Supernova kicks and misaligned Be star binaries

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ABSTRACT

Be stars are rapidly spinning B stars surrounded by an outflowing disc of gas in Keplerian rotation. Be star/X-ray binary systems contain a Be star and a neutron star. They are found to have non-zero eccentricities and there is evidence that some systems have a misalignment between the spin axis of the star and the spin axis of the binary orbit. The eccentricities in these systems are caused by a kick to the neutron star during the supernova that formed it. Such kicks would also give rise to misalignments. In this paper, we investigate the extent to which the same kick distribution can give rise to both the observed eccentricity distribution and the observed misalignments. We find that a Maxwellian distribution of velocity kicks with a low velocity dispersion, $σ_k \approx 15 \text{ km s}^{-1}$, is consistent with the observed eccentricity distribution but is hard to reconcile with the observed misalignments, typically $i \geq 25^\circ$. Alternatively, a higher velocity kick distribution, $σ_k = 265 \text{ km s}^{-1}$, is consistent with the observed misalignments but not with the observed eccentricities, unless post-supernova circularization of the binary orbits has taken place. We discuss briefly how this might be achieved.

Key words: accretion, accretion discs – stars: emission lines, Be – stars: neutron – X-rays: binaries.

1 INTRODUCTION

Be stars were discovered by Secchi (1867) who observed emission lines in γ Cas. These stars are rapidly rotating at about 70 per cent of their breakup velocity (Porter 1996). In fact, relative to their breakup velocities, they are the fastest rotating bodies observed. They are early type main-sequence stars which have shown Hα emission at least once. They are variable in brightness and spectra which show Balmer emission with sharp absorption lines of ionized metals and broad He i absorption and emission at either visual or ultraviolet wavelengths. It is found that the emission and so presumably the discs are only temporary and so Be stars become B stars and vice versa.

If the disc is viewed edge-on, the Be star is seen as a shell star. The spectra then show Balmer emission with sharp absorption cores, narrow absorption lines of ionized metals and broad He i absorption. Be-star discs vanish and re-appear on time-scales of a few hundred days. Dachs, Kiehling & Engels (1988) studied Balmer emission line profiles and concluded that the envelopes surrounding Be stars are in Keplerian motion within the disc. The disc shows optical and infrared (IR) emission lines and an IR continuum excess.

The stars γ Cas and 59 Cyg have shown two successive shell events. These were associated with a remarkably synchronous quasi-cyclic variation of the emission linewidth in all observed emission lines that has been called spectacular variation (Hummel 1998). The change in emission linewidth removes the correlation between the projected surface velocity, $v \sin i$, and the full width half-maximum, and so a circumstellar equatorial disc fails to explain the spectacular variations. The emission lines and shell Be stars are explained by differences in disc inclination to the line of sight, so transitions between the two were not expected. Hummel (1998) explains the spectacular variations by a Keplerian disc which is somehow tilted with respect to the equatorial plane of the star. The variation in emission linewidths and profile shapes is then due to the precession of the disc. He suggests that the sequence of alternating shell phases and narrow single-peak phases is due to the variation in disc inclination caused by precession. The idea that a disc might change its inclination to the line of sight is borne out by observations of 28 Tau (Pleione) by Hirata (2007). This star also changes between B star, Be star and shell star and in this star the intrinsic polarization angle changes in phase with these variations. The cause of the precession is also not clear but Hummel (1998) suggested that it might be induced by tides from a binary companion. The two systems γ Cas and 59 Cyg are binary. The system 28 Tau appears to show radial velocity variations, although there is no confirmed orbital period (Rivinius, Štefl & Baade 2006). In both γ Cas and 28 Tau, the misalignment angle between the stellar equator and the disc/orbital plane is thought to be around 25° (Hummel 1998; Hirata 2007).

In further support of this possibility, we note that the B-star binary PSR J0045–7319 has a spin–orbit misalignment suggested by its orbital plane precession (Lai, Bildsten & Kapsi 1995; Kaspi et al. 1996). This misalignment in a B-star orbit means that
misalignment in Be stars is not uncommon. In this case, the B star rotates retrogradely with respect to the orbit (Lai 1996a).

The standard model for Be-star discs is that they are decretion discs with the mass expelled from the neighbourhood of the Be star itself (Cassinelli et al. 2002). In this case, we expect the plane of the inner disc to be aligned with the spin axis of the Be star. There are two reasons for the disc to be found at an angle discussed in the literature. First, Porter (1998) suggested that the disc precession might be caused by a radiation-induced instability (Pringle 1996). Secondly, as we reported above, it is widely suggested that the warp and precession are caused by a misalignment between the spin axis of the Be star and the orbit of the binary companion. If the disc is a decretion disc, we expect the inner edge to be aligned with the equatorial plane of the B star and the outer edge to tend to be tidally aligned with the orbital plane. Thus, there must be a warp at some radius in the disc. Here, we focus on this second possibility and concentrate on the Be/X-ray binaries in which the companion stars are neutron stars.

Neutron stars, observed as radio pulsars, have space velocities much greater than their progenitors (Gunn & Ostriker 1970). The accepted explanation for this is that supernova explosions are asymmetric and give very large kicks to the newly formed neutron stars (Shklovskii 1970; Sutantyo 1978). Indeed, some supernova remnants show evidence for asymmetric explosions (Aschenbach, Egger & Trumper 1995; Morse, Winkler & Kirshner 1995). Thus, any system that contains a neutron star could have had a supernova kick. It is also found that Be/X-ray binaries have high eccentricities that cannot be explained without supernova kicks (Verbunt & van der Heuvel 1995).

In a Be-star binary system, prior to the supernova in which the core of its companion collapses to a neutron star, we expect the Be-star spin to be aligned with the orbit and for the orbit to be circular. The kick from the supernova has two effects, it makes the orbit eccentric, and perhaps even unbinds it, and misaligns the orbit with the spin axis of the Be star. Thus, information about the distribution of eccentricities in Be-star systems can in principle give us information about the distribution of spin–orbit misalignments.

Lyne & Lorimer (1994) analysed the known pulsar velocities and concluded that they were born with a mean speed of about 450 km s\(^{-1}\). Hansen & Phinney (1997) considered the selection effects, as a result of the flux limits, of the pulsar surveys and the accuracy of the proper motion determinations and found a mean birth speed of around 250–300 km s\(^{-1}\). More recently, Arzoumanian, Chernoff & Cordes (2002) found a best-fitting distribution with two Maxwellian components, one for 40 per cent of the pulsars with velocity kicks mainly in close binaries. Kramer & Stairs (2008) give an extensive discussion of the second kick in the double pulsar J0737–3039 and conclude that it probably had to be small. Theoretical studies such as those by Scheck et al. (2006) and Kitaura, Janke & Hillebrandt (2006) do not yet throw much light on this but it may be that there are more lower mass supernovae, with smaller kicks, in binary systems or that the binary orbit quenches the hydrodynamic instabilities which lead to a very asymmetric explosion.

In a binary system, if the kick is too strong, the system does not remain bound. Even a small velocity kick can lead to a large eccentricity and inclination between the old and new orbits (Brandt & Podsiadlowski 1995). We do not know how the angular momentum of the remnant is also affected by the supernova kick, so do not know how much of the misalignment of the neutron star now is caused by the orbital inclination. We can, however, expect the companion to continue spinning aligned with the pre-supernova orbit immediately after the explosion. There are several binary systems with neutron star companions that are observed to be misaligned.

Brandt & Podsiadlowski (1995) investigated some of the effects of high supernova kick velocities on the orbital parameters of post-supernova neutron star binaries. Here, we look at a variety of velocity kick distributions and consider the implications for the distribution of the inclinations between the orbit before and after the kick. After ensuring that we can reproduce the work of Brandt & Podsiadlowski (1995), we model Be-star systems with our preferred distributions for their progenitors including a range of masses. There are somewhat more data now available for comparison in the period–eccentricity plane and we find that, though all systems can be formed with our models, they tend to be more circular than expected with kicks distributed according to Hobbs et al. (2005). We investigate what kick distribution could lead to the observed eccentricity distribution and also how the eccentricity distribution might have changed since the supernova. We consider three types of kick distribution, a single-peaked Maxwellian velocity kick distribution, kicks which are direction limited and a double–Maxwellian distribution.

2 MISALIGNMENT PROBABILITY DISTRIBUTION

In this section, we consider the effect of a velocity kick on the orbital inclination. We start with a binary in a circular orbit. One star then has an asymmetric supernova explosion which gives it a kick with velocity \(0 \leq v_k < \infty\) in a direction given by the angle \(\phi\) out of the binary plane \((-\pi/2 \leq \phi \leq \pi/2)\) and an angle between the direction opposite to the instantaneous velocity of the star and the projection of the velocity kick into the binary orbital plane of \(0 \leq \theta < 2\pi\) (see Fig. 1). For now, we assume that no mass is lost.

We are interested in the misalignment angle of the system, \(i\), after the supernova kick. This is the angle between the old and new angular momenta of the orbits (Fig. 2). If \(0 \leq i < \pi/2\) then the system is closer to alignment than counter alignment, and if \(\pi/2 < i \leq \pi\) it is closer to counter alignment. Brandt & Podsiadlowski (1995) find this angle to be given by

\[
\cos i = \frac{v_{\text{orb}} - v_k \cos \omega \cos \phi}{[v_k^2 \sin^2 \phi + (v_{\text{orb}} - v_k \cos \omega \cos \phi)^2]^{1/2}},
\]

where \(v_{\text{orb}}\) is the initial orbital velocity of the system. The relative velocity of the stars after the supernova is

\[
v_n^2 = v_k^2 + v_{\text{orb}}^2 - 2v_{\text{orb}}v_k \cos \omega \cos \phi.
\]
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The system before the supernova. The two stars of mass $M_1$ and $M_2$ are in a circular orbit about their centre of mass at the origin. The orbital angular momentum is in the $z$-direction and the orbit is in the $xy$-plane. When star 2 explodes as a supernova it is travelling in the direction of the negative $x$-axis with speed $v_{\text{orb}}$ relative to star 1. It receives a kick of velocity $v_k$ at an angle $\phi$ to the plane of the binary orbit. The angle between the projection of the velocity kick on to the binary plane and the $x$-direction is $\omega$.

We can rearrange equation (1) to find

$$\cos \omega = \frac{v_{\text{orb}}}{v_k} \frac{1}{\cos \phi} \pm \frac{\tan \phi}{\tan i}$$

(3)

when $v_k \neq 0$, $\tan i \neq 0$ (so that $i \neq 0$, $\pi$) and $\cos \phi \neq 0$ (so that $\phi \neq -\pi/2$, $\pi/2$). Then, if $0 \leq i < \pi/2$ from equation (1) we have $v_{\text{orb}} > v_k \cos \omega \cos \phi$. With equation (3), this corresponds to

$$\frac{\pm \tan \phi}{\tan i} v_k \cos \phi > 0$$

(4)

and because $\tan i > 0$ we see

$$\mp \sin \phi > 0$$

(5)

Similarly, when $\pi/2 < i \leq \pi$, so that $\tan i < 0$, we find the same condition as above on $\sin \phi$. Now, we can rewrite equation (3) as

$$\cos \omega = \frac{v_{\text{orb}}}{v_k} \frac{1}{\cos \phi} - \frac{|\tan \phi|}{\tan i}$$

(6)

We consider where this equation has real valued solutions in the $\phi - v_k$ plane. In Fig. 3, we plot the locus of $\cos \omega = 1$ ($\omega = 0$),

$$v_+ = \frac{v_{\text{orb}}}{\cos \phi} \left(1 + \frac{|\tan \phi|}{\tan i}\right)^{-1}$$

(7)

and the locus of $\cos \omega = -1$ ($\omega = \pi$),

$$v_- = \frac{v_{\text{orb}}}{\cos \phi} \left(-1 + \frac{|\tan \phi|}{\tan i}\right)^{-1}$$

(8)

for four values of $i$. The region between the $v_+$ and $v_-$ contours is the region where we have real values of $\cos \omega$ and it represents the combinations of kick parameters which can lead to a misalignment of the chosen $i$.

3 ISOTROPIC MAXWELLIAN KICK DISTRIBUTION

As an illustration, we apply these results to a simple isotropic Maxwellian kick distribution. For an isotropic kick distribution,

$$P(\omega) \, d\omega = \frac{1}{2\pi} \, d\omega$$

(9)

and

$$P(\phi) \, d\phi = \cos \phi \, d\phi$$

(10)

We here choose the kick speed to have a Maxwellian distribution so that

$$P(v_k) \, dv_k = \sqrt{\frac{2}{\pi \sigma_k^2}} \frac{1}{\sigma_k^2} e^{-\frac{v_k^2}{\sigma_k^2}} \, dv_k$$

(11)

where $\sigma_k$ is the dispersion of the velocity, and recall that Hobbs et al. (2005) find $\sigma_k = 265 \, \text{km s}^{-1}$. In Sections 5.2 and 5.3, we shall consider alternative velocity kick distributions. Because $i = i(v_k, \phi, \omega)$, its probability distribution is

$$P(i) \, di = \int_{v_k=0}^\infty \int_{\phi=-\pi/2}^{\pi/2} \int_{\omega=-\pi/2}^{\pi/2} P(\phi) P(\omega) P(v_k) \, d\omega \, d\phi \, dv_k$$

(12)

We change variables from $(v_k, \phi, \omega)$ to $(v_k, \phi, i)$ and find

$$P(i) \, di = \int_i^{i+di} \int_i^{i+di} P(\phi) P(\omega) P(v_k) J \, d\phi \, dv_k$$

(13)

where $R = R_1 + R_2$ is the region in the $(\phi, v_k)$ plane where $\cos \omega$ is real valued. This is illustrated in Fig. 3 for different values of $i$. Outside of the region bounded by these curves, the given velocity kick and angle $\phi$ cannot produce a system misaligned by $i$ because then $|\cos \omega| > 1$. We consider this region for bound systems in Section 3.1. The Jacobian, $J$, for the change of variables is given by

$$dv_k \, d\phi \, d\omega = |J| \, dv_k \, d\phi \, di$$

(14)
Figure 3. Possible combinations of the ratio of star 2’s supernova kick velocity, \( v_k \), to the relative orbital velocity, \( v_{\text{orb}} \), and angle \( \phi \) between the kick direction and the orbital plane for four different inclinations \( i \) between the pre- and post-supernova orbital planes, top-left panel \( i = 17.2^\circ \), top-right panel \( i = 57.3^\circ \), bottom-left panel \( i = 114.6^\circ \) and bottom-right panel \( i = 171.9^\circ \). For angles \( i > 90^\circ \), the post-supernova orbit counter rotates with respect to the spin of star 1. For each value of \( \phi \), there is a range of values of \( v_k \) which can give rise to the required misalignment \( i \). This depends on the angle \( \omega \) shown in Fig. 1. In each panel, the solid line corresponds to \( \omega = 0 \) (and thus to the kick velocity \( v_+ \); equation 7) and the dashed line to \( \omega = \pi \) (kick velocity \( v_- \); equation 8). Note that \( \omega = 0, \phi = 0 \) corresponds to a kick directly opposed to the motion of star 2 and \( \omega = \pi, \phi = 0 \) corresponds to a kick in the direction of motion of star 2. In the bottom two plots, which correspond to post-supernova retrograde motion, it is not possible to achieve this with a prograde kick \( (\omega = \pi) \) for any value of \( \phi \) so that the dashed lines corresponding to \( \omega = \pi \) are absent. The dotted lines show the maximum velocity kick, \( v_{\text{bound}} \), (equation 26) as a function of \( \phi \), for which the system remains bound. Below this line, the system remains bound after the supernova and above it the binary is disrupted. In order to find the probability distribution \( P(i) \) of the misalignment angle \( i \) in equation (18), we integrate in \( (v_k/v_{\text{orb}}, \phi) \) space over the regions \( R = R_1 + R_2 \) between the contours of \( v_+ \) (solid line), \( v_- \) (dashed line). To find only the probability distribution \( P(i) \) for the bound systems alone, we integrate over the regions \( R_2 \) only.

\[
J = \begin{pmatrix}
\frac{\partial v_k}{\partial \phi} & \frac{\partial v_k}{\partial \omega} & \frac{\partial v_k}{\partial v_{\text{orb}}} \\
\frac{\partial v_\phi}{\partial \phi} & \frac{\partial v_\phi}{\partial \omega} & \frac{\partial v_\phi}{\partial v_{\text{orb}}} \\
\frac{\partial v_i}{\partial \phi} & \frac{\partial v_i}{\partial \omega} & \frac{\partial v_i}{\partial v_{\text{orb}}}
\end{pmatrix}
\]  

Because \( \omega, \phi \) and \( v_k \) are independently distributed, we find

\[
J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (\frac{\partial \omega}{\partial t})_{v_k,\phi} \end{pmatrix} \begin{pmatrix} \frac{\partial \omega}{\partial i} \\ 0 \\ (\frac{\partial \omega}{\partial i})_{v_k,\phi} \end{pmatrix}.
\]

We differentiate equation (6) to find

\[
J = \begin{pmatrix}
\frac{\partial \omega}{\partial i} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & (\frac{\partial \omega}{\partial i})_{v_k,\phi}
\end{pmatrix}.
\]

Now, we can write down the full probability density distribution for the misalignment as an integral

\[
P(i) \, di = \sqrt{\frac{2}{\pi \sigma_k^2}} \int_0^{\pi} \int_{-\infty}^{\infty} P(i) \, dv_k \, dv_{\text{orb}} \, d\phi.
\]
and the kinetic energy is
\[ E_{\text{kin}} = \frac{1}{2} \left( \frac{M_1 M_2}{M} \right) v^2_{\text{orb}}, \] (21)
in the centre of mass frame, where \( M = M_1 + M_2 \). In a circular orbit, we have
\[ -E_{\text{kin}} = \frac{1}{2} E_{\text{grav}}, \] (22)
and the total energy
\[ E_{\text{tot}} = -\frac{1}{2} \frac{GM_1 M_2}{a}. \] (23)

So,
\[ E_{\text{grav}} = -v^2_{\text{orb}} \frac{M_1 M_2}{M}. \] (24)

After the kick, the gravitational energy remains the same because in this case no mass is lost. The kinetic energy becomes
\[ E_{\text{kin}} = \frac{1}{2} \left( \frac{M_1 M_2}{M} \right) v^2_{\text{b}} , \] (25)
where \( v_b \) is given by equation (2). If \( E_{\text{kin}} > -E_{\text{grav}} \), the new system is unbound. The condition for the system to be unbound is \( v^2_{\text{b}} > 2v^2_{\text{orb}} \). We solve \( v^2_{\text{b}} = 2v^2_{\text{orb}} \) with equation (2) for the critical velocity of \( v_b \).

Because \( v_b > 0 \), we take the term with the positive sign. If \( v_b > v_{\text{bound}} \), then the system is unbound but if \( v_b < v_{\text{bound}} \) it remains bound after the supernova kick.

This condition for the system to be bound affects the region in the \( v_b - \phi \) plane that we integrate over to find the probability distribution for the misalignment angle. To integrate over all systems, bound or unbound, we integrate equation (18) over the region \( R = R_1 + R_2 \) shown in Fig. 3. There we also plot the upper limits on the velocity kicks for the system to remain bound as dotted lines, \( v_{\text{bound}} \). Below these lines, a system remains bound but above the kick is too strong and the two stars fly apart. The region in which we have bound systems, \( R_2 \), is much smaller than the region that can produce the given misalignment angle, \( R_1 + R_2 \). To find the probability distribution of the misalignment angle, \( P(i) \), for bound systems only we integrate expression (18) over this smaller region \( R = R_2 \).

We compute this numerically and plot it in Fig. 4 as the upper dashed line when \( v_{\text{orb}} = \sigma_k \). The lower dashed line is for \( v_{\text{orb}} = 0.5\sigma_k \). Most kicks unbind the systems but those that remain bound are somewhat more likely to be counter aligned than for smaller kicks. On the other hand, if \( v_{\text{orb}} > \sigma_k \) we find that few kicks are able to cause counter alignment.

As expected, the higher the misalignment angle of a system the lower the probability of it forming. We see that, by restricting to only bound systems, the number with small misalignment is greatly reduced whereas those closer to counter alignment are less so. The probability of a system closer to counter alignment than alignment becomes relatively high for small \( v_{\text{orb}} \).

### 3.2 Mass loss

So far, we have assumed that the mass lost from the system is negligible. We now allow the mass of star 2 to fall in the supernova to
\[ M_2' = M_2 - f M. \] (27)
where \( f \) is the fraction of mass lost relative to the total mass of the binary system so that the total mass of the system becomes
\[
M' = (1 - f)M. \tag{28}
\]
The gravitational energy of the system after the supernova is
\[
E_{\text{grav}} = -v_{\text{orb}}^2 \frac{M_1 M_2}{M}, \tag{29}
\]
where \( v_{\text{orb}} \) is the relative velocity that the stars would have in a circular orbit of separation \( a \) and
\[
v_{\text{orb}}^2 = \frac{G M'}{a} = v_{\text{orb}}^2 \frac{M'}{M} = (1 - f) v_{\text{orb}}^2 \tag{30}
\]
because the instantaneous separation \( a' = a \). Note that \( v_{\text{orb}}' \) is the orbital velocity for the equivalent circular orbit while the orbit itself is eccentric after the supernova. The new kinetic energy is
\[
E_{\text{kin}} = \frac{1}{2} v_{\text{orb}}^2 \frac{M_1 M_2'}{M'}, \tag{31}
\]
and so the condition for a bound system \( (E_{\text{kin}} < -E_{\text{grav}}) \) becomes
\[
v_{\text{orb}}^2 < 2(1 - f) v_{\text{orb}}^2. \tag{32}
\]
Thus, we find
\[
v_{\text{bound}} = -v_{\text{orb}} \sin \phi \tan i + \sqrt{(3 - 2 f) v_{\text{orb}}^2 + v_{\text{orb}}^2 \sin^2 \phi \tan^2 i}. \tag{33}
\]
In Fig. 5, we plot contours of \( v_{\text{bound}} \) for varying \( f \) with \( i = 0.3 \) rad = 17.2° and \( v_{\text{orb}} = \sigma_{k} \). The \( v_{i} \) and \( v_{j} \) contours and the top dotted line remain the same as in the top-left plot in Fig. 3. The more mass that is lost in the supernova, the lower is the limit on the kick velocity for a bound system and so the less likely a bound orbit with a given inclination becomes.

In Fig. 4, for \( v_{i} = \sigma_{k} \), we plot the probability distribution for bound systems for \( f = 0 \) (upper dashed line), 0.2 (upper dotted line) and 0.5 (lower dotted line). A larger \( f \) increases the likelihood of counteralignment in bound systems.

\section{4 Eccentricity Probability Distribution}

In the previous section, we discussed the effect of particular supernova kick distributions on the distribution of orbital misalignments. We now consider what kick distributions are most able to give rise to the observed Be-star eccentricity distribution. Given the sparsity of the data, the large number of free parameters and the unknown selection effects, we do not attempt to find a best fit to the periods and eccentricities of Be stars. Rather we look for a kick distribution consistent with these observations and then examine its consequences for the distribution of orbital misalignments.

The new semimajor axis of the orbit after the supernova can be found from
\[
v_n^2 = GM' \left( \frac{2}{a} - \frac{1}{a_n} \right), \tag{34}\]
where \( a_n \), the old semimajor axis, is the instantaneous separation. Combining this with equation (2), we can find \( a_n \). The new system has specific angular momentum
\[
h' = r \times v_n. \tag{35}\]
where \( r \) is the separation vector of the stars. We have
\[
GM_2 a_n(1 - e^2) = |r \times v_n|^2 \tag{36}\]
and so
\[
GM_2 a_n(1 - e^2) = a^2 \left[ v_i^2 \sin^2 \phi + (v_k \cos \omega \cos \phi - v_{\text{orb}})^2 \right] \tag{37}\]
which can be solved to find the eccentricity, \( e \), of the new system (Brandt & Podsiadlowski 1995). The binary system is unbound if \( e > 1 \), in which case \( a_n \leq 0 \). Although we could find eccentricity probabilities by direct integration in a similar way to the inclinations in the previous section, it becomes very complicated and we do not learn much new from the procedure. Instead, we use Monte Carlo methods to evaluate the integrals.

We note that \( e \) and \( i \) both depend only on \( v_{i}/v_{\text{orb}}, 1 - f \) and the two angles \( \phi \) and \( \omega \). For typical progenitors of Be stars, the dependence on masses, through \( v_{\text{orb}} \) and \( 1 - f \) as well as the radius of star 2, turns out to be weak so it can suffice to concentrate on only one set of masses initially. We choose a pre-supernova mass of \( M_2 = 5 M_{\odot} \) that leaves a neutron star of mass \( M_2' = 1.4 M_{\odot} \) and a companion mass \( M_1 = 15 M_{\odot} \). These masses were used by Brandt & Podsiadlowski (1995), and we have ensured that we can reproduce their results too.

We use the 
\textit{NAG} Library routine \texttt{g05caf} to generate pseudo-random numbers \( \{X_i\} \) uniformly distributed between 0 and 1. Then, for an isotropic kick distribution
\[
\sin \phi = X_{j1} \tag{38}\]
and
\[
\omega = 2\pi X_{j2}. \tag{39}\]
The distribution of orbital periods, \( P_i \), immediately before the supernova depends in a complex way on the previous evolution of the system. There are many as yet unquantified processes that contribute to this evolution (Hurley, Tout & Pols 2002), and so we stick with the relatively simple assumption that \( \log P_i \) is uniformly distributed between \( P_{\text{min}} \) and \( P_{\text{max}} \) so that
\[
\log P_i = \log P_{\text{min}} + X_{j1}(\log P_{\text{max}} - \log P_{\text{min}}). \tag{40}\]
We take \( P_{\text{min}} \) to be the period at which star 1 would fill its Roche lobe in a circular orbit if it has the main-sequence radius given by
Tout et al. (1996, 5\textit{R}_\odot for a 15\textit{M}_\odot star). Its Roche lobe radius \textit{R}_L is approximated by the formula of Eggleton (1983),
\[
\frac{\textit{R}_L}{a} = g(q) = \frac{0.49q^{2/3}}{0.6q^{2/3} + \log_2(1 + q^{1/3})}, \quad 0 < q < \infty, \tag{41}
\]

where \textit{q} = \textit{M}_1/\textit{M}_2. We take \textit{P}_{max} = 10^5\text{d} because beyond this almost all systems are disrupted.

For the post-supernova systems, Brandt & Podsiadlowski (1995) rejected any system that would have filled its Roche lobe if it were circular at its periastron separation. So, if
\[
(1 - \epsilon)a_k < \frac{\textit{R}_L}{g(q)}, \tag{42}
\]

they rejected the system. In practice, we expect that systems cannot actually survive down to this separation because tides enforce pseudo-synchronization of star 1 at periastron (Hut 1991) and so it ends up spinning up to about 1.16 times faster than it would in a circular orbit of the periastron separation. However, there is no equivalent potential theory in the eccentric orbit so we do not try to be any more precise than condition (42).

We can reproduce figs 4 and 5 of Brandt & Podsiadlowski (1995). They chose \textit{v}_k to be constant and used a period distribution which is uniform in \textit{P}, rather than log \textit{P}. In their figs 4–6, for a given \textit{x}-axis value, they found the median on the \textit{y}-axis of 10,000 runs and then worked out the regions in which 20, 40, 60, 80 and 98 per cent of systems lie away from that median. They found very high values of the mean inclination because they used a high single-value kick velocity.

Instead we integrate over \textit{v}_k distributed according to equation (11) up to 850 km s\textsuperscript{−1} using Simpson’s rule, and \textit{\omega}, \textit{\phi} and \textit{P}_i by the Monte Carlo method. In our figures, we prefer to plot contours of probability density in the two-dimensional space normalized so that the probability of lying in the plots is 1.

Rather than sticking to the fixed masses, we distribute the masses of the companion star from \textit{M}_{1min} = 5\textit{M}_\odot to \textit{M}_{1max} = 25\textit{M}_\odot according to a function
\[
N(\textit{M}_1) d\textit{M}_1 \propto \textit{M}_1^{−2.7} d\textit{M}_1, \tag{43}
\]

(Kroupa, Tout & Gilmore 1993) which can be generated from
\[
\textit{M}_1 = \left(\frac{\textit{X}_0 - \textit{X}_k}{\textit{k}}\right)^{−1/1.7}, \tag{44}
\]

where we find \textit{X}_0 and \textit{k} from the minimum and maximum masses. Because the mass range is limited, the resulting distributions are not very different from the fixed initial masses of \textit{M}_1 = 15\textit{M}_\odot and \textit{M}_2 = 5\textit{M}_\odot as used by Brandt & Podsiadlowski (1995) to represent a typical Be star binary. Before presenting our results, we discuss the observations with which we compare.

5 MODEL COMPARISON TO OBSERVED SYSTEMS

There is now a large number of Be star binary systems with measured periods and eccentricities. The data to which we shall apply our models are given in Table 1. The bulk of these comes from the catalogue of Be/X-ray binaries assembled by Raguzova & Popov (2005).\textsuperscript{1} We also include O/B stars that we have found in the literature that have no emission but must have formed in the same way. We can use these to look at the eccentricity distribution and remember that the disc in Be stars may come and go so that B stars can become Be stars and vice versa.

Because we are primarily interested in Be stars with discs, we also impose the condition that there must be enough room for a decetration disc of 4 \textit{R}_\odot or so inside the Roche lobe of star 1. We explain our choice in the appendix but note that there is one Be star in Table 1, 0535–668, that we do not include in our analysis because it cannot accommodate such a disc.

There are three systems for which the misalignment angle \textit{i} between the Be star spin and disc have been estimated. There is one more system, 28 Tau, which has no reliable orbital parameters but for which an inclination of \textit{i} = 25°–30° has been suggested.

5.1 Single-Maxwellian peak velocity kick distribution

We now consider how well the standard velocity kick distribution of Hobbs et al. (2005) fits the observed data. In the left-hand frame of Fig. 6, we plot the probability distribution contours of eccentricity against final period for these systems and in the right-hand panel we plot contours in the inclination–eccentricity plane. The probability of a highly inclined system is quite small and most systems end up closer to alignment than to counter alignment. Larger inclinations are more likely for the more eccentric systems.

To illustrate this further, the solid lines in Fig. 7 show the probabilities of \textit{e} and \textit{i} integrated over all systems. We also plot a histogram of the observed Be star systems’ eccentricities. It is evident, in line with previous findings on less substantial data sets, that the Hobbs et al. (2005) distribution gives a poor fit to the eccentricity distribution. It does, however, give a fairly flat distribution of misalignments. There are too few measured misalignments to use these as a test but the measured values of \textit{e} consistent with the standard Hobbs et al. (2005) kick distribution.

Because the final period distribution depends very strongly on our choice of initial period, we have not made use of it for a statistical comparison with observations except to say that the systems appear to fit well in Fig. 6 (and later in Fig. 10). This implies that our choice of period distribution is reasonable.

We use the Kolmogorov–Smirnov (KS) test to determine if two data sets are significantly different. It is non-parametric and distribution free. In Fig. 8, we plot the cumulative eccentricity distributions of the Be stars, of the B stars and of the B and Be stars combined. We also plot cumulative eccentricity distribution for our model predictions. We then perform a KS test between the observed data and our model prediction and give the results in Table 2. We find the largest deviation of the observed data from the model and use probability tables for the KS test to find the probability that the observed sample of stars came from the distribution predicted by the model. In Table 2, we give the probability that the observations of Be stars, B stars, B\textsc{\textsuperscript{2}}, and combined B and Be stars, \textit{P}_3, are consistent with our various models. Probabilities smaller than 10\textsuperscript{−4} are listed as zero. The very small probabilities for the standard Hobbs et al. (2005) distribution (the first line in Table 2) demonstrates that it is essentially impossible that the B and Be systems have formed as they are in this way, with a single-Maxwellian distribution with \textit{\sigma}_k = 265 km s\textsuperscript{−1}.

We cannot, however, immediately rule out a kick distribution in the form of a single-Maxwellian distribution with \textit{\sigma}_k = 265 km s\textsuperscript{−1}, because the current eccentricity distribution might not be representative of the eccentricity distribution immediately after the supernova. One possibility is that the systems have begun to circularize by some mechanism but, given that they are not completely circular, the time-scale on which this circularization operates is

\textsuperscript{1}http://xray.sai.msu.ru/~raguzova/BeXcat

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In Fig. 7, we also plot the dotted lines to show the eccentricity and inclination distributions for a single-Maxwellian peak with dispersion $\sigma_k = 15 \text{ km s}^{-1}$. We see that this curve appears to fit the eccentricity distribution much better than that with $\sigma_k = 265 \text{ km s}^{-1}$. However, for such low-velocity kicks, the misalignments tend to be small (right-hand panel of Fig. 7) and are hard to reconcile with the observed values.

### 5.2 On axis kicks

It is possible that the direction of the velocity kick in the supernova is restricted (Brandt & Podsiadlowski 1995). To investigate this, we consider the extreme case that the kick is always directed along the...
Figure 6. Left-hand panel: the stars listed in Table 1 in a period–eccentricity diagram. The solid squares are the Be/X-ray binaries and the open triangles are the binary B stars. The contours are lines of constant probability density \( P(P_f, e) \) when the dispersion of the velocity kick distribution in \( \sigma_k = 265 \, \text{km s}^{-1} \) and post-supernova binaries which are too tight to permit a disc of size \( 4 R_\star \) are excluded (Section 4). The probability density \( P \) is defined so that the probability of finding a system with \( e \) in the interval \((e, e + de)\) and with period \( \log P_f \) in the interval \((\log P_f, \log P_f + d\log P_f)\) is \( P \, de \, d\log P_f \). The area at small \( P_f \) is excluded by the models. The outermost contour is at \( P = 0.01 \). Moving inwards, the contour levels are \( P = 0.1, 0.2, 0.4, 0.6, 0.8 \) and 1. Right-hand panel: contours of equal probability density \( P(e, i) \) for the same models as in the left-hand panel. The contour levels are, starting from the left, \( P = 0.01, 0.1, 0.2, 0.4 \) and 0.6. The asterisks correspond to the systems given in Table 1 for which there are estimates of the misalignment angle \( i \).

Figure 7. Left-hand panel: the two-dimensional probability density \( P(P_f, e) \) contours, of which are shown in Fig. 6 (left-hand panel), integrated over period to give a one-dimensional probability density \( P(e) \). This is plotted as the solid curve (normalized so that \( \int_0^1 P(e) \, de = 1 \)) and for a kick distribution with \( \sigma_k = 265 \, \text{km s}^{-1} \). The dotted line gives the probability density \( P(e) \) for models constructed with \( \sigma_k = 15 \, \text{km s}^{-1} \). The histogram is the observed eccentricity distribution for the Be stars (the solid squares in Fig. 6). The lower value of \( \sigma_k \) gives a much better fit to the data. Right-hand panel: as for the left-hand panel but the probability distributions for misalignment angles \( i \) predicted by the models are plotted. The few measured misalignment angles which are known are not plotted but all exceed \( i \approx 25^\circ \). It is evident that the solid line (\( \sigma_k = 265 \, \text{km s}^{-1} \)) is consistent with a fairly uniform spread of misalignments while the models with \( \sigma_k = 15 \, \text{km s}^{-1} \) (dotted line), which provide a better fit to the eccentricity distribution, are not consistent with the observed misalignments. For this model, only 7 per cent have \( i > 15^\circ \) and only 1 per cent have \( i > 25^\circ \).

Spin axis of the star, perpendicular to the orbital plane, so that \( \phi = \pi/2 \) and \( \omega \) is undetermined. With equation (28) and \( v_{\text{orb}}^2 = GM/a \), we can express equation (34) as

\[
v_n^2 = v_{\text{orb}}^2(1 - f) \left( 2 - \frac{a}{a_1} \right)
\]

and then equation (37) becomes

\[
1 - e^2 = \frac{1}{1 - f} \left( 2 - \frac{v_n^2}{v_{\text{orb}}^2} \frac{1}{1 - f} \right) \times \left[ \frac{v_n^2}{v_{\text{orb}}^2} \sin^2 \phi + \left( \frac{v_n}{v_{\text{orb}}} \cos \omega \cos \phi - 1 \right)^2 \right].
\]
If the kick lies on the $z$-axis, we have $\sin \phi = 1$ and so the misalignment angle between the old and new orbital planes is

$$\cos i = \frac{1}{\sqrt{\frac{1}{\sigma_1^2} + 1}}.$$  \hfill (47)$$

We can then relate the eccentricity and the inclination by

$$1 - e^2 = \frac{1}{1 - f} \left( 2 - \frac{\sec^2 i}{1 - f} \right) \sec^2 i$$ \hfill (49)$$

and we plot this in the right-hand panel of Fig. 9. For $M_2 = 5 M_\odot$ and $M_1 = 1.4 M_\odot$, we have

$$1 - f = \frac{M'}{M} = \frac{1.4 + M_1/M_\odot}{5 + M_1/M_\odot}.$$ \hfill (50)$$

For illustration, we choose $M_1 = 5, 15$ and $25 M_\odot$ and so $f = 0.36, 0.18$ and $0.12$.

We plot the eccentricity distributions if the kick is parallel to the binary orbital axis in the left-hand panel of Fig. 9. We see that we cannot get highly misaligned systems or low-eccentricity systems with a kick in the $z$-direction. The low eccentricities could be explained by circularization, but the highly misaligned systems are ruled out in this case.

### 5.3 Bimodal distribution of velocities

A bimodal velocity kick distribution has been suggested to explain both the high-velocity neutron stars and also the fact that neutron stars appear to be easily contained in globular clusters (Katz 1975). The escape velocity of a neutron star from a globular cluster is about $30 \text{ km s}^{-1}$, and it is generally believed that about 10 per cent of neutron stars born within them are retained (Drukier 1996).

Arzoumanian et al. (2002) used observed properties of radio pulsars and other neutron stars to show that a two-component velocity kick distribution fits the data much better than any one component model. They used a distribution of velocities with two Maxwellian distributions

$$P(v) dv \propto \frac{v^2 - v_k^2}{\sigma_1^2} e^{-v^2/2\sigma_1^2} + (1 - w_1) \frac{v^2 - v_{k2}^2}{\sigma_2^2} e^{-v^2/2\sigma_2^2} dv,$$ \hfill (51)$$

where $\sigma_1 = 90 \text{ km s}^{-1}$, $\sigma_{k2} = 500 \text{ km s}^{-1}$ and $w_1 = 0.4 \pm 0.2$ is the weight of the first distribution. We perform the same KS test on this bimodal distribution and find that it does not fit our eccentricity distribution well at all (see Table 2). We note that almost all systems that fall into the higher peak are disrupted so that the poor fit is entirely due to the high $\sigma_{k1}$.

We found previously that the best-fitting distribution with one peak had $\sigma_k = 15 \text{ km s}^{-1}$. For a bimodal distribution with $\sigma_{k1} = 15 \text{ km s}^{-1}$ and $\sigma_{k2} = 265 \text{ km s}^{-1}$ with equal weight, we find that the fit is somewhat poorer than for the single peak at $\sigma_k = 15 \text{ km s}^{-1}$. However, it fits the data significantly better than a distribution with just $\sigma_k = 265 \text{ km s}^{-1}$.

The weight factor $w_1$ is unimportant if $\sigma_{k2}$ is large enough to disrupt most systems. For $\sigma_{k1} = 15 \text{ km s}^{-1}$ and $\sigma_{k2} = 500 \text{ km s}^{-1}$ with $w_1 = 0.4$, the fit is as good as the single peak at $\sigma_k = 15 \text{ km s}^{-1}$ (see Table 2). In Fig. 10, we plot the eccentricity–final-period and inclination–eccentricity contours for this velocity kick distribution. Most of the survival systems are from the $\sigma_{k1} = 15 \text{ km s}^{-1}$ part and so the inclinations are all low. Nearly all systems with kicks from $\sigma_{k2} = 500 \text{ km s}^{-1}$ are disrupted.

This bimodal distribution fits the distribution of eccentricities we find here just because the low-kick systems dominate those that remain bound. The effect of the larger kicks is minor simply because...
most systems with large kicks are disrupted but this would explain the high pulsar space velocities. Until we understand the nature of the supernovae explosions properly in three-dimensional, bimodal distributions of this kind will remain good but ad hoc solutions.

6 DISCUSSION

We have modelled the eccentricity distribution of observed B and Be stars and confirm the findings of others that the eccentricities tend to be lower than predicted by a standard Hobbs et al. (2005) kick distribution. The final periods depend on the uncertain choice of inner boundary and initial period distribution. So, to test the models, we compared only the probability distribution of the eccentricities (left-hand panel of Fig. 7). To quantify this, we performed a KS test (see Table 2) to see how well the B and Be stars fit our models. To obtain a good fit from a Maxwellian kick velocity distribution, we require \( \sigma_k = 15 \text{ km s}^{-1} \), much lower than the \( \sigma_k = 265 \text{ km s}^{-1} \) of Hobbs et al. (2005). This can be combined with a second Maxwellian as long as that has \( \sigma_k \) large enough to disrupt most systems. A combined low- and high-velocity kick in the bimodal distribution can...
then reproduce the high-space-velocity pulsars and the eccentricity distribution of the bound systems.

In Fig. 7, we show that the few estimates of misalignment angles $i$ between Be star spin axis and disc axis are too large to be easily reconciled with kicks with $\sigma_k$ as low as 15 km s$^{-1}$. They are much better accommodated by $\sigma_k = 265$ km s$^{-1}$ (Fig. 6). This is even more true for the B-star binary PSR J0045–7139 in which the B star spins retrogradely with respect to the orbit. However, although the observed misalignments are indicative of high kick velocities, we cannot yet base any firm conclusions on the inclinations because (i) there are only a few, rather uncertain, measurements, (ii) it is easier to measure a large inclination than a small one and (iii) misalignment angles of less than the disc opening angle, estimated to be around 13$^\circ$ (Hauschke 1996), would not be sufficient to easily give rise to change between Be star and shell star.

If on the other hand we believe that $\sigma_k$ must be larger, as in the distribution of Hobbs et al. (2005, $\sigma_k = 265$ km s$^{-1}$) and even the bimodal distribution of Arzoumanian et al. (2002, $\sigma_{k1} = 90$ km s$^{-1}$, $\sigma_{k2} = 500$ km s$^{-1}$), then we need an alternative explanation of the lower than expected eccentricities. We first consider whether this could be due to observational selection effects. We are less likely to observe systems with high eccentricity if we only see them when they are close to periastron. Such systems spend the majority of their orbital periods closer to apastron and so are less likely to have been observed. In the case of the Be stars, this may simply be due to the fact that the companion is much more likely to interact with the decretion disc at periastron. The same would not be true of the plain B stars which appear to follow a similar eccentricity distribution. However, there are fewer of them and they still may have been selected by radial velocity variations which would be larger at periastron.

If we assume that we have found all of the Be star systems in our lowest eccentricity bin of $0 < e < 0.105$ (see histogram in Fig. 7) then the relative number needs to be brought down by a factor of about 3.4. This would require the actual number of Be star systems in our Galaxy to be 3.4 times greater than what we have found, and the extra ones must all have high eccentricities. We have found measured eccentricities and periods for 20 Be star systems in our galaxy and so would need $3.4 \times 20 = 68$ systems to account for selection effects. There are 67 observed Be star systems in our galaxy in the Be/X-ray binary catalogue (Raguzova & Popov 2005) of which 52 do not have both a measured eccentricity and period. It is unlikely that the B stars without observed eccentricities will all turn out to have high eccentricity. However, it is important for observers to measure the eccentricities of more Be star systems because this is vital to rule out selection effects.

Alternatively, and perhaps more interestingly, it is possible that Be stars did form with an eccentricity distribution favouring large eccentricities of $e > 0.5$ or so. In this case, the systems must have subsequently circularized and the circularization time-scale must be similar to the lifetime of the Be phase. The periods of Be stars can be large and the tidal circularization time, for these stars with radiative envelopes, is much longer than their lifetimes. One possibility is that the neutron star interacts with the decretion disc of the B star at periastron passage. Such a mechanism might be self regulating in the sense that the decretion disc only has time to build up to a large radius when the system is very eccentric and the neutron star spends a long time far from the B star. So, this merits further investigation particularly if it can be established that supernovae kicks cannot have such a low dispersion of $\sigma_k = 15$ km s$^{-1}$.

7 CONCLUSIONS

The distribution of eccentricities in Be and B stars cannot be reproduced directly if supernova kicks have a Maxwellian distribution with $\sigma_k > 30$ km s$^{-1}$ or so. Our best fit requires $\sigma_k \approx 15$ km s$^{-1}$ though this may be combined to a bimodal distribution with a second Maxwellian with $\overline{\sigma_{k2}} \approx 500$ km s$^{-1}$, sufficient to disrupt most systems but to account for the high-space-velocity pulsars. We have also considered the distributions of the misalignment between Be star spin and orbit that would result from such a kick distribution. It is evident that larger kicks result in larger misalignments. We note that the data indicate that the low-velocity kicks required to give the current eccentricity distribution might not be consistent with the observed misalignments.

If such a low-velocity kick distribution is ruled out then either selection effects must severely limit the observed distribution to such an extent that we are only seeing about two in seven of the high-eccentricity systems or the systems must circularize on a time-scale similar to their lifetimes. Given that this circularization is not biased towards low periods, tides in the stars are not sufficient to be its cause in the wide systems. We postulate that a dissipative interaction between the neutron star and the decretion disc is a more likely mechanism. We have suggested that circularization of Be stars might actually be brought about by an interaction between the neutron star companion and the Be star’s decretion disc. This disc is free to grow in size while the neutron star is far away. It spends most of its time at apastron. Then by periastron the disc may have grown sufficiently that the neutron star passes through it and is slowed down. Such an interaction can dissipate energy and thus circularize the orbit. It cannot, however, change the angular momentum of the star nor can alter the inclination of the orbit because both these processes require transfer of angular momentum as well as dissipation of energy. The moment of inertia of the disc is smaller than that of the B star and much less than that of the orbit. The star–disc interaction can therefore easily align the outer parts of the disc with the orbit but any significant change in the angular momentum of the B star or of the orbit can only occur on the disc’s viscous time-scale. Meanwhile, orbital energy can still be dissipated. Thus, the misalignment should still contain information on the kick at formation even if the eccentricity no longer does.

Inclinations between the pre- and post-supernova orbits are affected by various factors. Higher kicks lead to fewer bound systems but a larger probability of counter alignment. An increase in the mass lost does the same. Systems with lower pre-supernova periods are less affected and so tend to be more aligned and more able to survive. For the standard Hobbs et al. (2005) kick distribution, Be stars ought to be about three times more likely to end up with aligned rather than counter-aligned orbits, but counter-aligned orbits should not be uncommon. On the other hand, if $\sigma_k = 15$ km s$^{-1}$ all orbits should be not far from alignment. In order to distinguish between these various possibilities, more information is required about the distribution of misalignment angles between the Be star spin axis and the orientation of the disc.

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APPENDIX A

Be stars have a decretion disc around them and theory suggests that Be stars lose their disc to become B stars. If the disc reappears, the star becomes a Be star again. If we take this into consideration, there must also be room for the decretion disc within the Roche lobe of

Figure A1. The solid squares are the Be stars and the triangles are the B stars without emission from Table 1. The contours show $\epsilon = 1 - a_{in}/a_{out}$ against $P_1$ with $C = 1, 2, 4$ (solid line), 6, 10 and 20 from left to right. The initial mass of the exploding star is $M_1 = 25 M_\odot$. 

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the B star so we insist that

\[(1 - e)a_n > C \frac{R_1}{g(q)},\]  

where \(C\) is the size of the disc in units of the stellar radius, to include a system in our analysis. From interferometry, Grundstrom & Gies (2006) find the size of the disc in \(\gamma\) Cas to be \(R_d = 8.1 \pm 1.1\) R*, where \(R_*\) is the radius of the star so \(C = 8.1\) and Grundstrom et al. (2007b) find the disc of X Per to be about six times larger than the stellar radius so \(C = 6\).

In Fig. A1, we plot the locus defined by equality in equation (A1) against \(P_f\) for \(R_1/g(q) = 6.9\) R⊙ for \(C = 1, 2, 4\) (solid line), 6, 10 and 20 in the case where the initial mass is the maximum of \(M_2 = 25\) M⊙. For each \(C\), the area to the left-hand side of the contour is invalid. Only one Be star system requires \(C\) as low as 1.6.

This system, 0535−668 is in the Large Magellanic Cloud which has low metallicity and hence stars have smaller radii. The remainder of Be star systems can be accommodated if \(C = 4\). The two B stars which lie to the left-hand side of the \(C = 4\) line do not have discs because they show no emission and so we do not include them in our comparisons.

One effect of increasing \(C\) from 0 to 4 is to reduce the number of high-eccentricity systems at low period. Our initial period distribution biases to low period so this in effect reduces \(P(e)\) for large \(e\) in what follows. Thus, if we were not to include this our conclusions in Section 5.1 would be even stronger.