First-year Sloan Digital Sky Survey-II supernova results: consistency and constraints with other intermediate-redshift data sets


1Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth PO1 3FX
2Center for Particle Astrophysics, Fermi National Accelerator Laboratory, PO Box 500, Batavia, IL 60510, USA
3Argelander-Institut für Astronomie, Universität Bonn, Auf dem Hügel 71, D-53121 Bonn, Germany
4Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7701, South Africa
5South African Astronomical Observatory, PO Box 9, Observatory 7935, South Africa
6School of Mathematics and Physics, University of Queensland, Brisbane QLD 4072, Australia
7Dark Cosmology Centre, Niels Bohr Institute, University of Copenhagen, DK-2100, Copenhagen, Denmark
8Department of Physics and Astronomy, Rutgers the State University of New Jersey, 136 Frelinghuysen Road, Piscataway, NJ 08854, USA
9Kavli Institute for Cosmological Physics, The University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637, USA
10Physics Department, University of Notre Dame, Notre Dame, IN 46556, USA
11Department of Physics and Astronomy, University of Pennsylvania, 209 South 33rd Street, Philadelphia, PA 19096, USA
12Department of Astronomy, AlbaNova, Stockholm University, SE-106 91 Stockholm, Sweden
13Department of Astronomy, University of Washington, Box 351580, Seattle, WA 98195, USA
14Department of Physics and Astronomy, Wayne State University, Detroit, MI 48202, USA
15Department of Astronomy, University of California, Berkeley, CA 94720-3411, USA
16Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA
17Department of Astronomy, MSC 4500, New Mexico State University, PO Box 30001, Las Cruces, NM 88003, USA
18Institute for Cosmic Ray Research, The University of Tokyo, 5-1-5 Kashiwa, Kashiwa City, Chiba 277-8582, Japan
19Department of Physics, Graduate School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan
20Physics Department, Rochester Institute of Technology, Rochester, NY 14623, USA
21Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218, USA
22Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218, USA
23Department of Astronomy and Astrophysics, 525 Davey Laboratory, Pennsylvania State University, University Park, PA 16802, USA
24Las Campanas Observatory, Carnegie Observatories, Casilla 601, La Serena, Chile
25Astronomy Department, University of Cape Town, Private Bag X3, Rondebosch 7701, South Africa
26Department of Astronomy, McDonald Observatory, University of Texas, Austin, TX 78712, USA
27Kavli Institute for Particle Astrophysics and Cosmology, Stanford University, CA 94305-4060, USA

Accepted 2009 October 9. Received 2009 October 2; in original form 2009 February 10

ABSTRACT

We present an analysis of the luminosity distances of Type Ia Supernovae (SNe) from the Sloan Digital Sky Survey-II (SDSS-II) SN Survey in conjunction with other intermediate-redshift (z < 0.4) cosmological measurements including redshift-space distortions from the Two-Degree Field Galaxy Redshift Survey (2dFGRS), the integrated Sachs–Wolfe (ISW) effect seen by the SDSS and the latest baryon acoustic oscillation (BAO) distance scale from both the SDSS and 2dFGRS. We have analysed the SDSS-II SN data alone using a variety of ‘model-independent’ methods and find evidence for an accelerating Universe at a >97 per cent level from this single data set. We find good agreement between the SN and

⋆E-mail: Hubert.Lampeitl@port.ac.uk
†Clay Fellow.
BAO distance measurements, both consistent with a \( \Lambda \)-dominated cold dark matter cosmology, as demonstrated through an analysis of the distance duality relationship between the luminosity (\( d_L \)) and angular diameter (\( d_A \)) distance measures. We then use these data to estimate \( w \) within this restricted redshift range (\( z < 0.4 \)). Our most stringent result comes from the combination of all our intermediate-redshift data (SDSS-II SNe, BAO, ISW and redshift-space distortions), giving \( w = -0.81^{+0.16}_{-0.18} \) (stat) \( \pm 0.15 \) (sys) and \( \Omega_M = 0.22^{+0.09}_{-0.08} \) assuming a flat universe. This value of \( w \) and associated errors only change slightly if curvature is allowed to vary, consistent with constraints from the cosmic microwave background. We also consider more limited combinations of the geometrical (SN, BAO) and dynamical (ISW, redshift-space distortions) probes.

**Key words:** supernovae: general – cosmology: observations – distance scale – cosmological parameters – large-scale structure of Universe.

1 INTRODUCTION

It is now widely believed that the late-time expansion of the Universe is accelerating. General Relativity (GR) implies that the acceleration is driven by ‘dark energy’ (DE) – an unknown energy component in the Universe with a negative effective pressure. Describing DE by an equation-of-state parameter of \( w = p/\rho \) requires that \( w < -1/3 \). Alternatively, accelerated expansion could be an indication that GR is not the correct theory of gravity or that we have applied GR incorrectly in a cosmological context (see recent reviews of DE by Peebles & Ratra 2003; Copeland, Sami & Tsujikawa 2006; Uzan 2007; Frieman, Turner & Huterer 2008b).

Over the last decade, the most direct way of studying this acceleration of the expansion of the Universe, and therefore DE, has been using Type Ia Supernovae (SNe Ia), as they have been shown by many authors to be well-calibrated ‘standard candles’ in the Universe, i.e. their relative distances can be determined from the dependence of their peak luminosity on the shape of the light-curve. This method was used to great effect by astronomers in 1998 to provide the first evidence for an accelerated universe (Riess et al. 1998; Perlmutter et al. 1999; see Filippenko 2005 for a review).

Briefly, an SN Ia occurs when a white dwarf star in a close binary system accretes enough mass from its companion to undergo a thermonuclear explosion in the core. Both Phillips (1993) and Hamuy et al. (1993) have shown that such explosions can serve as consistent light sources in the Universe to high accuracy. This is achieved by transforming the measured light-curve of the explosion into the rest frame of the SN (so-called K-corrections) and then correcting the luminosity at maximum as a function of the shape of the rest-frame light-curve.

Several techniques now exist for fitting SN light-curves known under different acronyms (\( \Delta m_{15} \), Hamuy et al. 1996; MLCS, Riess, Press & Kirshner 1996; stretch, Perlmutter et al. 1997; CMAGIC, Wang et al. 2003; BATM, Tonry et al. 2003; SALT, Guy et al. 2005;\( \Delta C_{12} \), Wang et al. 2006; SALT2, Guy et al. 2007; SIFTO, Conley et al. 2008). In this analysis, we consider distance modules obtained by the multicolour light-curve shape method in its updated version (MLCS2k2, Jha, Riess & Kirshner 2007; Riess, Press & Kirshner 1996), which is among the most commonly used and best tested.

Recently, several dedicated SN surveys have been carried out to confirm and extend the earlier detections of an accelerating universe [Hubble Space Telescope (HST), Riess et al. 2004, 2007; Supernova Legacy Survey (SNLS), Astier et al. 2006; Equation of State: Supernovae Trace Cosmic Expansion (ESSENCE), Wood-Vasey et al. 2007] as well as new compilations of existing SN data sets (Davis et al. 2007; Kowalski et al. 2008; Hicken et al. 2009). In addition to SNe, observations of baryon acoustic oscillations (BAO) can be used to measure distances in the Universe (Blake & Glazebrook 2003; Hui & Haiman 2003; Seo & Eisenstein 2003). The BAO are caused by sound waves in the early Universe which leave a preferred scale in the distribution of matter equal to the sound horizon at recombination. Today, this scale corresponds to \( \sim 100 h^{-1} \) Mpc (Hubble constant at present: \( H_0 = 100 h \) km s\(^{-1}\) Mpc\(^{-1}\)) and can thus be used as a ‘standard ruler’ throughout the Universe. The BAO signature has been detected in the clustering of galaxy clusters by Miller, Nichol & Batuski (2001), in the correlation of galaxies in the Sloan Digital Sky Survey (SDSS; York et al. 2000) by Eisenstein et al. (2001), Hütsi (2006), Padmanabhan et al. (2007) and Blake et al. (2007), and in the Two-Degree Field Galaxy Redshift Survey (2dFGRS; Colless et al. 2001) by Cole et al. (2005).

In addition to the geometrical methods discussed above, observations of the dynamical properties of matter can provide constraints on the matter density of the Universe and \( w \) (assuming that GR is the appropriate theory of gravity). For example, the growth rate of structure in the Universe can be observed via the coherent infall of galaxies into large clusters and superclusters of galaxies seen in redshift surveys (Kaiser 1987). Also, the growth of structure can be measured using the late-time integrated Sachs–Wolfe (ISW) effect (Sachs & Wolfe 1967), which has now been detected to high significance through the cross-correlation of galaxy surveys with the cosmic microwave background (CMB; see Giannantonio et al. 2008 and references therein). The ISW is sensitive to deviations from a matter-dominated, Einstein–de Sitter universe (\( \Omega_M = 1 \), where \( \Omega_M \) is the matter density at present divided by the critical density).

Taken together, the present combination of cosmological measurements suggests that we live in a flat universe, dominated by a cosmological constant (\( \Lambda \)) with the energy density in matter and \( \Lambda \) known to a statistical accuracy of better than a few per cent (see Dunkley et al. 2009). However, several of these cosmological measurements, especially SNe, are becoming limited by their systematic uncertainties which are now dominating; for example, Hicken et al. (2009) showed that the best combination of available SNe and BAO measurements provides \( 1 + w = 0.013^{+0.065}_{-0.068} \) but with a systematic uncertainty of 0.11. Therefore, it is clear that future cosmological surveys must resolve these systematic errors through new observations and better analysis methods to mitigate their effect.

This paper is one of three complementary papers focused on the cosmological analysis of a new sample of intermediate SN distances recently obtained by the SDSS-II SN Survey (see Section 2 for

© 2009 The Authors. Journal compilation © 2009 RAS, MNRAS 401, 2331–2342
Our analysis differs from those presented in our companion papers of Kessler et al. (2009) and Sollerman et al. (2009), as we first study the cosmological information obtained solely from the SDSS-II SN sample and then in combination with other cosmological probes over the same redshift range (z < 0.4). Alternatively, Kessler et al. (2009) presents a detailed examination of the impact of both statistical and systematic errors on deriving standard cosmological constraints based on the combination of the SDSS-II SN with most of the currently available high- and low-redshift SNe and which are all analysed in a consistent way. Sollerman et al. (2009) then use the same compilation of data to study an expanded set of exotic cosmological models, in combination with a wider variety of other cosmological information. Our approach is also complementary to the many other analyses in the literature (e.g. Davis et al. 2007; Kowalski et al. 2008; Hicken et al. 2009) that have used data from all possible sources.

In our approach, we concentrate on the information from cosmological measurements that cover the same range of redshifts as the SDSS SN sample. Our aim is not to derive the most stringent limit on cosmological parameters available, but rather to verify that if we restrict ourselves to probes coming from a small and similar redshift slice the results on cosmological parameters remain stable and consistent. This approach is warranted because of the growing emphasis on controlling systematic uncertainties in the analysis of cosmological data. There are clearly a number of systematic uncertainties that could affect the use of SNe as cosmological probes which likely depend on, or change with, redshift, including SN evolution (e.g. changes in the metallicity of progenitor stars; Timmes, Brown & Truran 2003; Howell et al. 2009; Sullivan et al. 2009), intergalactic dust (Conley et al. 2007; Holwerda 2008), Malmquist bias and the effects of gravitational lensing and peculiar velocities (PVs; Hui & Greene 2006). Moreover, the photometric uncertainties associated with combining SN data from multiple surveys, over a range of redshifts, are already seen as the main limitation in using presently available data sets (see Hicken et al. 2009). Our analysis addresses this issue by focusing exclusively on the SDSS SN sample, which is derived from a well-understood and stable photometric system. The SDSS has a relative photometric accuracy of better than 2 per cent in griz and 4 per cent for the u band (Padmanabhan et al. 2007), while the absolute calibration is also known to be of the order of 1 per cent, leading to a homogeneous set of SN light-curves with high photometric accuracy (see Holtzman et al. 2008). This set of data is robust to uncertainties in light-curve fitting. For example, the MLCS2k2 and SALT2 fits to the SDSS-only sample are shown to agree well (see section 10 in Kessler et al. 2009) which is not the case when the higher redshift SN samples are added.

The outline of this paper is as follows. In Section 2, we describe the SDSS-II SN data and use these data in Section 3 to study the cosmic acceleration in the Universe. Section 4 then compares the SDSS-II SN luminosity distances to the BAO distances from the SDSS and 2dFGRS, checking the distance duality relation. We then derive in Section 5 constraints on w by combining the best-fitting luminosity distances for SDSS-II SNe with the growth rate of structure measurements taken from Hawkins et al. (2003) and a new measurement of the ISW effect taken from Giannantonio et al. (2008). We conclude in Section 6.

2 THE FIRST-YEAR SDSS SN DATA

The SDSS-II SN Survey (Frieman et al. 2008a) was part of the SDSS-II project and was focused on constructing a large sample of intermediate-redshift SNe (0.045 < z < 0.42). One of the strengths of the SDSS-II SN Survey is that it builds upon the existing (and stable) infrastructure from the original SDSS (Fukugita et al. 1996; Gunn et al. 1998; York et al. 2000; Eisenstein et al. 2001; Hogg et al. 2001; Lupton et al. 2001; Strauss et al. 2002; Pier et al. 2003; Gunn et al. 2006). The SDSS-II SN Survey is based on repeat imaging of ‘Stripe 82’, a region of the original SDSS with significantly deeper photometry than the regular SDSS. This region is ∼120 deg long and 2.5 wide, centred along the celestial equator and extending from 20 deg to 45 deg in right ascension (passing through 0 deg).

The SDSS-II SN Survey was carried out in three observing campaigns from September through December in each of 2005, 2006 and 2007 (there were also some observations for a short period in 2004). The new imaging data were initially reduced using the standard SDSS pipelines (Smith et al. 2002; Stoughton et al. 2002; Ivezić et al. 2004; Tucker et al. 2006; Adelman-McCarthy et al. 2008; Padmanabhan et al. 2008), followed by specific image-subtraction software to identify transient objects (Sako et al. 2008). To determine the nature of these transients, the data were both visually inspected and fitted with SN models. Subsequently, objects with a high probability of being an SN Ia (Sako et al. 2008) were spectroscopically observed using a variety of telescopes around the world (Frieman et al. 2008a; Zheng et al. 2008).

In this paper, we only consider the 2005 observing campaign (the first year of operation) as the data from other years are still being collated and analysed. That year, the SDSS-II SN Survey discovered 130 spectroscopically confirmed SNe Ia and additional 16 spectroscopically probable SNe Ia. In Kessler et al. (2009), distance moduli are obtained for 103 of the spectroscopically confirmed SNe Ia that pass stringent light-curve quality cuts, using the MLCS2k2 light-curve fitting routine. In the upper panel of Fig. 1, we show for these 103 SNe the residuals of the distance modules with respect to equation (9). The two data points represent the BAO measurements from Percival et al. (2009).
(2009) for an extensive discussion of systematic effects caused by changing the various light-curve fitting parameters.

We also direct the reader to section 11 of Kessler et al. (2009) for a comparison of the MLCS2k2 and SALT2 (Guy et al. 2007) light-curve fitters. They show that for the SDSS-only data the systematic difference between these two light-curve fitting methods is only 0.04 in $w$, while fig. 42 of their paper shows that the two methods give consistent distance moduli for the same SDSS-II SNe. We also highlight that the two methods give similar contours in figs 26a and 35a of Kessler et al. (2009) when comparing the full cosmological fits to the SDSS-only data. This motivates the analysis in this paper and means that our results are unaffected by the choice of the light-curve fitter used. We restrict the analysis to the MLCS2k2 reduction taken from table 10 in Kessler et al. (2009), as these data include corrections for selection effects.

When using these SDSS-II SNe for cosmological fitting, we calculate the confidence intervals via the $\chi^2$ statistic:

$$\chi^2 = \sum_{i=1}^{N} \left( \frac{\langle \mu_{\text{LC}}(z_i) - \mu_{\text{model}}(z_i,x) \rangle}{\sigma_{\text{LC}}(z_i) + \sigma_{\mu} + \sigma_{\text{int}}(z_i,x)} \right)^2,$$

where $\mu_{\text{LC}}$ and $\sigma_{\text{LC}}$ are the distance moduli and errors, respectively, derived from the light-curve fitting method (see Kessler et al. 2009), and $\mu_{\text{model}}$ are the expected distance moduli according to parameters $x$ of the assumed cosmological model. Uncertainties in the measured spectroscopic redshift and PV of the SN are taken into account using

$$\sigma_{\mu} = \frac{5}{\ln(10)} \frac{(1+z)}{z(1+z/2)} \frac{\sqrt{(\Delta z)^2 + (\Delta v_p)^2/c^2}},$$

where $\Delta z$ is the measurement uncertainty in redshift and $\Delta v_p$ is the characteristic amplitude of the PVs, which we take to be 360 km s$^{-1}$ (see e.g. Masters et al. 2006). We investigated the effect of correlated PVs of SDSS-II SNe and found no detectable effect (see Appendix B). We therefore do not include PV correlations in further analyses. Besides PVs, we ignore further potential correlations between individual SNe and treat them according to equation (1) as statistically independent.

Following standard practice, we add an intrinsic dispersion in distance modulus, $\sigma_{\text{int}}$, in the denominator of equation (1) and determine it by setting $\chi^2/N_{\text{dof}} = 1$ for the best-fitting cosmological model. This term accounts for the fact that the errors on the distance modulus reported by the light-curve fitter could underestimate the real error if we assume that a smooth cosmology is the correct underlying model. For the best-fitting model with constant $w$, we find $\sigma_{\text{int}} = 0.088$ mag for our SDSS-only SN sample, while in Kessler et al. (2009) a value of $\sigma_{\text{int}} = 0.16$ derived from the nearby SN samples is used in combination with the SDSS data. This results in broader contours compared to the one shown in this paper (see their fig. 26a and appendix E for a possible explanation), but only marginally changes the most likely values of $w$ and $\Omega_M$. Similarly if we omit any intrinsic dispersion, we get narrower contours but only slight changes in $w$ and $\Omega_M$ well within the errors given in Table 1.

A further complication arises due to the uncertain calibration of the absolute magnitude (at the peak) of SNe Ia, leading to a degeneracy with the absolute value of $H_0$. To account for this, we marginalized over $H_0$. This procedure makes use of the relative distances reported by the light-curve fitter but not of their absolute value. In a recent paper, Riess et al. (2009) redetermined the Hubble constant $H_0$ or equivalently the absolute brightness of SN Ia. One could use this value obtained from measurements in the nearby Universe as a prior on $H_0$ in our analysis. But – as laid out in Section 1 – we want to limit our analysis to probes taken at comparable redshifts as the SDSS SNe and therefore refrain from using this additional information.

## 3 TESTING COSMIC ACCELERATION

Given the homogeneity of the SDSS-II SN data set, we begin our analysis by revisiting the original evidence for cosmic acceleration in the expansion of the Universe. In detail, if we simply assume that the Universe is homogeneous and isotropic, and is described by a Robertson–Walker metric with a scalefactor of $a(t)$, then the purely kinematic deceleration parameter, $q$, is defined by

$$q(z) \equiv -\frac{\ddot{a}}{a^2},$$

where $q < 0$ corresponds to acceleration (we also assume that light follows null geodesics and is therefore redshifted in the usual way). We relate $q$ to the Hubble parameter by

$$H(z) \equiv -\frac{\dot{a}}{a} = H_0 \exp \left( \int_0^z \frac{1 + q(z')}{1 + z'} \mathrm{d}z' \right).$$

Luminosity distance can then be calculated directly from the expansion history via

$$d_L(z) = c(1+z) \int_0^z \frac{\mathrm{d}z'}{H(z')}.$$

where we have assumed a flat universe. Thus, the magnitudes and redshifts of any SNe can be used to constrain $q(z)$ without choosing a particular DE model or even a particular theory of gravity. The assumption of flatness is necessary in practice since constraints on $q(z)$ degrade significantly when curvature is allowed to vary. A prior on curvature from CMB measurements would, of course, strengthen the constraint, but such a prior is based on the validity of GR, counter to the intention of the $q(z)$ analysis.

The simplest deceleration model we can fit is a constant, $q(z) = q_0$. In this case, the luminosity distance simplifies to

$$d_L(z) = c(1+z) \frac{H_0}{H_0 - q_0} \left[ 1 - (1 + z)^{-q_0} \right].$$

Fitting equation (6) to the SDSS-only SN data, we find a best fit of $q_0 = -0.34$ and $h = 0.636$. Marginalizing the joint probability

<table>
<thead>
<tr>
<th>Data set</th>
<th>Geo.</th>
<th>$w$</th>
<th>$\Omega_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN+BAO</td>
<td>Flat</td>
<td>$-0.74^{+0.17}_{-0.32}$</td>
<td>$0.37^{+0.16}_{-0.64}$</td>
</tr>
<tr>
<td>SN+RS</td>
<td>Flat</td>
<td>$-0.77^{+0.19}_{-0.25}$</td>
<td>$0.26^{+0.15}_{-0.12}$</td>
</tr>
<tr>
<td>SN+ISW</td>
<td>Flat</td>
<td>$-0.74^{+0.16}_{-0.22}$</td>
<td>$0.23^{+0.15}_{-0.12}$</td>
</tr>
<tr>
<td>SN+RS+ISW</td>
<td>Flat</td>
<td>$-0.76^{+0.17}_{-0.19}$</td>
<td>$0.23^{+0.10}_{-0.08}$</td>
</tr>
<tr>
<td>SN+RS+ISW+BAO</td>
<td>Flat</td>
<td>$-0.81^{+0.16}_{-0.18}$</td>
<td>$0.22^{+0.09}_{-0.08}$</td>
</tr>
<tr>
<td>SN+BAO</td>
<td>Curved</td>
<td>$-0.99^{+0.30}_{-0.59}$</td>
<td>$0.50^{+0.22}_{-0.13}$</td>
</tr>
<tr>
<td>SN+RS</td>
<td>Curved</td>
<td>$-0.82^{+0.26}_{-0.31}$</td>
<td>$0.31^{+0.15}_{-0.12}$</td>
</tr>
<tr>
<td>SN+ISW</td>
<td>Curved</td>
<td>$-0.78^{+0.16}_{-0.22}$</td>
<td>$0.27^{+0.13}_{-0.15}$</td>
</tr>
<tr>
<td>SN+RS+ISW</td>
<td>Curved</td>
<td>$-0.79^{+0.16}_{-0.20}$</td>
<td>$0.27^{+0.10}_{-0.09}$</td>
</tr>
<tr>
<td>SN+RS+ISW+BAO</td>
<td>Curved</td>
<td>$-0.85^{+0.17}_{-0.19}$</td>
<td>$0.27^{+0.10}_{-0.09}$</td>
</tr>
</tbody>
</table>
density function (PDF) over $h$, we find $q_0 = -0.34 \pm 0.18$, or $q_0 < 0$ with 97 per cent probability, i.e. the SDSS alone finds evidence for acceleration at 2$\sigma$ without concerns regarding the absolute calibration of the peak brightness of SN Ia and the relative calibration between SN surveys.

There is no reason to expect $q(z)$ to be constant; indeed, under $\Lambda$ cold dark matter (Λ CDM), $q$ evolves from 1/2 during matter domination to $-1$ when vacuum energy dominates. However, we find that including additional parameters in $q(z)$ (e.g. by Taylor expanding $q$) degrades our constraints to the point of being uninteresting.

3.1 Principal component analysis

When trying to reconstruct an unknown function from noisy data, there is the concern that particular features of the reconstruction are not indicative of the true underlying function, but an artefact of the chosen parametrization. This concern is magnified for a function like $q(z)$, which is related to the data $\mu(z)$ by two derivatives. Therefore, we need a more robust way to determine if the Universe has accelerated – a way that does not depend on the naïve assumption of a constant $q(z)$. A principal-component analysis (Huterer & Starkman 2003) can be used to address this issue in a parameter-independent way. Principal components are a unique set of orthogonal basis functions $(e_i(z))$, such that

$$q(z) = \sum_i \alpha_i e_i(z),$$

which allows us to specify $q(z)$ using the coefficients, $\alpha_i$. The principal components, or ‘modes’ $e_i(z)$, are explicitly constructed so that the coefficients can be measured independently of each other, i.e. they have uncorrelated error bars. To construct these modes, we start with a piecewise-constant parametrization of $q(z)$ in bins of $dz = 0.01$, and we use our data to calculate a Fisher matrix for this parameter set and the Hubble parameter $H_0$. After marginalizing over $H_0$, the modes $e_i(z)$ are given by the eigenvectors of the resulting matrix. We are free to normalize these functions so that

$$\int e_i(z) e_j(z) \, dz = \delta_{ij},$$

which now specifies each function up to an overall sign convention. This procedure is completely general; in the limit of $dz \to 0$, we can specify any continuous function using these modes. The procedure is parameter-independent in the sense that we do not specify the modes a priori: they are determined primarily by the data. We do choose $q(z) = 0$ as the fiducial model for our Fisher matrix calculation, but the resulting modes are insensitive to this choice.

The six modes constrained best by the SDSS-only SN data are shown in Fig. 2. Each mode is accompanied by an estimate of the error bar, $\sigma(\alpha_i)$, for its coefficient $\alpha_i$. The errors are uncorrelated, and we have ordered the modes according to the size of their error bars. Only the first six modes are shown.

![Figure 2](https://example.com/figure2.png)

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.082</td>
<td>0.256</td>
<td>0.517</td>
<td>0.912</td>
<td>1.46</td>
<td>1.98</td>
</tr>
</tbody>
</table>

The significance of our detection could be enhanced by combining the SDSS-II SNe with other SN data sets. This has already been done in part by Shapiro & Turner (2006) but could suffer from systematic uncertainties associated with combining data from different instruments and surveys. Again, the reader is referred to Kessler et al. (2009) for a detailed description of such combinations of the SDSS-II SN Survey with other SN data sets. We plan to repeat this analysis with the full 3-year SDSS-II SN data set.

4 COMPARISON OF DISTANCES

We next consider the comparison of our SDSS-II SN distances with other geometrical distance estimates over the same redshift range. This is motivated by the original results of Percival et al. (2007) who noted some tension ($\gg 2\sigma$) between the cosmological constraints derived from nearby BAO measurements ($z = 0.2$ and $z = 0.35$) and higher redshift SNe of Astier et al. (2006). The BAO provide a measure of distances in the Universe by relating the scale of the sound horizon at last scattering ($r_s$) to the scale of the corresponding correlations seen in the galaxy distribution. One such measurement of this ratio is given by the $A$-parameter in Eisenstein et al. (2005). This parameter is frequently used in combination with SN data to derive constraints on $w$ (Astier et al. 2006; Kowalski et al. 2008); for example, see Kessler et al. (2009) for the combination of the SDSS-II SN data with the $A$-parameter.

Here, we adopt the quantity

$$D_V(z) = \left[ \frac{D_M c z}{H(z)} \right]^{1/2},$$

defined in Eisenstein et al. (2005) where $D_M$ is the co-moving distance. Percival et al. (2007) showed by combining measurements of $r_s/D_V$ at redshifts of 0.2 and 0.35 from both the 2dFGRS and SDSS galaxy samples, that one can obtain the ratio of the distance between two different redshifts that is independent of both $r_s$ and $H_0$. This approach also avoids a large extrapolation between the redshift of recombination ($z_{CMB} = 1090$, Komatsu et al. 2009) and these intermediate-redshift measurements.
For the analysis presented in this paper, we have adopted the latest value of $D_V(z = 0.35)/D_V(z = 0.2) = 1.736 \pm 0.065$ taken from Percival et al. (2009). The inferred value of $D_V(z = 0.35)/D_V(0.2)$ from Percival et al. (2009) is lower than that of Percival et al. (2007), bringing it into better agreement with ΛCDM. This change was caused by a revised error analysis and a change in the methodology adopted, as well as the addition of more data. In this paper we do not use constraints on $r_s(z_d)/D_V(z)$, which depend on the sound horizon at the baryon drag epoch $r_s(z_d)$. We therefore avoid including CMB data, commonly used to model the sound horizon.

### 4.1 The sliding window method

In Fig. 1, we show the Hubble diagram for the SDSS-II SN data discussed in Section 2 compared with a variety of cosmological and non-parametric models discussed herein. We find a scatter of 0.14 mag around these models independent of the particular fitting method. The first non-parametric model we consider is a ‘sliding window’ method which allows us to investigate the general shape and smoothness of the Hubble diagram without assuming a cosmological model. We have thus fitted piece-wise Hubble parameters and luminosity distances in different redshift bins using a local redshift window following the approach of Daly & Djorgovski (2003). At each redshift $z$, we fit the SNe comoving distances, $D_V(z) = d_L/(1 + z)$, over a fitting window of $z_i - z < z_i < z_i + \Delta z$, by a polynomial of second order given by

$$f(z_i - z) = A_0 + A_1(z_i - z) + A_2(z_i - z)^2.$$  

(10)

The values of $A_i$ are determined separately via $\chi^2$ minimization in each redshift window, and we slide this window as a function of redshift in increments of 0.01 throughout the entire range. The best-fitting $D_V$ at $z$ is proportional to $A_0$, while the best-fitting $c/H$ is proportional to $A_1$, and $D_V$ is related to $[z A_0^2 A_1]^{1/3}$. Our results depend on the size of the redshift window, with a wider window allowing less flexibility but smaller errors, and vice versa.

We show in Fig. 1 the resulting non-parametric fit to the SDSS-II SN data as an example for a window size of $\Delta z = 0.15$ which demonstrates that the SDSS SNe data are fully consistent with the individual BAO measurements at $z = 0.2$ and 0.35 of Percival et al. (2009) (the cyan and green shaded regions indicate the 1σ and 2σ errors, which are highly correlated as they share the same data points in the overlapping fitting windows). Next, we derive the ratio $D_V(0.35)/D_V(0.2)$ and determine the covariances between the $A_i$ values within, and between, redshift bins using the observational errors. This is shown in Fig. 3 for different window sizes. It is interesting to note that the sliding window method tends to prefer large values (steeper slopes) for $D_V$ compared to $D_V$ calculated from a $q_0$ fit described in Section 3, or $D_V$ from the best-fitting $w-\Omega_M$ parametrizations given in Table 1. We also see that the sliding window method provides values for $D_V$ which are fully consistent with the BAO result. We also show in Fig. 3 the best-fitting ratios of $D_V$ when the SDSS-II SN data are simultaneously fitted with the BAO data. The SN constraints dominate these results because of their smaller uncertainty.

1 We actually allow for a tapered region at both ends of this redshift window where the weights (i.e. the inverse-squared errors) are reduced by a Gaussian function with a standard deviation of $\sigma_z = 0.02$. This leads to suppression of fluctuations due to the inclusion of individual data points into the window (see Daly & Djorgovski 2003).

Figure 3. Upper panel: measurements of the ratio of $D_V(0.35)/D_V(0.2)$ using different fits to the SN data, in comparison to or in combination with the BAO (solid circle) measurement. The points labelled $q_0$ indicate $D_V$ derived using the fit in Section 3. Points labelled $-\Omega_M$ are derived from the $w-\Omega_M$ parametrization marginalized over $0 < \Omega_M < 1$ and $-2 < w < 0$. Points shown as triangles include the BAO measurement as a prior whereas boxes are without the BAO. The points to the right show the values derived from the ‘sliding window’ method as a function of the window width. The grey points are for values of the sliding window that are comparable to the best-fitting approach the global fit. The dashed line indicates ΛCDM with $\Omega_M = 0.3$ and $\Lambda \Omega = 0.7$. Lower panel: given the above-mentioned parametrizations, we show the best-fitting value of $q_0$ defined in equation (13).

### 4.2 Testing the distance duality relation

In the following, we use the famous reciprocity relation (Etherington 1933; Ellis 1971) or distance duality to compare the SN and BAO distance scales. In detail, the angular diameter distance and luminosity distance are related by

$$\frac{d_L}{d_A} = (1 + z)^2$$  

(11)

(for a discussion, see e.g. Bassett & Kunz 2004). This relation relies on photon conservation, but holds for any geometry and any metric theory of gravity where photons follow null geodesics. Therefore, it is a general test of our underlying assumptions about the nature of our Universe.

One might have expected that the distance duality relation has already been tightly constrained by observations of the blackbody CMB spectrum from the Cosmic Background Explorer (COBE) Far Infrared Absolute Spectrometer (FIRAS) experiment (Mather et al. 1994). However, this observation does not constrain deviations from distance duality as the photon number may not be conserved (either through production or loss of photons) or, more radically, photons may not follow null geodesics. Also, a grey dust component that absorbed photons independent of frequency would not cause spectral distortions away from a blackbody in the CMB since all frequencies would be affected equally. However, this grey dust would cause strong deviations from distance duality since it would make the luminosity distance to any objects larger while leaving the angular diameter distance unchanged. Another way to hide the distance duality effects from CMB observations would be to affect the photon number only at much higher or lower frequencies than the
microwave. This is, for example, what was needed to make the axion–photon mixing proposal for the dimming of the SNe Ia consistent with CMB constraints (Csáki, Kaloper & Terning 2002).

Beyond the CMB, several other analyses, using similar data to that discussed herein, have reported evidence for violations of distance duality at the $\sim 2\sigma$ level (Bassett & Kunz 2004; Lazkoz, Nesseris & Perivolaropoulos 2008). We revisit this issue here using a methodology similar to that outlined by More, Bovy & Hogg (2009) and Avgoustidis, Verde & Jimenez (2009) that does not rely on the absolute calibration of the distances to compute the ratio $d_{l}(z)/(d_{A}(z)(1 + z)^2)$. Instead, we check the relative behaviour of this ratio as a function of redshift by testing the consistency of the ratio at two redshifts, $z = 0.2$ and $z = 0.35$, where we have now updated BAO measurements from Percival et al. (2009).

In the following, we parameterize the distance duality relation in what we call the $\alpha$ model as

$$\Delta l = (1 + z)^{2+2\alpha} d_{A} = (1 + z)^{(1+\alpha)} D_{M},$$

where $\alpha = 0$ represents the expected distance duality relation, and therefore $\alpha \neq 0$ indicates a possible violation. To quantify the discrepancy between the two measures, we replace $D_{M}$ in equation (9) with $d_{l}$ from equation (12), and derive the relation

$$\left(1 + z_{2}\right)^{2+2\alpha} \frac{d_{l}(z_{1})}{D_{l}(z_{1})} = \frac{d_{l}(z_{2})}{D_{l}(z_{2})} H(z_{1})$$

where $\alpha$ is the minimum ratio at one redshift given the ratio at the other redshift and is therefore not sensitive to any scaling proportional to $(1 + z)^2$ but would be sensitive to any other loss function.

For the second method, we use the sliding window technique to derive $D_{V, SN} \equiv [z A_{r}^{2} A_{l}^{3}]^{2}$ (see equation 10) and the corresponding covariance matrix at the two redshifts ($z = 0.2$ and $z = 0.35$) where we have BAO measurements. The best fit and error of $\alpha$ are calculated by applying the Bayes theorem. In detail, we model $D_{V, BAO} = \beta (1 + z)^{2/3} D_{V, SN}$ at the two redshifts based on equation (13) where $\beta$ is a free scale parameter absorbing $H_0$ and the scale of the sound horizon $r_s$ at recombination. We then calculate the likelihood of the BAO $D_{V}$ measurements for $\alpha$ and $\beta$, integrating over all possible SN $D_{V, SN}$ at $z = 0.2$ and $z = 0.35$ given the Gaussian prior $p(D_{V} | D_{V, SN})$ constructed from the results of the sliding window technique:

$$L(D_{V, BAO} | \beta, \alpha) = \int \mathcal{D}D_{V} \times \left( L(D_{V, BAO} | \beta(1 + z)^{2/3} D_{V}) p(D_{V} | D_{V, SN}) \right).$$

The covariance matrix used in $L(D_{V, BAO} | \beta(1 + z)^{2/3} D_{V})$ is taken from Percival et al. (2009). Applying this procedure and subsequently marginalizing over $\beta$, we calculate the best fit $\alpha$ and its error. The results are shown in the lower panel of Fig. 3, along with the result from the $\theta_0$ method.

For the $\theta_0$ parametrization, we find $\alpha = -0.55 \pm 0.45$, while for the sliding window scheme, we find that all results agree with $\alpha = 0$ within $1\sigma$, i.e. the errors on $\alpha$ are $\sim 0.5$ for most of the window sizes shown in Fig. 3. These results re-enforce our findings that the SN and BAO distance scales are now in good agreement over this redshift range (confirming the new findings of Percival et al. 2009).

5 CONSTRAINTING COSMOLOGICAL PARAMETERS

In contrast to the previous sections, which focused on kinematic constraints of the cosmic expansion, here we investigate the constraints on standard cosmological parameters using the SDSS SN data only in combination with dynamical measurements, e.g. from the BAO, redshift-space distortions and ISW. This will have less statistical power than using the combination of SN data sets presented in Kessler et al. (2009) and Sollerman et al. (2009), but our analysis is complementary to these companion papers and maximizes the impact of the homogeneity of the SDSS data (for both the SN and BAO).

In Fig. 4, we begin by showing our constraints on $w$ and $\Omega_M$ using only the SDSS SN data. This is achieved using the $\chi^2$ statistic according to equation (1) over a three-dimensional grid of 200 bins in $w$, 100 bins in $\Omega_M$ and 160 bins in $H_0$ ranging from 40 to 80 km s$^{-1}$ Mpc$^{-1}$. Subsequently, we convert the $\chi^2$ into a likelihood using $L = \exp(-\frac{1}{2}(\chi^2 - \chi_{min}^2))$, where $\chi_{min}^2$ is the minimum $\chi^2$ found in the parameter space and, in our case, is by definition close to the number of SNe in the data set (as we have added $\sigma_{int}$). We then marginalize over $H_0$ by summing the likelihoods over the $H_0$ bins. The shaded blue contours show the resulting confidence levels (68, 95 and 99 per cent) in the $w$–$\Omega_M$ plane under the assumption of a flat universe ($\Omega_{\Lambda} = 1 - \Omega_M$). These contours include statistical errors only; systematic errors are discussed in Section 5.2.

Fig. 5 is similar, but here – instead of assuming a flat universe – we allow for curvature according to the CMB shift parameter $R$. We do this by calculating for a given $(w, \Omega_M)$ combination the corresponding value of $\Omega_{\Lambda}$ according to the constraints from $R$ (see Appendix A for more details on the treatment of curvature). As discussed in Section 1, we see little effect on our results from allowing curvature to vary because of the relative small deviations from flatness allowed by the Wilkinson Microwave Anisotropy Probe (WMAP) data. For results, see Table 1.

For comparison with the blue SN contours, we also provide in Figs 4 and 5 the (red) contours for the BAO measurements from Percival et al. (2009). The BAO measurements are in reasonable agreement with the SDSS-only SN contours but still prefer $w < -1$ as originally discussed in Percival et al. (2007), although all
measurements are consistent with a cosmological constant. As the BAO and SN distances have reasonable overlap, we provide in Table 1 various constraints on \( w \) and \( \Omega_M \) derived from combinations of the SN data with intermediate-redshift dynamical measurements described in the following sections and the BAO. We note that the SN data dominate the width and position in the \( w \)-direction of the likelihood contours, as they have smaller errors compared to the BAO constraints, while adding the BAO data helps to curtail the large values of \( \Omega_M \) seen for the SDSS-only results.

5.1 Constraints from dynamical measurements

To improve the cosmological constraints, we can include a number of other low-redshift cosmological measurements thus providing a first measurement of the cosmological model within the local Universe. In particular, we consider constraints on \( w \) derived from measurements of the growth of structure in the Universe, including redshift-space distortions from the 2dFGRS and the ISW effect from the SDSS imaging survey. Both of these methods are particularly sensitive to \( \Omega_M \) and thus provide independent and orthogonal constraints to the SN data.

5.1.1 Redshift-space distortions

Under the assumption that galaxies are related to the large-scale dark-matter distribution, the anisotropy of the redshift-space correlation function depends on the parameter

\[
\beta(z) = f_s(z)/b(z),
\]

where \( b(z) \) is the linear bias relating the galaxies to the underlying dark matter and \( f_s(z) \) the growth rate of structure. Absolute deviations between the real-space and redshift-space correlation functions depend on the parameter combination \( f_s(z)\sigma_s(z) \), where \( \sigma_s(z) \) is defined as the root-mean square (rms) mass fluctuation in spheres of a radius of \( 8 \) \( h^{-1} \) Mpc, and provides a convenient way of normalizing the matter fluctuations (for a recent review, see Percival & White 2009).

To remove either the dependence on galaxy bias from a measurement of \( \beta(z) \) or equivalently the dependence on \( \sigma_s(z) \) from the measurement of \( f_s(z)\sigma_s(z) \), we need further cosmological information. In this paper, we adopt the central value of \( \beta(z = 0.15) = 0.49\pm0.09 \) calculated by Hawkins et al. (2003) from the 2dFGRS, which is an update of the measurement given by Peacock et al. (2001). We follow the procedure outlined by Guzzo et al. (2008), and convert from \( \beta \) to \( f_s \) by adding an additional uncertainty of 0.12 in quadrature to account for the uncertainty in galaxy bias, which is estimated to be close to unity. This error includes the cosmological dependence of the bias measurement. For further calculations, we thus use \( f_s(z = 0.15) = 0.49\pm0.15 \), assuming the weighted median redshift of \( z \approx 0.15 \) of the 2dFGRS.

We adopt the parametrization given by Linder (2005):

\[
g(a) = \frac{\delta}{a} = \exp \int_0^a dt a [\Omega_M(a)^{\gamma} - 1],
\]

which is related to the growth rate by

\[
f_s = \frac{\delta}{\gamma}.
\]

where \( \delta = \delta \rho_M / \rho_M \) describes the perturbations in the density of matter (\( \rho_M \)). For constant \( w > -1 \), the exponent \( \gamma \) in equation (16) can be approximated as

\[
\gamma = 0.55 + 0.05(1 + w),
\]

while for a phantom-like DE component (with \( w < -1 \)) the exponent is

\[
\gamma = 0.55 + 0.02(1 + w).
\]

Solving equation (17) numerically, we derive the (orange) contours shown in Figs 4 and 5. In Table 1, we present the constraints on \( w \) from the combinations of these data with the SDSS-II SN and BAO likelihoods, marginalized over \( \Omega_M \) and vice versa.

5.1.2 The late-time integrated Sachs–Wolfe effect

The ISW effect is caused by the change in the energy of CMB photons as they pass through a time-varying gravitational potential (Sachs & Wolfe 1967). In a flat, matter-dominated universe, we would not expect to see an ISW effect as the large-scale gravitational potentials do not change in conformal time. However, in a universe dominated by DE or curvature, we should detect a so-called late-time ISW effect, which provides a direct measure of these quantities at the redshift of the changing potentials, i.e. the effect does not depend on the previous history of the growth of structure.

The late-time ISW effect introduces additional secondary anisotropies on top of the primary CMB fluctuations and is therefore hard to detect directly. However, the ISW effect can be seen via the cross-correlation of the CMB with tracers in the large-scale structure of the Universe as outlined by Crittenden & Turok (1996). This has now been achieved by a number of authors using a host of different galaxy data sets (Fosalba, Gaztañaga & Castander 2003; Scranton et al. 2003; Afshordi, Loh & Strauss 2004; Boughn & Crittenden 2004; Fosalba & Gaztañaga 2004; Nolta et al. 2004; Padmanabhan et al. 2005; Cabrera et al. 2006; Giannantonio et al. 2006; Rassat et al. 2007; Granett, Neyrinck & Szapudi 2008; Ho et al. 2008).

In this paper, we exploit the recent analysis of Giannantonio et al. (2008) and focus on the subset of intermediate-redshift (\( z < 0.4 \)) SDSS data they used. Even this subset of data shows a detection of the ISW effect at the 3\( \sigma \) level (Giannantonio, private communication). The contours for this new determination of the ISW effect are plotted in Figs 4–6. In Table 1, we present the combination of this new ISW effect measurement with our SDSS-II SN and BAO data, using the same procedure as discussed in Section 5.1.1.
our analysis to intermediate-redshift probes and therefore do not include the CMB constraints, which results in larger uncertainties.

Our analysis of the MLCS2k2 systematic uncertainties discussed above does not include the large shift in $w$ discussed in Kessler et al. (2009) when the rest-frame $U$-band template is removed in the light-curve fitting. As seen in table 6 of Kessler et al. (2009), removing the rest-frame $U$ band results in a $-0.31$ shift in $w$, while we find a shift of $-0.43$ if we remove these data. This particular uncertainty would therefore give rise to a bimodal result either centred around $w \approx -0.8$ (with the $U$ band included) or $w = -1.2$ (without the $U$ band), yet both consistent with $w = -1$ within the errors.

We do not add the uncertainty due to excluding the rest-frame $U$ band to our systematic errors because we believe that it is incorrect to exclude these data from the light-curve fitting. Even though there is evidence for diversity in the UV spectra of SNe Ia (see Ellis et al. 2008; Foley et al. 2008), the removal of the rest-frame $U$-band data from the SDSS-only analysis results in the light-curve fitter using only two filters at $z < 0.2$ to constrain the colours of the SNe. This provides significant freedom to the MLCS2k2 fitter and then the priors on the fitted parameters become important. We note that $w$ is only shifted by $-0.1$ when using the SALT light-curve fitter (see table 8 in Kessler et al. 2009) on the SDSS-only sample with the rest-frame $U$ band excluded. This is the only noticeable difference between these two light-curve fitting methodologies when considering the SDSS-only sample, namely the error on $w$ when the rest-frame $U$ band is excluded. Finally, Kessler et al. (2009) also sees a clear jump in the SDSS Hubble diagram at $z \approx 0.2$ when the rest-frame $U$ band is excluded from the MLCS2k2 analysis (see their section 10.1.3 and fig. 30), indicating that a constant $w$ model is not a good fit in this case.

### 6 Conclusions

We present an analysis of the luminosity distances of SNe Ia from the SDSS-II SN Survey in conjunction with other intermediate-redshift ($z < 0.4$) cosmological measurements including redshift-space distortions from the 2dFGRS, the ISW effect and the BAO distance scale from both the SDSS and 2dFGRS. We have analysed the SDSS-II SN luminosity distances using several ‘model-independent’ methods, including fitting the data using a $q(z)$ parametrization, principal components and a non-parametric ‘sliding window’ method. We find consistent results between all these methods that provide evidence for an accelerating universe based solely on the first-year SDSS-II SN data. The strongest evidence we find comes when we make the strongest assumptions that $q_0$ is constant and the Universe is flat which gives probability for acceleration of $>97$ per cent.

We also compare our SDSS-II SN data with the local BAO measurements and find that they are in good agreement. This is in contrast with the findings of Percival et al. (2007), who found tension between the two distance measures, but confirms the new BAO analysis of Percival et al. (2009) who note that this tension has now lessened. Taking this observation further, we test the distance duality relation, i.e. for any metric theory of gravity, we expect $d_L/(d_A(1+z)^2) = 1$. We see no evidence for a discrepancy from this relation (at the 1$\sigma$ level) in contrast to previous claims for a potential violation on the 2$\sigma$ level as seen in Bassett & Kunz (2004) and Lazkoz et al. (2008). Finally, we present a new measurement of the equation-of-state parameter of DE using a combination of geometrical distances in the Universe and estimates for the growth rate of structure. Our strongest constraint comes from the combination of all four data sets discussed herein (SDSS-II SN, BAO,
redshift-space distortions, ISW) with $w = -0.81^{+0.16}_{-0.18}$ (stat) and
$\Omega_m = 0.22^{+0.09}_{-0.08}$ (stat) (assuming a flat universe). However, the combi-
nation of just the SDSS-II SNe and the ISW measurements alone is almost as powerful
in constraining these parameters (Table 1).

Our results only change slightly if we allow curvature to vary, con-
sistent with the CMB measurements (see Appendix A). We quote
a systematic uncertainty of $\Delta w = \pm 0.15$ based on the details of
the MLCS2k2 light-curve fitter (see Kessler et al. 2009 for a fuller
discussion).

Thus, we have shown that low-redshift cosmological probes give
a self-consistent picture of the distance–redshift relation. When
combined with growth of structure and ISW at the same epoch, this
picture is consistent with $\Lambda$CDM and re-enforces the complement-
arity amongst other data and analyses in the literature.

ACKNOWLEDGMENTS

We thank an anonymous referee for helpful comments on this paper
which greatly improved the content of the paper. RCN on behalf of
the authors would like to thank Mike Turner for helpful discussions
on the $q_0$ fit and Eric Aubourg for useful discussions on the distance
duality relation. We also thank Rick Kessler and Mark Sullivan for
extensive discussions about their work and papers. TG thanks Jussi
Valiviita for helpful suggestions. HL, CS and RCN are grateful to
the Science and Technology Facilities Council (STFC) for funding
this research with rolling grants ST/F002335/1 and ST/H002774/1.
H-JS is supported by the D.O.E at Fermilab. AVF is grateful for the
support of US NSF grant AST–0607485.

Funding for the creation and distribution of the SDSS and SDSS-
II has been provided by the Alfred P. Sloan Foundation, the Partici-
pat ing Institutions, the National Science Foundation, the U.S.
Department of Energy, the National Aeronautics and Space Admin-
istration, the Japanese Monbukagakusho, the Max Planck Society
and the Higher Education Funding Council for England. The SDSS
web site is http://www.sdss.org/.

The SDSS is managed by the Astrophysical Research Conser-
vium for the Participating Institutions. The Participating Institutions
are the American Museum of Natural History, Astrophysical Insti-
tute Potsdam, University of Basel, Cambridge University, Case
Western Reserve University, University of Chicago, Drexel University,
Fermilab, the Institute for Advanced Study, the Japan Participa-
tion Group, Johns Hopkins University, the Joint Institute for Nu-
clear Astrophysics, the Kavli Institute for Particle Astrophysics and
Cosmology, the Korean Scientist Group, the Chinese Academy of
Sciences (LAMOST), Los Alamos National Laboratory, the Max-
Planck-Institute for Astronomy (MPA), the Max-Planck-Institute
for Astrophysics (MPiA), New Mexico State University, Ohio State
University, University of Pittsburgh, University of Portsmouth,
Princeton University, the United States Naval Observatory and the
University of Washington.

This work is based in part on observations made at the fol-
lowing telescopes. The Hobby–Eberly Telescope (HET) is a joint
project of the University of Texas at Austin, the Pennsylvania State
University, Stanford University, Ludwig–Maximilians-Universität
München and Georg-August-Universität Göttingen. The HET is
named in honour of its principal benefactors, William P. Hobby
and Robert E. Eberly. The Marcario Low-Resolution Spectrograph
is named for Mike Marcario of High Lonesome Optics, who fab-
ricated several optical elements for the instrument but died before
its completion; it is a joint project of the HET partnership and the
Instituto de Astronomía de la Universidad Nacional Autónoma de
México. The Apache Point Observatory 3.5 m telescope is owned and
operated by the Astrophysical Research Consortium. We thank
the observatory director, Suzanne Hawley, and site manager, Bruce
Gillespie, for their support of this project. The Subaru Telescope
is operated by the National Astronomical Observatory of Japan.
The William Herschel Telescope is operated by the Isaac Newton
Group on the island of La Palma in the Spanish Observatorio del
Roque de los Muchachos of the Instituto de Astrofísica de Canarias.
The W. M. Keck Observatory is operated as a scientific partnership
among the California Institute of Technology, the University of
California and the National Aeronautics and Space Administration;
the observatory was made possible by the generous financial support
of the W. M. Keck Foundation.

REFERENCES

Boughn S., Crittenden R., 2004, Nat, 427, 45
Hack, 372, L23
Academic Press, New York, p. 104
Etherington I. M. H., 1933, Philos. Mag., 15, 761
Dwarfs: Cosmological and Galactic Probes. Springer, Dordrecht, p. 97
Frieman J. A. et al., 2008a, AJ, 135, 338
Fukugita M., Ichikawa T., Gunn J. E., Doi M., Shimasaku K., Schneider
Giannantonio T. et al., 2006, Phys. Rev. D, 74, 063520
Giannantonio T., Scranton R., Crittenden R. G., Nichol R. C., Boughn S. P.,
Gunn J. E. et al., 2005, ApJS, 159, 1
Ivezić Z. et al., 2004, Astron. Nachr., 325, 583
Linder E. V., 2005, Phys. Rev. D, 72, 043529

APPENDIX A: COSMIC CURVATURE

In most parts of this paper, we assume a flat universe, or ΩM + ΩΛ = 1. Observationally, the most robust constraints on curvature come from the distance to the last scattering surface of the CMB as determined by measurements of the CMB power spectrum. In cases where we consider curvature, allowing any combination of ΩM and ΩΛ, we include a prior on the ‘scaled distance to recombination’ R defined as

\[ R = \sqrt{\frac{\Omega_M H_0^2}{(1 + z_{\text{CMB}})}}, \]  

(A1)
given in Wang & Mukherjee (2007), with a value of \( R = 1.71 \pm 0.019 \) from Komatsu et al. (2009). We note that \( R \) is independent of \( H_0 \) because \( d_L \) scales linearly with \( 1/H_0 \). For fixed values of \( \Omega_M \) and \( w \), it is now possible to constrain \( \Omega_L \) from a measurement of \( R \). We show 68 per cent confidence levels in Fig. A1 calculated from equation (A1). As \( R \) does not depend on the Hubble constant \( H_0 \), or on the baryon density \( \Omega_b h^2 \), no further assumptions are needed on these quantities. From Fig. A1, it is obvious that using only \( R \), a curvature of the order of a few per cent cannot be neglected (depending on the value of \( w \)). We introduce the effects of curvature in the redshift-space distortion analysis by adding an additional \( y^2 \) term in the likelihood analysis calculated from equation (A1) and subsequently marginalize over \( \Omega_L \), whereas for the ISW effect we pick for a given \( (w, \Omega_M) \) combination the best-fitting value of \( \Omega_L \).

APPENDIX B: CORRELATED PECULIAR VELOCITIES

The PVs of SNe introduce an additional scatter on to the Hubble diagram (see equation 2). However, as pointed out by Hui & Greene

![Figure A1](https://academic.oup.com/mnras/article-abstract/401/4/2331/1124994/414231125694) by guest on 03 March 2019
(2006), we expect these PVs to be correlated, especially at low redshift, thus leading to significant covariance between pairs of SNe, i.e. a pair of SNe at radial positions $r_i, r_j$ has a projected velocity correlation function of

$$\xi(r_i, r_j) = \langle v(r_i) \cdot \hat{r}_i \rangle \langle v(r_j) \cdot \hat{r}_j \rangle.$$ 

We can calculate this function in linear theory using the matter power spectrum, the linear growth function and its derivative. This is interesting because, if this effect is detected, it may enable the SNe to constrain the parameters of structure formation, in addition to the standard background expansion.

The expression for the full covariance between SNe is given by Gordon, Land & Slosar (2007) and Abate & Lahav (2008):

$$C_v(i, j) = \left(1 - \frac{(1 + z^2)}{Hd_L i}\right) \left(1 - \frac{(1 + z^2)}{Hd_L j}\right) \xi(r_i, r_j).$$  

This can be compared with the standard diagonal random errors, which are

$$\sigma^2(i) = \left(\frac{\ln 10}{5}\right)^2 \left[\sigma_i^2 + \mu_{\text{err}}(i)^2\right] + \left(1 - \frac{(1 + z)^2}{Hd_L}\right) \sigma_v^2,$$

where the intrinsic magnitude and velocity scatters, $\sigma_m$ and $\sigma_v$, respectively, have been introduced as usual. A numerical evaluation shows that the two are comparable at low redshift. In particular, for a pair of SNe at $z = 0.05$ and zero angular separation, the covariance is $C_v(i, j) \approx 0.1\sigma(i)\sigma(j)$. This decreases at higher redshifts and greater separations.

This effect has been detected by Gordon et al. (2007) using a catalogue of 124 low-redshift SNe by Jha et al. (2007) at $\bar{z} = 0.017$, and it has been carried further to constrain parameters such as $\sigma_8$ and the growth factor $\gamma$ (Abate & Lahav 2008).

Here we repeated the analysis for the SDSS SNe, but since our minimum redshift is $z \approx 0.05$, we expect the effect of correlated PVs to be small. Indeed, we found that a likelihood study performed by a Monte Carlo Markov Chain (MCMC) analysis of the cosmological parameters yields no change in the results whether the full PV covariance matrix of equation (B1) is included or not. For example, if we set the intrinsic scatters $\sigma_m = \sigma_v = 0$, we find that the reduced $\chi^2/\nu$ for the best-fitting cosmology decreases by only 1 per cent when the PV covariance matrix is included.

Therefore, we are unable to detect the correlation of SN peculiar velocities with these data and will be safe if we ignore them in further analyses. This effect will become more important with larger samples of low-redshift SNe.

This paper has been typeset from a TEX/LATEX file prepared by the author.