-seismic constraints and Δν₀₂ of α Cen A and B, in which the masses of α Cen A and B are slightly modified and the age of the system is about 5.2–5.3 Gyr. For Z = 0.025, the closest masses we find to the observed masses are M₀ = 0.922 M⊙ and M₀ = 1.115 M⊙. The differences between these masses and the corresponding observed masses are about 0.01 M⊙.

Key words: Sun: fundamental parameters – Sun: interior – stars: evolution – stars: individual: α Cen – stars: interiors.

1 INTRODUCTION

Microscopic or macroscopic, solid or fluid of any kind, every object oscillates at frequencies determined by its structure. In many cases, the oscillations sound the structure of the visible and invisible parts of the object in which they are trapped. Among such objects are stars, particularly solar-like stars, which oscillate in a variety of modes so that one can infer physical conditions deep in the nuclear core (Tassoul 1980), which is not directly observable. In this context, Christensen-Dalsgaard (1988) developed a seismic Hertzsprung–Russell (HR) diagram in which one can deduce the size and evolutionary phase (age) of a star from its oscillation frequencies. In this diagram, the horizontal axis is the so-called large separation between frequencies of consecutive oscillations with order n (Δvᵣₙ = vᵣₙ − vᵣₙ₋₁), and the vertical axis is the small separation between frequencies of oscillations with different harmonics. As a result of nuclear evolution, the position of a star in the seismic HR diagram changes with time. Roxburgh & Vorontsov (2003, hereafter RV2003) argued that the ratio of the small separation to the large separation is more sensitive to time than the accurate value of the small separation, and proposed an alternative expression for it.

In many respects, and particularly because of its high-quality data, the Sun is an excellent object with which to study stellar interiors for calibration of the evolution code to be used. In other words, our success in modelling interiors of stars of different kinds depends on how successful we are in modelling the solar interior. The solar models with chemical composition given by Grevesse & Sauval (1998) are in very good agreement with seismic inferences (Bahcall, Pinsonneault & Wasserburg 1995; Christensen-Dalsgaard et al. 1996; Gabriel & Carlier 1997; Yıldız 2001). However, the agreement disappears if the recent solar chemical composition given by Asplund, Grevesse & Sauval (2005, hereafter AGS2005) is used (see Basu et al. 2007 and references therein). Although uncertainties in the abundances of heavy elements are discussed by Pinsonneault & Delahaye (2006) and found large enough to cover the old solar composition, an increase in the diffusion coefficient (Guzik, Watson

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& Cox 2005) and opacity (Bahcall et al. 2005) below the convective zone are suggested to restore the agreement between the models and helioseismic constraints.

In our previous paper on $\alpha$ Cen (Yıldız 2007), we obtained $8.9$ Gyr for the age of the system from the non-seismic constraints (NOS models in Yıldız 2007) and about $5.6$–$5.9$ Gyr from the seismic constraints (SIS models in Yıldız 2007; see also Miglio & Montalban 2005). There may be observational or theoretical reasons for this discrepancy. In this study, we investigate how well core structures of the Sun and $\alpha$ Cen A and B are represented by different forms of small separations and how precise their fundamental properties are.

As main-sequence (MS) stars evolve, their oscillation frequencies decrease because of the increase in their size. However, the amount of decrease in the frequencies depends also on how the sound speed changes in the cavity in which the oscillations are trapped. As a result of nuclear evolution, the sound speed gradually decreases in the nuclear core, and therefore frequencies of oscillations with $l = 0$ are much more reduced than are those with $l = 1$ and $l = 2$. Thus, the frequency ratio $v_{l=0}/v_{l=2}$ is a function of time and has in principle a diagnostic potential as the small separation for information about the age of solar-like stars. We also use this expression, despite its lower sensitivity to time than $\delta v_{l=2}$, in order to test if it gives the same age as the usual expressions (see Section 3).

In Yıldız (2007), the small separation $D_0$ is computed from $\delta v_{l=2}$ for $\alpha$ Cen A, and from $\delta v_{l=2}$ and $\delta v_{l=3}$ for $\alpha$ Cen B ($D_0 = (\delta v_{l=2}/6 + \delta v_{l=3}/10)/2$), because $\delta v_{l=2} = (v_{l=0} - (v_{l=1,1} + v_{l=1})/2$) values of models are quite different from the values found from the observed frequencies. In this study, using in addition the seismic data of $\alpha$ Cen A given by Bedding et al. (2004, BK2004), we consider if it is possible to fit model values of $\delta v_{l=2}/2$, $\delta v_{l=2}/6$ and $\delta v_{l=3}/10$ one by one to the corresponding values inferred from the seismic data of the Sun and $\alpha$ Cen A and B.

The remainder of this paper is organized as follows. In Section 2, the basic properties of solar models with old and recent chemical compositions are presented. The results are presented and discussed in Section 3. Finally, we make some concluding remarks in Section 4.

### 2 PROPERTIES OF THE SOLAR MODELS

The characteristics of the code used in the construction of models of stellar interiors are given in Yıldız (2007). Further details of the code are given in the references of that paper.

The models of $\alpha$ Cen A and B required for our analysis are given in Yıldız (2007). We construct new solar models, because significant changes have occurred in our knowledge of the chemical composition of the Sun (AGS2005).

The basic properties of the solar models with the recent solar composition given by AGS2005 are listed in Table 1. In the first row of this table, the solar model with $Z_0 = 0.016$, which is in very good agreement with the observed chemical composition of the Sun, is presented: its surface helium and heavy element abundances are $0.244$ and $0.0124$, respectively. However, the base radius of its convective zone $(0.733 R_\odot)$ is significantly greater than the value inferred from helioseismology $(0.713 \pm 0.001 R_\odot$, Basu & Antia 1997). In order to test the influence of the initial value of the heavy element abundance, another solar model is constructed with $Z_0 = 0.017$. This model is given in the second row of Table 1. The base radius of its convective zone $(0.730 R_\odot)$ is slightly smaller than that of the solar model with $Z_0 = 0.016$, but an agreement with the value inferred from helioseismology is not achieved. The age of these two models is taken as $4.6$ Gyr. For later usage, we also construct a solar model with $Z_0 = 0.016$ and age $= 4.9$ Gyr (see Section 3.4).

In Fig. 1, the relative sound speed difference between these solar models and the Sun is plotted with respect to the relative radius. The relative sound speed difference between the solar model with $Z_0 = 0.016$ and the Sun is represented with diamonds. The largest difference, about $1.7$ per cent, occurs in the region just below the base of the convective zone.

The larger the initial heavy element abundance, the smaller the relative sound speed difference in the

### Table 1. Properties of the solar models with the chemical composition given by AGS2005. The last line is for the observed values. The subscripts $c$ and $s$ represent the values at the centre and radius, respectively. The uncertainty in the bottom radius of the convective zone ($R_{bcz}$) is $0.001 R_\odot$ (Basu & Antia 1997). The observed values of the surface helium and metal abundances are taken from Basu & Antia (1995) and AGS2005, respectively.

<table>
<thead>
<tr>
<th>$X_0$</th>
<th>$Z_0$</th>
<th>$\alpha$</th>
<th>$\rho_s$</th>
<th>$T_s$</th>
<th>$X_s$</th>
<th>$Z_s$</th>
<th>$Y_s$</th>
<th>$R_{bcz}/R_\odot$</th>
<th>$t_\odot$ (Gyr)</th>
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<td>0.241</td>
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<td>0.246</td>
<td>0.713</td>
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</tbody>
</table>
outer part of the radiative interior. For $Z_0 = 0.017$, for example, the largest difference reduces to 1.4 per cent (the dotted line). A very similar profile for the relative sound speed difference occurs for the model with $Z_0 = 0.016$ and age $= 4.9 \, \text{Gyr}$ (solid line). For comparison, the model with $Z_0 = 0.02$ and mixture given by Grevesse & Sauval (1998) is also plotted in Fig. 1. It is in better agreement with the helioseismic results than the solar models with AGS2005.

3 STRUCTURE OF THE NUCLEAR CORE AND OSCILLATION FREQUENCIES

The acoustic oscillations are trapped in the stellar cavity. The dimension of this cavity depends on the degree ($l$) and frequency of oscillations. Whereas the cavity for the oscillations with $l = 0$ extends down to the centre, the oscillations with higher $l$ have shallower cavities. The turning point ($r_t$) of an oscillation with non-zero $l$ is approximately given as

$$c_l \approx \frac{\omega}{r_t} \sqrt{l(l+1)},$$

where $\omega$ is the frequency of oscillation and $c(r_t)$ is the sound speed at the turning point. The turning points of the oscillations with the observed frequencies of $\alpha$ Cen A and B are computed from SIS models and plotted with respect to $n$ for the modes with $l = 1$ and $l = 2$ in Fig. 2. Whereas the modes of $\alpha$ Cen A (thin solid line with diamonds) and B (thin dotted line with $\times$s) with $l = 2$ turns at nearly $r_t/R_\star \approx 0.1$, the modes with $l = 1$ (thick solid line with boxes for $\alpha$ Cen A and dotted line with filled circles for $\alpha$ Cen B) sink deeper into the nuclear core ($r_t/R_\star \approx 0.05$).

Whereas the small separation is a sensitive function of the physical conditions in the central regions, where the nuclear evolution occurs, the large separation, $\Delta \nu_0 = \nu_{0,0} - \nu_{0,1,0}$, is a measure of the mean density. The small separation between frequencies (Christensen-Dalsgaard 1988), $\delta \nu_0 = \nu_{0,0} - \nu_{0,1,2}$, is plotted as a function of $n$ in Fig. 3, using the observed frequencies of the Sun (thin solid line with diamonds; Chaplin et al. 1999), $\alpha$ Cen A (thick solid line with boxes; Bouchy & Carrier 2002, hereafter BC2002) and $\alpha$ Cen B (thin solid line with circles; Kjeldsen et al. 2005, hereafter KB2005). Its $n$ dependence is very similar to a straight line from BK2004. Although its average value is about 0.3 $\mu$Hz, its mean value (6.3 $\mu$Hz) is close to the value found from BC2002 (5.6 $\mu$Hz).

In Fig. 3, $\delta \nu_0$ of the Sun (Chaplin et al. 1999) is much smoother than the corresponding values for $\alpha$ Cen A and B. However, for the less accurate solar data of the early stages of helioseismology, this was not the case. In Fig. 4, $\delta \nu_0$ of the old (Grec et al. 1983; GF1983; dotted line with $\times$s) and the recent (Chaplin et al. 1999) data of the Sun are plotted with respect to $n$. For small values of $n$, despite the scattering of the GF1983 data, the two data sets are in agreement. However, for $n \geq 25$, there is a significant difference. Grec et al. (1983) state that the frequency resolution is 2 $\mu$Hz. The error in frequencies found by Chaplin et al. (1999) is about 0.05 $\mu$Hz for $n \leq 22$ and increases very rapidly for larger values of $n$: for $n = 26$, for example, it is about 0.3 $\mu$Hz. We also note in Fig. 4 the similarity between the old data of the Sun and the data of $\alpha$ Cen B. As our skill in detecting the seismic properties of $\alpha$ Cen A and B develops, as in the solar case, we may obtain a much smoother variation of $\delta \nu_0$ with respect to $n$.

On the theory side, however, uncertainty arises from the fact that model frequencies are computed assuming an adiabatic process for the oscillations (Christensen-Dalsgaard & Thompson 1997). The adiabatic approximation is valid for almost the entire interior, except for the near-surface regions. Therefore, the perturbation should also be applied to the energy equation, but the perturbation of energy equation is highly uncertain, at least for the convective flux...
(Christensen-Dalsgaard et al. 1996). However, use of either the difference between or the ratio of frequencies for comparison is a good way to minimize the effects of such a troublesome problem.

3.1 Sound speed profiles of the Sun and α Cen A and B and the time variation of frequencies

The variation of stellar oscillation frequencies with time is primarily a result of the global expansion of the star like a heated ball. The secondary effect on the time dependence of frequencies arises from the fact that the sound speed profile changes throughout the star. For the secondary effect, the most significant change occurs within the nuclear core. Therefore, this secondary effect is important for modes with low degree. Whereas variation of the large separation with time is a good measure of the first effect, the variation of the small separation is the result of the secondary effect. In Fig. 5, the sound speed profiles in the central regions of the Sun (thin dotted line, Basu, Chaplin & Christensen-Dalsgaard 1997), the solar model with $Z_0 = 0.016$ (stars), and SIS models of α Cen A (thick dotted line) and B (thin solid line) are plotted with respect to the relative radius. For comparison, the sound speed (thick solid line) of α Cen A (SIS model) near the zero-age main sequence (ZAMS) $(t = 0.1$ Gyr) is also plotted. As stated above, the most significant difference between the sound speed profiles occurs in the nuclear core: as hydrogen is converted to helium, the mean molecular weight increases and consequently the sound speed at the centre drops from $5.6 \times 10^7$ to $4.5 \times 10^7$ cm s$^{-1}$.

The oscillations with $l = 1$ are influenced much more than the oscillations with $l = 2$ because of this variation in the sound speed profile with time. It is well known that the turning point of oscillations depends mainly on the degree of oscillation. Whereas the turning points of oscillations with $l = 1$ are about $r_1 = 0.05R_\odot$, the mean value for the turning points of oscillations with $l = 2$ is about $r_1 = 0.10R_\odot$. As the nuclear evolution proceeds the sound speed decreases, and consequently the oscillations with $l = 1$ and $l = 2$ sink deeper, according to equation (1). From the ZAMS to the present time, the decrease in sound speed at point $r_1 = 0.05R_\odot$, with time for α Cen A, for example, is about $0.6 \times 10^7$ cm s$^{-1}$ (Fig. 5);

$$\delta \nu_{02} = (\nu_{n,0} - 2\nu_{n,1} + \nu_{n,2}).$$

(2)

The second difference is the difference between $2\delta \nu_{01}$ and $\delta \nu_{02}$. According to the asymptotic relation, $\delta \nu_{012}$ is very small, just like $\delta \nu_{02}$. In Fig. 6, $\delta \nu_{012}$ is plotted against $n$ at various evolutionary times of α Cen A, from the ZAMS to the terminal-age MS (TAMS). Close to the ZAMS (thin solid line with diamonds), $\delta \nu_{012}$ is negative and varies between $-2.0$ and $-1.5$ μHz. As the model evolves, $\delta \nu_{012}$ becomes positive at an age of nearly half of the life time of the star, and reaches an average value of about $7.0$ μHz at age $=7$ Gyr. $\delta \nu_{012}$ computed from the observed frequencies is also plotted in Fig. 6. From the comparison of the model and the observational $\delta \nu_{012}$, contrary to the result from $\delta \nu_{02} = \nu_{n,0} - \nu_{n,2}$, we deduce that α Cen A must be in the early phase of its MS evolution.

The solar model with $Z_0 = 0.016$ and age $= 4.6$ Gyr is also plotted in Fig. 6 (dotted line with circles). As for α Cen A, $\delta \nu_{012}$ of the Sun is independent of $n$. However, the model value of $\delta \nu_{012}$ for α Cen B is dependent on $n$ near the ZAMS (solid line with ×s) and the TAMS (thick solid line with stars) and has less $n$ dependence between these two cases (dotted line with filled circles for a model with age $= 10$ Gyr).

For the Sun, for which we have high-quality seismic data, the average value of $\delta \nu_{012}$ derived from the seismic data over $n$ is...
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3.3 Ratio of frequencies

Although the ratio $\nu_{n,0}/\nu_{n,2}$ is not a more sensitive function of time than $\delta \nu_{02}$, we want to test whether it gives the same age as $\delta \nu_{02}$. In Fig. 8, the ratio of $\nu_{n,0}$ to $\nu_{n,2}$ for α Cen A is plotted as a function of $n$ at 1 Gyr (thin solid line with diamonds), 3 Gyr (thin dotted line), 5 Gyr (thick dotted line) and 7 Gyr (thick solid line with boxes). The decrease in the ratio with time is noticeable. For comparison, the observed ratio is also plotted (solid line with filled circles). The observed ratio is in agreement with the curve of 5 Gyr, except for the ratio for $n=16$.

3.4 Age and metallicity of the Sun from the seismic constraints

The observed value of $\delta \nu_{02}$ for the Sun is found to be 9.84 ± 0.1 μHz from the seismic data of the Bison group. The value of $\delta \nu_{02}$ for a solar model with $Z_0 = 0.016$ is, however, 10.11 μHz; that is, it is slightly greater than the observed value. For the solar model with $Z_0 = 0.017$, $\delta \nu_{02} = 10.08$ μHz. The value of $\delta \nu_{02}$ for a solar model with $Z_0 = 0.016$ and age = 4.9 Gyr, 9.86 μHz, is in very good agreement with the observed value. From the ratio of frequencies ($\nu_{n,0}/\nu_{n,2}$), we confirm that the same age (4.9 Gyr) is more suitable than 4.6 Gyr. Similarly, from the comparison of the observed value of $\delta \nu_{02}$ (2.50 μHz) with that of the solar model with $Z_0 = 0.016$ and age = 4.6 Gyr (2.57 μHz), we find that the solar model with 4.9 Gyr is in better agreement with the helioseismic data.

3.5 Are values of $\delta \nu_{01}/2$, $\delta \nu_{02}/6$ and $\delta \nu_{13}/10$ compatible with each other

The small separation $D_0$ can be derived from $\delta \nu_{01}$, $\delta \nu_{02}$ and $\delta \nu_{13}$, if we have enough observational data:

$$D_0 = (\delta \nu_{01} + \delta \nu_{02} + \delta \nu_{13})/3 = (\delta \nu_{01}/2 + \delta \nu_{02}/6 + \delta \nu_{13}/10)/3.$$  \(3\)
As emphasized above, in Yıldız (2007), $D_0$ is computed from $\delta \nu_{\odot}$ for $\alpha$ Cen A and from $\delta \nu_{\odot}$ and $\delta \nu_{13}$ for $\alpha$ Cen B. Its values inferred from the observed seismic frequencies ($D_{02,\text{obs}}$ and $D_{13,\text{obs}}$) and models with an age of about 5.7–5.9 Gyr are in good agreement. The values of $D_{01,\text{obs}} = \delta \nu_{\odot}/2$ for both $\alpha$ Cen A and B are not used in the analysis in Yıldız (2007), because they are not in agreement with the $D_{01,\text{obs}}$ values derived from SIS models and are not compatible with $D_{02,\text{obs}}$ and $D_{13,\text{obs}}$ (see Table 2). In fact, the disagreement between the values of $\delta \nu_{\odot}$ computed from the models and the values inferred from the seismic data (see Section 3.2) arises from this inharmoniousness. Such an inharmoniousness does not exist for the Sun, and we may hope to remove it with observational seismic data for $\alpha$ Cen A and B that are much more precise than the present data.

We first consider the solar data and try to fit $D_{01,\text{obs}}$, $D_{02,\text{obs}}$, and $D_{13,\text{obs}}$ of the solar models to $D_{01,\text{obs}}(\odot)$, $D_{02,\text{obs}}(\odot)$, and $D_{13,\text{obs}}(\odot)$. In Table 2, $D_{01,\text{obs}}$, $D_{02,\text{obs}}$, and $D_{13,\text{obs}}$ of the solar models are greater than $D_{01,\text{obs}}(\odot)$, $D_{02,\text{obs}}(\odot)$, and $D_{13,\text{obs}}(\odot)$ (given in the eighth row, respectively). For comparison, the values of $D_{02,\text{obs}}(\odot)$ and $D_{13,\text{obs}}(\odot)$ derived from the recent seismic solar data of Basu et al. (2007) and their uncertainties are given in the ninth and tenth rows, respectively. In the fourth row, the solar model with an age of 4.9 Gyr and $Z_\odot = 0.016$ is given. Its $D_{02,\text{obs}}$ is the same as $D_{02,\text{obs}}$, and $D_{01,\text{obs}}$ and $D_{13,\text{obs}}$ are very close to $D_{01,\text{obs}}(\odot)$ and $D_{13,\text{obs}}(\odot)$, respectively. In order to find the optimum values of $Z_\odot$ and an age for the Sun we write three equations for each $D_{0j}$; for instance,

$$D_{01,\text{obs}} = D_{01,\text{obs}} + \frac{\Delta D_{01}}{\Delta Z_\odot} Z_\odot + \frac{\Delta D_{01}}{\Delta t} t,$$

and solve them for $Z_\odot$ and age ($t$). We compute the required derivatives such as given in equation (4) from the solar models discussed above and find the solution: $Z_\odot = 0.017$ and $t = 4.93$ Gyr. The solar model with these values is given in the sixth row of Table 2. We use this model as the reference model and obtain another solution: $Z_\odot = 0.0165$ and $t = 4.98$ Gyr. The solar model with the new results is presented in the seventh row. $D_{0j}$ of these solar models with about $Z_\odot = 0.0165–0.0170$ and age about 4.9–5.0 Gyr are very similar and are compatible with the results inferred from helioseismology.

In this context, we also consider $\alpha$ Cen A and B. Their models from Yıldız (2007) are given in the first three rows of their corresponding parts of Table 2. For our analysis of $\alpha$ Cen A and B, we write again three equations similar to equation (4) for $D_{0j}$ for each star but with three unknowns. In addition to $Z_\odot$ and $t$, we assume that the masses of the stars are also unknown. We obtain separate solutions for $\alpha$ Cen A and B. For $\alpha$ Cen B, $Z_\odot = 0.020, t = 3.50$ Gyr, and $M_B = 1.006 M_\odot$. For $\alpha$ Cen A, we obtain $t = 3.50$ Gyr and $M_A = 1.19 M_\odot$. However, the value of $Z_\odot$ for the solution for $\alpha$ Cen A is not a reasonable value and therefore we use the same value of $Z_\odot$ as obtained for $\alpha$ Cen B. Models of $\alpha$ Cen A and B (SIS2) with these values are given in the fourth row of their corresponding parts of Table 2. Although these values are the optimum values in order to fit model values of $D_{0j}$ to $D_{0j,\text{obs}}$, there are still significant differences between $D_{0j}$ and $D_{0j,\text{obs}}$ of each star. We construct similar models for $\alpha$ Cen A and B with the same values, but with masses ($M_A = 1.15 M_\odot$ and $M_B = 0.97 M_\odot$) that are averages of the masses we find and the masses given by Pourbaix et al. (2002). These models (SIS2p) are presented in the fifth row of the corresponding parts for $\alpha$ Cen A and B in Table 2. $D_{0j}$ values of the SIS2 and SIS2p models are significantly different. However, there is no simultaneous agreement between the model and the observed values of $D_{0j}$ for $\alpha$ Cen A and B as in the case of the Sun.

### Table 2. $D_{01,\text{obs}}$, $D_{02,\text{obs}}$, and $D_{13,\text{obs}}$ of the solar models and of models of $\alpha$ Cen A and B.

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<th>Star</th>
<th>$t$ (Gyr)</th>
<th>$Z_\odot$</th>
<th>$X_\odot$</th>
<th>$D_{01}(\mu\text{Hz})$</th>
<th>$D_{02}(\mu\text{Hz})$</th>
<th>$D_{13}(\mu\text{Hz})$</th>
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4 MASSES AND AGE OF α CEN A AND B

An agreement between the ages of α Cen A and B derived from their seismic and non-seismic constraints can be obtained by modification of their masses. To do this, we write down six equations (for luminosity, radius and $D_{\nu} = \delta\nu_{12}/\delta\nu_{02}$ of each component) similar to equation (4) and solve them to find $M_A$, $M_B$, $\alpha_A$, $\alpha_B$, $X$ and $t$ for fixed $Z$. These equations are not independent, and therefore we obtain a set of solutions. Some of these solutions are given in Table 3, for $Z = 0.025$ and $Z = 0.020$. For comparison, the values found from the observed data are also given (for α Cen A, Bazot et al. 2007 gives $D_0 = 1.15 \pm 0.07$ μHz). Different combinations of $M_A$ and $M_B$ are possible. In Fig. 9, $M_B$ is plotted against $M_A$. The closest masses to the observed masses are $M_B = 0.922 M_\odot$ and $M_A = 1.115 M_\odot$, for $Z = 0.025$. Whereas $M_B$ is less than the observed value by 0.012 $M_\odot$, $M_A$ is greater than the observed value by 0.010 $M_\odot$. We notice two common properties of these models.

(i) The age of the system is about 5.2–5.3 Gyr.
(ii) For each solution, except the one given in the first and the second rows, the convective parameter (fifth column in Table 3) of α Cen A is smaller than that of α Cen B. This result is in good agreement with the result given in fig. 5 of Yıldız (2007).

The large separations of the models of α Cen A and B whose radii are fitted to the observed radius are greater than the observed large separations. In order to fit the large separations of the models to the values inferred from asteroseismology, the radii of α Cen A and B should be $R_A = 1.230 R_\odot$ and $R_B = 0.875 R_\odot$.

The adiabatic oscillation frequencies of the best-fitted models for α Cen A and B with masses $1.115 M_\odot$ and $0.922 M_\odot$, respectively, are given in Table 4. The fundamental properties of these models are presented in the sixth and the seventh rows of Table 3.

5 CONCLUSIONS

Using solar models and models of α Cen A and B, the seismic properties of these stars are investigated in detail. Because of inconsistent results on the age of the α Cen system from the classical and seismic constraints, we test how well the small separation indicates age. In this context, we consider two expressions in place of the customarily used form of the small separation between the frequencies ($\delta\nu_{02}$ or $D_0$):

(i) the second difference, $\delta\nu_{012} = (\nu_{n,0} - 2\nu_{n,1} + \nu_{n,2})$; and
(ii) the frequency ratio $(\nu_{n,0}/\nu_{n,2})$.

For the Sun, these three expressions give consistent results: the solar models with age 4.9–5.0 Gyr and with $Z_0 \approx 0.0165$ are in better agreement with the helioseismic data than the solar model with $Z_0 = 0.0165$ and age 4.6 Gyr. For α Cen A and B the situation is complicated: although $\delta\nu_{012}$ gives about 5.7–5.9 Gyr for the age of the system (nearby the TAMS of α Cen A), we deduce from the comparison of the observed and model values of $\delta\nu_{012}$ that α Cen A and B are not so much evolved.

The expression we use for the small separation, $\delta\nu_{012}$, has several important advantages over the customarily used expression $\delta\nu_{02}$:
the resolution of the seismic HR diagram with $\delta v_{102}$ for massive solar-like stars (e.g. $\alpha$ Cen A) is higher than that with $\delta v_{102}$.

(ii) the evolutionary track of a star in the new seismic HR diagram is in agreement with its evolutionary track in the classical HR diagram (luminosity–effective temperature); and

(iii) $\delta v_{102}$ is nearly independent of $n$.

$\delta v_{102}$ deserves more detailed investigation in order to ascertain whether it has more specific advantages (Christensen-Dalsgaard, private communication).

From the three equations for $D_{ll} = \delta v_{10} / (4l + 6)$ for $\alpha$ Cen A and B, we obtain solutions for the initial heavy element abundance, age and mass of each star: for $\alpha$ Cen B, $Z_0 = 0.020, t = 3.50$ Gyr and $M_B = 1.006 M_\odot$; for $\alpha$ Cen A, $t = 3.50$ Gyr and $M_A = 1.19 M_\odot$. The $Z_0$ value found for $\alpha$ Cen A is not a reasonable value. Therefore, we adopt $Z_0 = 0.020$. The ratio of masses ($M_A/M_B$) is the same as the mass ratio derived from observations (Pourbaix et al. 2002). The heavy element abundance at the surface of the model of $\alpha$ Cen B with diffusion (SIS2) is $Z'_s = 0.0179$. This means that the average overabundance relative to the solar value ($Z_s = 0.0122$, AGS2005) is 0.17 dex.

Using the non-seismic constraints and $\delta v_{102}$ of $\alpha$ Cen A and B we find from the solution of six equations that the age of the system is about 5.2–5.3 Gyr. We also confirm that the mixing-length parameter of $\alpha$ Cen A is smaller than that of $\alpha$ Cen B. This is consistent with the result given in fig. 5 of Yıldız (2007), which shows that the mixing-length parameter of $\alpha$ Cen A is a decreasing function of time. Another interesting result is that the ages of the Sun and the $\alpha$ Cen system are very close to each other, about 5 Gyr.

The models of $\alpha$ Cen A and B that have the same luminosity and radius as the observed values, have large separations greater than the observed values. If $R_A = 1.230 R_\odot$ and $R_B = 0.875 R_\odot$, the large separations of the models are in good agreement with the values inferred from asteroseismology.

In comparing the old (Grec et al. 1983) and the recent (Chaplin et al. 1999) seismic solar data we find that the level of accuracy of the seismic data of $\alpha$ Cen A and B is very similar to that of the old solar data. We hope that the situation will be much better in the near future than it is at present.

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On the structure of the Sun and \(\alpha\) Cen A and B

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