The prospects for constraining dark energy with future X-ray cluster gas mass fraction measurements

David Rapetti,1,2* Steven W. Allen1,2 and Adam Mantz1,2

1Kavli Institute for Particle Astrophysics and Cosmology at Stanford University, 382 Via Pueblo Mall, Stanford, CA 94305-4060, USA
2Stanford Linear Accelerator Centre, 2575 Sand Hill Road, Menlo Park, CA 94025, USA

Accepted 2008 May 13. Received 2008 April 30; in original form 2007 October 1

ABSTRACT
We examine the ability of a future X-ray observatory, with capabilities similar to those planned for the Constellation-X or X-ray Evolving Universe Spectroscopy (XEUS) missions, to constrain dark energy via measurements of the cluster X-ray gas mass fraction, \( f_{\text{gas}} \). We find that \( f_{\text{gas}} \) measurements for a sample of \( \sim 500 \) hot \( (kT \gtrsim 5 \text{ keV}) \), X-ray bright, dynamically relaxed clusters, to a precision of \( \sim 5 \) per cent, can be used to constrain dark energy with a Dark Energy Task Force (DETF) figure of merit of 15–40, with the possibility of boosting these values by 40 per cent or more by optimizing the redshift distribution of target clusters. Such constraints are comparable to those predicted by the DETF for other leading, planned ‘Stage IV’ dark energy experiments. A future \( f_{\text{gas}} \) experiment will be preceded by a large X-ray or Sunyaev–Zel’dovich survey that will find hot, X-ray luminous clusters out to high redshifts. Short ‘snapshot’ observations with the new X-ray observatory should then be able to identify a sample of \( \sim 500 \) suitably relaxed systems. The redshift, temperature and X-ray luminosity range of interest has already been partially probed by existing X-ray cluster surveys which allow reasonable estimates of the fraction of clusters that will be suitably relaxed for \( f_{\text{gas}} \) work to be made; these surveys also show that X-ray flux contamination from point sources is likely to be small for the majority of the targets of interest. Our analysis uses a Markov Chain Monte Carlo method which fully captures the relevant degeneracies between parameters and facilitates the incorporation of priors and systematic uncertainties in the analysis. We explore the effects of such uncertainties for scenarios ranging from optimistic to pessimistic. We conclude that the \( f_{\text{gas}} \) experiment offers a competitive and complementary approach to the best other large, planned dark energy experiments. In particular, the \( f_{\text{gas}} \) experiment will provide tight constraints on the mean matter and dark energy densities, with a peak sensitivity for dark energy work at redshifts mid-way between those of supernovae and baryon acoustic oscillation/weak lensing/cluster number count experiments. In combination, these experiments should enable a precise measurement of the evolution of dark energy.

Key words: cosmological parameters – cosmology: observations – cosmology: theory.

1 INTRODUCTION
In the early 1990s, measurements of the baryonic mass fraction in X-ray luminous galaxy clusters provided compelling evidence that we live in a low-density universe. Under the assumption that large clusters provide approximately fair samples of the matter content of the Universe, X-ray observations require that the mean matter density, \( \Omega_m \), is significantly less than the critical value, with a best-fitting value \( \Omega_m \sim 0.2–0.3 \) (e.g. Fabian 1991; White & Frenk 1991; Briel, Henry & Boehringer 1992; White et al. 1993; David, Jones & Forman 1995; White & Fabian 1995; Ettori & Fabian 1999; Mohr, Mathiesen & Evrard 1999; Roussel, Sadat & Blanchard 2000; Grego et al. 2001; Allen, Schmidt & Fabian 2002; Ettori, Tozzi & Rosati 2003; Lin, Mohr & Stanford 2003; Sanderson & Ponman 2003; Allen et al. 2004, 2008; LaRoque et al. 2006). When combined with the expectation from inflation models, later confirmed by cosmic microwave background (CMB) studies (Bennett et al. 2003; Spergel et al. 2003, and references therein), that the Universe should be close to spatially flat, X-ray results on the cluster baryon mass fraction quickly lead to the suggestion that the mass-energy density of the Universe may be dominated by a cosmological constant (e.g. White et al. 1993).

The first direct evidence for late-time cosmic acceleration, as would be produced by a sizeable cosmological constant, was provided in the late 1990s by Riess et al. (1998) and Perlmutter et al.
The key to determining the nature of dark energy is to obtain precise measurements of its evolution with redshift, \( z \), or scalefactor, \( a = 1/(1+z) \). The Dark Energy Task Force (DETF) report (Albrecht et al. 2006) presented estimates of the constraints on dark energy parameters that should be achievable with a number of future proposed or planned dark energy experiments. In particular, the report forecasted the ability of these experiments, in combination with CMB data from the Planck satellite, to constrain a dark energy model of the form \( w(a) = w_0 + w_a(1-a) \), and defined a figure of merit (hereafter FoM) to allow for easy comparison of the constraints.

In this paper, we use the same dark energy parametrization and FoM to quantify the constraining power of future \( f_{\text{gas}} \) experiments, to be carried out with, for example, the Constellation-X or X-ray Evolving Universe Spectroscopy (XEUS) missions, in combination with CMB data. We show that the \( f_{\text{gas}} \) experiment is likely to provide comparable constraining power to the best other, contemporary space and ground-based experiments described by the DETF. When combined, future CMB, SNe Ia, baryon acoustic oscillation (BAO), weak lensing, cluster number count and \( f_{\text{gas}} \) experiments should provide precise, accurate constraints on \( w(z) \) and allow significant progress in understanding the origin of cosmic acceleration.

The structure of this paper is as follows: in Section 2 we define the dark energy model and the FoM. In Section 3 we describe the simulated \( f_{\text{gas}} \) and CMB data sets. For the \( f_{\text{gas}} \) data, we assume instrument characteristics appropriate for the baseline Constellation-X mission. The CMB data set approximates that expected from two years of Planck data. We also simulate a data set representative of that produced by follow-up observations of the Sunyaev–Zel’’dovich (SZ) effect in the clusters targeted for the \( f_{\text{gas}} \) work. Section 4 describes the Markov Chain Monte Carlo (MCMC) pipeline and details of the analysis method. Our main results are presented in Section 5. Section 6 summarizes our conclusions.

2 THE DARK ENERGY MODEL AND FOM

We characterize the evolution of dark energy by its energy density in units of the critical density, \( \Omega_m \), and its equation of state, \( w \). Following the DETF, we parametrize the evolution of the dark energy equation of state as \( w(a) = w_0 + w_a(1-a) \) (Chevallier & Polarski 2001; Linder 2003) for which a cosmological constant has \( w(a) = -1 \). In this model, the dimensionless Hubble parameter as a function of scalefactor has the form

\[
E(a) = \frac{H(a)}{H_0} = \sqrt{\Omega_m a^{-3} + \Omega_k f(a) + \Omega_b a^{-2}},
\]

where

\[
f(a) = a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)}.
\]

\( H_0 \) is the present-day value of the Hubble parameter and \( \Omega_m \) and \( \Omega_k \) are the mean matter density and curvature density in units of the critical density, respectively.

Using this parametrization, the DETF defines a FoM that is used to compare the constraining power of different dark energy experiments. Nominally, the FoM scales with the inverse of the area enclosed by the 95 per cent confidence contour in the \( w_0-w_a \) plane. However, the DETF showed that since there is little correlation in the \( w_0-w_a \) plane, the area is also proportional to the product of the standard deviations \( \sigma(w_0) \times \sigma(w_a) \), where \( w_a = w(a) \) is the pivot value of \( w(a) \), that is, the value of \( w(a) \) at which its uncertainty is minimized (Linder 2006). (Note that the standard error \( \sigma(w_a) \) approximately corresponds to the 68.3 per cent uncertainty in \( w \) that would be obtained for a constant \( w \) dark energy model).
leads to the definition
\[
\text{FoM} = \left( \frac{\sigma(w_p) \times \sigma(w_a)}{\sigma(w_a)} \right)^{-1}.
\]

For the DETF Fisher matrix analysis, the 1σ confidence region in the \(w_p-w_a\) plane forms an ellipse for which the semi-axes are the standard deviations of the Gaussians to either side of the peak, namely \(\sigma_{w_p}(w_p), \sigma_{w_a}(w_a)\) and \(\sigma_{w_a}(w_a)\). The area of such contours is equivalent to the area of an ellipse with semi-axes \(\hat{\sigma}(w_p) = [\sigma_{w_p}(w_p) + \sigma_{w_a}(w_a)]/2\) and \(\hat{\sigma}(w_a) = [\sigma_{w_a}(w_a) + \sigma_{w_{\hat{a}}}(w_a)]/2\). Thus, we calculate our FoM\(^{1}\) as the inverse of the product of the semi-axes \([\hat{\sigma}(w_p) \times \hat{\sigma}(w_a)]^{-1}\) which allows a direct comparison with the results reported by the DETF.

3 SIMULATED X-RAY DATA

3.1 A strategy for future \(f_{\text{gas}}\) work

We assume that a future \(f_{\text{gas}}\) experiment will be carried out by an X-ray observatory with capabilities comparable to those of Constellation-X, as summarized in Table 1. The major improvements of such a mission with respect to current X-ray observatories are in collecting area, which is a factor of \(\sim 100\) larger than that provided by the Chandra X-ray Observatory, and spectral resolution.\(^{2}\) We assume that the \(f_{\text{gas}}\) experiment will be preceded by, and will build upon, forthcoming X-ray and/or SZ cluster surveys\(^{3}\) that will scan a significant fraction of the sky and find a large number of hot, X-ray luminous, high-z clusters. These surveys will provide the initial target lists for the \(f_{\text{gas}}\) experiment as well as allowing an array of complementary cosmological tests based on the power spectrum and mass function of galaxy clusters (e.g. Albrecht et al. 2006).

\(^{1}\) To confirm the validity of our definition of the FoM we have explicitly measured the area contained by the fitted contours in the right-hand panel of Fig. 2. Dividing this area by both the geometric factor \(\pi\), which accounts for the conversion between the area of an ellipse and a quarter of its circumscribed rectangle, and the factor 2.3, which accounts for the change in the confidence levels from two to one degree of freedom, we successfully match the measured area to the value obtained by the product \(\hat{\sigma}(w_p) \times \hat{\sigma}(w_a)\).

\(^{2}\) For details on planned X-ray observatories see http://constellation.gsfc.nasa.gov/ and http://www.rssd.esa.int/index.php?project=XEUS.

\(^{3}\) Forthcoming X-ray survey missions include Spectrum-RG/eROSITA; see http://www.mpe-garching.mpg.de/projects.html/eerosita and http://www.mpe-garching.mpg.de/eerosita/MDD-6.pdf. Several large-area SZ surveys are already underway, including the South Pole Telescope (SPT) (e.g. Ruhl et al. 2004, see http://spt.uchicago.edu/), and the Atacama Cosmology Telescope (ACT) (e.g. Sehgal et al. 2007, see http://wwwphy.princeton.edu/act/).

\(^{4}\) The snapshot observations will also be of great benefit for a range of ancillary cluster science.

---

**Table 1.** Baseline X-ray observatory characteristics.

| Band pass | 0.3–10 keV |
| Spectral resolution | \(E/\Delta E \sim 2400\) (@6 keV) |
| Effective area | \(15\,000\,\text{cm}^2\) (@1.25 keV) |
| PSF | \(\leq 15\) arcsec (half power diameter) |
| FoV | \(\geq 5 \times 5\) arcmin\(^2\) |

---

From initial surveys of tens of thousands of clusters, the \(\sim 4000\) most X-ray luminous (or highest integrated SZ flux) clusters will be identified. The new X-ray observatory will then be used to take short snapshot exposures (\(\sim 1\) ks) of these clusters, to identify the most apparently dynamically relaxed systems that are most suitable for \(f_{\text{gas}}\) work (Allen et al. 2008). The selection of relaxed clusters is likely to be based primarily on X-ray morphology, but will also utilize the high spectral resolution capabilities to measure bulk gas motions.\(^{4}\) The most relaxed clusters will be re-observed with deeper exposures to measure the gas mass fraction to the required level of precision.

Current studies of the Massive Cluster Survey (MACS) (Ebeling, Edge & Henry 2001; Ebeling et al. 2007) show that at redshifts \(z \lesssim 0.5\) approximately 1/4 of the clusters are sufficiently relaxed for \(f_{\text{gas}}\) work (Allen et al. 2008). We (conservatively) calculate predicted cosmological constraints for two separate \(f_{\text{gas}}\) data sets, containing either \(\sim 500\) or 250 relaxed clusters, that is, we assume that only approximately 1/8 or 1/16 of the 4000 hottest, most X-ray luminous clusters detected in a future survey will be suitable for use in the \(f_{\text{gas}}\) experiment.

For the 500-cluster sample, we assume an average exposure time per cluster of \(\sim 20\) ks. For the 250-cluster sample, the typical exposure is \(\sim 40\) ks. In both cases, the total time required to complete the \(f_{\text{gas}}\) observations will be \(\lesssim 15\) Ms. For the assumed instrument characteristics, we expect statistical uncertainties in the \(f_{\text{gas}}\) measurements resulting from 20-ks exposures of \(\sim 5\) per cent, which corresponds to \(\sim 3.3\) per cent in distance. For typical exposures of 40 ks, we expect to measure \(f_{\text{gas}}\) to \(\sim 3.5\) per cent or distance to \(\sim 2.3\) per cent. In Section 5, we show that the constraints on dark energy from both the 500- or 250-cluster sample are comparable.

We adopt the 500-cluster sample with 5 per cent \(f_{\text{gas}}\) measurement uncertainties as our default data set.

3.2 The simulated \(f_{\text{gas}}\) data set

3.2.1 The luminosity function of clusters

To simulate the \(f_{\text{gas}}\) data set, we first need to predict the redshift distribution of clusters. We assume an X-ray flux-limited cluster survey similar to that expected to be produced by the Spectrum-RG/eROSITA mission, with a flux limit of \(F_{\text{lim}} = 3.3 \times 10^{-14}\) erg cm\(^{-2}\) s\(^{-1}\) in the 0.1-2.4 keV band and a uniform sky coverage of \(f_{\text{sky}} = 0.5\). We calculate the number of clusters expected to be observed, \(N_i\), in each redshift bin, \(z_i\), as (Mantz et al. 2008):

\[
N_i(z_i) = \int_{z_{i-1}}^{z_i} \frac{dV}{dz} dz \int_0^\infty \frac{dn(M,z)}{dM} Q dM ,
\]

where

\[
Q = \int_0^{L'} dL' \int_{\ln(L')}^{\infty} dL \int_{\ln(L)}^{\infty} dL' P(L' \mid M) P(L) ,
\]

Here, \(V\) is the comoving volume, \(n(M,z)\) is the comoving number density of haloes with a mass less than \(M\) at redshift \(z\), \(L'\) is the intrinsic luminosity of a galaxy cluster associated with a halo of mass \(M\), and \(L\) is its luminosity inferred from observations. \(P(L' \mid M)\) is the probability for a cluster of mass \(M\) to have an intrinsic luminosity \(L'\); \(P(L)\) is the probability for a cluster with intrinsic luminosity \(L'\) to be observed with luminosity \(L\); and \(\ln(L)\) is the luminosity limit.
function. We calculate the comoving volume element per redshift interval as (Hogg 1999):

$$\frac{dV}{dz} = 4\pi f_{\text{sky}} \frac{c}{H_0} \frac{(1+z)^2 d_A(z)^2}{E(z)},$$

(6)

where $c$ is the speed of light, and $d_A$ the angular diameter distance. Using $N$-body simulations, Jenkins et al. (2001) obtained the following fitting formula for the mass function of dark matter haloes:

$$\frac{dn(M, z)}{d\ln \sigma^{-1}} = \frac{\tilde{p}}{M} A \exp \left[-\ln \sigma^{-1} + B \left| \sigma \right|^\epsilon \right],$$

(7)

where $\tilde{p}$ is the comoving mean matter density of the Universe and $A, B$ and $\epsilon$ are fitted parameters. Here $\sigma^2(M, z)$ is the variance of the linearly evolved density field, smoothed by a spherical top-hat filter, $W(k; M)$. In Fourier-space representation,

$$\sigma^2(M, z) = \frac{D^2(z)}{2\pi^2} \int_0^\infty k^2 P(k) W^2(k; M) dk,$$

(8)

where $k$ is the wave number, $P(k)$ is the power spectrum of the linear density field extrapolated to redshift zero and $D(z)$ is the growth factor of linear perturbations normalized to be 1 when $z = 0$. We calculate the power spectrum using the CAMB code (Lewis, Challinor & Lasenby 2000). For halo finding algorithms tied to the mean mass density, Jenkins et al. (2001) showed that the values of $A, B$ and $\epsilon$ are almost invariant under both a broad range of cosmologies and redshift. However, these authors also showed that these parameters depend on the cluster algorithm. Here, we use $A = 0.316, B = 0.67, \epsilon = 3.82$ (Jenkins et al. 2001), which are appropriate for the spherical overdensity algorithm SO($\kappa = 324$) (Davis et al. 1985; Lacey & Cole 1994), where $\kappa$ is the mean overdensity of the halo with respect to the mean matter density of the Universe.

In equation (5) we have a lognormal probability distribution (Mantz et al. 2008)

$$P(L|M) = \frac{e^{\log_{10} L - \log_{10} L(M)^2/2\sigma^2}}{L(\log(10\sqrt{2\pi}\sigma))},$$

(9)

where $\tilde{L}(M)$ is the best-fitting luminosity for a given mass $M$, and $\sigma$ is its scatter, determined from the mass-luminosity data set of Reiprich & Böhringer (2002) using the relation

$$\log_{10} \left[ \frac{ME(z)}{h^{-2}_7 \odot} \right] = A + \sigma \log_{10} \left[ \frac{L_X(0.1 - 2.4 \text{ keV})}{10^{44}h^{-2}_7 \text{ erg s}^{-1}E(z)} \right],$$

(10)

for which $\sigma = 0.67$ and $A = \log_{10} [M_8/(h^{-2}_7 \odot)] = 14.49$, and $\sigma = 0.12$, as obtained by Mantz et al. (2008). In equation (5) we also have a Gaussian probability distribution

$$p(L|L') = \frac{e^{-(L-L')^2/2\sigma^2}}{\sqrt{2\pi}\sigma},$$

(11)

with standard deviation $\sigma_I = (\sigma_{n_{ph}}/n_{ph}) L$. Here $n_{ph}$ is the number of photons detected from a cluster in the survey and $\sigma_{n_{ph}} = \sqrt{n_{ph}}$ is the associated Poisson error. We assume that at the flux limit of the survey, $F_{\text{lim}}$, or equivalently at the luminosity limit $L_I = L(F_{\text{lim}}, z)$, the number of photons is $n_{\text{ph, lim}} \sim 20$. Using this, we have $\sigma_I = (\sqrt{L_I}/n_{\text{ph, lim}}) \sqrt{L}$.}

### Table 2. Parameter values of our fiducial cosmology, which is a flat ΛCDM cosmology.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_0$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>$0.27$</td>
</tr>
<tr>
<td>$H_0$</td>
<td>$72 \text{ km s}^{-1} \text{ Mpc}^{-1}$</td>
</tr>
<tr>
<td>$\Omega_k$</td>
<td>$0.046$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$0.95$</td>
</tr>
<tr>
<td>$\Omega_c$</td>
<td>$0.073$</td>
</tr>
<tr>
<td>$A_s$</td>
<td>$0.8$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.82$</td>
</tr>
<tr>
<td>$b$</td>
<td>$0.09$</td>
</tr>
</tbody>
</table>

3.2.2 Temperature selection

In order to minimize systematic scatter in the $f_{\text{gas}}$ experiment, Allen et al. (2008) restrict their analysis to dynamically relaxed clusters with mean gas mass-weighted temperatures measured within $r_{2500}$ $> 5$ keV. We impose the same temperature cut in this analysis, calculating the luminosity limit $L_i$ that corresponds to this temperature limit from the relation (Bryan & Norman 1998):

$$\log_{10} \left[ \frac{L_X(0.1 - 2.4 \text{ keV})}{10^{44}h^{-2}_7 \text{ erg s}^{-1}E(z)} \right] = A + B \log_{10} \left( \frac{kT_e}{\text{keV}} \right),$$

(12)

where $T_e$ is the emission-weighted X-ray temperature. Fitting the above relation (12) to the X-ray luminosity and temperature data of Reiprich & Böhringer (2002) using the linear regression BCES(Y|X) algorithm of Akritas & Bershady (1996), we obtain $A = -1.46 \pm 0.09$ and $B = 2.50 \pm 0.13$.

The limiting luminosity in equation (5) is then

$$L_{\text{lim}}(z) = \min[L_i, 4\pi F_{\text{lim}} n_{\text{ph, lim}}^2],$$

(13)

with the appropriate K-correction applied in calculating the $F_{\text{lim}}$ values.

3.2.3 The redshift distribution of $f_{\text{gas}}$ clusters

Table 2 summarizes the parameters describing our fiducial cosmology. For this cosmology, we have calculated the redshift distribution of galaxy clusters over the range $0 < z < 2$. Our fiducial cosmology approximately matches that used by the DETF, but includes updated values for $n_s$ and $\tau$ to better match the WMAP three-year and five-year results (Spergel et al. 2007; Dunkley et al. 2008). We also adopt a lower value for $\sigma_s = 0.8$, consistent with both the WMAP three-year and five-year results and the results of Mantz et al. (2008) from measurements of the X-ray luminosity function of galaxy clusters within $z < 0.7$.

Fig. 1 shows the redshift distribution (solid line) for clusters detected above the Spectrum-RG/eROSITA X-ray flux limit with mass-weighted temperatures $kT_{2500} > 5$ keV. A sky coverage, $f_{\text{sky}} = 0.5$ is assumed. Approximately 5000 clusters meet these criteria from which, following our observing strategy, 4000 will be observed by short snapshots. Assuming that approximately 1/8 of these clusters will also meet the relaxation criteria based on X-ray morphology (Allen et al. 2008; Million & Allen in preparation), a

6 $r_{2500}$ is the radius within which the mean density is 2500 times the critical density of the Universe at the redshift of the cluster.

7 $T_e$ scales with $T_{2500}$ as $kT_{2500} \sim kT_e/m_n$ with $\eta \sim 1.1 - 1.2$, based on MACS clusters spanning the redshift range $0.3 < z < 0.7$. Beyond redshift 0.7 the value of $\eta$ slowly decreases towards $\sim 1$. To be conservative, however, we ignore the difference between $T_e$ and $T_{2500}$, that is, we assume $\eta = 1$, at all redshifts.

8 We exclude objects from the Reiprich & Böhringer (2002) sample for which the temperature was estimated from the luminosity–temperature relation of Markevitch (1998) rather than directly measured. This leaves 88 data points in total.
sample of ~500 hot, X-ray luminous, dynamically relaxed clusters can be defined. Taking snapshot observations of the available ~5000 clusters instead of 4000, and assuming that approximately 1/8 of these clusters are relaxed, we will obtain a sample of ~625 \( f_{\text{gas}} \) targets. This allows us to either use a larger sample of clusters, assume an even more conservative ratio of relaxed clusters, or select a different redshift distribution for the \( f_{\text{gas}} \) sample of ~500 clusters. In Section 5.3, we discuss the latter case.

For comparison purposes, Fig. 1 also shows (dashed curve) the redshift distribution for the case of a luminosity limit \( \lim_{0.1–2.4 \text{ keV}} > 3.35 \times 10^{44} \, h^{-2}_{50} \, \text{erg s}^{-1} \) (no temperature cut) which gives a similar total number of clusters. The latter distribution has more high-\( z \) clusters.

We generate mock \( f_{\text{gas}} \) measurements for 500 clusters with the redshift distribution appropriate for the case of the temperature cut (solid curve, Fig. 1; in accordance with the selection criteria used for current \( f_{\text{gas}} \) work [Allen et al. 2008]). For each cluster, we assign a statistical error in the \( f_{\text{gas}} \) measurements of ~5 per cent. We have also generated a set of mock measurements for the case of 250 clusters observed with \( f_{\text{gas}} \) measurements accurate to 3.5 per cent. This latter data set is used to study the impact on the dark energy constraints in the case that the fraction of suitably relaxed clusters is less than 1/8 at high redshifts.

We stress that the predicted redshift distribution, which peaks around \( z \sim 0.65 \) in the case of the temperature cut, has already been probed, at least partially, over the luminosity and temperature range of interest, by the MACS survey (Ebeling et al. 2001); MACS covers the redshift range \( 0.3 < z < 0.7 \) to a flux limit of \( F_{\text{lim}} = 10^{-12} \, \text{erg cm}^{-2} \, \text{s}^{-1} \) in the 0.1–2.4 keV band. For MACS, approximately 1/4 clusters are found to be sufficiently relaxed for \( f_{\text{gas}} \) work (Allen et al. 2008). Therefore, our assumption that approximately 1/8 of the clusters detected in a future X-ray survey and meeting the X-ray flux and temperature criteria will be suitably relaxed, appears reasonable. Moreover, as discussed in Section 5.1, for the case of the 250-cluster sample (i.e. assuming that only approximately 1/16 of the clusters are relaxed) and using a similar total observing time to obtain individual \( f_{\text{gas}} \) measurements to ~3.5 per cent accuracy, we obtain very similar dark energy constraints (see Table 4).

A final important point regards contaminating point sources: for MACS clusters, the fraction of the measured 0.1–2.4 keV X-ray flux arising from contaminating point sources is small, typically of the order of a per cent (Mantz et al. 2008; this is also the case for the hottest, \( kT_e \geq 5 \, \text{keV} \), relaxed clusters at lower redshifts.) Therefore, we do not expect our target clusters, which have comparable X-ray temperatures and luminosities, to be severely affected by contaminating point sources, especially at \( z \lesssim 1 \). This alleviates the instrumental requirements on the point spread function. An instrument with capabilities similar to the baseline characteristics listed in Table 1 should be capable of making significant strides in dark energy work.

### 3.3 Follow-up SZ observations

The thermal SZ effect is a modification to the CMB spectrum caused by Compton scattering of CMB photons by hot electrons in the intracluster medium. The SZ flux measured at radio or submm wavelengths can be expressed in terms of the Compton \( y \)-parameter. For a given cosmology, the \( y \)-parameter can also be predicted from the same X-ray data used to determine the \( f_{\text{gas}} \) measurements, being proportional to the integral along the line-of-sight of the product of electron density and temperature, \( \int n_e T_e \, dl \).

We have examined the additional cosmological constraining power that can be achieved with follow-up radio/submm SZ observations of our sample of 500 clusters, assuming direct SZ flux measurements accurate to 2 or 5 per cent (a level of accuracy that should be straightforward for SZ detector technology available at the time of the experiment; see Muchvej et al. 2007, and references therein). The statistical uncertainties in the predicted Compton \( y \)-parameters will be comparable to those associated with the \( f_{\text{gas}} \) measurements: ~5 per cent for the 500-cluster sample. We generate our predicted \( y \)-parameter data set for the redshift distribution shown in Fig. 1 (solid curve).

---

\[ \text{Figure 1. The redshift distribution (solid curve) of clusters above the Spectrum-RGI/EROSITA X-ray flux limit with temperatures } kT_{200} > 5 \, \text{keV.} \]

---

\[ \text{The redshift distributions shown in Fig. 1 are sensitive to the mass-observable relation obtained by Mantz et al. (2008) using current data. See that work for details.} \]
3.4 Mock CMB data sets

We have used the camb code (Lewis et al. 2000) to generate auto and cross temperature and polarization angular power spectra, $C_{TT}^{\ell}$, $C_{EE}^{\ell}$, and $C_{TE}^{\ell}$, for the fiducial, flat $\Lambda$CDM cosmology described in Table 2. We follow Lewis (2005) and Lewis, Weller & Battye (2006) and assume that the temperature, $T$, and polarization $E$-fields are Gaussian and isotropic. We also assume that the polarization $B$-field is negligible.

Having $C_{TT}^{\ell}$, $C_{EE}^{\ell}$ and $C_{TE}^{\ell}$, we add a simple, isotropic noise power spectrum (Cooray, Hu & Tegmark 2000; Lewis 2005; Lewis, Weller & Battye 2006) to a precision of $\sigma$. For our analysis in the case of polarization contamination, Planck collaboration (2006). To account for the effects of polarization contamination, we consider a more conservative scenario where $\sigma = 0.01$ by enlarging by an order of magnitude the noise at low multipoles $\ell < 30$ in the polarization data.

For both scenarios, we use only the data from multipoles $2 \leq \ell \leq 2000$. For simplicity, we adopt the zero-contamination scenario as our default CMB data set.

4 DATA ANALYSIS METHOD

4.1 Markov Chain Monte Carlo code

Given the dark energy model described in Section 2 and the simulated $f_{\text{gas}}$ and CMB data sets described in Section 3, we use the Metropolis MCMC algorithm implemented in the cosmomc code to calculate CMB power spectra; this accounts for the effects of dark energy perturbations for evolving dark energy equations of state (Rapetti et al. 2005) (see Section 4.5 for details). Our modified version of the cosmomc code also incorporates the $f_{\text{gas}}$ analysis method described by Allen et al. (2008) (see also Rapetti et al. 2005, 2007).

Our choice to forecast parameter constraints using a full MCMC analysis has some advantages over the more widely used Fisher matrix formalism (see discussions in Perotto et al. 2006; Lewis et al. 2006). First, the shape of the mean log likelihood (see equation 22) (Lewis et al. 2006) in the MCMC analysis encapsulates all of the relevant degeneracies between parameters, which is crucial for non-Gaussian distributions. Secondly, the fact that our forecasts are made using the same cosmomc analysis code used to analyse current data (Allen et al. 2008) ensures consistency between present and future constraints. Finally, the MCMC method allows us to easily and efficiently introduce priors and allowances and thereby study the effects of systematic uncertainties.

4.2 X-ray gas mass fraction analysis

4.2.1 The $f_{\text{gas}}$ method

The X-ray gas mass fraction, $f_{\text{gas}}$, is defined as the ratio of the X-ray-emitting gas mass to the total mass of a cluster. This quantity can be determined from the observed X-ray surface brightness and the deprojected, spectrally determined gas temperature profile, under the assumptions of spherical symmetry and hydrostatic equilibrium.

To ensure that these assumptions are as accurate as possible, it is essential to limit the $f_{\text{gas}}$ analysis to the hottest, most X-ray luminous, dynamically relaxed clusters available [Section 3.1; for a detailed discussion of the method and current measurements see Allen et al. (2008) and references therein.]

In order to study dark energy, Allen et al. (2008) use $f_{\text{gas}}$ measurements for a sample of 42 hot ($kT_{2500} > 5$ keV), X-ray luminous, dynamically relaxed clusters. The $f_{\text{gas}}$ measurements are made within an angle $\theta_{2500}^2$ for each cluster, corresponding to $r_{2500}$ for a reference flat $\Lambda$CDM cosmology (with $\Omega_m = 0.3$ and $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$). The $f_{\text{gas}}$ measurements in the reference cosmology $f_{\text{gas}}^0$ are related to the true values $f_{\text{gas}}^0$ as

$$f_{\text{gas}}^0(z; \Omega_m^0) = f_{\text{gas}}^0(z; \Omega_m^0) \left( \frac{\Omega_h^2}{\Omega_m^2} \right)^{3/2}. \quad (16)$$

Non-radiative hydrodynamical simulations (Eke, Navarro & Frenk 1998; Nagai, Vikhlinin & Kravtsov 2007; Crain et al. 2007) suggest that $f_{\text{gas}}$ is likely to be approximately constant in redshift. Thus (Allen et al. 2008),

$$f_{\text{gas}}^0(z; \Omega_m^0) = \left( \frac{\Omega_h}{\Omega_m} \right) \left( \frac{b_0}{1 + s_0} \right). \quad (17)$$

where $s_0 = 0.16h^{0.5}_{70}$ (Lin & Mohr 2004; Gonzalez, Zaritsky & Zabludoff 2007) is the observed ratio of the mass in stars (both in galaxies and intracluster light) to the X-ray-emitting gas mass, and $b_0 = 0.82$ (Eke et al. 1998) is the depletion factor for the baryon fraction in clusters with respect to the cosmic mean value.

As discussed by Allen et al. (2008), an angular correction factor is also required to account for the fact that $f_{\text{gas}}^0(z; \Omega_m^0)$ need not
be exactly equal to \( f_{\text{gas}}^{\text{true}} (z; \Omega_{\Lambda}^{\text{CDM}}) \). Observations of large, relaxed clusters show that for the radial range of interest, \( 0.7 < r/r_{2500} < 1.2 \), the \( f_{\text{gas}}(r) \) profiles can be fitted by a shallow power-law model with slope \( \eta = 0.214 \pm 0.022 \).\(^{13}\) Thus, we have

\[
f_{\text{gas}}^{\text{true}} (z; \Omega_{\Lambda}^{\text{CDM}}) = \frac{f_{\text{gas}}^{\text{true}} (z; \Omega_{\Lambda}^{\text{CDM}})}{f_{\text{gas}}^{\text{true}} (z; \Omega_{\Lambda}^{\text{CDM}})} \eta.
\]

(18)

where \( \theta_{2500} = r_{2500}/d_A \), and

\[
\frac{dA_{\text{CDM}}}{df_{\text{gas}}^{\text{true}} (z; \Omega_{\Lambda}^{\text{CDM}})} \eta = \left( \frac{H(z) d_A(z)}{[H(z) d_A(z)]^{\Omega_{\Lambda}^{\text{CDM}}}} \right).
\]

(19)

This correction factor is small and can be neglected for most analyses of current data, although its inclusion leads to slightly tighter constraints on dark energy (Allen et al. 2008). However, for future experiments of the precision being considered here, the inclusion of the angular correction term becomes important.

4.2.2 Allowances for systematic uncertainties

Following Allen et al. (2008), we modify equation (17) to account for systematic uncertainties in the \( f_{\text{gas}} \) analysis:

\[
f_{\text{gas}}^{\text{true}} (z; \Omega_{\Lambda}^{\text{CDM}}) = \gamma K \left( \frac{\Omega_h}{\Omega_m} \right) \left[ b(z) \right] \left[ 1 + s(z) \right].
\]

(20)

Here \( \gamma \) allows for departures from the assumption of hydrostatic equilibrium, due to non-thermal pressure support; \( K \) is a normalization uncertainty relating to instrumental calibration and certain modelling issues; \( b(z) = b_0(1 + \alpha_b z + \beta_b z^2) \) accounts for uncertainties in the cluster depletion factor, both in the normalization, \( b_0 \), and possible linear, \( \alpha_b \), and quadratic, \( \beta_b \), evolution with redshift;\(^{14}\) \( s(z) = s_0(1 + \alpha_s z + \beta_s z^2) \) accounts for uncertainties in the stellar gas mass fraction.\(^{15}\)

Using hydrodynamic N-body simulations, Nagai et al. (2007) show that for measurements at \( r_{2500} \) in large, relaxed clusters, non-thermal pressure support is unlikely to exceed 8 per cent. Furthermore, if, as suggested by some current X-ray data (Fabian et al. 2003, 2005; Reynolds et al. 2005), the gas viscosity is higher than that included in current simulations, then non-thermal pressure support could be even lower. Based on these findings, we adopt by default a uniform prior such that non-thermal pressure support lies in the range 0–8 per cent (although a more pessimistic range of 0–16 per cent is also considered). Since the use of an asymmetric prior would bias the analysis, leveraging \( \Omega_m \) above the fiducial value, we employ an equivalent, rescaled symmetric prior such that \( 1 - (a/2) < \gamma < 1 + (a/2) \), where \( a = [1 - 1.08]/1.04 \).

The depletion parameter, \( b_0 \), reflects the thermodynamic history of the X-ray-emitting cluster gas. Using non-radiative simulations of hot, massive clusters of comparable size to the real clusters to be used in the \( f_{\text{gas}} \) experiment, Eke et al. (1998) (see also Allen et al. 2004; Nagai et al. 2007; Crain et al. 2007) obtained \( b_0 = 0.82 \pm 0.03 \) at the radius of the measurements \( r_{2500} \approx 0.25r_{2500} \) and found no evidence for redshift evolution: \( \alpha_b = 0.00 \pm 0.03 \) for measurements made at \( r \sim 0.5r_{2500} \), spanning the redshift range \( 0 < z < 1 \). As discussed by Allen et al. (2008), however, systematic uncertainties are associated with current predictions for \( b(z) \), due to limitations in the accuracy of the physical approximations employed in the simulations. Estimating the residual uncertainties in the prediction of \( b(z) \) that will be appropriate at the time of a future \( f_{\text{gas}} \) data set (\( \sim 2015 - 2020 \)) is difficult. We have chosen to use a range of values that extend from optimistic to pessimistic scenarios (see Table 3).

Current optical and near-infrared data for low-to-intermediate redshift clusters give \( s_0 = 0.16b_0^{0.5} \) (Fukugita, Hogan & Peebles 1998; Lin & Mohr 2004; Gonzalez, Zaritsky & Zabludoff 2007). Although, at present, the constraints on \( s(z) \) for clusters at \( z \gtrsim 0.5 \) are sparse, we expect the form of \( s(z) \) to be relatively well understood by the time of the \( f_{\text{gas}} \) experiment.

In order to keep the interpretation of our results simple, we present results for three sets of systematic allowances: for the parameters, \( K, b_0, \alpha_b, \beta_b, s_0, \alpha_s, \beta_s \), we employ allowances of either \( \pm 2 \) per cent (optimistic), \( \pm 5 \) per cent (standard), or \( \pm 10 \) per cent (pessimistic). In all cases, we employ uniform priors with the exception of \( K \) and \( s_0 \), for which Gaussian priors are more appropriate and therefore used. As noted above, a uniform allowance of \( \pm 4 \) per cent on \( \gamma \) is included by default, although the effects of doubling the uncertainty in this parameter are also examined. We stress that whether \( \gamma = 1 \) precisely, or \( \alpha_s, \beta_s \), etc., are precisely zero, is not of primary importance to a future analysis: if known, the exact values can be incorporated into the default model. It is the uncertainties in the values that affect the accuracy and precision of the dark energy constraints.

4.3 Analysis of the SZ data: the XSZ experiment

For the true, underlying cosmology, the measurement of the Compton \( \gamma \)-parameter from both the X-ray and SZ data should match (e.g. Molnar, Birkhinshaw & Mushotzky 2002; Schmidt, Allen & Fabian 2004; Bonamente et al. 2006). For a given cosmology, the \( \gamma \)-parameter predicted by X-ray data depends on the square root of the angular diameter distance to the cluster, \( d_A^{0.5} \), whereas the observed SZ flux at radio or submm wavelengths is independent of the cosmology assumed. Combining the \( \gamma \)-parameter results, we can measure the distances to the clusters as a function of redshift and, therefore, constrain dark energy.

\[
y^{\Lambda_{\text{CDM}}} = y^{\text{SZobs}} k(z) \left( \frac{d_A}{d_A^{0.5}} \right)^{1/2}.
\]

(21)

Here \( y^{\Lambda_{\text{CDM}}} \) is the X-ray measurement of the \( \gamma \)-parameter for the reference cosmology and \( y^{\text{SZobs}} \) is the radio/submm observation.\(^{16}\) Following a similar approach to that adopted with the \( f_{\text{gas}} \) data, we incorporate systematic allowances into equation (21): \( k(z) = k_0(1 + \alpha_k z) \) accounts for the combined systematic uncertainties in the X-ray and SZ data \( \gamma \)-parameter measurements due to calibration, geometric effects, gas clumping, etc., and their evolution. We employ Gaussian priors on \( k_0 \) of size 2 (optimistic) or 5 (standard/pessimistic) per cent and uniform priors on \( \alpha_k \) of size 2 (optimistic), 5 (standard) or 10 (pessimistic) per cent.

We note that the best clusters to observe for the XSZ experiment are the same systems used for the \( f_{\text{gas}} \) experiment: the largest, most dynamically relaxed clusters. These are the clusters for which

\(^{13}\) Note that even using two very different reference cosmologies such as SCDM (\( \Omega_m = 1, H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \)) and \( \Lambda \text{CDM} \) (\( \Omega_m = 0.3, H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \)), Allen et al. (2008) obtained similar values for \( \eta \) around \( r_{2500} \).

\(^{14}\) Note that the allowances on \( \alpha_b \) and \( \beta_b \) can also be assumed to encompass the combined uncertainties in the redshift evolution of \( \gamma, K \) and \( b \), which have the same effect on equation (20).

\(^{15}\) Working with current data, Allen et al. (2008) use only the linear order of the redshift expansions for their systematic allowances, that is, \( \alpha_b \) and \( \alpha_s \).

\(^{16}\) The limitations of existing SZ data have to date restricted the XSZ experiment to measurements of the Hubble constant (e.g. Bonamente et al. 2006, and references therein).
the SZ signals are strongest and for which systematic uncertainties associated with geometry and thermodynamic structure are minimized. Note also that no additional X-ray observations are required to carry out the XSZ experiment, once the $f_{\text{gas}}$ data are in hand.

### 4.4 Incorporating the CMB data

In addition to the dark energy model parameters and the $f_{\text{gas}}$ parameters discussed in Section 4.2, we vary the following eight CMB-related parameters in the MCMC analysis: the mean physical baryon density, $\Omega_b h^2$; the mean physical cold dark matter density, $\Omega_c h^2$; the (approximate) ratio of the sound horizon at last scattering to the angular diameter distance (Kosowsky, Milosavljevic & Jimenez 2002), $\theta$; the optical depth to reionization (assumed to occur in a sharp transition), $\tau$; the mean curvature density of the Universe, $\Omega_k$; the scalar adiabatic spectral index, $n_s$; and the scalar adiabatic amplitude, $A_s$, at $k = 0.05 \text{ Mpc}^{-1}$. We employ a uniform prior on $\ln(A_s)$. The combination of $\theta_s$ and $\ln (A_s)$, as parameters, rather than $H_0$ and $A_s$, leads to a more Gaussian probability density distribution which, in turn, aids sampling (Kosowsky et al. 2002; Lewis et al. 2006).

The degeneracies between dark energy model parameters and $\Omega_k$ are of particular importance in the analysis (Rapetti, Allen & Weller 2005; Clarkson, Cortes & Bassett 2007).\footnote{Allen et al. (2004, 2008) and Rapetti et al. (2005) showed that the combination of $f_{\text{gas}}$ plus CMB data allows one to drop both the assumption of flatness and the priors on $\Omega_b h^2$ and $h$ that would otherwise be required for the $f_{\text{gas}}$ analysis. The $f_{\text{gas}}$+CMB data combination also alleviates other important parameter degeneracies, for example, between $\Omega_b h^2$, $n_s$ and $r$.} For their forecasts, the DETF include Planck priors in their Fisher matrix analysis, approximating the role of future CMB constraints as well as the degeneracies between the dark energy parameters and $\Omega_k$. Here, we account fully for the degeneracies between parameters, and the complementarity of the data sets.

Given the vector, $\epsilon$, of CMB-related parameters, we sample the exponential of the following mean logarithmic likelihood (Lewis et al. 2006):

$$
(\ln P(\epsilon | \theta)) = -\frac{1}{2} \left[ Tr(\mathbf{C}(\epsilon)\mathbf{C}(\epsilon)^{-1}) + \ln |\mathbf{C}(\epsilon)| \right],
$$

where $\epsilon$ is the vector formed by the corresponding fiducial values of Table 2. Note that where the posterior is non-Gaussian, the marginalized constraints on individual parameters need not peak exactly at the fiducial values, although for all cases considered here the differences are very small.

### 4.5 Dark energy clustering

For a dark energy model with a constant equation of state, $w$, Weller & Lewis (2003) and Bean & Dore (2004) showed that dark energy clustering can have a non-negligible impact on the constraints, driven primarily by the effect of such perturbations on the ISW effect. Spergel et al. (2007) showed that accounting for dark energy clustering has a large effect on current dark energy constraints derived from CMB data alone. Since, for constant-$w$ models, combining the CMB data with, for example, distance measurements from SNe Ia or X-ray galaxy clusters leads to tight constraints on $w$ and a result consistent with a cosmological constant $(w = -1$, for which no dark energy clustering occurs), the importance of accounting for dark energy perturbations is reduced (Weller & Lewis 2003; Rapetti et al. 2005; Spergel et al. 2007). However, when one considers more general models in which $w$ evolves, Rapetti et al. (2005) showed that even with the best current data combinations, accounting for the effects of dark energy perturbations is important: the constraints on $w(a)$ increase by a factor of $\sim 2$ with respect to the case where dark energy clustering is (wrongly) ignored.

For our analysis, we assume that dark energy is an imperfect fluid where dissipative processes generate entropy perturbations. As suggested by quintessence scenarios, we assume a constant, general (non-adiabatic) sound speed $c^2_s = 1$ in the comoving frame of the fluid (denoted by the circumflex). This is the only frame for which the general sound speed is gauge-invariant (Bean & Dore 2004). Following Weller & Lewis (2003) and Bean & Dore (2004), Rapetti et al. (2005) extended the dark energy perturbation equations to account for an evolving dark energy equation of state, $w(a)$. As in Rapetti et al. (2005) we calculate the density, $\delta$, and velocity, $v$, perturbation equations (Ma & Bertschinger 1995) in the synchronous gauge

$$
\dot{\delta} = -3H (\dot{\epsilon}_s^2 - w) \delta - (1 + w)(k v + 3 \dot{B}) + \mathcal{E}(w),
$$

$$
\dot{v} = -H (1 - 3\epsilon_s^2) v + \frac{k \dot{\epsilon}_s \dot{\delta}}{1 + w},
$$

where both derivatives, denoted by dots, and the Hubble parameter, $H$, are with respect to conformal time. $B = ba/a$ is the metric perturbation and $\delta$ is the density perturbation in the comoving frame of the dark energy fluid. The density perturbation, $\delta$, can be recast into the CDM comoving frame density and velocity perturbations,
single classical scalar field cannot evolve from a quintessence-like, a super-accelerated phase \((w < -1)\). Woodard (2002, 2004) proposed a single scalar field model where the equation of state of two combined scalar fields, one with \((w < -1)\), can cross the cosmological-constant boundary, \(w = -1\), as it evolves in time. Other models that allow \(w(a)\) to cross this boundary have also been proposed. However, for practical purposes, using an effective, evolving dark energy equation of state produces a well-known divergence in equation (24) when \(w(a) = -1\). This divergence can be avoided (Hu 2004; Caldwell & Doran 2005) by imposing \(\dot{\delta} = 0\) and \(\dot{v} = 0\) within the logarithmic singularity region, \(w = -1 + |\epsilon|\), where \(\epsilon\) is infinitesimally small. Inaccuracies in this approximation have a negligible impact on the resulting CMB power spectra (Rapetti et al. 2005; Xia et al. 2006, 2007).

In what follows, we present results for cases where dark energy clustering is either accounted for or ignored in the CMB analysis (Section 5.4). Dark energy clustering does not have a significant impact on the \(f_{\text{gas}}\) analysis.

### 5 RESULTS

As described in Section 2, to enable a direct comparison with the predicted dark energy constraints for other planned experiments, we parametrize our results in terms of the DETF FoM. We present results for a fiducial \(f_{\text{gas}}+\text{CMB}\) data set, incorporating the statistical uncertainties and systematic allowances described above, and with zero scatter about the fiducial curves. The absence of scatter in the simulated samples ensures that the peaks of the posterior probability distributions occur at the expected values, in the same way that the DETF Fisher matrix analysis does, that is, in order to compare our results with the DETF, we select the same realization as the DETF. Note also that the use of a zero-scatter realization does not affect the FoM. We have explicitly confirmed this, and that using other realizations does not have a large impact on the FoM, by comparing the FoM for the fiducial realization to a series of Monte Carlo simulations, in which appropriate scatter about the fiducial \(f_{\text{gas}}\) curve was included.

#### 5.1 Constraints on the FoM

In the first case, we determine constraints for our ‘default’ analysis: this involves \(f_{\text{gas}}\) data for 500 clusters measured to 5 per cent accuracy and CMB data with negligible foreground contamination. We ignore the effects of dark energy clustering (as do the DETF) and allow linear evolution in \(b(z)\) and \(s(z)\). No follow-up SZ data are included.

The constraints from the default analysis are shown in Figs 2, 4 and 5. The left-hand panel of Fig. 2 shows the well-known degeneracy between \(w_0\) and \(w_a\). The right-hand panel of that figure shows the constraints in the \(w_p−w_s\) plane. The results from the MCMC analysis confirm that \(w_p\) and \(w_s\) are approximately uncorrelated, which facilitates the simple calculation of the FoM = \([\delta(w_p)\times\delta(w_s)]^{-1}\) as described in Section 2.

Table 4 summarizes the results on \(\Omega_m\), \(\Omega_b\), \(w_0\), \(w_s\), \(w_p\) and the FoM for the default analysis and 2, 5 or 10 per cent systematic allowances. Also included in the table are results for five further slightly modified, interesting scenarios: for the case where we include dark energy clustering (Section 4.5); for the case where we use the more conservative CMB data set (Section 3.4); for the case where we allow quadratic redshift evolution in the gas depletion...
Table 4. The 1σ uncertainties on the dark energy parameters and FoM. Systematic allowances of 2 per cent (optimistic), 5 per cent (standard) or 10 per cent (pessimistic) have been used. Results are presented for the default model and for six other cases, described in the text. We obtain a FoM in the range 34–43, for the optimistic allowances, in the range 21–33, for the standard, and in the range 15–29, for the pessimistic.

<table>
<thead>
<tr>
<th>Run</th>
<th>Model</th>
<th>FoM/FoM (percentage)</th>
<th>ΔFoM/FoM (percentage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Default</td>
<td>0.014</td>
<td>0.011</td>
</tr>
<tr>
<td>2</td>
<td>DE clustering</td>
<td>0.014</td>
<td>0.010</td>
</tr>
<tr>
<td>2</td>
<td>Quadratic</td>
<td>0.015</td>
<td>0.011</td>
</tr>
<tr>
<td>2</td>
<td>Half sample</td>
<td>0.014</td>
<td>0.011</td>
</tr>
<tr>
<td>2</td>
<td>Double γ</td>
<td>0.019</td>
<td>0.013</td>
</tr>
<tr>
<td>2</td>
<td>Adding XSZ</td>
<td>0.013</td>
<td>0.010</td>
</tr>
<tr>
<td>5</td>
<td>Default</td>
<td>0.022</td>
<td>0.016</td>
</tr>
<tr>
<td>5</td>
<td>DE clustering</td>
<td>0.020</td>
<td>0.016</td>
</tr>
<tr>
<td>5</td>
<td>CMB conservative</td>
<td>0.023</td>
<td>0.017</td>
</tr>
<tr>
<td>5</td>
<td>Quadratic</td>
<td>0.022</td>
<td>0.016</td>
</tr>
<tr>
<td>5</td>
<td>Half sample</td>
<td>0.022</td>
<td>0.016</td>
</tr>
<tr>
<td>5</td>
<td>Double γ</td>
<td>0.024</td>
<td>0.018</td>
</tr>
<tr>
<td>5</td>
<td>Adding XSZ</td>
<td>0.021</td>
<td>0.015</td>
</tr>
<tr>
<td>10</td>
<td>Default</td>
<td>0.033</td>
<td>0.024</td>
</tr>
<tr>
<td>10</td>
<td>DE clustering</td>
<td>0.029</td>
<td>0.022</td>
</tr>
<tr>
<td>10</td>
<td>CMB conservative</td>
<td>0.038</td>
<td>0.027</td>
</tr>
<tr>
<td>10</td>
<td>Quadratic</td>
<td>0.033</td>
<td>0.025</td>
</tr>
<tr>
<td>10</td>
<td>Half sample</td>
<td>0.033</td>
<td>0.024</td>
</tr>
<tr>
<td>10</td>
<td>Double γ</td>
<td>0.034</td>
<td>0.025</td>
</tr>
<tr>
<td>10</td>
<td>Adding XSZ</td>
<td>0.028</td>
<td>0.020</td>
</tr>
</tbody>
</table>

5.2 Comparison with DETF results

Comparing our results on the FoM with those reported by the DETF (page 77 of the DETF report; Albrecht et al. 2006), we find that the $f_{\text{gas}}$ experiment has similar dark energy constraining power to other leading, future (DETF stage IV) ground- or space-based experiments.

Fig. 4 shows the 95 per cent confidence constraints in the $\Omega_{\text{de}}$–$w_a$ plane for the default model and optimistic (2 per cent), standard (5 per cent) and pessimistic (10 per cent) allowances. The size of this confidence region is inversely proportional to the FoM. The DETF

18 The $f_{\text{gas}}$ experiment described here would fall under the category of stage IV experiments, as defined by the DETF.
distribution we form an \( \Omega_{1} \) distribution on the dark energy constraints. In particular, the power of the \( f_{\text{gas}}(+\text{Planck}) \) experiment in constraining \( \Omega_{0} \) is evident.

### 5.3 The relevance of the redshift distribution

To calculate the dark energy results presented in Table 4, we employ a redshift distribution of clusters drawn from a simulated X-ray luminosity function (based on the work of Mantz et al. 2008, as discussed in Section 3.2.1) for both a given fiducial cosmology and a future, planned X-ray cluster survey. As discussed in Section 3.2.2, we select clusters from this distribution using the same criterion (\( kT > 5 \) keV) than we use for current data (Allen et al. 2008). To apply this selection criterion to the simulated data set we use a luminosity-temperature relation obtained from present-day data (Reiprich & Böhinger 2002). Note that in this relation the temperatures are emission-weighted instead of mass-weighted, as they are in Allen et al. (2008), which makes our selection even more conservative.

Finally, we select only relaxed clusters by scaling the distribution with a factor of 1/8 (or 1/16 for the half sample scenario). This factor is especially conservative at low redshifts (\( z \lesssim 0.5 \)), where the MACS survey has shown that 1/4 of the clusters are sufficiently relaxed for \( f_{\text{gas}} \) work. This suggests that selecting a distribution with more clusters at low redshifts is a plausible alternative. Furthermore, after all these conservative cuts we obtain a total number of clusters suitable for \( f_{\text{gas}} \) work larger than 500. This offers us additional freedom to build an alternative redshift distribution. In this section, we use two alternative distributions to assess the impact that choosing a particular distribution has on the dark energy constraints.

Within the limits of the analysis described above, we design two test distributions such that their peaks are at a lower redshift, \( z \sim 0.5 \), than that of the original distribution, \( z \sim 0.65 \). From each test distribution we form an \( f_{\text{gas}} \) sample of \( \sim 500 \) clusters, and \( \sim 5 \) per cent \( f_{\text{gas}} \) measurement errors per cluster, as we did for the original distribution. The first test distribution is approximately gaussian, with a large number of clusters at low redshift, a small tail at high redshifts, and very few clusters beyond redshift 1. The second test distribution is similar to the original, but shifted towards lower redshifts. At high redshifts this distribution has less clusters than the original but significantly more clusters than the first test distribution.

Using our default dark energy model, and the 2 per cent set of systematic allowances, the first test distribution provides an increase in the FoM of \( \sim 10 \) per cent with respect to the original distribution. As mentioned in Section 3.2.3, the pivot redshift, \( z_{p} \), is important for the DETF FoM criterion. Fig. 5 shows that for the \( f_{\text{gas}} \) experiment, \( z_{p} \sim 0.25 \). Interestingly, the second test distribution provides an increase in the FoM of \( \sim 40 \) per cent, which suggests that having enough high-redshift clusters is also important for the DETF FoM. Note also that to obtain the \( f_{\text{gas}} \) measurements for each of the test samples, we will require a shorter total exposure time than that estimated for the original sample (\( \sim 15 \) Ms). Thus, using the same exposure time, an even larger increase in the FoM should be achievable with such samples.

These results indicate that a further analysis is required to determine the optimal redshift distribution of clusters with which to carry out future \( f_{\text{gas}} \) experiments. Such analysis is beyond the scope of this paper, but we will pursue it in a forthcoming publication.

### 5.4 The evolution of dark energy

Fig. 5 shows the evolution of the dark energy equation of state as a function of scalefactor \( w(a) \). The pivot scalefactor, \( a_{p} \), is the scalefactor at which we obtain the tightest constraint on \( w \) [that measurement being \( 
\sigma(w_{p}) \)]. As shown in Fig. 5, for the \( f_{\text{gas}} \) experiment we measure a pivot scalefactor of \( a_{p} \sim 0.8 \), which corresponds to a pivot redshift \( z_{p} \sim 0.25 \). Interestingly, these values lie between the pivot scalefactors/redshifts reported by the DETF for SN Ia experiments \( (a_{p} \sim 0.93/z_{p} \sim 0.075) \) and galaxy cluster number counts, weak lensing and BAO experiments \( (a_{p} \sim 0.65/z_{p} \sim 0.54) \). Pinning down the evolution of \( w \) over a wide redshift range will be a crucial

---

**Figure 4.** The 95 per cent confidence contours in the \( \Omega_{0} - w_{p} \) plane for the default dark energy model and optimistic (2 per cent; blue, solid contour), standard (5 per cent; dashed contour) and pessimistic (10 per cent; red contour) allowances. The axes are scaled to cover the same region as the figures presented by the DETF.

**Figure 5.** The 1σ confidence constraints on the evolution of the dark energy equation of state as a function of scalefactor \( w(a) \). Results are shown for the default model (Table 1) using the optimistic (2 per cent; shaded, purple region), standard (5 per cent; dashed line) and pessimistic (10 per cent; solid, red line) systematic allowances. The tightest constraints on \( w(a) \) occur at the pivot scalefactor, \( a_{p} \sim 0.8 \) (\( z_{p} \sim 0.25 \)).
The combination of dotted lines show the constraints from the dark energy at recombination. CMB data (Page et al. 2003) and is highly sensitive to the amount of $f$, for our experiment, dark energy is constrained primarily by $w_a$ changes drastically if the early dark energy density exceed the matter plus radiation density (Wright 2005; Upadhye, Ishak & Steinhardt 2005; Wright 2007). 19 At late times, for the experiment, dark energy is constrained primarily by the $f_{gas}$ data, with a small contribution from the ISW effect in the CMB.

A simple exercise provides further insight into how the CMB data help in constraining dark energy. For this, we re-examine the constraints in the $w_0$–$w_a$ plane obtained from the $f_{gas}$+CMB data; the 68 and 95 per cent confidence contours for the default model with 5 per cent allowances are shown (dashed curves) in Fig. 6. The combination of $f_{gas}$+CMB data provides tight constraints on $\Omega_b h^2, \Omega_{dm} h^2$ and $w_a$ (driven primarily by the CMB data), and on $h$ (driven by the combination of both data sets). Using these constraints as priors, we examine the constraints in the $w_0$–$w_a$ plane that can be obtained from the $f_{gas}$ data alone; the results are shown as the red, solid curves in Fig. 6. We see that the priors encompass some of the CMB constraining power, in particular in defining the characteristic upper boundary in the $w_0$–$w_a$ plane. However, they do not contain the full information on, for example, the covariance of $\Omega_b h^2, \Omega_{dm} h^2$ and $h$ (Rapetti et al. 2005; Wright 2007) which is also important in constraining dark energy at later times.

We note that the prior on $w_a$ provides a tight constraint on the curvature. The blue dotted curves in Fig. 6 show the constraints obtained from the $f_{gas}$ data alone, using only the priors on $\Omega_b h^2$ and $h$ and assuming flatness.

5.6 The importance of the XSZ experiment

The XSZ technique provides a complementary and independent experiment to measure dark energy. Although the inclusion of constraints from the XSZ experiment leads to only modest formal improvements in the FoM with respect to the results for the $f_{gas}$+CMB data (Table 4; as can be expected given the relatively weak dependence on dark energy in equation 21), it is important to note that the XSZ experiment relies on different assumptions and has different systematic uncertainties. In particular, the XSZ experiment is independent of assumptions regarding hydrostatic equilibrium, the depletion factor, and the stellar mass fraction. Thus, the combination of data from the $f_{gas}$ and XSZ techniques can help to ensure robustness in the results. In principle, the inclusion of XSZ data can also allow some of the priors in the $f_{gas}$ experiment to be relaxed.

6 CONCLUSIONS

We have examined the ability of a future X-ray observatory, with capabilities similar to those planned for Constellation-X, to constrain dark energy via the $f_{gas}$ experiment. We find that $f_{gas}$ measurements for a sample of 500 hot ($kT_{2500} \gtrsim 5$ keV), X-ray bright, dynamically relaxed clusters, with a precision of $\sim 5$ per cent, can be used to constrain dark energy with a FoM of 15–40. These constraints are comparable to those predicted by the DETF (Albrecht et al. 2006) for other leading, planned (DETF Stage IV) dark energy experiments. We also find that, for the $f_{gas}$ experiment, the FoM can be boosted up by at least $\sim 40$ per cent by selecting an optimal redshift distribution of suitable clusters on which to carry out the $f_{gas}$ observations. Interestingly, the optimal redshift distribution of $f_{gas}$ measurements appears to be shifted towards low redshifts.

As discussed in the text, a future $f_{gas}$ experiment will need to be preceded by a large X-ray or SZ cluster survey that will find hot, X-ray luminous clusters out to high redshifts. A survey such as that planned with the Spectrum-RG/eROSITA mission should find several thousands of such clusters. Short ‘snapshot’ follow-up observations of the clusters with a new, large X-ray observatory should be able to identify a sample of ~500 suitable systems for $f_{gas}$ work. Attaining a precision of $\sim 5$ per cent with individual $f_{gas}$ measurements should be straightforward for an observatory with characteristics similar to Constellation-X, requiring exposure times of $\sim 20$ ks on average. We note that the population of galaxy clusters in the redshift, temperature and X-ray luminosity range of interest has already been partially probed by the MACS survey (Ebeling et al. 2001; Chandra observations of MACS clusters are used extensively in current $f_{gas}$ studies (Allen et al. 2004; LaRoque et al. 2006; Allen et al. 2008). The low level of X-ray flux contamination from point sources observed in MACS clusters also alleviates the

19 The presence of the boundary in the $w_0$–$w_a$ plane (Rapetti et al. 2005; Upadhye et al. 2005; Wright 2007) makes it important to consider, as we do here, simulations that account fully for measurement uncertainties but which do not scatter about the fiducial curve. Otherwise, scatter towards the CMB boundary would increase the FoM, and scatter away would decrease it, complicating the interpretation of results.
requirements on the instrumental PSF for dark energy work via the $f_{\text{gas}}$ method.

In determining the predicted dark energy constraints, we have employed the same MCMC method used to analyse current data. The MCMC method encapsulates all of the relevant degeneracies between parameters and allows one to easily and efficiently incorporate priors and allowances in the analysis. We have included an array of such systematic allowances, with tolerances ranging from optimistic to pessimistic. Our technique differs from the DETF (Albrecht et al. 2006), who use a simpler Fisher matrix approach in the prediction of dark energy constraints. Despite these differences, we have endeavored to make our calculations of the FoM (Section 2) as comparable as possible.

Benchmarking our results against those of the DETF for other future ‘Stage IV’ dark energy experiments, that is, large, long-term missions, we find that the $f_{\text{gas}}$ experiment should provide a comparable FoM to future ground-based SNe Ia (FoM = 5–55), space-based SNe Ia (FoM = 19–27), ground-based BAO (FoM = 5–55), space-based BAO (FoM = 20–42) and space-based cluster counting (FoM = 6–39) experiments. Formally, the predicted FoM for the $f_{\text{gas}}$ experiment is comparable to ‘pessimistic’ scenarios for weak lensing experiments discussed by Albrecht et al. (2006), although the value falls short of the most optimistic DETF weak lensing predictions. The tight constraints on $\Omega_{\text{m}}$ and $\Omega_{\text{r}}$ for the $f_{\text{gas}}$ experiment will be of importance when used in combination with other techniques. Interestingly, the ‘pivot point’ for the $f_{\text{gas}}$ experiment lies between those of the SN Ia and BAO/weak lensing/cluster number count experiments, offering excellent redshift coverage in an attempt to pin down the evolution of dark energy.

We conclude that the $f_{\text{gas}}$ experiment offers a powerful approach for dark energy work, which should be competitive with and complementary to the best other planned dark energy experiments.

ACKNOWLEDGMENTS

We thank the members of the Constellation-X Facility Science Team (FST) for detailed discussions relating to the technical capabilities of the mission, especially N. White, H. Tananbaum and R. Mushotsky. DR thanks the NASA Goddard Space Flight Center for hospitality during the 2006 December Con-X FST meeting. We are grateful to A. Jenkins for sharing with us his code to calculate the mass function of dark matter haloes, and thank S. Church and J. Weller for discussions. We also thank G. Morris for technical support. The computational analysis was carried out using the KIPAC XOC and Orange computer clusters at SLAC, and the SLAC UNIX compute farm. SW acknowledges support from the National Aeronautics and Space Administration through Chandra Award Number DD5-6031X issued by the Chandra X-ray Observatory Centre, which is operated by the Smithsonian Astrophysical Observatory for and on behalf of the National Aeronautics and Space Administration under contract NAS8-03060. This work was also supported in part by the US Department of Energy under contract number DE-AC02-76SF00515. AM was additionally supported in part by a William R. and Sara Hart Kimball Stanford Graduate Fellowship.

REFERENCES


Clarkson C., Cortes M., Bassett B. A., 2007, JCAP, 8, 11
LaRoque S. J., Bonamente M., Carlstrom J. E., Joy M. K., Nagai D., Reese

Seljak U. et al., 2005, Phys. Rev. D, 71, 103515

This paper has been typeset from a TeX/LaTeX file prepared by the author.