What is the best way to measure baryonic acoustic oscillations?

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ABSTRACT
Oscillations in the baryon–photon fluid prior to recombination imprint different signatures on the power spectrum and correlation function of matter fluctuations. The measurement of these features using galaxy surveys has been proposed as a means to determine the equation of state of the dark energy. The accuracy required to achieve competitive constraints demands an extremely good understanding of systematic effects which change the baryonic acoustic oscillation (BAO) imprint. We use 50 very large volume N-body simulations to investigate the BAO signature in the two-point correlation function. The location of the BAO bump does not correspond to the sound horizon scale at the level of accuracy required by future measurements, even before any dynamical or statistical effects are considered. Careful modelling of the correlation function is therefore required to extract the cosmological information encoded on large scales. We find that the correlation function is less affected by scale-dependent effects than the power spectrum. We show that a model for the correlation function proposed by Crocce & Scoccimarro, based on renormalized perturbation theory, gives an essentially unbiased measurement of the dark energy equation of state. This means that information from the large-scale shape of the correlation function, in addition to the form of the BAO peak, can be used to provide robust constraints on cosmological parameters. The correlation function therefore provides a better constraint on the distance scale (∼50 per cent smaller errors with no systematic bias) than the more conservative approach required when using the power spectrum (i.e. which requires amplitude and long-wavelength shape information to be discarded).

Key words: methods: N-body simulations – theory large-scale structure of Universe.

1 INTRODUCTION
Prior to recombination, the ionized plasma of electrons and protons was coupled to the radiation in the Universe through the electromagnetic interaction. The pressure exerted by the photons worked against the gravitational collapse of perturbations in the density of baryons. This led to oscillations in the photon–baryon fluid. These ripples are imprinted on the temperature power spectrum of the cosmic microwave background (CMB), as detected convincingly for the first time around the turn of the millennium (de Bernardis et al. 2000; Hanany et al. 2000). A series of acoustic peaks has now been measured with impressive precision by the Wilkinson Microwave Anisotropy Probe satellite (Bennett et al. 2003; Hinshaw et al. 2003, 2007, 2008) and the Arcminute Cosmology Bolometer Array Receiver (Reichardt et al. 2008). The positions and relative heights of these peaks can be used to place tight constraints on the values of the fundamental cosmological parameters, particularly when the CMB measurements are combined with measurements of the galaxy power spectrum (Efstathiou et al. 2002; Percival et al. 2002; Spergel et al. 2003; Tegmark et al. 2004; Seljak et al. 2005; Sánchez et al. 2006; Seljak, Slosar & McDonald 2006; Spergel et al. 2007).

The acoustic oscillations are also imprinted on the matter power spectrum, albeit with different phases from the features seen in the CMB spectrum and with a reduced amplitude, due to the small fraction of the total mass in the Universe believed to be in the form of baryons (Sugiyama 1995; Eisenstein & Hu 1998; Meiksin, White & Peacock 1999). Recently, these features have attracted a great deal of interest as a potential route to measure the equation of state of the dark energy, $v_{\text{DE}} = P_{\text{DE}}/\rho_{\text{DE}}$, where $P_{\text{DE}}$ is the pressure of the dark energy and $\rho_{\text{DE}}$ is its density (Blake & Glazebrook 2003; Hu & Haiman 2003; Linder 2000; Seo & Eisenstein 2003; Wang 2006; Guzik, Bernstein & Smith 2007; Seo & Eisenstein 2007; Seo et al. 2008). The acoustic oscillations in the matter power spectrum are related to the sound horizon scale. If we know the size of the sound horizon through measurements of the CMB temperature fluctuations, we can use this scale as a standard ruler. The apparent size of this ruler depends upon the parameter $v_{\text{DE}}$, as this influences the angular diameter distance out to a given redshift.

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In this paper, we focus our attention on the two-point correlation function which is the Fourier transform of the power spectrum. The series of acoustic oscillations seen in the power spectrum translates into a bump or broad spike in the correlation function (Matsubara 2004). Such a feature was detected for the first time in the correlation function of luminous red galaxies (Eisenstein et al. 2005; Estrada, Sefusatti & Frieman 2008; Okumura et al. 2008). Evidence has also been found for baryon oscillations in the power spectrum of galaxy clustering (Cole et al. 2005; Hütisi 2006; Padmanabhan et al. 2007; Percival et al. 2007b). The appearance of the acoustic oscillations in the power spectrum was considered in an earlier companion paper to this one by Angulo et al. (2008).

Whilst the bulk of the literature on acoustic oscillations has focused on the power spectrum, there has been renewed interest recently in the use of the correlation function to extract the sound horizon scale. Angulo et al. (2005) showed that the correlation function of a sample of clusters at \( z \sim 1 \), as anticipated from the Sunyaev–Zeldovich survey proposed with the South Pole Telescope (Ruhl et al. 2004) or a red sequence photometric survey with Visible and Infrared Survey Telescope for Astronomy (VISTA) (http://www.vista.ac.uk), could potentially constrain the sound horizon scale to an accuracy of \( \sim 2 \) per cent, in turn fixing the dark energy equation of state to better than 10 per cent.

For the acoustic oscillation approach to provide competitive estimates of the value of \( \Omega_{m0} \), one needs to extract the sound horizon scale from the measured correlation function to sub-per-cent accuracy. To achieve this goal, it is essential to quantify any systematic deviations of the scale recovered from the size of the sound horizon. In order to do so in a way that avoids the introduction of new biases or systematic errors, a complete understanding of all the processes that shape the observed correlation function is required.

Based on the fitting formula of Smith et al. (2003) and using the position of the acoustic peak in the correlation function as an estimator of the sound horizon at recombination, Guzik et al. (2007) concluded that non-linear evolution biases the constraints by less than 0.3 per cent. Eisenstein, Seo & White (2006a) argue that any possible shift of the acoustic scale is unobservably small even at \( z = 0 \). On the other hand, the more recent analyses of Smith, Scoccimarro & Sheth (2008) and Crocce & Scoccimarro (2008) have shown that both large volume numerical simulations and theoretical predictions based on renormalized perturbation theory (RPT) indicate that the position of the peak in the correlation function shows important shifts with respect to the linear theory prediction. If unaccounted for, these shifts bias the constraints obtained by using baryonic acoustic oscillation (BAO) measurements as a standard ruler.

In this paper, we analyse some of the different tests that have been proposed to extract cosmological information from the correlation function on large scales, with particular emphasis on the possible systematic errors that can be introduced in their application. We first explain the relationship between the size of the sound horizon and the location and form of the peak in the correlation function in Section 2. We next determine the accuracy of various means of computing the matter two-point correlation function according to linear perturbation theory for a fixed set of cosmological parameters (Section 3).

Besides the linear evolution of density perturbations, there are other processes that will affect the measurement of the correlation function obtained from the new large galaxy redshift surveys: non-linear evolution of the density field, redshift–space distortions and halo bias. It is extremely important to test our ability to model these processes in order to firmly assess if the precision claimed by some future experiments is attainable with our current understanding of these effects. In Section 4, we describe the ensemble of simulations we used to model these effects, and the methodology implemented to estimate the two-point correlation function. In Section 5, we describe the details of our modelling of the correlation function and its ability to reproduce the results from the N-body simulations with respect to non-linear evolution, redshift–space distortions and scale-dependent halo bias. The measurement of the imprints of the BAOs in the power spectrum and correlation functions is affected in different ways by these problems. In Section 6, we compare the performance of these statistics to see which one offers the most advantages as a tool to recover unbiased constraints on cosmological parameters from BAO measurements. Finally, in Section 7, we give a summary of our main results.

2 WHAT IS THE RELATION BETWEEN THE SOUND HORIZON SCALE AND THE PEAK IN THE CORRELATION FUNCTION?

The two-point correlation function has a bump on scales in excess of 100 \( h^{-1} \) Mpc (Eisenstein et al. 2005). It is often assumed that the position of this bump coincides exactly with the sound horizon scale. In this section, we explore the connection between the sound horizon, the nature of the acoustic oscillations imprinted on the matter power spectrum and the position and form of the acoustic bump in the correlation function. We will demonstrate that, at the level of accuracy required by forthcoming measurements of distances using BAO, the assumption that the sound horizon scale is equal to the position of the peak in the correlation function is incorrect and introduces a systematic error in the distance measurement which is in excess of the expected random errors.

The correlation function is the Fourier transform of the power spectrum (see equation 1 in the next section). The acoustic oscillations of the baryon–photon fluid are imprinted on the matter power spectrum as a series of waves. If it was the case that these oscillations had a fixed wavelength and amplitude, then there would be a sharp feature in the correlation function at a scale centred on the wavelength of oscillation. However, it turns out, for various physical reasons, that the acoustic oscillations in the power spectrum have neither a fixed wavelength nor a fixed amplitude. This leads to a more complicated relationship between the sound horizon scale and the position of the peak in the correlation function than the naive assumption of equality. Furthermore, the size of the discrepancy between the peak location and the sound horizon scale depends upon the values adopted for the cosmological parameters, such as the matter density.

The physical reasons behind the appearance of the acoustic oscillations are set out clearly by Eisenstein & Hu (1998, hereafter EH98), who discuss the form of the power spectrum of density fluctuations in a universe containing cold dark matter (CDM) and baryons. There are two phenomena which have a direct impact on the nature of the acoustic oscillations. The easiest to understand is the reduction in the amplitude of the oscillations with increasing wavenumber. This is due to an imperfect coupling between photons and baryons as the end of recombination is approached; photons diffuse out of perturbations and the remaining coupling with the baryons causes a Compton drag which leads to some baryons being pulled out of perturbations. This leads to a reduction in the amplitude of the density fluctuation known as Silk damping (Silk 1968). The explanation of the change in the wavelength of the oscillation with harmonic, at wavenumbers for which \( k s \leq 10 \), where \( s \) is the sound horizon, is more subtle. After recombination, the baryons can...
‘slip’ past the photons. This motion of the baryons, called the velocity overshoot, generates new perturbations (Sunyaev & Zeldovich 1970; Press & Vishniac 1980). During recombination, EH98 argue that the velocity overshoot is not the dominant contribution to the growth of the baryonic perturbation, which leads to the ‘node shift’ in the baryonic transfer function.

The physically motivated fitting formula for the matter power spectrum presented by EH98 can be used to show the impact of the phenomena described above on the form of the acoustic bump in the correlation function. The consequences of Silk damping and the impact of velocity overshoot can be readily identified in this formula and can therefore be switched on or off to investigate their impact on the correlation function. We consider three cases.

(1) The fully consistent power spectrum, with Silk damping and a consistent treatment of velocity overshoot.

(2) A toy model in which there is no Silk damping and velocity overshoot is dominant at all \( k \). This corresponds to setting \( k_{\text{silk}} \rightarrow \infty \) and \( \bar{s} = s \) in equation (21) of EH98 (\( \bar{s} \) is defined in equation 22 of EH98).

(3) A second toy model with Silk damping (the appropriate value of \( k_{\text{silk}} \) is used) but with velocity overshoot dominant for all \( k \) (again, retaining \( \bar{s} = s \) in equation 21 of EH98).

We then Fourier transform the resulting power spectrum in each case to obtain the correlation function of matter fluctuations in linear perturbation theory.

The results of considering the above test cases are presented in Fig. 1, where we assume the background cosmological parameters to be \( \Omega_0 = 0.25, \Omega_b = 0.041 \) and \( h = 0.7 \), where \( H_0 = 100 \, h \, \text{km s}^{-1} \, \text{Mpc}^{-1} \). The sound horizon scale for these cosmological parameters is \( s = 113.8 \, h^{-1} \, \text{Mpc} \) (using equation 6 of EH98), and the correlation function corresponding to \( P(\bar{k}) \) (dot–dashed line), as a function of \( \bar{s} \), is marked by the vertical line in Fig. 1. Using the full linear theory \( P(k) \) (Case 1) to compute \( \xi(r) \) yields a broad peak with a maximum at \( r = 111.0 \, h^{-1} \, \text{Mpc} \). The correlation function corresponding to Case 2 has a much sharper bump with a higher amplitude than in Case 1. The maximum of this feature is at the true sound horizon scale. Turning on Silk damping (Case 3) shifts the peak to \( r = 113.0 \, h^{-1} \, \text{Mpc} \). The shaded area around the vertical line in Fig. 1 represents a 2 per cent error on the sound horizon measurement. This is the size of the random error forecast for the ongoing WiggleZ experiment (Glazebrook et al. 2007; Angulo et al. 2008). The peak of the correlation function bump lies outside the shaded region. Therefore, approaches which advocate measuring the location of the bump in the correlation function will make significant systematic errors if this scale is mistakenly identified as the sound horizon scale.

The size of the discrepancy between the location of the acoustic bump in the correlation function and the sound horizon scale depends upon the cosmology. Fig. 2 shows the variation of the sound horizon, \( s \), which is shown by the central solid line.

![Figure 1](https://academic.oup.com/mnras/article-abstract/390/4/1470/979645/1)

*Figure 1.* The correlation functions computed by Fourier transforming the power spectrum obtained using the EH98 formula for \( P(k) \) for three cases: solid line – a fully consistent linear theory \( P(k) \), dot–dashed line – a \( P(k) \) with no Silk damping and with dominant velocity overshoot on all wavenumbers and dashed line – a \( P(k) \) with no Silk damping but a standard velocity overshoot. The arrows mark the location of the peak in the acoustic bump, defined as the local maximum. The shaded region indicates a 2 per cent error on the sound horizon scale, which is shown by the central solid line.

![Figure 2](https://academic.oup.com/mnras/article-abstract/390/4/1470/979645/2)

*Figure 2.* The true sound horizon, \( s \) (solid line), and the position of the acoustic peak in the correlation function, \( r_p \) (dot–dashed line), as a function of \( \Omega_m \), for a fixed value of \( \Omega_b = 0.041 \). The dotted lines indicate a 2 per cent spread in the value of the sound horizon. The dashed line shows the location of the acoustic bump when the EH98 formalism for \( P(k) \) is replaced by a more accurate calculation made with CAMB (see the text for details).
In the next section, we will see that there are differences between the correlation functions computed using the EH98 fitting formula and that obtained using CMBFAST (Seljak & Zaldarriaga 1996) and CAMB (Lewis, Challinor & Lasenby 2000). The value of $r_p$ inferred using the EH98 approximation for $P(k)$ is biased towards smaller scales (see the dot–dashed line in Fig. 2). Nevertheless, the qualitative description of the processes that shift the location of the bump in $\xi(r)$ peak away from $s$ is still correct.

The conclusion from this simple analysis is that, even when using linear perturbation theory, it is wrong to assume that the position of the peak in the correlation function corresponds exactly with the value of the sound horizon. Such an assumption will lead to biased constraints on cosmological parameters. The correct approach is to model the full shape of the correlation function which we pursue in the following sections.

### 3 THE LINEAR THEORY TWO-POINT CORRELATION FUNCTION

In Section 2, we demonstrated that it is not possible to use the position of the acoustic peak in the correlation function as a proxy for the sound horizon scale. In order to obtain useful constraints on the cosmological parameters from the correlation function, it will be necessary to model the acoustic bump. In this section, we consider the form of the matter correlation function on large scales in linear perturbation theory. For a realistic survey, the impact on the correlation function of the non-linear growth of perturbations, redshift–space distortions and bias will have to be taken into account. We introduce measurements of the correlation function from $N$-body simulations in Section 4 and then discuss a full model of the correlation function in Section 5.

The first step to a model of $\xi(r)$ is the reproduction of the matter correlation function in linear perturbation theory. We obtain $\xi(r)$ by Fourier transforming the linear theory mass power spectrum

$$\xi(r) = \int_{-\infty}^{\infty} \Delta^2(k) j_0(kr) d\ln(k),$$

where $\Delta^2(k) = P(k)k^3/(2\pi^2)$ is the dimensionless power spectrum and $j_0(x)$ is the spherical Bessel function.

An accurate correlation function in linear perturbation theory therefore requires an accurate calculation of the power spectrum. The most commonly used codes to compute the power spectrum are as follows: (i) the approximate formula of EH98, (ii) CMBFAST (Seljak & Zaldarriaga 1996) and (iii) CAMB (Lewis et al. 2000). Seljak et al. (2003) tested the accuracy of CMBFAST against full Boltzmann codes and found that the relative error in the matter power spectrum is below $10^{-3}$. This comparison did not include CAMB although a similar accuracy is claimed for this code. The last release of CMBFAST was in 2003; CAMB is under continual development and so we adopt the view that amongst these three alternatives, CAMB provides the benchmark. As we have already mentioned in Section 2, EH98 produced a physically motivated functional form for the power spectrum, which they calibrated against CMBFAST. EH98 found that their formalism reproduced the results of CMBFAST at the per cent level, over a restricted range of wavevectors. EH98 provided subroutines for their fitting formulae, which are now in common use and are faster than running CMBFAST or CAMB.

Fig. 3 shows the ratio of the power spectra obtained using the EH98 fitting formula (dot–dashed line) and CMBFAST (solid line) to the power spectrum obtained using CAMB for a flat cosmological model with $\Omega_m = 0.237$, $\Omega_b = 0.041$ and $h = 0.735$ (which corresponds to the best-fitting parameters from Sánchez et al. 2006). There are clear differences between the $P(k)$ of EH98 and CAMB on the scales relevant to the acoustic oscillations that can be as large as 5 per cent. For this particular model, CAMB and CMBFAST show impressively good agreement up to $k \sim 1$ h Mpc$^{-1}$.

The consequences for the correlation function of these differences in the predicted linear theory power spectrum can be seen in Fig. 4. The upper panel of Fig. 4 shows the linear theory correlation functions obtained by Fourier transforming the power spectra shown in Fig. 3 according to equation (1). It can be clearly seen that while the correlation functions derived from the $P(k)$ computed with
CAMB and CMBFAST are in very good agreement, the use of the approximate $P(k)$ of EH98 produces a correlation function with a quite different shape and peak position. The lower panel of Fig. 4 shows the residuals of the correlation functions obtained from the EH98 and CMBFAST power spectra from the CAMB result. For the case of the EH98 formula, these residuals are comparable to or larger than the variance expected in future galaxy surveys (see Section 4.3). These results show that the use of the EH98 formula to model the shape of $\xi(r)$ can introduce strong biases on the obtained constraints. Our findings are consistent with those of Sánchez & Cole (2008) who showed that the use of the EH98 fitting formula to model the shape of the galaxy power spectrum introduces changes in the recovered value of $\Omega_m h^2$ of the order of 1σ. The EH98 formalism is excellent for providing physical insight into the form of the power spectrum in a CDM universe and is the simplest and quickest of the prescriptions listed above to use to generate large numbers of model spectra. However, it was never intended to supersede the more accurate calculations from codes like CMBFAST. In order to attain the level of precision demanded by forthcoming BAO analyses, the EH98 formalism should be replaced by the more accurate calculation of CAMB when modelling power spectra or correlation functions.

4 NUMERICAL MODELLING OF THE CORRELATION FUNCTION

A number of effects can alter the form of the two-point correlation function from the linear perturbation theory predictions presented in the last section: the non-linear growth of perturbations, redshift–space distortions and bias (Guzik, Bernstein & Smith 2007; Smith, Scoccimarro & Sheth 2007; Angulo et al. 2008; Crocce & Scoccimarro 2008; Smith, Scoccimarro & Sheth 2008). In this section, we model the impact of these processes on the two-point correlation function using $N$-body simulations. The ensemble of simulations used is described in Section 4.1. The estimation of the correlation function in a large number of huge-volume simulations could be prohibitively expensive without an efficient algorithm which we describe in Section 4.2. Finally, the use of an ensemble of simulations allows a direct estimate of the errors on the measured correlation function, which we set out in Section 4.3.

4.1 The ensemble of $N$-body simulations

We use an ensemble of 50 moderate-resolution, large-volume $N$-body simulations called L-BASICC II (Angulo et al. 2008). These simulations are analogous to the L-BASICC ensemble of simulations employed by Angulo et al. to assess the detectability of the acoustic oscillations in power spectrum measurements from future galaxy surveys. The only difference is that the L-BASICC II runs are based on a different choice of the cosmological parameters. We adopt a $\Lambda$CDM cosmology consistent with current constraints from the CMB data and large-scale structure measurements (Sánchez et al. 2006; Spergel et al. 2007). We assume a flat $\Lambda$CDM cosmological model with matter density parameter, $\Omega_m = 0.237$, baryonic density parameter, $\Omega_b = 0.041$, scalar spectral index, $n_s = 0.954$, a normalization of density fluctuations, $\sigma_8 = 0.77$, and Hubble constant, $h = 0.735$.

Each of the L-BASICC II simulations covers a comoving cubical region of side $1340h^{-1}$ Mpc, with the dark matter following 448$^3$ particles. This gives a particle mass comparable to that employed in the Hubble Volume simulation (Evrard et al. 2002). The equivalent Plummer softening length in the gravitational force is $\epsilon = 200h^{-1}$ kpc. The volume of each computational box, $2.41h^{-3}$ Gpc$^3$, is almost 20 times that of the Millennium Simulation (Springel et al. 2005), and more than three times the volume of the catalogue of luminous red galaxies from the Sloan Digital Sky Survey (SDSS) used to make the first detection of the acoustic peak by Eisenstein et al. (2005). The total volume of the ensemble is $120h^{-3}$ Gpc$^3$, more than four times that of the Hubble Volume. The position and velocity of every particle are stored at three output times ($z = 0.0, 0.5$ and $1.0$). We produce a friends-of-friends halo catalogue at each redshift of objects with 10 or more particles (corresponding to a mass limit of $1.75 \times 10^{13} h^{-1} M_\odot$). Due to their limited mass resolution, it is not feasible to populate these simulations with galaxies using semi-analytic models.

The initial conditions for the simulations were generated by perturbing particles from a glass-like distribution (Baugh, Gaztanaga & Efstathiou 1995). The input power spectrum of density fluctuations in linear perturbation theory is calculated using the CAMB package of Lewis et al. (2000). A different random seed is used for each member of the ensemble. The simulations were started at a redshift of $z = 63$. Angulo et al. (2008) have shown that for this choice of the starting redshift, the scales relevant for the analysis of acoustic oscillations are unaffected by any transients introduced by the method used to generate the initial conditions.

4.2 The practical estimation of the two-point correlation function

The number of operations required to calculate the correlation function of $N_p$ particles by direct pair counting scales as $N_p^2$. This is infeasible for the large number of particles in our simulations on the large scales considered in our analysis. The situation is further exacerbated by the fact that we need to repeat the calculation many times, estimating the correlation function for three outputs in 50 simulations.

We follow the approach introduced by Barriga & Gaztanaga (2002) and Eriksen et al. (2004) to speed up the estimation of the correlation function. The first step is to construct the density field of the simulation on a grid of $N_{\text{grid}}$ cells using the nearest grid point mass assignment scheme. Using this density grid, the correlation function can be estimated using

$$\xi(r) = \frac{1}{N_{\text{pairs}}} \sum_{ij} \delta_i \delta_j,$$

where $\delta_i = (n_i - \langle n \rangle)/\langle n \rangle$ is the density fluctuation in the $i$ th bin of the grid and the sum extends over the $N_{\text{pairs}}$ cells separated by distances between $r - \Delta r/2$ and $r + \Delta r/2$. This procedure scales as $N_{\text{grid}}^2$ which is a big reduction in time since usually $N_{\text{grid}} \ll N_p$. We use $N_{\text{grid}} = 240$. This method gives an accurate estimate of the correlation function on scales larger than a few grid cells. In this paper, we focus on pair separations in excess of 60$h^{-1}$ Mpc, which corresponds to just over 10 cells. This approach could easily be adapted to work on smaller scales by using different sized grids to tabulate the density field.

To further speed up the estimation of the correlation function, we first compute and store the indices of the $N_{\text{neigh}}$ cells which contribute to a given bin of pair (cell) separation. This list of indices can then be translated to different locations on the density grid to find out which cells contribute to the estimate of the correlation function in each bin of pair separation. In this way, no CPU time is wasted in recomputing cell separations. This reduces the number of operations from $N_{\text{grid}}^2$ to $N_{\text{grid}}N_{\text{neigh}}$. A further speed up is possible as this algorithm can be naturally divided between processors and run in parallel.
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... on a much expanded scale as plotted in Fig. 5(b), which shows the difference \( \xi_{\text{grid}}(r) - \xi_{\text{full}}(r) \) for the different values of \( N_{\text{grid}} \). It is clear that on using a value of \( N_{\text{grid}} = 240 \), the correlation function obtained gives an accurate description (at the level of a few parts in \( 10^3 \)) of \( \xi_{\text{full}}(r) \) over the range of scales relevant to our analysis. In particular, these deviations have no impact on the recovered values of \( k_* \) and \( \alpha \) (see Section 5).

An alternative approach to make a rapid estimate of the correlation function is to use a diluted sample of particles selected at random from the simulation. Fig. 5(a) also shows the results obtained using a sample containing a fraction of 5 per cent (dot–long-dashed line), 20 per cent (long-dashed line) and 40 per cent (dashed line) of the total sample. The deviations with respect to the full estimate obtained using this approach are larger than in the previous cases in which a grid is used. This can be seen more clearly in Fig. 5(c) which shows the difference \( \xi_{\text{random}}(r) - \xi_{\text{full}}(r) \) for the different values of \( N_{\text{ran}} \). Even for a value of \( N_{\text{ran}} = 0.4 \), the deviations from the full measurement are larger than in the grid-based measurement, showing that the grid-based approach is the preferred one when dealing with large samples where the full approach is not applicable. More importantly, the correlation function estimated from the diluted samples displays a systematic distortion from \( \xi_{\text{full}} \), as it is always below the full estimate and shows a different shape. This systematic deviation might introduce a small bias when constraining cosmological parameters.

We apply the algorithm described above to compute the two-point correlation function in real and redshift space for the three output redshifts \( (z = 0, 0.5 \) and \( 1) \) for each of the L-BASIIIC II simulations. Fig. 6(a) shows the mean correlation function of the dark matter from the ensemble at \( z = 0 \). The error bars show the variance in the correlation function estimated from the different realizations (see Section 4.3). To highlight the acoustic peak, we plot \( \xi(r) \times r^2 \) instead of \( \xi(r) \). The results in redshift space are divided by the Kaiser (1987) ‘boost factor’ (see Section 5.4 for further details on the redshift–space distortion). The solid lines in Fig. 6 show the theoretical prediction for the real–space dark matter correlation function computed as described in Section 5.1.

We have also measured the correlation function for haloes in three mass bins at each redshift. Halo sample 1 includes haloes with masses \( 1.75 \times 10^{12} < (M/\text{h}^{-1}\,\text{M}_\odot) < 3.5 \times 10^{13} \), Halo sample 2 covers the mass range \( 3.5 \times 10^{13} < (M/\text{h}^{-1}\,\text{M}_\odot) < 5.95 \times 10^{13} \) and Halo sample 3 is all objects with mass in excess of \( M = 5.95 \times 10^{12} \,\text{h}^{-1}\,\text{M}_\odot \). At \( z = 0 \), sample 1 accounts for roughly 50 per cent of all resolved haloes. The correlation functions measured for these halo samples are shown in Fig. 6(b), (c) and (d). The results for the different samples have been scaled by the respective bias factors \( b^2 = 2.78, 3.48 \) and 6.38 (for samples 1, 2 and 3) as described in the figure caption (see Section 5.5 for further details on the distortions due to halo bias).

4.3 The variance in \( \xi(r) \)

In this section, we compare the variance in the correlation function estimated from the ensemble of simulations with a theoretical prediction.

The starting point in the analytical estimate is the assumption that the variance in the power spectrum is that expected for Gaussian fluctuations with a shot-noise component arising from the finite number of objects used to trace the density field (Feldman, Kaiser & Peacock 1994):

\[
\sigma_P(k) = \sqrt{\frac{2}{V} P(k) \left( \frac{1}{n} \right) },
\]

(3)
where \( V \) is the simulation volume, and \( \bar{n} \) is the mean density of the objects considered (dark matter particles or haloes). Angulo et al. (2008) found good agreement between this expression and the variance in \( P(k) \) measured from numerical simulations.

The covariance of the two-point correlation function is defined by (Cohn 2006; Smith et al. 2008)

\[
C_\xi(r, r') = \langle \xi(r) - \bar{\xi}(r) \rangle \langle \xi(r') - \bar{\xi}(r') \rangle
\]

\[
= \int \frac{dk}{2\pi^2} j_0(kr) j_0(kr') \sigma_\xi^2(k),
\]

where the last term can be replaced by equation (3). The variance in the correlation function is simply \( \sigma_\xi^2(r) = C_\xi(r, r) \). The direct application of equation (5) would, however, lead to a substantial overprediction of the variance, since it ignores the effect of binning in pair separation which reduces the covariance in the measurement (Cohn 2006; Smith et al. 2008).

An estimate of the correlation function in the \( i \)th pair separation bin \( \hat{\xi}_i \) corresponds to the shell averaged correlation function

\[
\hat{\xi}_i = \frac{1}{V_i} \int_{V_i} \xi(r) \, d^3r,
\]

where \( V_i \) is the volume of the shell. The covariance of this estimate is given by

\[
C_{\hat{\xi}_i}(i, j) = \frac{1}{V_i V_j} \int d^3r \int d^3r' C_\xi(r, r')
\]

\[
= \int \frac{dk}{2\pi^2} \tilde{j}_0(k, i) \tilde{j}_0(k, j) \sigma_\xi^2(k),
\]

where

\[
\tilde{j}_0(k, i) = \frac{1}{V_i} \int_{V_i} j_0(kr) \, d^3r.
\]

The dotted lines in each panel of Fig. 6 show the estimates of the variance for the various samples computed using equation (8). For the halo samples, the variance is rescaled by the same bias factors as the measured correlation functions.

Figure 6. Mean correlation functions at \( z = 0 \) from our ensemble of simulations in real space (circles) and redshift space (triangles) for dark matter (panel a) and haloes in samples 1, 2 and 3 (panels b, c and d). The error bars show the variance from the estimates in the different realizations. To highlight the acoustic peak, we show \( \xi(r) \times r^2 \). The results in redshift space are divided by the Kaiser (1987) boost factor (see Section 5.4). The results for the halo correlation functions were scaled by the bias factors shown in the annotations in each panel (see Section 5.5). The solid lines show the fits to the simulation data with the model for the real–space dark matter correlation function described in Section 5.1. The dotted lines in each panel show the estimates of the variance for each sample computed using equation (8). For the halo samples, the variances are rescaled by the same bias factors than the measured correlation functions.
coming from the term involving $1/n^2$ as
\[
\frac{2}{V n^2} \int \frac{dk}{(2\pi)^3} J_0(kr_i) J_0(kr_j) = \delta_{ij} \frac{2}{V n^2} V_r,
\] (10)
which corresponds to the exact result from equation (8) in the Poisson clustering limit ($P(k) \ll 1/n$). This correction includes the effect of binning only in the last term of equation (5) and ignores the effect of the shell average in the first two terms. Fig. 8 shows a comparison of the variance measured from our ensemble of simulations (solid line histogram) for the halo correlation functions in halo samples 1, 2 and 3 with the predictions from equations (8) and (10). It can be clearly seen that even for the value of $\Delta r/r = 0.03$, equation (10) slightly overpredicts the true variance, while the full expression from equation (8) gives a more accurate description.

Equation (8) is very useful for predicting the error and full covariance matrix of two-point correlation functions estimated from galaxy samples. This is a valuable tool to use in forecasts of the likely constraints attainable on cosmological parameters from present and future galaxy surveys.

5 MODELLING THE FULL SHAPE OF $\xi(R)$

5.1 Non-linear evolution

The non-linear evolution of density perturbations changes the shape of the power spectrum and the correlation function due to the coupling between different Fourier modes. Numerical simulations have been used to model this effect (Efstathiou et al. 1988; Hamilton et al. 1991; Peacock & Dodds 1994; Smith et al. 2003). Based on a combination of the Hamilton et al. (1991) scaling relations and the halo model, Smith et al. (2003) proposed a fitting function to describe the effects of non-linear evolution on the shape of the matter power spectrum. This formula is able to reproduce the non-linear power spectra for pure CDM models to an accuracy of around 7 per cent and is the basis of the commonly used HALOFIT code.

Huff et al. (2007) and Crocce & Scoccimarro (2008) showed that the behaviour of the non-linear power spectrum predicted by HALOFIT fails to describe an important distortion to the BAOs seen in numerical simulations. Besides the change in the overall shape of $P(k)$, non-linear evolution washes out the acoustic oscillations by erasing the higher harmonic peaks (Meiksin et al. 1999; Angulo et al. 2005; Eisenstein et al. 2005, 2006b; Springel et al. 2005; Angulo et al. 2008). This effect can be modelled by computing a damped or ‘dewiggled’ power spectrum (Eisenstein et al. 2006b; Tegmark et al. 2006):
\[
P_{dw}(k) = P_{lin}(k)G(k) + P_{nw}(k)[1 - G(k)],
\] (11)
where $P_{lin}(k)$ is the linear theory power spectrum and $P_{nw}(k)$ is a smooth, linear theory, CDM only power spectrum, with the same shape as $P_{lin}(k)$ but without any baryonic oscillations (which can be computed using the fitting formula of Eisenstein & Hu 1999). The function $G(k) = \exp[-(k/k_0)^2]$ regulates the transition from large scales ($k \ll k_0$), where $P_{dw}(k)$ follows linear theory to small scales ($k \gg k_0$) where the acoustic oscillations are completely damped. The upper panel of Fig. 9 compares $P_{lin}(k)$ (dashed line)
and \( P_{\text{d}w}(k) \) (solid line) together with the different terms in equation (11). The damping of the BAO in the final power spectrum can be more clearly seen in the lower panel of Fig. 9 which shows the ratios of the linear and dewiggled power spectra to \( P_{\text{d}w}(k) \).

The degree of damping of the BAO is extremely sensitive to the value adopted for \( k_s \). We treat \( k_s \) as a free parameter in the model for the form of the correlation function. Some authors have attempted to compute the value of \( k_s \). Based on the analysis of Eisenstein et al. (2006b), Tegmark et al. (2006) computed the value of the damping scale as a function of \( \Omega_m^0 \) and the primordial amplitude of scalar fluctuations, \( A_s \). These authors expressed their results in terms of \( \sigma_8 \), where \( k_s = 1/\sigma_8 \):

\[
\sigma_8 = s_0 D(1 + f)^{1/3} \sqrt{A_s / 0.6841}. \tag{12}
\]

Here, \( s_0 = 12.4 \, h^{-1} \, \text{Mpc} \) is a reference scale, \( D \) is the growth factor (which is where \( \Omega_m^0 \) enters), \( f = d \log D / d \log a \) and \( A_s \) follows the normalization convention of Spergel et al. (2007).

Equation (11) gives a purely phenomenological description of the damping of the acoustic oscillations found in numerical simulations. Using RPT, Crocce & Scoccimarro (2006, 2008) give a theoretical justification of the ability of equation (11) to describe non-linear evolution. According to RPT, the first term on the right-hand side of equation (11) describes the growth of a single mode, quantified by the propagator function \( G(k) \). In the high-\( k \) limit, the propagator is given by the Gaussian form with \( k_s \) given by (Crocce & Scoccimarro 2006; Matsubara 2008)

\[
k_s = \left[ \frac{1}{3 \pi^2} \int \frac{dk}{P_{\text{lin}}(k)} \right]^{-1/2}. \tag{13}
\]

In the next section, we will compare the value of \( k_s \) obtained from equations (12) and (13) with the best-fitting value obtained by requiring our model for the correlation function to reproduce measurements made from the N-body simulations.
What is the best way to measure BAO?

Figure 10. An illustration of how the appearance of the correlation function changes after applying differing degrees of damping to the acoustic oscillations. The undamped, linear theory correlation function is shown by the solid line. The damped or ‘dewiggled’ correlation functions are shown by the dot–dashed lines, for different values of \( k_s \). For small damping (large \( k_s \)), \( \xi_{\text{dw}}(r) \) deviates by a small amount from \( \xi_{\text{lin}}(r) \). As the damping of the acoustic oscillations becomes stronger (smaller \( k_s \)), the peak is gradually erased and approaches the zero-baryon or ‘no-wiggle’ correlation function, \( \xi_{\text{nw}}(r) \) (dashed line) as \( k_s \to 0 \).

A full description of the non-linear two-point correlation function must include both the overall change in shape of the correlation function and the damping of the acoustic oscillations. In order to assess which of these effects is the more important for the analysis of the acoustic peak in \( \xi(r) \), Fig. 11 compares the mean \( z = 0 \) real–space correlation function of the dark matter measured from the ensemble of simulations (open points) with the following models for the correlation function: (i) the linear theory correlation function \( \xi_{\text{lin}}(r) \) (solid line), (ii) a non-linear correlation function \( \xi_{\text{nl}}(r) \) computed using \textsc{halofit}, without any damping of the acoustic oscillations (short-dashed line), (iii) the dewiggled linear theory correlation function \( \xi_{\text{dw}}(r) \), computed as described by equation (14) (long-dashed line), and (iv) a dewiggled correlation function non-linearized using \textsc{halofit} \( \xi_{\text{dw}}^{\text{nl}}(r) \) (dot–dashed line). The error bars indicate the variance between the correlation functions measured from the different realizations in the simulation ensemble.

It is clear from Fig. 11 that the acoustic peak in the two-point correlation function at redshift \( z = 0 \) shows strong deviations from the predictions of linear theory. The \textsc{halofit} fitting formula fails to describe these deviations since its application only produces a small decrease in the amplitude of the peak without shifting its position. On the contrary, the linear theory dewiggled correlation function from equation (14) gives a very good description of the results of our numerical simulations, showing that the damping of the oscillations is the most important effect to include in the modelling of the real–space correlation function on large scales. The incorporation of the full change in the shape of \( P(k) \) due to non-linear evolution produces very little difference in the shape of the acoustic peak in the correlation function. However, as we will see later (see Section 5.3), this effect might be important on intermediate scales (\( r \approx 70\, h^{-1}\, \text{Mpc} \)).

5.2 The model in practice

We now test whether or not the model for the correlation function described in Section 5.1 returns unbiased constraints on cosmological parameters. In particular, we are interested in the constraints on the dark energy equation of state parameter \( v_{\text{DE}} \). We consider a very simple case in which we assume that the values of all cosmological parameters are known, apart from \( v_{\text{DE}} \), and analyse the constraints on this parameter.

The change in the power spectrum due to variations in \( v_{\text{DE}} \) in this simple case is described in detail by Angulo et al. (2008). In order to measure the power spectrum of galaxy clustering, it is necessary to convert the angular positions and redshifts of the galaxies into comoving spatial separations. This requires a choice to be made for the values of the cosmological parameters, including \( v_{\text{DE}} \). The effect of a change in the value of \( v_{\text{DE}} \) from its true value to \( v_{\text{DE}} + \delta v_{\text{DE}} \) is to modify the separations between pairs of galaxies, which leads to a change in the appearance of the power spectrum. For small perturbations away from the true equation of state, the alteration in the measured power spectrum can be represented by a rescaling of the wavenumber from \( k_{\text{true}} \) to \( k_{\text{app}} \).

\[
\alpha = \frac{k_{\text{app}}}{k_{\text{true}}}. \tag{15}
\]

In the two-point correlation function, there will be an equivalent shift from scale \( r_{\text{true}} \) to \( r_{\text{app}} = r_{\text{true}}/\alpha \). We note that the variant models...
which arise from perturbing the equation of state in this way do not match observations such as the location of the Doppler peaks in the CMB (see Angulo et al. 2008). Nevertheless, our primary goal here is to compare the constraints on the stretch parameter obtained from the correlation function with those which result from the power spectrum, under the same idealized conditions.

We analyse the constraints on the stretch parameter using the two-point correlation functions in real space measured from the different L-BASIIC II realizations, as described in Section 4.2, as well as from the mean correlation function from the ensemble. In doing so, we use only information from the shape of the correlation function, and not its overall amplitude, which for real observational data can be affected by bias and redshift-space effects (see Sections 5.4 and 5.5). We also consider \( k_c \) as a free parameter, instead of fixing its value according to equation (12). To explore this simple parameter space, we constructed Monte Carlo Markov Chains. The constraints obtained in this way for redshifts \( z = 0, 0.5 \) and \( 1 \) are summarized in Table 1. The range of pair separations used in the fit is \( 60 \lesssim (r/h^{-1}\text{Mpc}) < 180 \); the results are not sensitive, in detail, to the choice of limits in \( r \). Unless otherwise stated, the quoted allowed ranges for the constrained parameters correspond to the 68 per cent confidence level according to the variance from our ensemble of simulations.

The solid lines in Fig. 12 show the two-dimensional constraints on \( \alpha \) and \( k_c \) obtained using the mean dark matter correlation function in real space measured from the ensemble of simulations. The constraints on \( k_c \) can be compared with the predictions of equation (12) which for this cosmology gives \( k_c = 0.095 h \text{Mpc}^{-1} \) at \( z = 0 \). Here, we find that at \( z = 0 \), the shape of the mean correlation function can be more accurately described by \( k_c = 0.115 \pm 0.009 h \text{Mpc}^{-1} \), which is 2\( \sigma \) away from the prediction of equation (12). On the other hand, this result is in excellent agreement with the prediction from equation (13) which gives \( k_c = 0.117 h \text{Mpc}^{-1} \) at \( z = 0 \). The lower panel of Fig. 13 shows the histogram of the values of \( k_c \) obtained for the \( z = 0 \)-correlation functions of each L-BASIIC II realization, which is completely consistent with the constraints obtained from the mean correlation function of the ensemble. The dotted line shows the value of \( k_c \) computed using equation (12) which, as we mentioned before, predicts a stronger damping of the acoustic oscillations than we find in the numerical simulations for this cosmology. This implies that if one applies equation (12), this may result in systematic errors in the values of the derived cosmological parameters. The dashed line shows the prediction from equation (13) which is remarkably consistent with the mean value of \( k_c \) recovered from our ensemble of simulations.

At higher redshift, the acoustic oscillations are less damped and higher values of \( k_c \) (corresponding to weaker damping) are favoured, with \( k_c = 0.140 \pm 0.010 h \text{Mpc}^{-1} \) returned at \( z = 0.5 \) and \( k_c = 0.159 \pm 0.013 h \text{Mpc}^{-1} \) at \( z = 1 \). The constraints on \( k_c \) also broaden since, as can be seen in Fig. 4, for higher values of \( k_c \), the shape of the acoustic peak becomes less sensitive to variations in this parameter.

At redshift \( z = 0 \), the mean correlation function gives tight constraints on the stretch factor, with \( \alpha = 0.996 \pm 0.006 \), showing that it is better described with a value of \( \alpha \) slightly less than 1. The upper panel of Fig. 13 shows the histogram of the values of \( \alpha \) recovered from the different realizations, which is centred on the value \( \alpha = 1 \), but which also shows a small tendency towards \( \alpha < 1 \) which can be detected by the slight skewness of the distribution. At higher redshifts, the position of the acoustic peak can be more precisely determined since it is less affected by the damping of the higher harmonic oscillations. This results in tighter constraints on \( \alpha \), with \( \alpha = 0.998 \pm 0.004 \) at \( z = 0.5 \) and \( \alpha = 0.997 \pm 0.003 \) at \( z = 1 \).

The origin of this slight bias towards \( \alpha < 1 \) can be understood by analysing the effect of variations in \( k_c \) and \( \alpha \) on the model correlation function. Fig. 14(a) shows the correlation functions corresponding to different values of \( k_c \) for a fixed stretch parameter, \( \alpha = 1 \) (dotted lines). To increase the dynamic range, we plot the residuals \( \xi(r) - \xi_{\text{dwe}}(r) \), where \( \xi_{\text{dwe}}(r) \) is the dewiggled correlation function for the best-fitting value of \( k_c = 0.115 h \text{Mpc}^{-1} \). For this reference value, \( \xi_{\text{dwe}} \) accurately describes the shape of the acoustic peak in the mean
correlation function measured from the ensemble of simulations. There are, however, differences between the model of equation (14) and the measured correlation function both on scales smaller and larger than the acoustic peak which present a systematic structure.

At scales around $r \simeq 150 h^{-1}$ Mpc, equation (14) slightly overestimates the true value of the correlation function, which follows more closely the result from linear perturbation theory. On these scales, the value of $\xi_{dw}$ is almost completely determined by the first term of equation (14), with the second term giving only a slight correction. This difference may indicate that the function $G(k)$ that controls the damping of the oscillations is not exactly Gaussian, thereby changing the shape of the first term of equation (14), or that the second term in this equation is not a good approximation to $\xi_{mc}(r)$, and underestimates the full contribution of this term.

The most important deviations between the model and the measured correlation function are found on scales smaller than the position of the peak, with $r \lesssim 90 h^{-1}$ Mpc. On these scales, the model underestimates the amplitude of the measured correlation function. Fig. 14(b) shows the correlation functions obtained for different values of $\alpha$, fixing $k_*=0.115 h$ Mpc$^{-1}$ (dot–dashed lines). As stated before, for $\alpha=1$ the model fails to reproduce the measured correlation function. Using a value of $\alpha$ slightly less than 1 re-scales the position of the peak and increases the amplitude at small scales in the model correlation function in a way that gives better agreement with the measurements from the simulations, causing the slight bias of the model towards $\alpha < 1$.

Crocce & Scoccimarro (2008) showed that $\xi_{mc}(r)$ introduces an additional shift in the acoustic peak which is not taken into account by the second term in the right-hand side of equation (14). At $z=0$, this shift can lead to a bias towards $\alpha > 1$ of the order of 0.8 per cent if this parameter is fitted using only the position of the peak (see their fig. 7). This apparent contradiction with our result of a small bias towards $\alpha < 1$ is due to the fact that our constraints are based on the full shape of the correlation function on large scales, and not only on the position of the peak. In this case, as shown above, the biggest difference between the models and the measured correlation functions is not given at the scales of the peak, but at smaller scales, where non-linear evolution increases the amplitude of $\xi(r)$ with respect to the prediction of equation (14). This difference is responsible for the shift in the preferred value of $\alpha$.

5.3 Improving the model

Although the model described by equations (11) and (14) gives a good description of the distortions in the shape of the peak in the
correlation function due to the damping of the acoustic oscillations, it is slightly biased towards $\alpha < 1$. This bias is smaller than the sample variance predicted in the correlation function for the volume of the L-BASIC simulation or expected in ongoing galaxy surveys. However, the much smaller sample variance anticipated in future galaxy surveys, such as Euclid (Robberto et al. 2007) or ADEPT, will mean that this level of inaccuracy in the modelling of the correlation function may lead to the introduction of systematic errors in the recovered dark energy equation of state parameter $\omega_{DE}$. In this section, we explore different ways to correct for this small bias.

The model of equation (11) can be extended to account for the change in the overall shape of the power spectrum due to non-linear evolution by

$$P_{nl}(k) = \left( G - \frac{1}{1 + \xi} \right) P_{lin}(k) = f(k)P_{lin}(k).$$

The factor $f(k)$ could also be used to model a scale-dependent bias factor. This model for non-linear evolution is based on the $Q$-model of Cole et al. (2005), modified by the addition of a new parameter, $B$, with the aim of achieving a better description of the behaviour of the non-linear power spectrum at high $k$. The value of the new parameter $B$ depends on the choice of $Q$. In practice, we fixed the value of $B$ using the relation $B = Q/10$, which gives the approximate behaviour of the non-linear power spectrum at large $k$, although the correlation function is almost insensitive to this choice. This scheme is an alternative means to describe the non-linear growth to using HALOFIT. This expression can be Fourier transformed to obtain a non-linear correlation function $\xi_{nl}(r)$, whose shape is only weakly dependent on the precise value of $B$.

Fig. 15(a) shows the quantity $\xi_{nl}^\alpha - \xi_{lin} \otimes \hat{G}$ for different values of the parameter $Q$ fixing $\alpha = 1$ and $k_x = 0.115 h \text{Mpc}^{-1}$ (dot–dashed lines). For $Q = 13$ (thick dot–dashed line), the non-linear correction factor from equation (16) changes the shape of the model correlation function, improving the agreement with the measurements from the simulations on scales $r \lesssim 90 h^{-1} \text{Mpc}$. On large scales, the measurements follow more closely the predictions from linear perturbation theory, and the models overpredict the amplitude of the correlation function.

Table 1 also shows the constraints on $\alpha$ and $k_x$ obtained by including the scale-dependent correction factor defined by equation (16), on allowing $Q$ to vary as a free parameter. As the model correlation function is fairly insensitive to the value of $Q$, the constraints on this parameter are weak. However, there is a tendency towards lower values of $Q$ with increasing redshift. As can be seen in Fig. 15, varying $Q$ changes the amplitude of the acoustic peak. This is why when this parameter is included in the analysis, the best fits shift towards lower values of $k_x$. This reflects the fact that when the non-linear correction of equation (11) is not included, the value of $k_x$ recovered from the correlation function contains some information on the distortion to the overall shape of $P(k)$ and not just the way in which the oscillations are damped.

The constraints on $\alpha$ are essentially unchanged upon the incorporation of the non-linear shape correction. The best-fitting results, as before, show a small bias towards $\alpha < 1$. Hence, although the model presented in equation (16) gives the freedom to correct for differing degrees of non-linear evolution or even a scale dependence of bias, it is still unable to describe the form of the measured correlation functions to correct for this small systematic shift in $\alpha$.

The solid curve in Fig. 15 shows the quantity $\xi_{nl}^\alpha - \xi_{lin} \otimes \hat{G}$, that is, the second term on the right-hand side of equation (14). As pointed out in Section 5, this term is a smooth function with no information about the acoustic peak. On the other hand, Crocce & Scoccimarro (2008) showed that the term $\xi_{lin}(r)$ does contain information about the acoustic scale. This fact imposes a limitation on the ability of a model based on equation (14) to reproduce the full shape of the non-linear correlation function.

Using standard perturbation theory, Crocce & Scoccimarro (2008) showed that on the scale of the acoustic peak the main contribution to $\xi_{lin}(r)$ will be of the form

$$\xi_{lin}(r) \propto \xi_{lin}^{(1)}(r),$$

Figure 15. Panel (a): the quantity $(\xi_{nl}^\alpha - \xi_{lin} \otimes \hat{G})$ for $Q = 1, 7, 13, 19$ and 26, fixing $\alpha = 1$ and $k_x = 0.115 h \text{Mpc}^{-1}$ (dot–dashed lines). For $Q = 13$ (thick dot–dashed line), the non-linear correction factor from equation (16) changes the shape of the model correlation function, improving the agreement with the measurements from the simulations on scales $r \lesssim 90 h^{-1} \text{Mpc}$. Panel (b): the quantity $(\xi_{nl}^\alpha - \xi_{lin} \otimes \hat{G})$ computed using equation (19) with $\lambda_{mc} = 0.15, 0.25, 0.37, 0.45$ and 0.55, fixing $\alpha = 1$ and $k_x = 0.115 h \text{Mpc}^{-1}$ (dot–dashed lines). This modelling gives a better description of the residuals of the measured correlation functions with respect to $(\xi_{lin} \otimes \hat{G})$ close to the acoustic peak. The solid lines on both panels show the difference $(\xi_{nl}^\alpha - \xi_{lin} \otimes \hat{G})$, that is the second term on the right-hand side of equation (14). The arrows indicate the effect of increasing $Q$ and $\lambda_{mc}$ on the model correlation functions. The error bars show the variance in the mean correlation function, that is $\sigma_\xi = \sigma_\xi / \sqrt{\text{50}}$. © 2008 The Authors. Journal compilation © 2008 RAS, MNRAS 390, 1470–1490
Table 1. Results of applying the general fitting procedure described in Section 5.1 to the correlation function of the different samples analysed in real space and redshift space. The first column gives the label of the sample, as defined in Section 2. The second column gives the redshift output. Column 3 gives the real–space bias factors computed as described in Section 5.5. Column 4 gives the value of the redshift–space boost factor computed according to equation (20). Columns 5 and 6 give the constraints on $k_s$ and $\alpha$ obtained by fitting $\xi_{\text{nl}}$ as in equation (14) in real space while Columns 7 and 8 show the results obtained using the redshift–space measurements. All quoted values correspond to the mean and 68 per cent c.l. of the respective parameter.

<table>
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<tr>
<th>Sample id</th>
<th>$z$</th>
<th>$b^2$</th>
<th>$S$</th>
<th>$k_s/(\text{h Mpc}^{-1})$</th>
<th>Real space</th>
<th>$\alpha$</th>
<th>$k_s/(\text{h Mpc}^{-1})$</th>
<th>Real space</th>
<th>$\alpha$</th>
<th>Redshift space</th>
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<tr>
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<td>0.988 ± 0.004</td>
<td>1.070 ± 0.005</td>
<td>0.995 ± 0.006</td>
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</table>

where $\xi'_{\text{lin}}$ is the derivative of the linear theory correlation function and

$$
\xi'^{(1)}_{\text{lin}}(r) = \hat{r} \cdot \nabla^{-1} \xi_{\text{lin}}(r) = 4\pi \int P_{\text{lin}}(k) j_1(kr) k \, dk.
$$

(18)

Based on this result, Crocce & Scoccimarro proposed an approximate phenomenological model to describe the effect of the mode coupling on the correlation function near to the acoustic scale, in which $\xi_{\text{mc}}(r) \approx A_{\text{mc}} \xi_{\text{lin}}$ (since $\xi'_{\text{lin}}(r)$ varies little on these scales).

We tested this ansatz by comparing the results from our simulations against the non-linear correlation function given by

$$
\xi_{\text{nl}}(r) = \xi_{\text{lin}}(r) \otimes \tilde{G}(r) + A_{\text{mc}} \xi_{\text{lin}} \xi'^{(1)}_{\text{lin}}(r),
$$

(19)

where $A_{\text{mc}}$ is a free parameter. Fig. 15(b) shows the non-linear correlation functions obtained for different values of $A_{\text{mc}}$ fixing $\alpha = 1$ and $k_s = 0.115 \, \text{h Mpc}^{-1}$ (dot–dashed lines). It can be seen that this model can describe the shape of the residuals of the measured correlation functions with respect to $\xi_{\text{lin}} \otimes \tilde{G}$ close to the acoustic peak. At smaller scales, where the approximation is not so accurate, the model underestimates the correlation functions.

Table 2 also shows the results obtained by applying this modelling. The constraints obtained on $k_s$ are slightly smaller than those obtained using equation (14). The most important effect of the incorporation of the second term of equation (19) is that it helps to alleviate the problem of the small bias towards $\alpha < 1$ found in Section 5.2 yielding $\alpha = 1.003 \pm 0.008$ at $z = 0$, with a slight increase in the allowed region. At higher redshifts, the constraints are tighter, with $\alpha = 1.002 \pm 0.005$ at $z = 0.5$ and $\alpha = 1.000 \pm 0.003$ at $z = 1$, in better agreement with $\alpha = 1$. The ability of this simple ansatz to improve the obtained constraints might indicate that the implementation of a full calculation of $\xi_{\text{mc}}$ using RPT over the full range of scales included in the analysis could help to improve the constraints even further.

5.4 Redshift–space effects

When dealing with spectroscopic surveys, the distance to a galaxy is inferred from its measured redshift. As the redshift is affected by the peculiar motion of the galaxy along the line of sight, the radial distance so obtained does not correspond to the true distance to the galaxy. This changes the clustering pattern of galaxies, leading to differences between the power spectrum and correlation function measured in redshift space, $P_s(k)$ and $\xi_r(r)$, and their true real–space counterparts. These differences are called redshift–space distortions. As the peculiar velocity field of galaxies is induced by the underlying matter distribution, it is possible to model the distortions that will arise in a given cosmological model.

On large scales, where linear theory is applicable, the peculiar velocities take on the form of coherent bulk flows towards overdense regions. This leads to an increase in the amplitude of $P_s(k)$ and $\xi_r(r)$ compared to the real–space values. Under the assumption of linear perturbation theory and the plane-parallel approximation, the scale-independent boost in power produced by this effect is given by (Kaiser 1987)

$$
S = \frac{\xi_r(r)}{\xi_{\text{lin}}(r)} = \left( 1 + \frac{1}{5} \beta + \frac{2}{5} \beta^2 \right),
$$

(20)

where $\beta = f/b$ and $b$ is the bias factor (which for the dark matter is simply $b = 1$). The same factor applies for the power spectrum.

On small scales, peculiar velocities are dominated by the random motions inside virialized structures. This makes bound structures such as dark matter haloes to appear elongated when mapped in redshift space, an effect commonly known as ‘fingers of god’. This smearing of structure causes a damping of the power spectrum and the correlation function on small scales. A complete description of redshift–space distortions must include both of these regimes (for an empirical description, see Peacock & Dodds 1994).

Scoccimarro (2004) predicted that in the case of the power spectrum, the boost factor of equation (20) is reached asymptotically on very large scales ($r > 100 \, \text{h}^{-1} \, \text{Mpc}$) and confirmed this with intermediate-scale $N$-body simulations. Similar conclusions were reached with much larger simulations by Angulo et al. (2008). For smaller scales, the power is damped by random motions in a way that can be approximated by (Cole, Fisher & Weinberg 1994; Park et al. 1994; Peacock & Dodds 1994)

$$
P_s(k) = \left( 1 + k^2 \sigma^2 \right)^{-1},
$$

(21)

where $\sigma$ is a free parameter connected to the pairwise velocity dispersion. This implies that in order to use information contained in
the full shape of the power spectrum to obtain constraints on cosmological parameters, a detailed model of redshift–space distortions must be implemented.

The open triangles of Fig. 6(a) show the mean two-point correlation function measured in redshift space from the ensemble of simulations, rescaled by the boost factor $S$ from equation (20). This accurately corrects the amplitude of $\xi(r)$ to that of the real space for a given scale. The most important effect on the correlation function is the most significant in power on smaller scales (as described by the extra factor in equation 21) is to produce an extra damping of the acoustic oscillations. This can be accurately modelled by a smaller value of $k_s$.

Hence, the treatment of redshift–space distortions in the correlation function is somewhat simpler than for the power spectrum, at least for the relatively large scales considered here. If the amplitude of the correlation function is marginalized over when constraining cosmological parameters, the presence of the boost factor $S$ is irrelevant. In this case, redshift–space distortions simply produce a stronger damping of the acoustic oscillations, which is reflected in the fit measurements preferring smaller values of $k_s$. This can be seen in Fig. 12, which shows the two-dimensional constraints on $\alpha$ and $k_s$. In redshift space, the measurements from the simulation prefer lower values of $k_s$ than in real space. The constraint on $k_s$ is also tighter in redshift space, with $k_s = 0.097 \pm 0.004 \text{hMpc}^{-1}$ at $z = 0$. This can also be seen in the lower panel of Fig. 13, which shows the distribution of the values of $k_s$ recovered from the $z = 0$ redshift–space correlation function of the different realizations (dashed histogram). The best value for $k_s$ in redshift space is in better agreement with the prediction of equation (12). The same situation is found at higher redshifts, where the preferred values for $k_s$ are $k_s = 0.107 \pm 0.005 \text{hMpc}^{-1}$ at $z = 0.5$ and $k_s = 0.124 \pm 0.005 \text{hMpc}^{-1}$ at $z = 1$, in both cases lower than the corresponding values in real space.

The constraints on the stretch factor found using redshift–space clustering are weaker than in real space, with $\alpha = 1.002 \pm 0.009$ at $z = 0$, showing a considerable increase in the width of the allowed region for this parameter. However, this result does not show the small bias seen in $\alpha$ in real space, since redshift–space distortions alleviate the problem of non-linear evolution at small scales. At higher redshift, the preferred values show again a small bias towards $\alpha < 1$, with $\alpha = 0.995 \pm 0.006$ at $z = 0.5$ and $\alpha = 0.994 \pm 0.005$ at $z = 1$, with $\alpha = 1$ on the edge of the 68 per cent c.l. regions.

When the non-linear correction of equation (16) is applied, the constraint on $\alpha$ remains unaltered. As when using real–space measurements, the same trend towards lower values of $Q$ with increasing redshift is found, and the mean values for $k_s$ are lower than the ones found without applying this non-linear correction.

Table 2 also shows the constraints obtained comparing the model of equation (19) against the redshift–space dark matter correlation functions. The mean values for $k_s$ recovered in this case are in excellent agreement with those obtained applying equation (14). On the other hand, the constraint on $\alpha$ shows much better agreement with the value $\alpha = 1$, especially at higher redshifts. This implies that, as suggested by Crocce & Scoccimarro (2008), the ansatz given
by equation (19) can also correct for the small bias towards $\alpha < 1$ when using redshift–space information.

5.5 Halo bias

According to the current picture of galaxy formation, galaxies are hosted within dark matter haloes. Hence, a model for the clustering of dark matter haloes is a necessary first step towards an understanding of galaxy bias. Moreover, as discussed in Angulo et al. (2005), galaxy clusters can be a useful tracer of BAO in their own right, offering an important crosscheck of the results obtained from galaxy samples. Clusters are more strongly clustered than galaxies, thereby increasing the amplitude of the correlation function signal on the very scales relevant to BAO, on which the amplitude of the typical galaxy correlation function is very low. Future cluster catalogues will map in a homogeneous way volumes large enough for an accurate determination of the signal of the acoustic peak (Ruhl et al. 2004; Bartlett et al. 2008). For these reasons, the analysis of the way in which the imprints of the acoustic oscillations on the halo two-point correlation function change as clustering evolves is of great importance.

Smith et al. (2008) performed a detailed analysis of the halo correlation function on the scales relevant to studies of acoustic oscillations. They focused on the determination of the existence of shifts in the peak of the correlation function from the predictions of linear perturbation theory. If the peak in the halo correlation function is identified with the sound horizon at recombination in order to constraint the dark energy equation of state parameter, these shifts introduce important biases on the obtained constraints. As we have discussed in Section 2, the position of the peak in the correlation function should not be considered as a precise cosmological tool since it does not correspond to the exact value of the sound horizon $s$. Instead, a detailed modelling of the full shape of the correlation function is required. Smith et al. (2008) also proposed an analytic model for the halo two-point correlation function in real space based on solutions to the pair conservation equation using characteristic curves and a model of the pairwise velocity distribution of the haloes. This model is able to reproduce the main trends found in numerical simulations, showing that it contains the most relevant physical processes that control the deviations from linear theory. Here, we follow a more practical approach and analyse the biases or systematic effects introduced in the constraints on cosmological parameters if the modelling of Section 5.1 is applied to describe the shape of the correlation function of different halo samples.

On the very large scales, where halo biasing is deterministic, a simple linear relation is expected to hold between the correlation function of dark matter haloes, $\xi_0(r)$, and the linear theory matter correlation function, with an effective bias factor, $b_{\text{eff}} = \xi_0/\xi_{\text{lin}}$, independent of scale. This effective bias can be computed by taking the weighted average of the bias $b(m)$ as a function of halo mass $m$, over the selected halo sample:

$$b_{\text{eff}} = \frac{\int_0^\infty \psi(m) b(m) n(m) \, dm}{\int_0^\infty \psi(m) n(m) \, dm}, \quad (22)$$

where $n(m)$ is the mass density of dark matter haloes, which gives the space density of haloes in the mass interval $m$ to $m + dm$ (e.g. Press & Schechter 1974; Bond et al. 1991; Bower 1991; Sheth, Mo & Tormen 2001; Jenkins et al. 2001), $b(m)$ gives the bias as a function of halo mass $m$ (Mo & White 1996; Mo, Jing & White 1996; Jing 1998; Sheth, Mo & Tormen 2001; Seljak & Warren 2004) and $\psi(m)$ represents the mass selection function applied to construct the halo sample. For the cases analysed here, $\psi(m)$ is simply given by the limits of the mass bins, i.e. $\psi(m) = 1$ if $m_1 \leq m \leq m_2$ and $\psi(m) = 0$ otherwise.

The effects of halo bias on the two-point correlation function can be seen in Fig. 6. The correlation functions of the different halo samples can be rescaled using a constant bias factor, $b^2$, to agree remarkably well with the measurement for the dark matter. This means that the halo bias is independent of scale for the pair separations plotted. For each halo sample and redshift analysed, we computed the bias factors $b^2$ that maximize the likelihood of the best-fitting $\xi_0$ model to the dark matter correlation function. The results are shown in Table 1. For $z = 0$, these values are $b = 1.68$, $1.84$ and $2.54$ for the halo samples 1, 2 and 3, respectively. We used these bias values to rescale the halo–halo correlation functions in Fig. 6, and to compute the theoretical variances for each sample as described in Section 4.3. These values can be compared with those predicted by equation (22). Using the recipe for the mass function from Jenkins et al. (2001) and the bias function of Sheth et al. (2001) (with the modified parameters of Sheth & Tormen 2002), we find that equation (22) gives a good description of the bias factors with $b_{\text{eq}} = 1.62, 1.89$ and $2.53$ for the same halo samples. Using the bias prescription from Seljak & Warren (2004), the disagreement with the estimates from the simulations is bigger. Angulo et al. (2008) measured the bias factor as a function of halo mass directly from the BASICC simulation and found good agreement with the analytic prescription we use here, for the modest peak heights corresponding to our halo samples.

In order to analyse any possible biases or systematic effects that can be introduced when using information from halo samples to constrain cosmological parameters, we obtained constraints on $k_s$ and $\alpha$ by applying the model described in Section 5 to our measurements of the halo two-point correlation function. Our results are summarized in Table 1. Besides the overall constant bias factors, which are ignored (marginalized over) by our analysis procedure, the estimates of $\xi_0(r)$ show a change in the shape and position of the acoustic peak with respect to $\xi_{\text{lin}}(r)$, similar to the changes seen in the dark matter correlation function.

Fig. 16 shows the two-dimensional constraints on $\alpha$ and $k_s$ obtained using the mean halo–halo correlation functions for halo samples 1, 2 and 3 in real (solid lines) and redshift space (dashed lines) measured from the ensemble of simulations. As a consequence of the larger shot noise, the allowed regions for these parameters increase with respect to the ones obtained using the dark matter correlation function. The upper panel of Fig. 17 shows the histogram of the values of $\alpha$ obtained for the $z = 0$ halo–halo correlation functions for halo sample 1 of each L-BASIIIC II realization, which is completely consistent with the constraints obtained from the mean correlation function of the ensemble. This distribution is consistent with $\alpha = 1$ though it presents the same small tendency towards values of $\alpha < 1$ found in Section 5.2 with the dark matter.

Smith et al. (2008) performed a similar analysis, fitting the halo correlation function for different halo samples with the linear theory $\xi(r)$ smoothed with a Gaussian filter. They found a slight tendency towards larger shifts in the position of the acoustic peak, which implies smaller values of $k_s$, with increasing mass. We do not reproduce such a tendency here. The lower panel of Fig. 17 shows the histogram of the values of $k_s$ recovered from each realization. As a reference, we also show the theoretical predictions from equations (12) and (13). Although the peak of the obtained distribution for this parameter is consistent with the one found for the dark matter, there is a tail towards higher values of $k_s$. The same behaviour is seen in the results obtained for the remaining halo samples. This tail causes the obtained values for $k_s$ to be larger.
than for the dark matter, indicating that on average the halo BAO experience weaker damping than we find for the dark matter. This tail is more important for the higher mass bins which then creates a tendency towards higher mean values of $k_\star$ with increasing halo mass, even though the peak of the likelihood functions for this parameter is always consistent with the ones found for the dark matter.

As we saw with the dark matter, the incorporation of $Q$ as a free parameter of the model does not alter the constraints on $\alpha$. The values of $Q$ preferred by the halo measurements decrease at higher redshifts, when non-linear distortions are not as strong as at $z = 0$. On the other hand, the constraints on the stretch parameter obtained by applying equation (19) are completely consistent with $\alpha = 1$ showing that this modelling is able to correct the small bias towards $\alpha < 1$ present in the previous results also in the case of halo samples.

The amplitude of the halo correlation function, like the matter correlation function, is affected by the redshift–space distortions. These effects can be correctly described by equation (20) with the value of $S$ computed using the bias factors described above for the different halo samples. The values of $S$ obtained in this way are shown in Table 2 and were used to rescale the redshift–space halo correlation functions plotted in Fig. 6. As we found for the dark matter, besides this increase in the amplitude, the halo correlation function in redshift space shows a stronger damping of the acoustic signal than in real space. This is reflected in the allowed region for $k_\star$, which shows a preference for lower values than are obtained for real–space data. The constraints on $\alpha$ show similar behaviour to those in real space, with some deviation from $\alpha = 1$, which, again, is unaltered upon the inclusion of the non-linear correction of equation (16). As in the previous cases, this small bias is corrected by the implementation of the modelling of equation (19).

6 POWER SPECTRUM VERSUS CORRELATION FUNCTION

The next generation of galaxy surveys will cover significantly larger volumes than current surveys (e.g. Euclid, Robberto et al. 2007, or ADEPT). For the first time, systematic effects on the scale of the BAO will become comparable to the sampling errors. The power spectrum and correlation function of clustering are affected in different ways by effects such as non-linear evolution, bias and redshift–space distortions. In this section, we compare the constraints on the dark energy equation of state from these two statistics, to assess which one yields the smallest random and systematic errors.

We first recap the power spectrum and correlation function approaches to constraining the equation of state to highlight where differences may lie between them. Various power spectrum approaches have been proposed (e.g. Blake & Glazebrook 2003; Dolney, Jain & Takada 2006; Wang 2006; Percival et al. 2007a; Seo & Eisenstein 2007; Seo et al. 2008). Angulo et al. (2008) developed the method introduced by Percival et al. (2007a), in which no information is used from either the amplitude or large-scale shape of the power spectrum. A reference spectrum is defined from the measured power spectrum, by applying a spline fit to the spectrum after rebinning into coarser bins.
The measured power spectrum is then divided by this reference spectrum to form a ratio which emphasizes the appearance of the BAO. The philosophy behind this conservative approach is that de-viding the measured power by a reference which is defined directly from the measurement reduces the impact of any long-wavelength gradients in $P(k)$ induced by non-linear effects or redshift–space distortions, at the expense of losing some of the information contained within the power spectrum. Such gradients do exist in the bias factor and redshift–space distortions measured in simulations (Scoccimarro 2004; Smith et al. 2007; Angulo et al. 2008). A further advantage of taking a ratio of power spectra is that this removes the need for an accurate model of such effects. The measured ratio is then compared to a ratio generated from a linear perturbation theory spectrum. In this case, a new reference spectrum is generated for each linear theory $P(k)$. The linear theory ratio is ‘de-wiggled’ in a similar way to equation (11). Angulo et al. used the wavenumber range $k = 0 \sim 0.4 \, h \, \text{Mpc}^{-1}$ in their fits.

In the correlation function approach described in Section 5, we model the full shape of $\xi(r)$. The key step is the damping or ‘de-wiggling’ of the BAO in the power spectrum, as described by equation (11), before taking the Fourier transform of the power spectrum to obtain the correlation function. We also apply two empirical non-linear distortions to the form of the power spectrum, using equations (16) and (19). The first approach, based on the $Q$-model of Cole et al. (2005), has little impact on the quality of the fit on the scales used, $r = 60 \sim 180 \, h^{-1} \, \text{Mpc}$. On the other hand, the second model, proposed by Crocce & Scoccimarro (2008) based on an ansatz inspired by RPT, gives improved constraints and avoids the small bias towards $\alpha < 1$ found in the original modelling. We have demonstrated that bias and redshift–space distortions do not change the general shape of the correlation function on these scales but can change its amplitude. These effects are more apparent in the power spectrum approach. The range of wavenumbers used in fitting models to the power spectrum ratio described in the previous paragraph is wider than we use for the correlation function, which gives a longer baseline for any gradients to become apparent. Also, when forming the correlation function from the power spectrum, different scales are mixed together, which weakens any gradients in $\xi(r)$. The only relevant impact of redshift–space distortions for the case of the correlation function is to increase the damping of the acoustic peak compared with real space, which results in a lower value of the smoothing scale $k_s$ being returned by the fitting procedure. The correlation function approach also ignores the absolute amplitude, but does use the shape of the correlation function around the acoustic peak, as well as the shape and form of the peak itself.

We now compare how well the two approaches work in practice. As we have already seen in Fig. 14, a detailed comparison of the predictions of equation (14) with the mean correlation function measured from the ensemble of simulations shows small discrepancies. These are the source of the small bias towards $\alpha < 1$ found in Section 5.2. On scales larger than the acoustic peak, the model slightly overestimates the amplitude of $\xi(r)$ which follows more closely the predictions from linear perturbation theory. On these scales, $\xi_{\text{gal}}(r)$ is almost entirely determined by the first term of equation (14) (Crocce & Scoccimarro 2008; Smith et al. 2008). These differences may indicate that $G(k)$, the function that controls the damping of the harmonic oscillations, may not be correctly described by a Gaussian on these scales. The largest differences between the model and the results from the simulations are found on scales of $r \leq 90 \, h^{-1} \, \text{Mpc}$. These discrepancies can be related to the change in the shape of the power spectrum due to non-linear evolution. The inclusion of the scale-dependent correction factor of equation (16) can alleviate these differences, but it is not enough to correct for the small bias in the constraints on $\alpha$. This implies that the implementation of this modelling when dealing with real observational data may lead to a slight bias in the constraints on the dark energy equation of state $w_{\text{DE}}$. On the other hand, the model based on RPT proposed by Crocce & Scoccimarro (2008) yields tighter constraints and does not suffer from biases towards $\alpha < 1$.

Fig. 18 shows the effectiveness of the same model applied to $P(k)$. The upper panel shows the mean real–space dark matter power spectrum measured from our ensemble of simulations (open points). The dotted lines indicate the variance over the different realizations. Discrepancies between the linear theory power spectrum (dot–dashed line) and the ‘dewiggled’ power spectrum from equation (11) (solid lines) are evident. Non-linear evolution distorts the shape of $P(k)$ in a way that can be correctly described by equation (16) (dashed line) with $Q = 13$ and $A = 1.5$. The lower panel of Fig. 18 shows the ratio of these power spectra to $P_{\text{gal}}(k)$. The results from the numerical simulations have also been divided by the non-linear distortion factor. Although $P_{\text{gal}}(k)$ does a good job of reproducing the full shape of the power spectrum and the damping of the oscillations, there are small discrepancies that show the limitations of the model.

In order to assess which of the two approaches is superior when used as a tool to recover unbiased constraints from BAO measurements, we have analysed the mean real–space power spectrum from the ensemble of simulations for the dark matter and the same halo samples as defined in Section 4.2 using the method of Angulo et al. (2008). The constraints on $\alpha$ and $k_s$ obtained in this way are listed in Table 3. Angulo et al. (2008) defined the mean bias factor as $b^2 = \langle P_{\text{halo}}(k) / P_{\text{gal}}(k) \rangle$. The values of $b^2$ obtained for the different halo samples analysed are also listed in Table 3, and are in close agreement with those we quote in Table 1 for the correlation function. For the case of the dark matter, the value of $k_s$ obtained from the power spectrum shows the same tendency to increase with redshift as was found in the analysis of $\xi(r)$. For the halo samples, the trend of $k_s$ with redshift is less evident, perhaps due to the
Figure 18. Upper panel: a comparison of the real–space dark matter power spectrum averaged over the simulation ensemble (open points) with the linear theory power spectrum (dot–dashed line), the ‘dewiggled’ power spectrum from equation (11) (solid line) and its non-linear version from equation (16) (dashed line) computed with $Q = 13$. The dotted lines indicate the variance on $P(k)$ estimated from the ensemble. Lower panel: the ratio of these power spectra to $P_{\text{true}}(k)$. The results from the numerical simulations have also been divided by the non-linear growth factor given by equation (16).

Table 3. The results of applying the general fitting procedure described in Angulo et al. (2008) to the real–space power spectra of the dark matter and the halo samples defined in Section 4.2. The first column gives the label of the sample. The second column gives the redshift output. Column 3 gives the mean bias factor $\xi^2 = \langle P_{\text{true}}(k) / P_{\text{true}}(k) \rangle$. Columns 4 and 5 give the obtained constraints on $\alpha$ and $\alpha$. All quoted values correspond to the mean and 68 per cent c.l. on the respective parameter.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>$\bar{z}$</th>
<th>$\xi^2$</th>
<th>$k_c/(h\text{ Mpc}^{-1})$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM</td>
<td>0.0</td>
<td>1</td>
<td>0.118 ± 0.010</td>
<td>1.006 ± 0.008</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>1</td>
<td>0.142 ± 0.011</td>
<td>1.002 ± 0.007</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1</td>
<td>0.172 ± 0.015</td>
<td>1.000 ± 0.006</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>2.796</td>
<td>0.141 ± 0.043</td>
<td>0.997 ± 0.019</td>
</tr>
<tr>
<td>Haloes 1</td>
<td>0.5</td>
<td>1</td>
<td>0.141 ± 0.044</td>
<td>1.004 ± 0.019</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>12.04</td>
<td>0.132 ± 0.039</td>
<td>0.992 ± 0.020</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>3.323</td>
<td>0.143 ± 0.049</td>
<td>1.009 ± 0.020</td>
</tr>
<tr>
<td>Haloes 2</td>
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<td>7.306</td>
<td>0.143 ± 0.053</td>
<td>0.972 ± 0.020</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>17.20</td>
<td>0.132 ± 0.046</td>
<td>0.994 ± 0.027</td>
</tr>
<tr>
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<td>0.156 ± 0.040</td>
<td>1.002 ± 0.015</td>
</tr>
<tr>
<td>Haloes 3</td>
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<td>0.157 ± 0.046</td>
<td>1.009 ± 0.015</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>31.61</td>
<td>0.132 ± 0.054</td>
<td>1.008 ± 0.023</td>
</tr>
</tbody>
</table>

scale-dependent bias seen in $P(k)$. There is also a tendency towards higher values of $k_c$ with increasing mass which is also suggested in the results of Section 5.2. The precise preferred values of $k_c$ obtained in this case are in agreement with the ones obtained using the correlation function.

The values for $\alpha$ recovered from both methods are similar, but the allowed region for this parameter is bigger when only the information from the oscillations in $P(k)$ is used. The extra information from the shape of the correlation function helps to improve the constraints on $\alpha$ resulting in a smaller allowed region for this parameter. This test implies that the simple model for the correlation function described in Section 5.1 can perform better than the methods in which the scale of the acoustic oscillations is extracted from the power spectrum. In a more realistic situation, when other cosmological parameters are included in the analysis, the extra information contained in the shape of the correlation function would make this approach seem even more attractive, since it can also be used to improve constraints on parameters, such as $\Omega_m$. In the analysis of CMB data, this parameter shows a strong degeneracy with $\theta_{DE}$, and therefore a better constraint on the matter density would also result in tighter bounds on the dark energy equation of state.

Finally, the mean values of the stretch parameter recovered from the dark matter power spectrum show a small bias towards $\alpha > 1$ which decrease with increasing redshifts. Crocce & Scoccimarro (2008) estimated the mode-coupling shifts that would affect the constraint on $\alpha$ using the method of Angulo et al. (2008). Their predictions are in good agreement with our results, suggesting that this bias might be attributed to the systematic effects introduced by the mode-coupling power spectrum $P_{\text{true}}(k)$. This deviation from $\alpha = 1$ is smaller than the sampling variance for the volume of the individual L-BASICC II realizations. Future surveys like Euclid or ADEPT will cover volumes that are close to the total volume simulated by our ensemble ($V = 120\text{ Gpc}^3 h^{-3}$). Rescaling the obtained covariance matrix by a factor of 1/50 (which then corresponds to the covariance of the mean correlation function from our ensemble), we can gauge the effect of this bias for these surveys. In this case, the error associated with the value of $\alpha$ at $z = 0$ will be $0.008/\sqrt{50} \approx 0.001$, which then implies a $5\sigma$ detection of the mode-coupling shifts for the dark matter. Instead, the results obtained from the correlation function applying the modelling of equation (19) are more consistent with $\alpha = 1$, even after re-scaling the error to the full volume of our ensemble. This shows that with this modelling, the correlation function is more likely to produce unbiased constraints than the power spectrum. In principle, the simple ansatz of equation (19) can be extended to the analysis of the power spectrum. Nevertheless, due to the fact that in Fourier space, redshift–space distortions and halo bias do show important scale dependence, it will be necessary to follow an approach similar to Angulo et al. (2008), dividing the measured power spectrum by a smooth function. This sacrifice of information will always have consequences for the obtained constraints.

7 CONCLUSIONS

In this paper, we have addressed two main questions. (1) What is the relation between the ‘acoustic peak’ in the two-point correlation function and the sound horizon scale? (2) Which is the better statistic to use to constrain the dark energy equation of state, the correlation function or the power spectrum? Future galaxy surveys will cover volumes one or more orders of magnitude larger than existing surveys. For the first time, systematic effects in the appearance of the BAO and in their interpretation will be comparable to the random or sampling variance errors on the measurement of the two-point statistics. It is therefore essential to understand these systematics and to improve theoretical modelling of the BAO so that they can be used to provide optimal and unbiased estimates of the values of cosmological parameters.

It is a common misconception that the location of the acoustic peak in the correlation function is identical to the size of the sound horizon at recombination. Indeed, several recent studies (Guzik et al. 2007; Crocce & Scoccimarro 2008; Smith et al. 2008) have focused on showing that the acoustic peak in the correlation function is shifted and distorted relative to the prediction of linear
perturbation theory. What has not been widely appreciated before is that even in linear perturbation theory, the centroid or maximum of the peak in $\xi(r)$ does not coincide with the sound horizon scale at the level of accuracy required to fully exploit the constraints that can be derived from forthcoming galaxy surveys. The size of the error in the distance scale which would result from making this incorrect assumption is of the order of 1 per cent, which is already close to the random error in the distance scale forecast for ongoing surveys.

In order to use the correlation function on large scales to constrain the values of the cosmological parameters, it is therefore necessary to model the form of the correlation function. By using a physically motivated model, a natural and consistent connection is made between the cosmological parameters and the form of the correlation function. There are several effects which have to be taken into consideration to produce a complete model: the non-linear growth of fluctuations, the possible scale dependence of the bias between the chosen tracer of the density field and the underlying matter fluctuations and the distortions introduced by using distance measurements which are inferred from redshifts, commonly referred to as redshift–space distortions. In order to assess the impact of these effects on the correlation function, we used an ensemble of 50 very large volume $N$-body simulations, the L-BASICC II. These simulations are analogous to the L-BASICC ensemble used by Angulo et al. (2008) for a similar analysis of the acoustic oscillations in the power spectrum. We measured the correlation function of the dark matter and of different halo samples in both real and redshift space in each of the realizations in the ensemble.

It turns out that bias, general non-linear evolution and redshift–space distortions are much simpler to deal with in the case of the correlation function than they are for the power spectrum. On the scales we consider ($60 < r/h^{-1}\text{Mpc} < 180$), these effects primarily result in a change in the amplitude of the measured correlation function and do not alter its shape. By ‘general’ non-linear evolution, we mean the change in the overall ‘coarse grained’ shape of the power spectrum. The crucial effect to include in the model is the damping of the higher order harmonic oscillations due to non-linear evolution, as given by equation (11). Whilst we find that a simple damping of the linear perturbation theory power spectrum is able to reproduce the correlation function we measure from the simulations, some discrepancies remain between the model and the simulation results on scales both larger and smaller than the acoustic peak. These differences lead to slightly biased constraints on the stretch factor $\alpha$, which in turn will yield small biases in the estimates of the dark energy equation of state parameter $v_{DE}$. The model is only marginally improved on incorporating an empirical model for non-linear distortions (which could also be used to describe a scale-dependent bias). This Q-model, introduced by Cole et al. (2005), is widely used in the literature but needs to be refined to extract the full information from the correlation function. On the other hand, the implementation of the model of equation (19) helps to bring the constraints on the stretch parameter in closer agreement with the value $\alpha = 1$. This parametrization was proposed by Crocce & Scoccimarro (2008) and is based on RPT.

To address the question of which two-point statistic is the most powerful for extracting the BAO, we repeated the power spectrum analysis carried out by Angulo et al. for several of the samples we used to measure correlation function. The analysis of Angulo et al. can be viewed as a conservative approach (see also Percival et al. 2007a,c). The measured power spectrum is divided by a ‘wiggle-free’ reference spectrum which is defined using the measured spectrum itself, without any modelling. No information is used about the amplitude of the spectrum or its overall shape. By taking a ratio in this way, the BAO are isolated and the impact of scale-dependent effects in the power spectrum is minimized. Similar ratios generated using linear perturbation theory, and applying some ‘de-wigging’ or damping of the BAO, are compared to the measured power ratio. We compared the constraints obtained from the model for $\xi(r)$ described in Section 5.1 and the ones obtained by applying the fitting procedure of Angulo et al. (2008). This simple test shows that a method that uses the full large-scale shape of the correlation function can provide tighter constraints on $v_{DE}$ than one in which the scale of the acoustic oscillations is extracted from the power spectrum. The extra information contained in the shape of the correlation function can also be extremely useful to constrain other cosmological parameters, such as $\Omega_m$. In the analysis of CMB data, $\Omega_m$ shows a strong degeneracy with $v_{DE}$, and so the use of the correlation function would result in tighter bounds on the dark energy equation of state.

The constraints on the stretch parameter obtained with the power spectrum show a small bias towards $\alpha > 1$ which is consistent with the predictions of Croce & Scoccimarro (2008) based on the effect of the mode-coupling power spectrum. This bias will become much more important for future galaxy surveys which will have much smaller random errors. Instead, the constraints obtained using equation (19) to model the correlation function are much more consistent with $\alpha = 1$ even for the large volumes that will be covered by these surveys.

The ability of equation (19) to improve the constraints obtained from the correlation function suggests that the recent advances in the modelling of the acoustic oscillations using RPT (Crocce & Scoccimarro 2008) may provide a basis for a more accurate theoretical model of the full shape of the correlation function and power spectrum. None the less, the problem is aggravated by the need to model scale-dependent galaxy bias, which may have already become the strongest limitation on the use of large-scale structure information to obtain constraints on cosmological parameters (Sánchez & Cole 2008). An accurate model capable of describing all sources of distortions must be implemented if the determinations of the galaxy correlation function and power spectrum from future surveys are to achieve competitive constraints on the dark energy equation of state. It is clear that realistic numerical simulations which are able to model the growth of structure in the dark matter and which can incorporate a physical model for galaxy formation will be essential to guide the development of such models. The use of both of these statistics will give the complementary information required to improve the model of equation (11), and will hopefully allow the full potential of BAO measurements as probes of the nature of dark energy to be reached.

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