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Is an 11 eV sterile neutrino consistent with clusters, the cosmic microwave background and modified Newtonian dynamics?

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ABSTRACT

In this paper, we show that if a single sterile neutrino exists such that $m_{\nu_s} \sim 11$ eV, it can serendipitously solve all outstanding issues of the Modified Newtonian Dynamics. We focus on fitting the angular power spectrum of the cosmic microwave background (CMB) in detail which is possible using a flat Universe with $\Omega_{\nu_s} \sim 0.23$ and the usual baryonic and dark energy components. One cannot match the CMB if there is more than one massive sterile neutrino, nor with three active neutrinos of 2 eV. This model has the same expansion history as the Λ cold dark matter (ΛCDM) model and only differs at the galactic scale, where the modified dynamics outperform ΛCDM comprehensively. We discuss how an 11 eV sterile neutrino can explain the dark matter of galaxy clusters without influencing individual galaxies and potentially match the matter power spectrum.

Key words: gravitation – neutrinos – cosmic microwave background.

1 INTRODUCTION

If one seeks a simple algorithm to explain the dynamics of galaxies, then one is ultimately drawn to Milgrom's modified Newtonian dynamics (MOND; see Milgrom 1983; Sanders & McGaugh 2002; Bekenstein 2006; Milgrom 2008b; Sanders 2008b). From the drastically different scales of Local Group dwarf galaxies (Milgrom 1995; Angus 2008) and tidal dwarf galaxies (Gentile et al. 2007; Milgrom 2007; see also Bournaud et al. 2007) to globular clusters or from low surface brightness (McGaugh & de Blok 1998; Milgrom & Sanders 2007) to high surface brightness spirals (Famaey & Binney 2005; Sanders & Noordermeer 2007; McGaugh 2008), as well as giant ellipticals (Milgrom & Sanders 2003; Angus et al. 2008), it works extraordinarily well.

Not only are the dynamics of the galactic systems well matched from the MOND prediction, but they also fall precisely on the Tully–Fisher relation (McGaugh et al. 2000; McGaugh 2005b) which correlates total enclosed mass and the fourth power of the asymptotic velocity, unless they are satellites of a larger galaxy.

The only alternative theory uses cold dark matter (CDM) particles with a severe lack of experimental motivation (CDMS Collaboration 2008) in massive, triaxial haloes (Hayashi & Navarro 2006) to provide the additional gravity needed to boost the rotation velocities of the systems with an acceleration discrepancy. There are several well-documented and as yet unresolved issues for this framework at the scales of galaxies such as the fine-tuning problem of dark matter (DM) haloes (Milgrom & Sanders 2005; McGaugh 2005a), the cusp problem (de Blok & McGaugh 1998; McGaugh & de Blok 1998; Gnedin & Zhao 2002; Gentile et al. 2004; Gilmore et al. 2007), the missing satellites problem (Klypin et al. 1999; Moore et al. 1999) and, more recently, the acceleration discrepancy in tidal dwarf galaxies (Milgrom 2007; Gentile et al. 2007; see also Bournaud et al. 2007). Nevertheless, it is a generally accepted model.

One might find this astounding except for the fact that it has unchallenged success at cosmological scales where MOND predictions are sketchy. Furthermore, clusters of galaxies require large quantities of dark matter in MOND (Aguirre, Schaye & Quataert 2001; Sanders 2003; Pointecouteau & Silk 2005; Clowe et al. 2006; Angus et al. 2007; Sanders 2007; Angus, Famaey & Buote 2008). This might sound like a contradiction, but it is perfectly sensible as long as an obvious contraint is satisfied, i.e. any dark matter in MOND has a free streaming scale that is larger than galaxies and the relative density is low. Otherwise, the dark matter would manifest itself in ordinary galaxies which would destroy the consistency of the baryonic Tully–Fisher relation (McGaugh et al. 2000; McGaugh 2005b).

It was postulated (Sanders 2003, 2007) that the active neutrinos (at $m_\nu = 2$ eV; very close to the experimental upper limit of 2.2 eV) can provide the dark mass of clusters. Neutrinos conform to certain scaling relations in clusters such as the proportionality of the electron density in the cores of clusters to $T^{3/2}$. Nevertheless, recent studies (Pointecouteau & Silk 2005; Clowe et al. 2006; Bradač et al. 2006; Angus et al. 2007, 2008; Diaferio & Ostorero 2008) have shown that, even under very favourable circumstances,

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a second species of dark matter would be necessary to explain the dynamics of the central 100 kpc of clusters and groups of galaxies, which more or less rules out the active neutrinos as good candidates.

We should be aware that there is no limit on the dark matter being baryonic since the necessary dark matter in clusters of galaxies is at most a few per cent of the big bang nucleosynthesis (BBN) baryons, of which only 20 per cent or so are observed at low redshifts (Silk 2007; McGaugh et al. 2007), the remainder is presumed to exist in a warm–hot intergalactic medium (Bregman 2007). This led Milgrom (2008a) to propose the dark matter in clusters to be a cold, molecular gas of approximately Jupiter mass. These are naturally difficult to detect, but might have the serendipitous fortune of resolving the cooling-flow problem (Fabian, Johnstone & Daines 1994).

Unfortunately, even if the cluster dark matter problem were resolved, there remains the issue of cosmological dark matter. To begin with, there is compelling evidence that the Universe consists of a form of dark energy (like a cosmological constant) that forces the expansion of the Universe to accelerate at late times (Schmidt et al. 1998; Perlmutter et al. 1999). Currently, we have no idea what this dark energy is (Diaferio 2008) from a particle physics point of view, although perhaps the coincidence between the acceleration constant of MOND $a_o = 3.6\,(\mathrm{km\,s^{-1}})^2\,\mathrm{pc^{-1}} = 1.2^{-10}\,\mathrm{ms^{-2}}$ and cH_o or $c(\Lambda/3)^{1/2}$ is a strong indication of a link (Milgrom 2002, 2008b).

With the presence of this dark energy, in order for the Universe not to expand too rapidly, there needs to be some form of matter (independent of the well-fixed quantity of baryons) to endow the Universe with additional inertia. This additional matter serves several purposes: it allows for structures to form more rapidly out of the expanding Universe which is shown by the matter power spectrum at large scales (Tegmark et al. 2004). It drives the collapse of the photon–baryon fluid to form fluctuations in the cosmic microwave background (CMB) at well-measured angular scales (White, Scott & Silk 1994) and it gives the correct distance–redshift relation (expansion history).

The underlying theory of MOND is still unknown. Moreover, it is only a classical framework, so a huge effort was made to extend MOND to the relativistic regime, which was assumed to be the Holy Grail. In 2004, a giant leap was made in this direction by Bekenstein (2004), and others have taken to thrashing out the predictions of other MOND-inspired relativistic theories (Sanders 2005; Skordis et al. 2006; Zlosnik, Ferreira & Starkman 2006, 2007, 2008). Sadly, the predictions for cosmology are not clear and there seems to be too much freedom, in contradiction to the *almost* absolute predictiveness of MOND in galaxies. Of course, in MOND, there is still a small freedom in the mass-to-light ratio assumed for the stars and also the free-function of MOND which connects the r^{-2} Newtonian gravity regime with the r^{-1} MONDian one. Fortunately, the form of this free function has no impact upon the relatively strong gravity of the CMB anisotropies and only affects structure formation for the short period of time during which a perturbation has a gravity around a_o (see Famaey & Binney 2005; Famaey et al. 2007; McGaugh 2008 for more details regarding the free-function).

In this paper, I show the predictions of coupling MOND with sterile neutrino dark matter using the *ansatz* employed by McGaugh (2004) when matching the CMB with MOND i.e. no MOND effects are present before recombination. A simple argument supporting this is that at a redshift of $z \sim 1080$ the angular-diameter distance to recombination $D_A = 14$ Gpc so the angular scale, θ, of the first (and largest) peak is 1° or 0.017 rad. Thus, the physical size of the first peak $r = \theta D_A$ is ∼240 Mpc. Since the average overdensity δ is only 1 part in 10^5 of the critical density $\rho_c(z)$, the typical gravities at a radius r from the centre of one of these overdensities are $g = G\delta M(r) r^{-2} = \frac{4\pi}{3} G\delta\rho_c(z) r$ where $\rho_c(z) = \frac{3H(z)^2}{8\pi G}$ and $H(z)^2 = H_o^2[\Omega_\mathrm{m}(1+z)^3 + \Omega_\Lambda]$. Compiling all this gives

$$g(r) \sim \frac{1}{2}\delta H_o^2\left[\Omega_\mathrm{m}(1+z)^3 + \Omega_\Lambda\right] r, \qquad (1)$$

which at $r = 240$ Mpc yields

$$g = \frac{1}{2} \times 10^{-5} \times 7.1 \times 10^{-5}\,[0.27 \times 1081^3 + 0.73]\,240\,\mathrm{Mpc} \sim 570 a_o.$$

Typical accelerations so many times greater than a_o are completely unaffected by MOND gravity, and therefore no MOND effects should influence the CMB. However, as z drops, so does $\rho_c(z)$ and thus peculiar accelerations can slide into the MOND regime. Thus, the matter power spectrum can be affected by MOND.

It is often forgotten when looking at MOND cosmology that *no* CDM exists. Therefore, we must relax many of the constraints that are set by CDM cosmology. The most important and obvious one is that there is now a large gap in the energy-density budget (to make $\Omega = 1$) since CDM is not present. It is perfectly reasonable to fill this gap with hot dark matter like neutrinos. The constraints on neutrino masses, for which cosmology is still the most stringent, must be reanalysed in the light of MOND.

Still, the empirical evidence from supernovae data (Schmidt et al. 1998; Perlmutter et al. 1999) strongly suggests that the universe is expanding at an accelerated rate due to the existence of dark energy, Ω_Λ. Furthermore, the baryon budget is strongly constrained by the well-understood physics of BBN to be around $\Omega_\mathrm{b} h^2 \sim 0.015$–$0.025$ (Boesgaard & Steigman 1985; Burles, Nollett & Turner 2001; McGaugh 2004), but this still leaves a large amount of latitude in the energy budget for DM.

Any DM, however, must be compatible with clusters of galaxies, the well-understood lack of DM in galaxies in MOND and the anisotropies in the angular power spectrum of the CMB. Furthermore, it must allow for the formation of cosmic structures at the correct rate on all scales. The best candidates for this hot DM are sterile neutrinos for the simple reason that they are already incorporated into simple extensions of the standard model of particle physics to explain the origin of neutrino masses and oscillations.

2 NEUTRINOS

2.1 Active neutrinos

The three active neutrinos (ν_μ, ν_e and ν_τ) from the standard model of particle physics have been shown to mix between flavours by atmospheric and solar neutrino experiments (Ahmad et al. 2001; Ashie et al. 2004). However, the exact masses of the three active neutrinos are not yet known, only their squared mass differences. Nevertheless, the masses of all three are known to be less than 2.2 eV from the Mainz–Troitz experiments (Kraus et al. 2005).

The maximum density that a neutrino species can produce after gravitational collapse is given by the Tremain–Gunn limit (Tremaine & Gunn 1979),

$$\frac{\rho_{\nu,i}^{\max}}{7 \times 10^{-5}\,\mathrm{M_\odot\,pc^{-3}}} = \left(\frac{T}{1\,\mathrm{keV}}\right)^{1.5}\left(\frac{m_{\nu,i}}{2\,\mathrm{eV}}\right)^4, \qquad (2)$$

for each of the three species; where $i = e, \tau, \mu$. Thus, the density is greatly dependent on the mass of the neutrinos. However, a recent, comprehensive study by Angus et al. (2008) has shown that the groups and clusters of galaxies analysed in the studies of Buote et al. (2007), Gastaldello et al. (2007), Humphrey et al. (2006) and

Zappacosta et al. (2006) ranging from $T = 0.7$ to 8.9 keV have dark matter that is much denser than can be produced by the active neutrinos even at the maximum mass of 2.2 eV. If the dark matter is indeed a neutrino-like species, it must be heavier than 8 eV.

There is a further problem with neutrinos at 2.2 eV since the contribution they make to the energy density of the Universe is given by

$$\Omega_\nu = 0.0205 m_\nu, \tag{3}$$

meaning that at 2.2 eV the three neutrinos make a 13.6 per cent contribution to the energy density of the Universe, but the maximum density is relatively low (see equation 2). Such a huge contribution would be easily detectable in the angular power spectrum of the fluctuations in the CMB as shown, for example, in Fig. 2. Therefore, the active neutrinos are a very poorly motivated candidate.

2.2 An 11 eV sterile neutrino and the CMB

As mentioned above, the three active neutrinos are known to have mass. Another oddity arising from this is that the active neutrinos are solely left handedly chiral, whereas all other fermions are ambidextrous. The easiest way to incorporate this into the standard model of particle physics is to introduce a right-handed ‘sterile neutrino’. This is not only for aesthetical reasons, the introduction of a single sterile neutrino was preferred from an analysis of the Miniboone experiment by Giunti & Laveder (2008) (see also Aguilar et al. 2001; Maltoni & Schwetz 2007) with a mass in the range 4–18 eV to explain the disappearance of electron neutrinos from the beam at low energies.

In the simplest model, if the mixing angle of the sterile neutrino is low enough, then thermalization in the early Universe can balance the abundance of the sterile and active neutrinos. In this case, the cosmological density is exactly related to their mass, just like the active ones (equation 3).

With the hypothesis that all the DM in MOND comes from a single sterile neutrino, we used the freely available CMB anisotropy code CAMB (Lewis, Challinor & Lasenby 2000) and incorporated it into a χ^2-minimization routine comparing with the data from the *Wilkinson Microwave Anisotropy Probe 5* (*WMAP5*) data release (Dunkley et al. 2008) and the Arcminute Cosmology Bolometer Array Receiver (ACBAR) 2008 data release (Reichardt et al. 2008). We allowed variation of Ω_b, Ω_{ν_s}, n_s, $\mathrm{d}n_s/\mathrm{d}\ln k$, τ, H_o, and fixed the Universe to be flat meaning $\Omega_\Lambda = 1 - \Omega_\mathrm{b} - \Omega_{\nu_s}$.

Obviously, in this MOND-inspired model there is no CDM by definition, but since the CDM model works well at producing the CMB anisotropies, we began the search by simply transferring Ω_cdm to Ω_{ν_s}. Furthermore, the three active neutrinos are taken, for simplicity, to be massless.

The parameters for the best fit are given in Table 1 which also contains the parameters for the *WMAP*5 fit from Dunkley et al. (2008), and a comparison of the two fits is shown in Fig. 1. All parameters are consistent with experimental bounds and are not significantly different to the Λ CDM model, which is sensible since the Λ CDM model of the CMB anisotropies is a good one.

Table 1. List of parameters used in the figures. The Λ CDM values come from Dunkley et al. (2008) but n_s has been scaled from the quoted 0.963 to 0.979 for a better match to the data.

Parameter	Single ν_s	ΛCDM	Active	Two ν_s
H_o(km s^{-1} Mpc^{-1})	71.5	72.4	71.5	71.5
$100\Omega_\mathrm{b}\,h^2$	2.4	2.27	2.4	2.4
$\Omega_\nu\,h^2$	0.117	0.0	0.07	0.117
$\Omega_\mathrm{cdm}\,h^2$	0.0	0.108	0.0	0.0
n_s	0.965	0.979	0.856	0.939
No. massless ν	3	3	0	3
No. massive ν	1	0	3	2

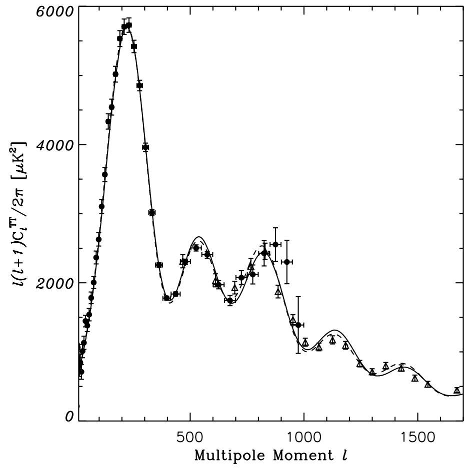

Figure 1. The data of the CMB as measured by the *WMAP* satellite year five data release (filled circles, Dunkley et al. 2008) and the ACBAR 2008 (Reichardt et al. 2008) data release (triangles). The lines are the ΛCDM max likelihood (dashed) and the 11 eV sterile neutrino model (solid) with parameters given in Table 1. The n_s for the Λ CDM model has been scaled from the quoted 0.963 in Dunkley et al. (2008) to 0.979 here to better match the data.

The mass of the sterile neutrinos inferred from the best-fitting value of $\Omega_{\nu_s} h^2 = 0.117$ is $m_{\nu_s} \approx 11$ eV. This mass range of sterile neutrino has never before been considered in the literature because it is excluded by cosmological data if we assume that Newton’s law is correct (Dodelson, Melchiorri & Slosar 2006; Seljak et al. 2006). This is for the simple reason that they cannot influence galaxy rotation curves because they would have a free-streaming scale (cf. Sanders 2007) of more than $R_c = 1.3\ (m_\nu/1\,\mathrm{eV})^{-4/3}$ $(V_r/200\,\mathrm{km\,s^{-1}})^{1/3} = 50$ kpc in a Milky Way type galaxy, for $V_r = 200\,\mathrm{km\,s^{-1}}$. The total mass this would create within 8 kpc is $\sim 5 \times 10^9\,\mathrm{M}_\odot$ which is about 10 per cent of the total mass and would actually help MOND fits to the Milky Way’s rotation curve (Famaey & Binney 2005; Gentile, Zhao & Famaey 2008; McGaugh 2008), but leaves a Newtonian Milky Way in turmoil.

It is not feasible to have a pair of very massive (>0.5 eV) sterile neutrinos because splitting the Ω_{ν_s} between two or more neutrinos reduces the available mass to each neutrino thus detrimentally lowering its Tremaine–Gunn limit ($\rho_\nu^\mathrm{max} \propto m_\nu^4$) and thus the gravity available to drive the collapse of the baryons prior to recombination on small scales, in particular, the third acoustic peak of the CMB. This is highlighted in Fig. 2 where the comparison is made between one sterile neutrino and two, as well as 3 × 2.2 eV active neutrinos.

This 11 eV sterile neutrino would have a similar contribution to the energy density as required from CDM ($\Omega_{\nu_s} h^2 = 0.117$; $\Omega_\mathrm{cdm} h^2 = 0.108$) and leave the matter power spectrum at large scales (>50 h^{-1} Mpc) unaltered. This is shown in Fig. 3 which compares the observed matter power spectrum with that predicted by the sterile neutrino model here, but with the Newtonian instead of MONDian gravity. The redshift by which scales as large as 50 h^{-1} Mpc are deep in the MOND regime (i.e. $g \sim \frac{a_o}{10}$) is roughly

$$z \sim \left(\frac{2g}{\delta H_o^2 \Omega_\mathrm{m} r}\right)^{1/3}, \tag{4}$$

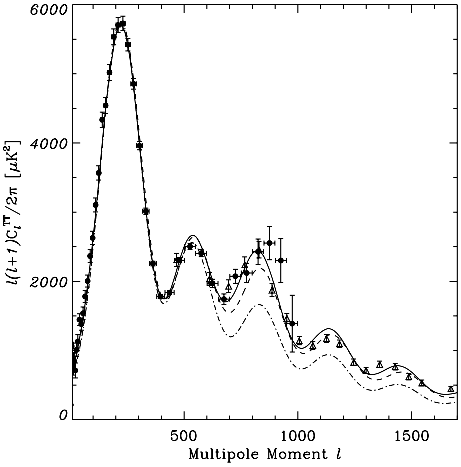

Figure 2. The same as Fig. 1 with the solid line again showing the single 11 eV sterile neutrino fit with parameters given in Table 1, but the dashed line is the fit with two sterile neutrinos sharing Ω_{ν_s} and the dotted line is for the maximum (3×2.2 eV) active neutrino contribution with $\Omega_\nu = 0.136$ and Ω_Λ compensated for a flat universe. We reduced n_s to 0.856 and 0.939, respectively, to match the amplitude of the first acoustic peak, but the second and third peaks are badly matched because there is not enough neutrino DM density on small scales because of the Tremaine–Gunn limit.

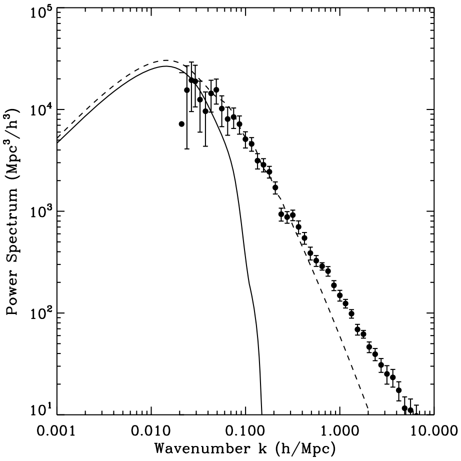

Figure 3. The filled circles are the data points from the SDSS (Tegmark et al. 2004), the solid line is the power spectrum generated by the single 11 eV sterile neutrino model but with the Newtonian instead of MONDian gravity. The dashed line is the linear ΛCDM model. It is predicted that the non-linear growth of structure in MOND will reconcile the model with the data as per Skordis et al. (2006). The P(k) matches at large scales because these scales have not had enough time (during the Hubble time) to be perturbed by MOND.

which for 70 Mpc is $z \approx 100$. Many authors (Nusser 2002; Knebe & Gibson 2004; Sanders 2008a) have shown that structures can form very quickly in MOND even without CDM, and galaxy size objects can be in place as early as $z \approx 10$.

Note that the ΛCDM power spectrum goes non-linear at scales of $\sim 0.3\,h\,\mathrm{Mpc}^{-1}$, and the sterile neutrino model does so at $0.02\,h\,\mathrm{Mpc}^{-1}$ (or at least it must do). The sterile neutrino model, however, has a non-linearly growing density perturbation that is not entirely due to the perturbation approaching unity, but rather because the non-linear gravity of MOND causes it everywhere to grow non-linearly. The reason scales larger than $0.02\,h\,\mathrm{Mpc}^{-1}$ suffer no increase in power is because there is simply not enough time, during the Hubble time, to perturb them sufficiently, since the Universe is not causally connected on large scales.

It is unfortunate that the tools to perform the full matter power-spectrum analysis are currently not available for MOND (although see Llinares, Knebe & Zhao 2008), nor standard dynamics, since it crucially depends on hydrodynamics. Nevertheless, the work of Skordis et al. (2006) comes relatively close to the sterile neutrino model. He took an Einstein–Boltzmann code and modified it to compute the non-linear evolution of density perturbations in a TeVeS (Bekenstein 2004) inspired model with 3×2.2 eV neutrinos. There exist some additional free parameters in the theory, but the basic premise of MOND-like gravity influencing regions with accelerations below a_o is sustained. Furthermore, the 2.2 eV neutrinos have $\Omega_\nu = 0.136$, close to the 11 eV sterile neutrino's $\Omega_{\nu_s} = 0.23$. From his fig. 4 (solid line), the neutrinos in combination with the MOND gravity appear to smooth the usual baryon only oscillations seen in the model presented by Sanders (2001) and impressively flows through most of the points taken from the Sloan Digital Sky Survey (SDSS) power spectrum. There seems to be a slight lack of power at $0.04\,h\,\mathrm{Mpc}^{-1}$, which may well be supplemented by the significantly smaller free streaming scale of the sterile neutrinos in addition to their greater impact on the matter density in general.

On the other hand, it may be the case that whereas sterile neutrinos at 11 eV are of the utmost importance for matching the details of the CMB power spectrum, the mere presence of a dominant dark matter component with fluctuations allowed to grow under MOND gravity is enough to match the matter power spectrum on large scales.

Assuming that including the modified dynamics enables a match to the matter power spectrum at all scales, the only conceivable ways of distinguishing between MOND and ΛCDM (if missing satellites, the lack of cusps in DM haloes and tidal dwarf galaxies is ignored) are in the complex modelling of galaxy formation, or the unambiguous detection of the hot or cold DM particles.

3 DISCUSSION

Certain analyses (Giunti & Laveder 2008) of neutrino-mixing experiments seem to require an additional, sterile neutrino with a mass in the range $4\,\mathrm{eV} < m_{\nu_s} < 18\,\mathrm{eV}$. Even if collider experiments detect a CDM candidate with mass of 300 GeV, this will give us virtually no information about its cosmological abundance and therefore brings us no closer to solve the dark matter problem. The great thing about sterile neutrinos is that if we can find the mass from laboratory experiments then this effectively fixes the cosmological abundance and the contribution the neutrinos can make to clusters of galaxies and the matter power spectrum at large can be strictly constrained. Many of the upcoming neutrino experiments cannot rule out the 11 eV sterile neutrino because of systematic errors on the electron neutrino fluxes, but the upcoming T2K experiment (Linder et al. 2008) has the potential to do this.

We have shown that a single massive sterile neutrino of 11 eV appears consistent with the current level of precision in the measurements of the CMB anisotropies. It is also able to clump together with densities surpassing the maximum density of the DM in groups and clusters of galaxies where MOND requires dark matter of some form. As discussed in Angus et al. (2008), there appears to be a scale at which MOND begins to poorly describe the dynamics of astrophysical systems. This is highlighted by Romanowsky et al. (2003), Milgrom & Sanders (2003), Angus et al. (2008) and O'Sullivan, Sanderson & Ponman (2007) and also the recent weak lensing study by Tian, Hoekstra & Zhao (2008) which shows that no dark matter is necessary to explain the detailed dynamics of relatively low mass groups of galaxies, ellipticals and systems smaller. This is expected for sterile neutrino dark matter because it would have a free streaming length significantly larger than any galaxy (∼50 kpc for the Milky Way). However, just as numerical simulations of clusters of CDM were necessary to show that the CDM haloes are a poor match to observed galaxies (de Blok & McGaugh 1998; McGaugh & de Blok 1998; Gnedin & Zhao 2002; Gentile et al. 2004; Gilmore et al. 2007), the equilibrium distribution of the sterile neutrino DM must be checked to be consistent with groups and clusters of galaxies (see Sanders 2007).

For agreement, the three active neutrinos should probably have masses well below 0.5 eV. Otherwise, it will become difficult to match the CMB power spectrum because the angular scale of the peaks prefers $\Omega_\nu h^2 = 0.117$. But, since $\Omega_\nu \propto m_\nu$, increasing the mass of another neutrino reduces the mass of the sterile neutrino and the amplitude of the third peak of the CMB diminishes due to the rapidly decreasing maximum density ($\rho_\nu^{\max} \propto m_\nu^4$). Herein lies the great advantage of CDM which places no limit on the phase-space density of the particles allowing unchallenged freedom to condense on any scale.

The work of Skordis et al. (2006) suggests that the matter power spectrum as measured by the SDSS may also be consistent with this model (they showed a similar model with three 2.2 eV active neutrino, and MOND-like gravity was in a reasonably good agreement). Since its inception, the most common (and to be honest well-justified) gripe about MOND is its lack of cosmological predictions. It could be argued that the cosmological model presented here with a basis of matching the CMB power spectrum gives the option of moving MOND from a position of having no concrete predictions for cosmology to having fixed predictions allowing, in principle, structure formation simulations in MOND with all the ingredients.

One serious flaw remains, however, that dark energy is still used as a fudge factor. Recall that there are several cosmological accelerations near a_o (discussed above), in addition to the unexpected coincidence in the order of magnitude of $\Omega_{\rm b}$, $\Omega_{\rm m}$ and Ω_Λ as well as the current lack of a physical basis for Λ. These facts hint towards a possible relation between MOND and dark energy. For instance, it seems logical that the extra gravity MOND 'takes' from the vacuum comes at a price, as the laws of thermodynamics dictate. While the field that binds galaxies and clusters together is attractive on small scales, it might be repulsive at large scales. This is reminiscent of the strong force which binds protons together in the nucleus of an atom, but is hapless to prevent the protons repelling each other outside the nucleus. It may be the case that this extra energy associated with the MOND acceleration is actually the cause of dark energy. Whatever the true story, any fundamental theory of MOND must link the dark energy and extra gravity afforded by weak accelerations in a manner analogous to the mixed dark matter models of Bertacca et al. (2008).

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