Simulations of momentum feedback by black hole winds

Sergei Nayakshin$^{1,2}$ and Chris Power$^1$

$^1$Department of Physics & Astronomy, University of Leicester, Leicester LE1 7RH

$^2$Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1, D-85741, Garching, Germany

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ABSTRACT

The observed super-massive black hole (SMBH) mass–galaxy velocity dispersion ($M_{\text{bh}} – \sigma$) correlation may be established when winds/outflows from the SMBH drive gas out of the potential wells of classical bulges. Here we present numerical simulations of this process in a static isothermal potential. Simple spherically symmetric models of SMBH feedback at the Eddington luminosity can successfully explain the $M_{\text{bh}} – \sigma$ and nuclear cluster mass $M_{\text{NC}} – \sigma$ correlations, as well as why larger bulges host SMBH while smaller ones host nuclear star clusters. However, these models do not specify how SMBH feed on infalling gas whilst simultaneously producing feedback that drives gas out of the galaxy.

More complex models with rotation and/or anisotropic feedback allow SMBH to feed via a disc or regions not exposed to SMBH winds, but in these more realistic cases it is not clear why a robust $M_{\text{bh}} – \sigma$ relation should be established. In fact, some of the model predictions contradict observations. For example, an isotropic SMBH wind impacting on a disc (rather than a shell) of aspect ratio $H/R \ll 1$ requires the SMBH mass to be larger by a factor of $\sim R/H$, which is opposite to what is observed. We conclude that understanding how an SMBH feeds is as important a piece of the puzzle as understanding how its feedback affects its host galaxy.

Finally, we note that in aspherical cases the SMBH outflows induce differential motions in the bulge. This may pump turbulence that is known to hinder star formation in star-forming regions. SMBH feedback thus may not only drive gas out of the bulge but also reduce the fraction of gas turned into stars.

Key words: accretion, accretion discs – Galaxy: centre – galaxies: active.

1 INTRODUCTION

It is believed that the centres of most galaxies contain super-massive black holes (SMBHs) whose mass $M_{\text{bh}}$ correlates with the velocity dispersion $\sigma$ of the host galaxy (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002). Similarly, there is a correlation between $M_{\text{bh}}$ and the mass of the bulge $M_{\text{bulge}}$ for large SMBH masses (Magorrian et al. 1998; Häring & Rix 2004; Gültekin et al. 2009). Observations also suggest that the masses of nuclear star clusters (NC) ($10^5 M_\odot \lesssim M_{\text{NC}} \lesssim 10^8 M_\odot$) correlate with the properties of their host dwarf ellipticals (Ferrarese et al. 2006; Wehner & Harris 2006) in a manner that is analogous to the one between SMBH and their host ellipticals.

These empirical relations can be explained in a very natural way if the growth of host galaxies and their central SMBHs or NCs are linked by feedback. This was first pointed out by Silk & Rees (1998), who highlighted the potential importance of SMBH heating and outflows before any robust observational evidence for such a link had been found. Subsequently, Fabian (1999) argued that radiation pressure acting on cold gaseous clouds in the bulge could give rise to the observed correlations. King & Pounds (2003) argued for the existence of sub-relativistic outflows from the very central regions of active galactic nuclei (AGN), which prompted King (2003, 2005) to study the motion of a shell of gas swept up by a wind/outflow from a central black hole in a galactic isothermal dark matter potential. King (2005) demonstrated that the shell will be expelled from the potential provided the black hole mass exceeds a critical value that, as a function of $\sigma$, turns out to be close to the observed $M_{\text{BH}} – \sigma$ relation.

The main result of King (2003, 2005) can be deduced using a simpler order of magnitude ‘weight argument’. According to this argument, the SMBH luminosity is assumed to be limited by the Eddington value. Radiation pressure drives a wind. King & Pounds (2003) argue that the wind velocity is comparable to the escape velocity from the inner accretion disc, e.g. $v \sim 0.1c$. The momentum outflow rate is assumed to be

$$P_{\text{SMBH}} \approx \frac{L_{\text{Edd}}}{c} = \frac{4\pi G M_{\text{BH}}}{\kappa},$$

(1)
where $\kappa$ is the electron scattering opacity and $M_{\text{BH}}$ is the SMBH mass. This result is natural to the order of magnitude: $L_{\text{Edd}}/c$ is the radiation momentum flux which is presumably passed to the wind as radiation accelerates the outflow (cf. King & Pounds 2003). The argument also assumes that the black hole wind is optically thin at the point of interaction with the ambient gas. Because the cooling time of the shocked gas is short on scales appropriate for observed bulges (King 2003, 2005), the bulk energy of the outflow is thermalized and quickly radiated away. It is then only the momentum push (equation 1) of the outflow on the ambient gas that is important since it is this that produces the outward force on the gas (as in the earlier model by Fabian 1999). The weight of the gas is $W(R) = GM(R)/M_{\text{total}}(R)/R^2$, where $M_{\text{gas}}(R)$ is the enclosed gas mass at radius $R$ and $M_{\text{total}}(R)$ is the total enclosed mass including dark matter. For an isothermal potential, $M_{\text{gas}}(R)$ and $M_{\text{total}}(R)$ are proportional to $R$, so the result is

$$W = \frac{4f_g\sigma^4}{G}.$$  

(2)

Here, $f_g$ is the baryonic fraction and $\sigma^2 = GM_{\text{total}}(R)/2R$ is the velocity dispersion in the bulge. By requiring that momentum output produced by the black hole just balances the weight of the gas, it follows that

$$M_g = \frac{f_g\kappa}{\pi G^2}\sigma^4,$$  

(3)

which is consistent with the observed $M_{\text{BH}} - \sigma$ relation.

To order of magnitude, relation (2) should hold for any potential at the virial radius. Therefore, the model by King (2003, 2005) appears to be a promising explanation of the observed correlations between SMBHs and their host galaxies. However, additional complications, not considered in King (2003, 2005), may be important. Amongst these are (1) the self-gravity of the gas, because gas that accumulates at the centre of the potential may begin to dominate it and therefore alter the result; (2) finite angular momentum of the gas may lead to the formation of a disc and (3) collimated and variable outflows.

The goal of this paper is to verify that the predictions of the analytical model of King (2003, 2005) hold and to explore more realistic settings, of the kind just described. Note that we concentrate on the early stages of galaxy evolution, when we would expect galaxies to be gas-rich and the baryonic mass within dark matter haloes is dominated by gas rather than stars. At later times, when the SMBH is likely to be less luminous and the galaxy is less gas-rich, we would expect different forms of feedback to become important, such as relativistic jets and the associated radio bubbles (e.g. Churazov et al. 2002) or pre-heating due to the inverse Compton effect (e.g. Sazonov et al. 2005). However, we do not include these forms of feedback in our current simulations.

2 FIXED BLACK HOLE MASS RUNS

2.1 Numerical method

Nayakshin, Cha & Hobbs (2009a) developed a new method for radiation transfer in smoothed particle hydrodynamics (SPH) based on Monte Carlo packets, which can also be used to simulate winds in the momentum-conserving phase. As discussed in Section 1, one can consider only the momentum transfer from the outflow to the ambient gas in this regime. In addition, for the problem at hand, the typical velocity of the ambient gas is of the order of the velocity dispersion in the bulge, which is less than a few 100 km s$^{-1}$. In contrast, the black hole outflow is much faster, $v \sim 0.1c$, which implies that the mass of the black hole wind is negligibly small compared with that of the gas in the bulge at the point where the latter is driven away. Hence, we can neglect the mass of the black hole wind in comparison to that of the ambient medium.

Accordingly, we use massless particles moving with velocity $v = 0.1c$ to simulate the black hole wind. The wind particles propagate in straight lines until they encounter one or more SPH particles at which point they transfer their momentum to the particles. For more information on the method and validation of the code, see Nayakshin et al. (2009a), especially their section 3.

For spherically symmetric tests, we use 35 000 SPH particles. Each of the wind particles carries momentum $p_{\text{wind}} = 0.1m_{\text{qph}}\sigma$. This satisfies the requirement $p_{\text{wind}} < \rho_{\text{ph}}$, where $\rho_{\text{ph}}$ is the typical SPH particle momentum ($\sim m_{\text{qph}}$ here), and ensures that Poisson noise from our Monte Carlo scheme is small enough not to compromise our results (see Nayakshin et al. 2009a). The particles are emitted by the black hole at the rate

$$N_{\text{wind}} = \frac{L_{\text{Edd}}}{c p_{\text{wind}}}.$$  

(4)

where $L_{\text{Edd}}$ is the Eddington luminosity for the black hole. They are ejected isotropically unless stated otherwise.

We perform our simulations in a static singular isothermal sphere potential with velocity dispersion $\sigma = 147$ km s$^{-1}$. The units of mass and distance are $M_\odot = 10^{10}$ $M_{\odot}$ and $R_\odot = 1$ kpc respectively; the unit velocity is $V_\odot = \sqrt{GM_\odot/R_\odot} = \sqrt{2}\sigma$. Equation (3) yields the expected value for the critical mass at which gas should be expelled:

$$M_\odot = \frac{f_g\kappa}{\pi G^2}\sigma^4 = 1.1 \times 10^9 M_\odot.$$  

(5)

To check this prediction, we first limit our simulations to spherically symmetric initial conditions. We work with a finite extent shell since that allows us to explore interesting regions of parameter space. The initial radial distribution of gas follows the distribution of dark matter, assumed to dominate the potential: $\rho(R) \propto R^{-2}$ for $R \ll R_\odot < R < R_{\text{out}}$. For all the tests in this paper, $R_{\text{in}} = R_{\text{out}}/2$. As in King (2003, 2005), the normalization of gas density, and hence the mass of the shell, is set by the cosmological baryon mass fraction, $f_\odot = \rho_f/\rho_{\text{total}} \simeq 0.16$, where $\rho_{\text{total}} = \rho_{\text{DM}} + \rho_g$ is the total mass density and $\rho_g$ and $\rho_{\text{DM}}$ are the gas and dark matter densities, respectively. For the isothermal potential, $\rho_{\text{total}}(R) = \sigma^2/(2\pi G R^2)$. Because our focus is momentum feedback, we adopt the isothermal equation of state for the gas, assuming $T = 3 \times 10^5$ K. This is lower than the virial temperature, $T_{\text{vir}} \approx 10^6$ K, for the potential we are using, preventing small-scale gas fragmentation. The latter process would lead to star formation and feedback from star formation, in both heating and momentum-driven forms. We plan to include these processes in our future work, but for now we will neglect them in comparison with the SMBH feedback altogether.

We set $R_{\text{acc}} = 0.5$ as the inner boundary of our computational domain. SPH particles that cross inside this inner boundary are removed. Note however that this scale is still too large to associate it with SMBH accretion directly, and there is no model-independent parameter-free way of connecting this ‘sunken’ mass with SMBH accretion and feedback. For simplicity, we take the pragmatic approach in this paper and consider only tests in which the black hole feedback is either constant or increases at a rate fixed by the Eddington limit. This allows us to concentrate on gas dynamics without worrying about the complexities of the non-linear and model-dependent accretion-feedback link. The ‘accreted’ gaseous mass is thus considered to be mainly used in star formation inside $R_{\text{acc}}$, and the feedback from that is neglected.
Before we begin the discussion of results, we show in Fig. 1 two typical snapshots from the spherically symmetric calculations. These show gas column densities in an angle-slice projection. Specifically, the gas column density shown in Fig. 1, and similarly in Figs 8–15, is calculated by

\[ \Sigma(x, y) = \int_{-\tan(\xi)}^{\tan(\xi)} \rho(x, y) \, dz, \]

where the limits of the integration are given by \( z(x, y) = r \tan(\xi) \), and \( r = (x^2 + y^2)^{1/2} \). The angle \( \xi \) is chosen to be \( \tan(\xi) = 1/4 \) for this paper. This projection method is convenient as it permits an unobstructed view into the central regions where the black hole resides. Had we used a constant thickness \( z(x, y) = \text{constant} \) projection method, then either the innermost region would not have been resolved sufficiently or an insufficient number of the SPH particles in the outermost regions would have been sampled for a statistically meaningful figure.

The tests shown in Fig. 1 are described in Sections 2.3 (left-hand panel) and 2.4 (right-hand panel). Briefly, the former has \( M_{\text{bh}} = 2 \times 10^8 M_\odot \) whereas the latter has \( M_{\text{bh}} = 4 \times 10^8 M_\odot \). In both cases, the gas is falling in with the radial velocity \( v_r = -1 \) (in code units) initially. In the lighter SMBH case, the black hole momentum outflow is insufficient to reverse the shell’s infall and it engulfs the black hole. In the right-hand panel, on the other hand, the initially thick shell is first compressed to a thinner shell by the opposing actions of the black hole outflow and the inward inertia of the outer layers, and then expelled to infinity.

### 2.2 Initially static shell tests

We begin with tests in which gas is initially at rest (even though this is unrealistic for the chosen temperature). The initial outer radius of the shell is \( R_{\text{out}} = 40 \) kpc for these runs. Fig. 2 shows results of three such tests, where we vary the black hole mass from \( M_{\text{bh}} = 5 \times 10^7 M_\odot \) to \( M_{\text{bh}} = 2 \times 10^8 M_\odot \) by factors of 2. We define a mean radius and velocity of the gas by averaging over all the SPH particles in a simulation. Defined in this way, the ‘shell’ radius is plotted in units of the initial mean radius whereas the velocity is shown in code units (\( \sqrt{2} \sigma \)). See Section 2.2 for more detail.

**Figure 2.** Mean radius (upper panel) and radial velocity (lower panel) of the gas as a function of time for three different black hole masses: \( M_{\text{bh}} = 2 \times 10^8 M_\odot \) (solid), \( M_{\text{bh}} = 10^8 M_\odot \) (dotted) and \( M_{\text{bh}} = 5 \times 10^7 M_\odot \) (dashed). The radius is plotted in units of the initial mean radius whereas velocity is shown in code units (\( \sqrt{2} \sigma \)). See Section 2.2 for more detail.
The additional initial inertia of the shell $M_C$, except that the radial velocity of gas is now non-zero, with $v_r = -1$. The additional inertia of the shell suggests that the critical $M_\sigma$ mass should be higher in this case, which turns out to be true.

Fig. 3 shows the mean shell radius, $R_{sh}$, in units of its initial value (dash–dotted green curve) as a function of time. The inward motion of the shell cannot be prevented in this simulation, as is clearly indicated by the monotonically decreasing mean radius. The solid curve shows the mean radial velocity of the gas scaled to its initial value. The shell is slightly decelerated initially by the momentum of the SMBH outflow, i.e. $|v_r|$ decreases until $t \sim 40$ Myr. However, at $t \sim 40$ Myr the shell starts to accelerate and falls ‘on to’ the SMBH, completely enveloping it. To follow the evolution of the shell from this point would require much greater numerical resolution and, more importantly, the physics of star formation.

The additional inward acceleration of the shell at $t \sim 40$ can be understood if one considers the self-gravity of the gas. We calculate the velocity dispersion corrected for the shell’s self-gravity:

$$\sigma_{sh}^2(R) = \frac{G}{2R} (M_{DM}(R) + M_{sh}) = (1 - f_{sh}) \sigma_0^2 + \frac{GM_{sh}}{2R}, \quad (7)$$

where $R = R_{sh}$. The second term increases for a constant $M_{sh}$ and a decreasing $R_{sh}$. It is convenient to define a ‘running’ value of the expected $M_\sigma$ mass based on the mean values of $\sigma_{sh}$ and $R_{sh}$ using equation (3). Further, we use the dimensionless variable $M_{\sigma}$:

$$M_{\sigma} = \frac{M_\sigma(R_{sh}, \sigma_{sh})}{M_{\sigma 0}} = \frac{f_x}{f_{sh 0}} \left( \frac{\sigma_{sh}}{\sigma_0} \right)^4, \quad (8)$$

where $M_{\sigma 0} = M_\sigma(R_0, \sigma_0)$ is the initial value of the $M_\sigma$ mass as a function of the initial values of shell radius $R_0$ and velocity dispersion $\sigma_0$, and $f_x$ is the current mass fraction of gas inside radius $R$. The function $M_\sigma$ is plotted in Fig. 3 with the dashed curve representing the present simulation. It becomes immediately apparent that the SMBH will find it harder to retard the shell once the shell has fallen in sufficiently deep into the potential and the gas starts to dominate the gravitational potential. If the SMBH mass is below the critical mass, then a runaway radial contraction of the shell occurs in this simple model.

![Figure 3](https://example.com/figure3.png)  
**Figure 3.** Mean radial velocity (solid) and radius (dash–dotted) of the gaseous shell as a function of time for $M_{bh} = 2 \times 10^8 M_\odot$. The dashed curve shows the expected dimensionless $M_\sigma$ mass, as defined by equation (8). The simulation is described in Section 2.3.

### 2.3 Initially infalling shell, $M_{bh} = 2 \times 10^8 M_\odot$

This simulation is identical to that presented in Section 2.2 with $M_{bh} = 2 \times 10^8 M_\odot$, except that the radial velocity of gas is now non-zero, with $v_r = -1$. The additional inertia of the shell suggests that the critical $M_\sigma$ mass should be higher in this case, which turns out to be true.

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$$M_{\sigma} = \frac{M_\sigma(R_{sh}, \sigma_{sh})}{M_{\sigma 0}} = \frac{f_x}{f_{sh 0}} \left( \frac{\sigma_{sh}}{\sigma_0} \right)^4, \quad (8)$$

where $M_{\sigma 0} = M_\sigma(R_0, \sigma_0)$ is the initial value of the $M_\sigma$ mass as a function of the initial values of shell radius $R_0$ and velocity dispersion $\sigma_0$, and $f_x$ is the current mass fraction of gas inside radius $R$. The function $M_\sigma$ is plotted in Fig. 3 with the dashed curve representing the present simulation. It becomes immediately apparent that the SMBH will find it harder to retard the shell once the shell has fallen in sufficiently deep into the potential and the gas starts to dominate the gravitational potential. If the SMBH mass is below the critical mass, then a runaway radial contraction of the shell occurs in this simple model.

![Figure 4](https://example.com/figure4.png)  
**Figure 4.** As in Fig. 3 but now the black hole mass is a factor of 2 larger (Section 2.4). The gaseous shell is now completely expelled from the potential well to infinity.

### 2.4 Initially infalling shell, $M_{bh} = 4 \times 10^8 M_\odot$

Fig. 4 shows the result of exactly the same calculation but now with an SMBH mass twice as large, i.e. $M_{bh} = 4 \times 10^8 M_\odot$. The behaviour of the shell in this case is radically different. The average radius of the shell decreases only during the first 40 Myr, during which time the outermost layers of the shell continue to fall in. However, at $t \sim 40$ Myr the shell briefly decelerates before quickly accelerating nearly linearly ($v_r \propto t$) out of the potential well.

It is worth noting how distinct the gas dynamics is between the two cases illustrated in Figs 3 and 4, respectively. This may be somewhat surprising given that the SMBH mass and hence the outward acceleration differ between the cases only by a factor of 2. However, the key difference is that in the first case the shell fell in far enough to become self-gravitating, with the self-gravity-corrected $\sigma$ increasing rapidly, increasing further the inward pull of gravity. This effect, missing in the analytical theory of King (2003, 2005), might actually increase the $M_\sigma$ value above that given by equation (3).

Comparing the two initially infalling shell tests with those for an initially static shell (Section 2.2), we note that the critical black hole mass is increased by a factor of 2 or so to $M_{bh} \approx 3 \times 10^8 M_\odot$. This is natural as the shell needs to be accelerated to a positive velocity $v_r \gtrsim 1$ to be driven out of the potential, and this is comparable to the initial negative velocity of the infalling tests. The required outward acceleration should then be roughly doubled compared with the initially static tests. This shows that variations in the initial radial velocity of the shell are unlikely to change the critical black hole mass by more than a factor of a few.

### 2.5 Scale-free nature of fixed BH mass solutions

Equation (3) shows that the critical $M_\sigma$ mass is independent of the initial location of the shell, $R_0$, as long as the initial gas fraction $f_{sh 0} = M_{sh}(R_0)/M_{tot}(R_0)$ does not vary with radius. We now show that the same is true for time-dependent solutions. This can be seen from the equation of motion for gas in the one-zone approximation, which is quite reasonable for a thin shell. We have

$$M_{sh} \frac{dv_r}{dt} = \frac{L_{edd}}{c} - \frac{G}{R^2} (M_{DM}(R) + M_{sh}) [M_{sh}], \quad (9)$$
where $v_i = dR/dt$. Let us assume $R(t) = R_0 \xi(t)$, where $\xi = t/t_0$, $t_0 = R_0/\sigma$. Obviously, we require $g(0) = 1$. We can now rewrite the above equation in this form:

$$R_0 \frac{d^2 \xi}{dt^2} = \frac{L_{\text{Edd}}}{c M_{\text{bh}}} - \frac{2 \sigma^2 (1 - f_{\text{g}})}{R_0 \sigma} - \frac{G M_{\text{bh}}}{R_0^2 \sigma^2}.$$  \hspace{1cm} (10)

Only the dimensionless time variable $\xi$ remains, implying a scale-free solution.

This conclusion is confirmed in Fig. 5, which shows the result of a simulation identical to the one presented in Fig. 4, except that the initial outer shell radius is $R_{\text{out}} = 10$ kpc rather than $R_{\text{out}} = 40$ kpc. The shell mass is also reduced by a factor of 4 to keep $M_{\text{sh}}$ the same. It is readily apparent that the two figures are identical, with the only difference evident in the time coordinate.

This scale-free nature of the King (2003, 2005) model in the isothermal potential has interesting observational implications. The model predicts a slope and normalization for the mass relation, such as the finite extent of astrophysical potential wells, or the amount of angular momentum in the gas. We shall now consider the first issue in relation to the finite maximum growth rate of the black holes.

### 3 EDDINGTON RATE – LIMITED GROWTH MODELS

In Section 2, the black hole mass was assumed fixed for simplicity. In this section, we allow the SMBH to grow at its Eddington accretion rate. In this case, the SMBH mass increases by the factor of $e$ in the Salpeter time:

$$t_{\text{Salp}} = \frac{M_{\text{bh}}}{M_{\text{Edd}}} \approx 4.5 \times 10^7 \text{ yr},$$  \hspace{1cm} (11)

Figure 5. As in Fig. 4, but now the initial shell radius is 10 kpc rather than 40 kpc. Note that the result is identical to Fig. 4 save for the time axis shrinking by a factor of 4. This confirms the scaling properties of the model discussed in Section 2.5.

\[\text{Figure 6. Gas mean velocity (solid), scaled mean radius (dot–dashed), scaled } M_{\text{g}} \text{ (dashed) and the ratio of the SMBH mass to the initial } M_{\text{g}} = M_0 = 3 \times 10^8 M_{\odot} \text{ mass (dotted) for an Eddington-limited simulation (Section 3.1). The initial SMBH mass is } 10^8 M_{\odot}, \text{ one-third of the } M_{\text{g}} \text{ mass. However, due to rapid accretion the SMBH catches up with the growing (contracting) bulge and expels the gas when } M_{\text{bh}} \approx 1.3 \times 10^9 M_{\odot}.\]

where $M_{\text{Edd}} = L_{\text{Edd}}/\epsilon c^2$ is the Eddington accretion rate and the radiative efficiency $\epsilon$ is set to 0.1.

The initial value of the outer radius, $R_{\text{out}}$, defines another important time-scale of the problem – the dynamical time:

$$t_{\text{dyn}} = \frac{R_{\text{out}}}{V_U V_0},$$  \hspace{1cm} (12)

where $V_U = \sqrt{GM/R} \approx 208 \text{ km s}^{-1}$ is the velocity unit for the potential.

As emphasized by Nayakshin, Wilkinson & King (2009b), there are two distinctly different regimes. If $t_{\text{Salp}} \ll t_{\text{dyn}}$, then the SMBH can grow arbitrarily quickly if provided with enough fuel. If there is an SMBH feeding-feedback link that can limit the SMBH mass, such as in the King (2003, 2005) model, the latter then grows to the appropriate $M_{\text{g}}$ and remains there. If $t_{\text{Salp}} \gg t_{\text{dyn}}$, then the SMBH is unable to grow sufficiently quickly to reach its maximum (i.e., $M_{\text{g}}$), even if it is provided with ample fuel during the dynamical time of the system. We now present two simulations that explore these two regimes.

#### 3.1 SMBH growth in a ‘large’ bulge

The initial condition used in this test is same as in Section 2.3, except that the initial SMBH mass is smaller, $M_{\text{bh}} = 10^8 M_{\odot}$. As we found in Section 2.4 for this initial condition, the $M_{\text{g}}$ mass is about $M_{\text{g}} = 3 \times 10^8 M_{\odot}$. The black hole thus needs to increase its mass by about a factor of 3 to drive the shell out. The dynamical time of the shell is $t_{\text{dyn}} \approx 200 \text{ Myr}$, which gives the SMBH plenty of time to grow.

Fig. 6 shows the mean radial velocity of gas (solid curve), the mean radius of the shell (dash–dotted), the self-gravity-corrected velocity dispersion of the gas (dashed) and, finally, the ratio of the black hole to bulge mass (dotted). The early phase of gas dynamics is quite similar to the underweight fixed mass case considered in Section 2.3. The shell is contracting and the gas velocity dispersion grows with time, increasing the $M_{\text{g}}$ value (see the dashed curve in the figure). However, the
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Figure 7. Similar to the simulation shown in Fig. 6 but with a smaller initial radius of $R_{\text{out}} = 10 \text{kpc}$ and a larger initial SMBH mass of $2 \times 10^8 M_\odot$. Note that the feedback from the growing SMBH is unable to prevent the contraction of the shell in this simulation.

SMBH grows even faster. At around 90 Myr, the gas suddenly gets decelerated and then accelerated to positive velocity.

While this is as expected based on simple analytical expectations (Nayakshin et al. 2009b), the SMBH mass at the time when gas velocity becomes positive is about $1.3 \times 10^9 M_\odot$, about a factor of 4 higher than the initial configuration value, $M_\sigma = 3 \times 10^9 M_\odot$. At the same time, the shell radius is much smaller at that moment than the initial value, increasing the self-gravity-corrected $M_\sigma$ value by a factor of about 2.5 to $7.5 \times 10^6 M_\odot$. If our models included star formation and if a good fraction of gas was turned into stars, then the resulting bulge, assuming that the stars remain bound as the gas is blown away, would satisfy the $M_{\text{bh}} - \sigma$ relation within a factor of 2. However, the bulge velocity dispersion would be higher than that of the underlying isothermal potential value.

3.2 SMBH growth in a ‘small’ bulge

We now repeat the run of Section 3.1 but shrinking the shell’s outer radius by a factor of 4 to $R_{\text{out}} = 10 \text{kpc}$ and increasing the SMBH’s initial mass to $M_{\text{bh}} = 2 \times 10^8 M_\odot$. This is about two-thirds of the initial $M_\sigma$ mass, and hence the black hole needs to increase in mass by only a small fraction to reverse the inflow of gas. However, as Fig. 7 demonstrates, the black hole’s growth is too slow for this configuration of gas. As the SMBH mass grows, so does the required $M_\sigma$ mass, since the shell contracts. In fact, when the shell’s mass exceeds the local dark matter mass the shell becomes self-gravitating and the increase in $M_\sigma$ accelerates, leaving no chance for the SMBH to catch up.

The results of these experiments confirm that the Salpeter time should be sufficiently short compared to the dynamical time of the system in order for the SMBH feedback to compensate for the shell contraction. However, if star formation is included, removing some of the available gas and producing its own feedback, this requirement could be relaxed somewhat.

4 FEEDBACK ON GAS WITH ANGULAR MOMENTUM

So far we have neglected the angular momentum of the gas; instead, we were looking at problems in which the motion of the shell is purely radial. However, we expect the gas to have non-zero angular momentum and this will be as important for the impact of the SMBH feedback, if not more so, as how the gas is distributed or what its thermodynamical properties are. If all the gas in the bulge possesses non-zero angular momentum with respect to the black hole, then its feeding must proceed through a disc. As efficiency of disc accretion is not well understood in the presence of star formation in massive cold discs (e.g. Goodman 2003; Nayakshin, Cuadra & Springel 2007), the exact distribution of gas in the angular momentum space prior to disc formation is very important. Furthermore, on larger scales the angular momentum of gas determines the resulting scale of the galactic disc, and perhaps the bulge.

The parameter space of this problem is very large, and so we can only begin to scratch the surface of the problem here. We choose to study a case of the same initially spherical shell infalling with $v_t = -1$ at $t = 0$, $R_{\text{out}} = 40 \text{kpc}$ (e.g. Section 2.3), but now rotating around the z-axis. In the spherical coordinates, in which $z = r \cos \theta$, $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, the rotation law is given by

$$v_\phi = v_{\text{rot}} \sin \theta,$$

where $v_\phi$ is the $\phi$-component of velocity and $v_{\text{rot}} = 0.3$. Thus, material near the pole rotates slowly while material near the equator ($\theta = 0$) rotates at the maximum for the test. We expect these tests to be geometrically more complicated than the spherical shell tests. Therefore, we significantly improve the resolution of these simulations by employing $N_{\text{grid}} \approx 4 \times 10^7$ particles, i.e. by more than a factor of 10.

4.1 Gas dynamics without feedback

We begin by studying the dynamics of the gas in the absence of feedback from the black hole. Fig. 8 shows two snapshots of the simulation viewed edge-on. We also show velocity vectors of the gas. In this figure and in all others showing velocity information, the normalization of velocity vectors is not to unit velocity but to the largest velocity found in the snapshot. This is convenient as a subset of gas can have a velocity much larger than unity in the more complex cases considered below, and its velocity vectors then clutter the figures.

As expected, a gaseous disc forms with an outer radius of about $R \sim R_{\text{out}}/3 \sim 13$, appropriate for the rotation velocity equal to about a third of the circular velocity in the isothermal potential. The inner radius of the disc goes all the way to our inner boundary of the simulation domain, $R = 0.5 \text{kpc}$, since the regions close to the polar axis have small angular momentum. By late times, about 4 per cent of the gas disappear inside the inner boundary. The disc undergoes radial oscillations for many orbits before the gas settles on circular orbits. Signatures of these oscillations can be observed in the right-hand panel of Fig. 8, where the disc vertical structure is not completely relaxed. In particular, one can see some low density regions having a small positive radial velocity in the corners of the figure.

Due to the high gas temperature selected for the simulations, $T = 3 \times 10^4 \text{K}$, the vertical pressure scaleheight of the final disc is relatively large, $H/R \sim 0.3$. Realistic discs are expected to be cooler and thus could be geometrically thinner and denser. However, star formation in these discs and especially feedback due to star formation may make such discs vertically more extended than one would get from simple estimates based on their temperature only. In any case, our deliberate choice of a rather geometrically thick disc implies that our main conclusions on the role of angular momentum,
4.2 Feedback and disc formation

We now turn to simulations with black hole winds. In order to isolate the main effects in the interplay between the feedback and angular momentum, we consider the simplest case of a constant and isotropic feedback. The momentum flux from the black hole is fixed at the value appropriate for a black hole mass of $M_{\text{bh}} = 2 \times 10^8 M_\odot$, as in Section 2.3.

Fig. 9 shows two snapshots from the simulation. The left-hand panel of Fig. 9 demonstrates a rather obvious point – that inflow is slower in the presence of feedback than in its absence (Fig. 8); in fact, the inner region is devoid of gas. As in the spherically symmetric feedback cases, a shell is formed. The shell is not spherical. The equatorial regions are held back by the centrifugal force more than the polar regions. Whilst this is occurring, gas also starts to accumulate in the mid-plane just like it does when there is no feedback. Because there is a competition between feedback and centrifugal forces, on one hand, and gravity, on the other, a ring – rather than a disc – is seen to form just beyond the shock front in the right-hand panel of Fig. 9. Also note that the cavity is not perfectly azimuthally symmetric at later times (right-hand panel), due to the development of a combination of instabilities and the self-gravity of the gas.

Fig. 10 shows two snapshots at a later time. A bipolar initially bubble-like structure develops around but not exactly on the symmetry axis. The bubble quickly bursts and a cavity is opened along

![Figure 8](image1.png)  
![Figure 9](image2.png)
the symmetry axis. In that region, some gas is driven sideways by
the centrifugal force and some to infinity by the black hole wind.
At later times, the central region $R \lesssim 10\text{–}15$ is devoid of gas except
for the black hole wind. Gas on the interface between the void and
the higher density gas is continuously stripped away, feeding the
(secondary) outflow.

The centrifugal force rarefies the region of the shell near the
poles and the black hole winds then evacuate that region by pushing
the gas away. In the equatorial plane, on the other hand, the disc
becomes too dense for the black hole feedback to have much of an
effect. The disc manages to shadow its interior from the black hole
influence.

Analysing the net effect of the finite angular momentum of the
shell on the feedback–shell interaction, we note that the polar re-
geions can now be expelled more easily. If these regions are to
ultimately provide the SMBH with fuel, then this would reduce the
required $M_\text{s}$ mass. However, it is now much more difficult for the
SMBH to affect the self-shielding disc. The mean density in the disc
is $\sim R/H$ times higher than the density for the same mass spread in a
spherical volume. If it is the disc that provides the SMBH with fuel,
then one would predict a value for the $M_\text{s}$ mass to be at least higher
by a factor of $R/H$ than that given in equation (3). Indeed, in the case
of isotropic feedback, the fraction of SMBH wind intercepted by
the disc is $\sim H/R$. If feedback is anisotropic and beamed away from
the disc, as would seem natural for a ‘grand design’ disc, i.e. one
extending from galaxy scales of kpc to sub-pc scales, then there is
a further inefficiency in feedback delivery to the disc. This appears
to contradict the recent observations of ‘pseudo-bulges’, e.g. bulges
that are disc or bar-like. The SMBH masses in such bulges appear
to be smaller rather than larger than those in classical bulges at the
same velocity dispersion (Hu 2009).

In fact, it is hard to see why there would exist any $M_\text{bh}\text{–}\sigma$ relation
at all if SMBHs were fed by large-scale discs that are immune to
the SMBH feedback.

5 COLLIMATED FEEDBACK

There are good reasons to believe that feedback may be collimated
rather than being isotropic. Unlike stars that radiate due to inter-
nal energy sources, black holes need a continuous fuel supply in
the form of accreting gas. Thus, there must be regions where feed-
back is relatively inefficient and net inflow dominates. Most likely,
such inflow takes the form of a standard accretion disc (Shakura &
Sunyaev 1973) on scales of Mpc and less. The outflow is then
probably stronger along the axis of symmetry of the disc. In addi-
tion, the putative molecular torus is probably quite a massive and
geometrically thick structure (e.g. Krolik & Begelman 1988) that
may restrict the black hole outflow to the direction perpendicular
to the mid-plane of the torus (if one can be defined). Note that the
inner accretion flow does not have to be coaligned with the torus,
especially if the black hole is rapidly spinning (King et al. 2005).

If the direction of the flow fluctuates rapidly compared with the
time-scales of bulge formation, $\sim 10^8$ yr, then we arrive at possibly
quite a complex picture of black hole outflows – non-stationary,
non-spherical and with a fluctuating direction.

How is the momentum feedback picture developed for the spher-
ical model (King 2003, 2005) modified when these collimation
effects are taken into account? The answer is probably very com-
plicated and depends on the setting/environment. Here we present
simulations for two relatively simple situations, designed to moti-
vate further discussion rather than give final answers.

5.1 Outflow misaligned with the disc

Our first simulation uses a fixed black hole mass $M_\text{bh} = 2 \times 10^8 M_\odot$
for simplicity and the same rotating shell initial condition used in
the simulations presented in Section 4.2. However, rather than assum-
ing isotropic feedback, the black hole feedback is now uniformly
distributed within the angle $0.7 \lesssim \cos \theta' \lesssim 1$ around the sym-
metry axis $z'$ (see below). Note that within this conical region,
the momentum flux density carried by the black hole wind is $1/0.3 \approx
3$ times higher than in the isotropic case studied in Section 4.2.

We anticipate that the effect of collimated feedback when the
axis of symmetry coincides with the $z'$-axis should be qualitatively
similar to the effect of isotropic feedback on gas with non-zero
angular momentum, of the kind presented in Section 4.2. Therefore
we consider a less trivial case in which the direction of the axis
of symmetry of the outflow, $z'$, is inclined by angle $\pi/4$ from the
z-axis defined by the direction of the angular momentum vector of the shell. For convenience of presentation, we choose the angular momentum vector of the shell and the outflow symmetry axis to lie in the z-y plane.

Fig. 11 shows angle-slice projections of the gas surface density at times $t = 23$ Myr (left-hand panel) and $t = 59$ Myr (right-hand panel). The projections are done along the x-axis, as before. In this projection, both the disc axis of rotation (z-axis) and the outflow axis of symmetry are in the plane of the figure. It is apparent from Fig. 11 that the outflow drives strong non-spherically symmetric motions in the infalling shell, sending gas to different non-circular orbits inclined to the z-axis at various angles.

The evolution of the shell is different from anything seen in the previous tests. Initially (see the right-hand panel of the figure), regions of the shell directly exposed to the feedback are compressed into a thin shell, which is slowly driven outwards. However, because the shell is rotating, gas that was not previously exposed to feedback continuously replenishes the outflow’s cone, and most of the shell eventually collapses inwards rather than being driven outwards, as is seen in the right-hand panel of Fig. 11.

At the same time, the segments of the shell not exposed to feedback at all – regions along the diagonal line from the upper left corner to the lower right corner of the figure – approach the black hole initially almost freely falling (the right-hand panel). At later times, their radial motion is stopped by the centrifugal force. Some parts of the gas then appear to be pushed into the feedback cone by the ram pressure of the continuous inflow. These parts are blown away into the outflow.

This geometrically complicated interplay of feedback and rotation results in a very strongly warped rotating structure visible in Fig. 12 at time $t = 100$ Myr. Here we show two projections, one along the x-axis (left-hand panel) and the other along the z-axis (right-hand panel), for clarity. Note the circularizing motions of gas at larger radii, settling into the disc. These regions of the disc are shielded from the black hole outflow by the inner disc regions. The inner disc regions are highly disturbed, however, with tangential, outflowing and inflowing motions present (see the central part of the right-hand panel of Fig. 12).

Fig. 13 shows two later stages in the evolution of the system. The left-hand panel of the figure is for time $t = 160$ Myr, whereas the right-hand panel shows $t = 400$ Myr. Apparently, the feedback is eventually able to clear the directions in which it is operating to enforce a strong outflow in those directions. In short, parts of the disc perpendicular to the outflow do survive, whereas the less fortunate parts, in the path of the black hole wind, are blown away by the wind. This occurs on roughly the dynamical time of the original shell ($R_{\text{out}}/\sigma \approx 190$ Myr). However, this is probably a function of the assumed disc temperature. A cooler and thicker disc could be more difficult to affect; more simulations of this are needed in the future.

The resulting ‘truce’ between the black hole outflow and the inertia of the disc is not an easy or natural one. The disc is warped and is still an evolving structure, with gas orbits in the inner part being nearly circular while those in the outer parts being more eccentric. The outflow also has faster and slower parts, with the denser regions moving more slowly. Not all parts of the outflow reach escape velocities before they are shielded, and some fall back along directions not exposed to the black hole outflow.

Comparing this simulation to the one with isotropic black hole wind (Section 4.2), it is apparent that collimated feedback leads the separation of the gas into disc and outflow regions to occur much later in time. Furthermore, this separation is not as distinct here as it is in Section 4.2. In addition, while no gas was able to reach the accretion radius of $R_{\text{acc}} = 0.5$ kpc in the spherically symmetric feedback simulation, the misaligned feedback run resulted in about 1 per cent of gas crossing the inner boundary. This value is still significantly less than the 4 per cent of gas accreted in the control simulation with no feedback (Section 4.1).

5.2 Outflow with a rotating axis of symmetry

The final simulation that we present here has a set-up similar to the previous simulation of misaligned collimated feedback, except that in this case the feedback axis is rotating around the x-axis according
to
\[ \theta_{\text{out}} = \Omega_{\text{out}} t, \]  
where \( \Omega_{\text{out}}^{-1} = 30 \) Myr (the simulation in Section 5.1 corresponds to a constant \( \theta_{\text{out}} = \pi/4 \)). Thus, the outflow axis becomes perpendicular to its original position in 15 Myr. Realistically, we might expect changes in the direction of black hole rotation to occur on even shorter time-scales (King & Pringle 2006), and so this simulation is probably still less complex than realistic bulges.

Fig. 14 shows the angle-slice projections of the simulation at times \( t = 47 \) Myr (left-hand panel) and \( t = 83 \) Myr (right-hand panel). Fig. 15 shows the same but at times \( t = 120 \) (left-hand panel) and \( t = 165 \) (right-hand panel). There are certain similarities in gas dynamics between the present simulation and that with the misaligned collimated outflow (Section 5), but there are also significant differences. One similarity is the presence of gas inflow along directions not presently exposed to the black hole outflow and outflow along some directions. However, there does not appear to be a settled structure in this case at all. Very strong velocity gradients and transient shocks occur in the present simulation. Such time-dependent shocks may be efficient drivers of gas turbulence.

6 FEEDBACK AND TURBULENCE

Fig. 16 shows the radial velocity structure for the SPH particles in the isotropic feedback simulation in which the shell had some angular momentum at a time of \( t = 236 \) Myr. As we remarked in Section 4.2, the feedback and the disc achieve a natural division of
the available space on the spheres of influence. Along the directions close to the pole, feedback dominates. In the mid-plane, most of the disc particles are shielded by the inner edge of the disc. The intermediate region is taken up by the wind driven off from the inner edge of the disc by the feedback. These features are clearly observable in Fig. 16 in terms of the radial velocity component.

We now consider the radial velocity plots for the simulation with a misaligned (stationary axis) outflow (see Section 5.1). Fig. 17 shows that initially the radial velocity distribution is quite complex, but this settles into the warped disc–outflow structure fairly quickly; that is, already by the time \( t = 59 \) (lower panel) of the figure. With time, this configuration evolves into a progressively more quasi-stationary state.

In contrast, when the outflow is rotating (Fig. 18), no clear-cut steady state is reached. The radial velocity pattern keeps evolving on time-scales comparable to the rotation period of the jet at relatively late times. One observes development of occasional transient shocks such as the vertical feature in the bottom panel of Fig. 18. The SMBH feedback thus provides momentum input into complicated differential gas motions, probably pumping turbulence in the bulge.

We believe that this is in fact another way in which the SMBH can be affecting its host galaxy. The current consensus in the field of star formation points to the key role of turbulence in providing support against the gravitational collapse of gas on large scales while allowing star formation to proceed on smaller scales in dense filaments (for recent reviews, see Mac Low & Klessen 2004; McKee & Ostriker 2007). As turbulent motions are not equivalent to an isotropic pressure support (Dobbs, Bonnell & Clark 2005), the main effect of the turbulence is to delay or slow down star formation. While future higher resolution studies are needed to confirm the

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**Figure 14.** Simulations with a rotating ‘jet’ outflow. The left-hand panel is for time \( t = 47 \) Myr and the right-hand panel is for \( t = 83 \) Myr.

**Figure 15.** Same as Fig. 14 but for later times: \( t = 120 \) Myr and \( t = 165 \) Myr for the left-hand and the right-hand panels, respectively.
point, it appears intuitively clear that AGN-driven turbulence in the bulge can also hinder star formation in its host bulge. In this case, the host is affected by the SMBH feedback in two ways: the gas is blown away and the fraction of gas turned into stars is reduced.

7 DISCUSSION

7.1 Spherically symmetric models: successful feedback

Spherically symmetric models can be considered a success to a certain degree since earlier analytical work is confirmed by the more detailed calculations here. First, the analytical estimate of the critical $M_\sigma$ mass obtained by King (2003, 2005) agrees well with our numerical simulations, especially for an initially static shell (see Section 2.2). If the shell is already falling in, then the inwardly directed inertia of the shell added raises the $M_\sigma$ mass by a factor of a few at most (Sections 2.3 and 2.4).

The complications arising from a finite system size were recently pointed out by Nayakshin et al. (2009b) who noted that dynamical time in the bulge, $R_b/\sigma$, where $R_b$ is the scale radius of the bulge, plays a key role in what happens. Black holes growing at the Eddington rate, $\dot{M}_{\text{Edd}}$, e-fold their mass on the Salpeter time, $t_{\text{Salp}} = M_{\text{BH}}/\dot{M}_{\text{Edd}} \approx 4.5 \times 10^7$. Observationally,

$$t_{\text{dyn}} = \frac{R_b}{\sigma} = 17 \text{ Myr} \left[ \frac{\sigma}{150 \text{ km s}^{-1}} \right]^{2.1}.$$  

Thus ‘large’ bulges, $\sigma \gtrsim 200 \text{ km s}^{-1}$, have dynamical times comparable to or larger than $t_{\text{Salp}}$. Under the assumption that the duration of the black hole feeding event is comparable to the dynamic time of
the bulge, these ‘large’ bulges allow sufficient time for their SMBHs to grow significantly. These bulges would then have ‘fully grown’ black holes saturated at their $M_{\ast}$ mass. On the other hand, lower mass bulges with velocity dispersions of $\sigma \lesssim 100$ km s$^{-1}$ would evolve too rapidly for their SMBHs to ‘catch up’. Therefore, feedback from these SMBHs would be overpowered by the inflow of gas from the bulge. The central regions of these lower mass bulges could be sites of copious star formation, leading to the birth of NC. The latter then reach their $M_{\ast}$ masses (see Nayakshin et al. 2009b), replacing the SMBH black holes as the dominant objects in the bulges.

Our simulations (Section 3) confirm this effect. The numerical experiment in Section 3.1 has shown that the black hole immersed in a 40 kpc bulge is able to grow enough to expel the gaseous shell when started from the initial mass of $M_{\text{bh}} = 10^8 M_{\odot}$ (Fig. 6). At the same time, a black hole that is initially twice as massive in a 10 kpc bulge is unable to expel the shell (Section 3.2, Fig. 7) that is less massive than that in Section 3.1.

One complication not considered by analytical models yet is that if the mass fraction of gas is significant, i.e. larger than the universal cosmological fraction of $f_g \approx 0.16$, then the self-gravity of the gas can dominate the potential. The weight argument of course applies to self-gravitating shells as well, and they too can be unbound by a sufficiently strong black hole wind (e.g. see Fig. 6). However in this case our models are inconclusive as they do not include star formation. When gas is blown away, the velocity dispersion of the self-gravitating shell should decrease. A detailed calculation including star formation is needed to establish whether or not the black hole in the remaining bulge follows the $M-\sigma$ relation.

### 7.2 Spherically symmetric models: inconsistent SMBH feeding

There are at least two major worries about the general usefulness of spherically symmetric models. Large-scale cosmological simulations usually produce very complicated gas density and velocity flows on to the centres of dark matter haloes (e.g. Kereš et al. 2005) and so it is not clear whether or not such spherically symmetric flows ever take place.

There is also a conceptual difficulty with the spherically symmetric models. In a self-consistent model, where black hole feedback is linked to the accretion of gas on to the hole, one cannot have accretion and outflow at the same time. Yet both are needed. We not only need accretion to grow and power the SMBH, but also need outflow to eventually curtail the SMBH growth and expel the gas from the galaxy.

Having accretion before, e.g. while the SMBH has not reached the $M_{\ast}$ mass, and outflow later, after it has done so, is not a comfortable option. Indeed, to change inflow to outflow for most of the gas in a bulge, we require the momentum input of

$$P \sim M_{\text{gas}}v_{\text{esc}} \approx M_{\text{bulge}}\sigma,$$

where $M_{\text{gas}}$ is the mass of the gas which we assumed to be of the order of the bulge mass $M_{\text{bulge}}$ and the escape velocity $v_{\text{esc}} \approx \sigma$. The minimum amount of mass that the black hole needs to accrete to produce this much momentum outflow is

$$\Delta M_{\text{in}} = \frac{P}{\epsilon \sigma}.$$  

For fiducial numbers $\sigma = 150$ km s$^{-1}$ and $\epsilon = 0.1$, we have

$$\Delta M_{\text{in}} = 0.005M_{\text{bulge}},$$

which is comparable to the observed $M_{\text{bh}}$ masses for most bulges (Häring & Rix 2004). Now, if this gaseous mass is accumulated somewhere near the SMBH, e.g. in a disc that is immune to the feedback so that it can power the SMBH, then the question is: why could this mass not be much higher or much lower?

In other words, if SMBH feeding is local to a small-scale region (e.g. inside the SMBH influence radius, which is typically between a pc and a few tens of pc), where enough material is stored to feed the SMBH while most of the gas is driven away, the casual link between feedback and accretion is broken, and there does not appear a reason to have an $M_{\text{bh}}-\sigma$ relation. Thus spherically symmetric models, while providing excellent fits to the observed relations, do not naturally explain how the SMBH gets its fuel or how it knows to stop growing once the many-kpc-scale shell starts to be blown away.

### 7.3 Non-spherical models: clear mode for feeding but no feedback

The second type of simulations that we considered here does not have spherical symmetry for one or more reasons. SMBH feeding in these models can be achieved (1) by features denser than the mean, i.e. discs or filaments, or (2) through regions not exposed to feedback at all. In the former case, if the SMBH-feeding gas has a column density (as seen from the black hole) much higher than the mean for the bulge, then it is only weakly affected by the momentum feedback. In both these cases, there can be inflow to feed the SMBH and outflow to expel most of the gas in the bulge simultaneously. However, if these dense features feed the SMBH despite it producing feedback at the maximum rate, then it is not clear why there should be a feedback-mediated SMBH – galaxy link.

This problem is already apparent in the simplest non-spherical case – the simulation of a rotating initially spherical shell of gas (Section 4.2). In that simulation, gas separates out in two well-defined regions – the polar region where the black hole outflow dominates and the disc mid-plane where gas is shielded from the black hole outflow by the inner edge of the disc. The edge of the disc is slowly ‘evaporating’ into an outflow. The rest of the disc is hardly affected, and the orbits there are nearly circular.

We have also set a numerical experiment in Section 5.1 where feedback outflow is confined to a broad cone misaligned with the rotation axis of the shell. The shell initial condition was identical to that explored in Section 4.2. We found, not surprisingly, that the resulting pattern of gas dynamics is even more complex than that of an isotropic feedback with a rotating shell. The inflow persists along directions in which black hole winds are not emitted and the outflow prevails inside the feedback cone. The collimated rotating black hole outflow (Section 5.2) is even more complex geometrically, with the black hole feedback probably driving turbulence into the bulge gas.

### 7.4 Shortcomings and comparison to other numerical work

There is a substantial body of work on numerical simulations of SMBH feedback in cosmological simulations, e.g. Springel, Di Matteo & Hernquist (2005), Di Matteo, Springel & Hernquist (2005) and Sijacki et al. (2007). This paper complements this work. While addressing similar issues, the scales involved in studying SMBH feedback in a cosmological context are very different. Cosmological simulations have clear strengths – they naturally account for hierarchical build-up of galaxies assuming realistic initial conditions and include recipes for physical processes that we expect to be important for galaxy formation such as cooling, star formation and feedback. Therefore, they can provide a powerful tool with
which to explore SMBH feedback. In comparison, our simulations are more modest in terms of the number of physical processes studied, making use of idealized initial conditions in static analytical potentials.

On the other hand, the all-inclusive nature of cosmological simulations means that ‘individual’ processes, such as black hole accretion and feedback, cannot be modelled in sufficient detail and require sub-grid models to be assumed. As far as we are aware, all the current cosmological simulations rely on the Bondi & Hoyle (1944) and Bondi (1952) formulation for accretion of gas. The latter formulation is physically inconsistent if accreting gas has angular momentum as gas simply cannot reach the SMBH due to the centrifugal barrier. Furthermore, at least the ‘quasar’ mode of feedback is done in a very sketchy manner, with a fixed fraction of energy released by the quasar being passed to the SMBH neighbour particles. The right fraction is found a posteriori to fit the observed $M_{\text{sh}}/\sigma$ relation and other observational constraints.

Therefore, modest ‘small-scale’ simulations such as ours, especially when extended to include more physics and to reach in as close as possible to the SMBH, may offer another important route to solving the mystery of SMBH and galaxy growth. Ideally, the small-scale and cosmological simulations should be merged to some degree in the future to allow as self-consistent a study of SMBH feedback as possible.

8 CONCLUSIONS

Our main conclusions are as follows.

(i) Predictions of spherically symmetric models with black hole feedback tied to the Eddington limit luminosity as in the models of King (2003, 2005) are confirmed numerically. Self-gravity of the gas complicates the evolution of the system, and a self-consistent treatment of star formation and its feedback is necessary for further progress.

(ii) As suggested earlier based on analytical arguments, it is the dynamical time in the bulge, $R_e/\sigma$, that determines whether or not the SMBH can reach its limiting $M_\text{sh}$ value. Central regions of smaller bulges, where the dynamical time is observed to be shorter than the Salpeter time, could be smeothered with infalling gas despite the SMBH feedback. This process may be the origin of the NC in ‘smaller’ galaxies.

(iii) A net angular momentum in the shell is essential in determining the fate of the shell and the SMBH feeding. There is no well-defined $M_\text{sh}$ mass in that case since the momentum thrust required is different in different directions. Gas near the symmetry axis is blown out easier than in the spherically symmetric case, whereas gas settled into a disc requires $\sim R/H$ more thrust than in the latter case.

(iv) If the SMBH is fed through a several-kpc-scale disc, the SMBH mass would have to be $\sim R/H$ larger than in the spherical case to expel the feedback-resistant self-shielding disc. This directly contradicts the recent observations of Hu (2009) that show that pseudo-bulges have lighter SMBHs than their classical counterparts. This may imply that black holes are not fed by large (kpc or larger) discs but rather by flows with a small specific angular momentum.

(v) We also noted that SMBH outflows in a realistic, i.e. aspherical, situation may pump turbulence (differential motions) in the bulge. Turbulence is known to hinder star formation. Therefore, we believe that AGN feedback not only expels the gas from the galaxy but may also reduce the amount of mass turned into stars during bulge formation.

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NOTE ADDED IN PRESS

The authors have recently learned that R. Kurosawa and D. Proga (e.g. Kurosawa & Proga 2009), in a series of articles, have explored preheating and radiation pressure feedback from AGN on slowly rotating accretion flows. While different in detail, their study also finds multi-phase inflows and outflows near bright AGN.

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