Estimating the impact of recombination uncertainties on the cosmological parameter constraints from cosmic microwave background experiments

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ABSTRACT
We use our most recent training set for the rico code to estimate the impact of recombination uncertainties on the posterior probability distributions which will be obtained from future cosmic microwave background experiments, and in particular the Planck satellite. Using a Monte Carlo Markov Chain (MCMC) analysis to sample the posterior distribution of the cosmological parameters, we find that Planck will have biases of $-0.7$, $-0.3$ and $-0.4$σ for $n_S$, $\Omega_B h^2$ and $\log(10^{10} A_S)$, respectively, in the minimal six-parameter $\Lambda$ cold dark matter model, if the description of the recombination history given by rico is not used. The remaining parameters (e.g. $\tau$ or $\Omega_{dm} h^2$) are not significantly affected. We also show that the cosmology dependence of the corrections to the recombination history modelled with rico has a negligible impact on the posterior distributions obtained for the case of the Planck satellite. In practice, this implies that the inclusion of additional corrections to existing recombination codes can be achieved using simple cosmology-independent ‘fudge functions’.

Finally, we also investigated the impact of some recent improvements in the treatment of hydrogen recombination which are still not included in the current version of our training set for rico, by assuming that the cosmology dependence of those corrections can be neglected. In summary, with our current understanding of the complete recombination process, the expected biases in the cosmological parameters inferred from Planck might be as large as $-2.3$, $-1.7$ and $-1$σ for $n_S$, $\Omega_B h^2$ and $\log(10^{10} A_S)$, respectively, if all those corrections are not taken into account. We note that although the list of physical processes that could be of importance for Planck seems to be nearly complete, still some effort has to be put into the validation of the results obtained by the different groups.

The new rico training set as well as the fudge functions used for this paper are publicly available on the rico webpage.

Key words: cosmic microwave background – cosmological parameters – cosmology: observations.

1 INTRODUCTION

The cosmic microwave background (CMB) is nowadays an essential tool of theoretical and observational cosmology. Recent advances in the observations of the CMB angular fluctuations in temperature and polarization (e.g. Hinshaw et al. 2009; Komatsu et al. 2009) provide a detailed description of the global properties of the Universe, and the cosmological parameters are currently known with accuracies of the order of few per cent in many cases. The experimental prospect for the Planck satellite (The Planck Collaboration 2006), which was launched on 2009 May 14, is to achieve the most detailed picture of the CMB anisotropies down to angular scales of $\ell \sim 2500$ in temperature and $\ell \sim 1500$ in polarization. These data will achieve sub-per cent precision in many cosmological parameters. However, those high accuracies will rely on a highly precise description of the theoretical predictions for the different cosmological models.
Currently, it is widely recognized that the major limiting factor in the accuracy of angular power spectrum calculations is the uncertainty in the ionization history of the Universe (see Hu et al. 1995; Seljak et al. 2003).

This has motivated several groups to re-examine the problem of cosmological recombination (Zeldovich, Kurth & Syunyaev 1968; Peebles 1968), taking into account detailed corrections to the physical processes occurring during hydrogen (e.g. Dubrovich & Grachev 2005; Rubiño-Martín, Hernández-Monteagudo & Sunyaev 2005; Chluba & Sunyaev 2006b; Kholupenko & Ivanikh 2006; Rubiño-Martín, Chluba & Sunyaev 2006; Chluba, Rubiño-Martín & Sunyaev 2007; Chluba & Sunyaev 2007; Hirata 2008; Karshenboim & Ivanov 2008; Chluba & Sunyaev 2009a,c,e; Hirata & Forbes 2009; Jentschura 2009; Labzowsky, Solovyev & Plunien 2009) and helium recombination (e.g. Kholupenko, Ivanikh & Varshalovich 2007; Wong & Scott 2007; Hirata & Switzer 2008; Kholupenko et al. 2008; Rubiño-Martín, Chluba & Sunyaev 2008; Switzer & Hirata 2008a,b; Chluba & Sunyaev 2009b). Each one of the aforementioned corrections individually leads to changes in the ionization history at the level of $\geq 0.1$ per cent, in such a way that the corresponding overall uncertainty in the CMB angular power spectra exceeds the benchmark of $\pm 3\sigma$ at large $\ell$ (for more details, see Fendt et al. 2009, hereafter FCRW09), thus biasing any parameter constraints inferred by experiments like the Planck, which will be cosmically variance limited up to very high multipoles.

The standard description of the recombination process is provided by the widely used RECFAST code (Seager, Sasselov & Scott 1999), which uses effective three-level atoms, for both hydrogen and helium, with the inclusion of a conveniently chosen fudge factor which artificially modifies the dynamics of the process to reproduce the results of a multilevel recombination code (Seager, Sasselov & Scott 2000).

The simultaneous evaluation of all the new effects discussed above make the numerical computations very time-consuming, as they currently require the solution of the full multilevel recombination code. Moreover, some of the key ingredients in the accurate evaluation of the recombination history (e.g. the problem of radiative transfer in hydrogen and the proper inclusion of two-photon processes) are solved using computationally demanding approaches, although in some cases semi-analytical approximations (see e.g. Hirata 2008) might open the possibility of a more efficient evaluation in the future.

In order to have an accurate and fast representation of the cosmological recombination history as a function of the cosmological parameters, two possible approaches have been considered in the literature. The first one consists of the inclusion of additional fudge factors to mimic the new physics, as recently done in Wong, Moss & Scott (2008) (see RECFAST v1.4.2), where they include an additional fudge factor to modify the dynamics of helium recombination. The second approach is the so-called risco code (FCRW09), which provides an accurate representation of the recombination history by using a regression scheme based on risco (Fendt & Wandelt 2007a,b). The risco code smoothly interpolates the $X_\alpha(z; p)$ function on a set of pre-computed recombination histories for different cosmologies, where $z$ is the redshift and $p$ represents the set of cosmological parameters.

In this paper, we present the results for parameter estimations using risco with the most recent training set presented in FCRW09. This permits us to accurately account for the full cosmological dependence of the corrections to the recombination history that were included in the multilevel recombination code which was used for the training of risco (see Section 2.1 for more details). With this tool, we have evaluated the impact of the corrections on the posterior probability distributions that are expected to be obtained for the Planck satellite, by performing a complete Monte Carlo Markov Chain (MCMC) analysis. The study of these posteriors has shown that the impact of the cosmology dependence is not very relevant for those processes included in the current risco training set. Therefore, by assuming that the cosmology dependence of the correction in general can be neglected, we have also investigated the impact of recent improvements in the treatment of hydrogen recombination (see Section 2.2). The basic conclusion is that, with our current understanding of the recombination process, the expected biases in the cosmological parameters inferred from Planck might be as large as $1.5-2.5\sigma$ for some parameters such as the baryon density or the primordial spectral index of scalar fluctuations, if all these corrections to the recombination history are neglected.

The paper is organized as follows. Section 2 describes the current training set for risco, and provides an updated list of physical processes during recombination which were not included in FCRW09. Section 3 presents the impact of the recombination uncertainties on cosmological parameter estimation, focusing on the case of the Planck satellite. Section 4 further extends this study to account for the remaining recombination uncertainties described in Section 2. Section 5 presents the analysis of present-day CMB experiments, for which the effect is shown to be negligible. Finally, the discussion and conclusions are presented in Sections 6 and 7, respectively.

### 2. Updated List of Physical Processes During Recombination

In this section, we provide an updated overview on the important physical processes during cosmological recombination which have been discussed in the literature so far. We start with a short summary of those processes which are already included in the current training set (FCRW09) of risco (Section 2.1). The corresponding correction to the ionization history close to the maximum of the Thomson electron visibility function is shown in Fig. 1.

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**Figure 1.** Recombination uncertainties with respect to the standard RECFAST v1.4.2 code. For the fiducial cosmological model, we show the correction to the ionization history which is incorporated in the new risco training set (dotted line); the correction due to Ly$\alpha$ radiative transfer effects (Chluba & Sunyaev 2009a; Chluba & Sunyaev 2009c; Chluba & Sunyaev 2009e) (dot-dashed); the effect of Raman scattering (Hirata 2008) (dashed); and the combination of all previous effects (solid line).
cosmological ionization history (see Fig. 1) that is not yet included in the current training set of rico. However, as we explain below (Section 3.4) it is possible to take these corrections into account (Section 4), provided that their cosmology dependence is negligible. Our computations show that this may be a valid approximation (Section 3.4).

We end this section mentioning a few processes that have been recently addressed but seem to be of minor importance in connection with parameter estimations for Planck. Overall it seems that the list of processes that could be of importance in connection with Planck is nearly completed. However, still some effort has to go into cross-validation of the results obtained by different groups.

2.1 The current training set for RICO

As demonstrated in FCRW09, RICO can be used to represent the recombination history of the Universe, accurately capturing the full cosmology dependence and physical model of the multilevel recombination code that was used in the computations of the RICO training set.

For the current RICO training set, we ran our full recombination code using a 75-shell model for the hydrogen atom. The physical processes which are included during hydrogen recombination are described in detail in FCRW09: the induced 2s–1s two-photon decay (Chluba & Sunyaev 2006b); the feedback of Lyα photons on the 1s–2s absorption rate (Kholupenko & Ivanchik 2006); the non-equilibrium populations in the angular momentum substates (Rubíno-Martín et al. 2006; Chluba et al. 2007); and the effect of Lyman series feedback (Chluba & Sunyaev 2007). For helium recombination we took into account: the spin-forbidden He I 2^3P_1–1^1S_0 transition (Dubrovich & Grachev 2005); and the acceleration of helium recombination by neutral hydrogen (Switzer & Sunyaev 2008a). Furthermore, we also updated our physical constants according to the National Institute of Standards and Technology (NIST) data base, including the new value of the gravitational constant and the helium to hydrogen mass ratio (Wong & Scott 2007).

A more detailed description of the physical processes that were taken into account in the current RICO training set can be found in FCRW09. This RICO training set is now publicly available at http://cosmos.astro.uiuc.edu/rico.

2.2 Updated radiative transfer calculations during hydrogen recombination

As already pointed out earlier (e.g. Chluba & Sunyaev 2008b), in particular, a detailed treatment of the hydrogen Lyα radiative transfer problem including two-photon corrections is expected to lead to an important additional modification during hydrogen recombination. An overview of the relevant physical aspects in connection with this problem was already given in sections 2.3.2 and 2.3.3 of FCRW09. However, at that time the problem was still not solved at full depth, but recently several important steps were taken, which here we now want to discuss briefly (for additional overview see also Sunyaev & Chluba 2009).

2.2.1 Partial frequency redistribution due to line scattering

Recently, the effect of partial frequency redistribution on the Lyα escape rate and the ionization history during hydrogen recombination was independently studied in detail by Chluba & Sunyaev (2009a) and Hirata & Forbes (2009). As shown by Chluba & Sunyaev (2009a), the atomic recoil effect leads to the dominant contribution to the associated correction in the ionization history, and the result for this process alone seems to be in good agreement with the one obtained earlier by Grachev & Dubrovich (2008), leading to ∆N_e/Ne ≃ −1.2 per cent at z ∼ 900. Also the computations by Hirata & Forbes (2009) seem to support this conclusion.

However, Chluba & Sunyaev (2009a) also included the effect of Doppler broadening and Doppler boosting, which was neglected in the analysis of Grachev & Dubrovich (2008). Doppler boosting acts in the opposite direction as atomic recoil and therefore decelerates recombination, while the effect of Doppler broadening can lead to both an increase in the photon escape probability or a decrease, depending on the initial frequency of the photons (see Chluba & Sunyaev 2009a, for more detailed explanation). The overall correction to the recombination history due to line scattering amounts to ∆N_e/Ne ≃ −0.6 per cent at z ∼ 900. The results of Chluba & Sunyaev (2009a) seem to be rather similar to those of Hirata & Forbes (2009), however a final comparison will become necessary in order to reach full agreement on the final correction. In the computations presented below we will use the results of Chluba & Sunyaev (2009a).

2.2.2 Two-photon transitions from higher levels

Initially, the problem of two-photon transitions from highly excited levels in hydrogen and helium was proposed by Dubrovich & Grachev (2005). However, for hydrogen recombination only very recently this problem has been solved convincingly by Hirata (2008) and Chluba & Sunyaev (2009c), using two independent, conceptually different approaches. Also until now, Chluba & Sunyaev (2009c) took the main contribution coming from the 3d–1s and 3s–1s two-photon profile corrections into account, while Hirata (2008) also included the ns–1s and nd–1s two-photon profile corrections for larger n.

In the analysis of Chluba & Sunyaev (2009c), three independent sources for the corrections in connection with the two-photon picture were identified (we will discuss the other two processes in Sections 2.2.4 and 2.2.3). As they explain, the total modification coming from purely quantum mechanical aspects of the problem (i.e. corrections due to deviations of the line profiles from the normal Lorentzian shape, as pointed out by Chluba & Sunyaev (2008b)] leads to a change in the free electron number of ∆N_e/Ne ∼ −0.4 per cent at z ∼ 1100. It also seems clear that the remaining small difference (at the level of ∆N_e/Ne ∼ 0.1–0.2 per cent) between the results of Hirata (2008) and Chluba & Sunyaev (2009c) for the total correction related to the two-photon decays from excited states is because Chluba & Sunyaev (2009c) only included the full two-photon profiles of the 3s–1s and 3d–1s channels. Below, we will use the results of Chluba & Sunyaev (2009c) for this process.

2.2.3 Time-dependent aspects in the emission and absorption of Lyα photons

One of the key ingredients for the derivation of the escape probability in the Lyα resonance using the Sobolev approximation (Sobolev 1960) is the quasi-stationarity of the line transfer problem. However, as shown recently (Chluba & Sunyaev 2009c, e) at the


2 They used a Fokker–Planck approximation (e.g. Rybicki 2006) for the frequency redistribution function.
per cent level, this assumption is not justified during the recombi-
nation of hydrogen, since (i) the ionization degree, expansion rate of
the Universe and Lyω death probability change over a character-
istic time \( \Delta z / z \approx 10 \) per cent, and (ii) a significant contribution to
the total escape probability is coming from photons emitted in the
distant wings (comparable to \( 10^{-2} \)–\( 10^{3} \) Doppler widths) of the Lyω
resonance. Therefore, one has to include time-dependent aspects in
the emission and absorption process in the line transfer problem,
leading to a delay of recombination by \( \Delta N_e / N_e \approx +1.2 \) per cent
at \( z \sim 1000 \). Below, we will use the results of Chluba & Sunyaev
(2009c).

2.2.4 Thermodynamic asymmetry in the Lyω emission
and absorption profile

As explained by Chluba & Sunyaev (2009c), the largest correction
related to the two-photon formulation of the Lyω transfer problem
is due to the frequency-dependent asymmetry between the emis-
sion and absorption profiles around the Lyω resonance. This asym-
metry is given by a thermodynamic correction factor, which has
an exponential dependence on the detuning from the line centre,
ic.e. \( f_v \propto \exp[\alpha(v-v_\nu)/kT \gamma] \), where \( v_\nu \) is the transition frequency
for the Lyω resonance. Usually this factor can be neglected, since
for most astrophysical problems the main contribution to the num-
ber of photons is coming from within a few Doppler widths of the
line centre, where the thermodynamic factor indeed is very close
to unity. However, in the Lyω escape problem during hydrogen re-
combination contributions from the very distant damping wings are
also important (Chluba & Sunyaev 2009c,e), so that there \( f_v \neq 1 \)
has to be included.

As explained in Chluba & Sunyaev (2008b), the thermodynamic
factor also can be obtained in the classical picture, using the de-
tailed balance principle. However, in the two-photon picture this
factor has a natural explanation in connection with the absorp-
tion of photons from the CMB blackbody ambient radiation field
(Chluba & Sunyaev 2009c; Sunyaev & Chluba 2009). This process
leads to a \( \sim 10 \) per cent increase in the Lyω escape probabil-
ity, and hence accelerates hydrogen recombination. For the cor-
rection to the ionization history, Chluba & Sunyaev (2009c) obtained
\( \Delta N_e / N_e \sim -1.9 \) per cent at \( z \sim 1100 \). Note also that in the anal-
ysis of Chluba & Sunyaev (2009c) the thermodynamic correction
factor was included for all \( ns–1s \) and \( nd–1s \) channels with \( 3 < n < 10 \).
Below, we will use the results of Chluba & Sunyaev (2009c) for
this process.

2.2.5 Raman scatterings

Hirata (2008) also studied the effect of Raman scatterings on the
recombination dynamics, leading to an additional delay of hydro-
gen recombination by \( \Delta N_e / N_e \sim 0.9 \) per cent at \( z \sim 900 \). Here,
in particular, the correction due to 2s–1s Raman scatterings is im-
portant. Again it is expected that a large part of this correction can
be attributed to time-dependent aspects and the correct formulation
using detailed balance, and we are currently investigating this pro-
cess in detail. In the computations presented below, we will use the
results of Hirata (2008) for the effect of Raman scatterings.

2.3 Additional processes

There are a few more processes that here we only want to mention
very briefly (although with this the list is not meant to be absolutely
final or complete), and which we did not account for in the computa-
tions presented here. However, it is expected that their contribution
will not be very important.

2.3.1 Effect of electron scattering

The effect of electron scattering during hydrogen recombination
was also recently investigated by Chluba & Sunyaev (2009c) using
a Fokker–Planck approach. This approximation for the frequency
redistribution function may not be sufficient towards the end of hy-
drogen recombination, but in the overall correction to the ionization
history was very small close the maximum of the Thomson visibility
function, so that no big difference is expected when more accurately
using a scattering Kernel approach. Very recently Haimoud, Grin
& Hirata (in preparation) showed that this statement indeed seems
to be correct.

2.3.2 Feedback of helium photons

Very recently Chluba & Sunyaev (2009b) investigated the feedback
problem of helium photons including the processes of \( \gamma(He \\bar{I}) \rightarrow
\text{He} \\bar{I} \), \( \gamma(He \text{II}) \rightarrow \text{H} \text{I} \), \( \gamma(He \text{II}) \rightarrow \text{He} \\bar{I} \) and \( \gamma(He \text{II}) \rightarrow \text{H} \text{I} \) feedback.
They found that only \( \gamma(He \\bar{I}) \rightarrow \text{He} \\bar{I} \) feedback leads to some small
correction (\( \Delta N_e / N_e \approx +0.17 \) per cent at \( z \approx 2300 \)) in the ionization
history, while all the other helium feedback induced corrections are
negligible. This is because the \( \gamma(He \\bar{I}) \rightarrow \text{H} \text{I} \), \( \gamma(He \text{II}) \rightarrow \text{He} \\bar{I} \) and
\( \gamma(He \text{II}) \rightarrow \text{H} \text{I} \) feedback processes all occur in the pre-
recombinational epochs of the considered species, where the popu-
lations of the levels are practically in full equilibrium with the free
electrons and ions.

The \( \gamma(He \\bar{I}) \rightarrow \text{He} \\bar{I} \) feedback process was already studied by
Switzer & Hirata (2008a), but the result obtained by Chluba & Sunyaev
(2009b) seems to be smaller. However, it is clear that any
discrepancy in the helium recombination history at the 0.1–0.2 per
cent level will not be very important for the analysis of future CMB
data.

We would also like to mention that although the \( \gamma(He \text{II}) \rightarrow
\text{H} \text{I} \), \( \gamma(He \text{II}) \rightarrow \text{He} \text{I} \) and \( \gamma(He \text{II}) \rightarrow \text{H} \text{I} \) feedback processes do not
affect the ionization history, they do introduce interesting changes
in the recombinational radiation, increasing the total contribution
of photons from helium by \( 40–70 \) per cent (Chluba & Sunyaev
2009b).

2.3.3 Other small corrections

Recently, several additional processes during hydrogen recombi-
nation were discussed. These include: the overlap of Lyman series res-
onances caused by the thermal motion of the atoms (Haimoud et al.,
in preparation); quadrupole transitions in hydrogen (Grin, Haimoud
& Hirata, in preparation); hydrogen deuterium recombination (Fung
& Chluba, in preparation; Kholupenko, in preparation) and 3s–2s
and 3d–2s two-photon transitions (Chluba & Sunyaev 2008b). All
these processes seem to affect the ionization history of the Universe
at a level (well) below 0.1 per cent.

Furthermore, one should include the small re-absorption of
photons from the 2s–1s two-photon continuum close to the Lyω
resonance, where our estimates show that this leads to another
\( \Delta N_e / N_e \approx 0.1–0.2 \) per cent correction.
Figure 2. Same as Fig. 1, but for the angular power spectrum. For the fiducial cosmological model, we show the correction to the angular power spectrum due to the recombination history which is incorporated in the new \textsc{rco} training set (grey lines), and the correction due to the additional inclusion of \textsc{Ly} \alpha radiative transfer effects (Chluba & Sunyaev 2009a,c,e) and Raman scattering (Hirata 2008) (dark lines).

2.4 Overall correction

Fig. 1 shows the current best estimate of the remaining overall correction to the recombination history, while Fig. 2 translates these corrections into changes of the angular power spectrum. To describe the \textsc{Ly} \alpha transfer effects, we adopt the curve presented in Chluba & Sunyaev (2009a), which includes the results of the processes investigated by Chluba & Sunyaev (2009c) and Chluba & Sunyaev (2009e). We also include the effect of Raman scattering on hydrogen recombination as described in Hirata (2008).

It seems that the remaining uncertainty due to processes that were not taken into account here can still exceed the 0.1 per cent level, but likely will not lead to any significant addition anymore. However, it is clear that a final rigorous comparison of the total result from different independent groups will become necessary to assure that the accuracy required for the analysis of the \textsc{Planck} data will be reached. Such detailed code comparison is currently under discussion.

3 IMPACT OF RECOMBINATION UNCERTAINTIES ON COSMOLOGICAL PARAMETER ESTIMATION

Here, the \textsc{rco} code together with the latest training set described above is used to evaluate the impact of the recombination uncertainties in the cosmological parameter constraints inferred from CMB experiments.

All our analyses use the software packages \textsc{camb}\(^3\) (Lewis, Challinor & Lasenby 2000) and \textsc{cosmomc}\(^4\) (Lewis & Bridle 2002). \textsc{camb} is used to calculate the linear-theory CMB angular power spectrum, and has been modified here to include the recombination history as described by \textsc{rco}. \textsc{cosmomc} uses a MCMC method to sample the posterior distribution of the cosmological parameters from a given likelihood function which describes the experimental constraints on the CMB angular power spectrum.

The default parametrization which is included inside \textsc{cosmomc} exploits some of the intrinsic degeneracies in the CMB angular power spectrum (see e.g. Kosowsky, Milosavljević & Jimenez 2002), and uses as basic parameters the following subset:

\[
\mathbf{p}_{\text{old}} = \{ \Omega_0 h^2, \Omega_{\text{dm}} h^2, \theta, \tau, n_s, \log(10^9 A_s) \}.
\]  

Here, \(\Omega_0, h^2\) and \(\Omega_{\text{dm}}, h^2\) are the (physical) baryon and dark matter densities, respectively, where \(h\) stands for the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\); \(\theta\) is the acoustic horizon angular scale; \(\tau\) is the Thomson optical depth to reionization; and \(n_s\) and \(A_s\) are the spectral index and the amplitude of the primordial (adiabatic) scalar curvature perturbation power spectrum at a certain scale \(k_0\).

For the computations in this paper, we will use \(k_0 = 0.03\) Mpc\(^{-1}\).

It is important to note that the above parametrization \(\mathbf{p}_{\text{old}}\) makes use of an approximate formula for the sound horizon (Hu & Sugiyama 1996), which used in its derivation some knowledge on the recombination history provided by \textsc{recfast}. Therefore, the use of this parameter is not appropriate if we are changing the recombination history because \(\theta\), as computed inside \textsc{cosmomc}, is no longer keeping the meaning of the acoustic horizon angular scale, and thus we are imposing a flat prior in a parameter which is non-physical. In principle, this might introduce artificial biases. For this reason and to explore this issue, in this paper, we have modified \textsc{cosmomc} to use a new parametrization \(\mathbf{p}_{\text{new}}\), which is defined as

\[
\mathbf{p}_{\text{new}} = \{ \Omega_0 h^2, \Omega_{\text{dm}} h^2, H_0, \theta, \tau, n_s, \log(10^9 A_s) \}.
\]

where we have replaced \(\theta\) by \(H_0\) as a basic parameter. In this way, we still exploit some of the well-known degeneracies (e.g. that between \(\tau\) and \(A_s\)), but we eliminate the possible uncertainties which may arise from the use of \(\theta\), at the expense of decreasing slightly the speed at which the chain converges. As we show below, both the shape of the posteriors and the confidence levels for the rest of the parameters are practically unaffected by this modification in the parameterization for the minimal six-parameter model, despite the fact that the \textsc{cosmomc} code now assumes a flat prior on \(H_0\) instead of a flat prior on \(\theta\). In practice, this means that we can still use the default parametrization inside \textsc{cosmomc} for this minimal model, but keeping in mind that \(\theta\) is no longer a physical parameter. A modified version of the \textsc{params} subroutine inside \textsc{cosmomc} which uses this new parametrization from equation (2) is also available from the \textsc{rco} webpage.

Finally, concerning the convergence of the chains, all computations throughout this paper were obtained using at least five independent chains; those chains have been run until the Gelman & Rubin (1992) convergence criterion \(R - 1\) yields a value smaller than 0.005 for the minimal (six-parameter) case, and values smaller than 0.02–0.2 for those cases with a larger number of parameters.

3.1 Impact on parameter estimates for \textsc{Planck} alone

For the case of the \textsc{Planck} satellite, the mock data are prepared as follows. We assume a Gaussian symmetric beam, and the noise is taken to be uniform across the sky. For definiteness, we have adopted the nominal values of the beam and pixel noise which correspond to the 143-GHz \textsc{Planck} band, as described by The \textsc{Planck} Collaboration (2006). Thus, we have \(\theta_{\text{beam}} = 7.1\) arcmin, \(w_{\text{sky}}^{-1} = \sigma_{\text{sky}}^{-1} \Omega_{\text{sky}} = 1.53 \times 10^{-4} \mu K^2\) and \(w_p^{-1} = 5.59 \times 10^{-4} \mu K^2\), where \(w_{\text{sky}}\) and \(w_p\) indicate the intensity and polarization sensitivities, respectively. Note that using a single frequency channel is implicitly assuming that the remaining \textsc{Planck} frequencies have been used to fully remove the foreground contamination from this reference 143 GHz channel. For a complete discussion on which combination of \textsc{Planck} channels is more appropriate in terms of the parameter constraints, see Colombo, Pierpaoli & Pritchard (2009).
The fiducial cosmological model used for these computations corresponds to the WMAP5 cosmology (Hinshaw et al. 2009), and has parameters \( \Omega_b h^2 = 0.02273, \Omega_d h^2 = 0.1099, h = 71.9, \tau = 0.087, n_s = 0.963 \) and an amplitude \( 2.41 \times 10^{-9} \) at \( k = 0.002 \text{Mpc}^{-1} \) (or equivalently, \( A_s = 2.14 \times 10^{-9} \) at \( k = 0.05 \text{Mpc}^{-1} \) or \( \log(10^{10} A_s) = 3.063 \)). In addition, we use \( T_0 = 2.726 \text{K} \) and \( Y_{\text{He}} = 0.24 \). The mock data are then produced using the cosmological recombination history as computed with our complete multilevel recombination code for this particular cosmology (using \( n = 75 \) shells to model the hydrogen atom). The shape of the likelihood function adopted here corresponds to the exact full-sky Gaussian likelihood function given in Lewis (2005), which is implemented in \textsc{cosmomc} using the all_{,exact} format for the data.\footnote{See also http://cosmomc.info/viewtopic.php?t=231 for more details.} We note that for the mock \textit{Planck} data we use not a simulation but the actual (fiducial) angular power spectrum.

We run \textsc{cosmomc} for the case of \textit{Planck} mock data alone and the minimal model with six free parameters. For each set of runs, we consider two cases for the recombination history: one is the current version of \textsc{recfast} (v1.4.2), and the second one is the \textsc{rcco} code with our latest training set. The main result is summarized in Table 1, and the corresponding posterior distributions are shown in Fig. 3. We note that the sizes of the error bars on these parameters are similar to those obtained for \textit{Planck} by other authors (e.g. Bond et al. 2004; Colombo et al. 2009). For the six-parameter case, the largest biases do appear in \( n_s \) (\( -0.7 \sigma \)), \( \Omega_b h^2 \) (\( -0.3 \sigma \)) and \( \log(10^{10} A_s) \) (\( -0.4 \sigma \)).

The sign of the correction in \( n_s \), which is the parameter having the largest bias, can be understood as follows. The physical effects on the recombination history included in \textsc{rcco} produce a slight delay of the recombination around the peak of the visibility function, i.e. an excess of electrons with respect to the standard computation (see Fig. 1). This in turn produces a slightly larger Thomson optical depth, which increases the damping of the anisotropies at high multipoles. In order to compensate this excess of damping, the analysis which uses the standard \textsc{recfast} code gives a lower value of \( n_s \).

The biases on the other three parameters, i.e. \( \tau \) (which is well constrained by large-scale polarization measurements), \( \Omega_d h^2 \) and \( H_0 \), are negligible (i.e. less than \( 0.1 \sigma \)). However, if the analysis is repeated with the standard parametrization using \( \theta \) as basic parameter, then we would find a \( +1.8 \sigma \) bias in \( \theta \), while for the rest of the parameters the posteriors remain the same as before. For illustration of these facts, Fig. 4 shows a comparison of the posteriors for \( n_s \) obtained with the two different parametrizations, as well as the posteriors obtained for \( \theta \) if using the standard parametrization.

Finally, we have checked that the inclusion of lensed angular power spectra on the complete procedure modifies neither the shape of the posteriors nor the biases. Therefore, for the rest of the paper, we perform the computations in the case without lensing. This decreases the computational time by a significant factor.

### 3.2 Importance of the cosmology dependence of the correction

One of the questions that can be explored with \textsc{rcco} and the new training set is the importance of the cosmology dependence of the corrections to the recombination history. In order to obtain a simplified description of the recombination history, it is important to evaluate if the cosmology dependence of the corrections plays a significant role in determining the final shape of the posteriors. To explore this issue, we have modified the standard \textsc{recfast} code by introducing the following (cosmology independent) correction:

\[
\chi^{\text{new}}(z; \{\text{cosmology}\}) = \chi^{\text{recfast}}(z; \{\text{cosmology}\}) f(z),
\]

where this function \( f(z) \) is computed as

\[
f(z) = 1 + \frac{\Delta x_{\ell}}{x_{\ell}},
\]

for a certain fiducial cosmological model. By introducing this modification inside \textsc{cosmomc}, we have compared the posterior distributions obtained in this case with those obtained using the full \textsc{rcco} code with the new training set. The results of this comparison are shown in Fig. 5. The fact that the differences in the posterior are negligible indicates that there is no significant cosmological dependence in the corrections to the ionization history included in the \textsc{rcco} training set. Therefore, and for the case of the \textit{Planck} satellite, one can in principle use cosmology-independent ‘fudge functions’, such as the one presented in equation (4), to accommodate additional corrections to the recombination history.

As a final check in this section, we have explored the sensitivity of the fudge function \( f(z) \) to the cosmological model which is used as fiducial model for the full computation of the recombination history. Taking as a reasonable range of variation the \( 2 \sigma \) confidence interval which is obtained from the analysis of WMAP5 data (Dunkley et al. 2009), we have compared the \( f(z) \) functions obtained from cosmological models which differ \( 2 \sigma \) with respect to the actual fiducial model that is used in this paper. The result is that the changes in \( f(z) \) with respect to the solid curve presented in Fig. 1 are below 1 per cent, thus giving a negligible correction to the main
Figure 3. Impact on parameter estimates for the Planck satellite, in the six-parameter case. The dark solid line shows the 1D and 2D posterior distributions which are obtained when the RICO code is used to describe the recombination history. The light dashed solid line shows the same posteriors obtained with the RECFAST code. The biases are evident in $n_S$, $\Omega_b h^2$ and $\log(10^{10} A_S)$.

4 ESTIMATING THE CORRECTIONS FROM THE REMAINING RECOMBINATION UNCERTAINTIES

Although a full recombination code which includes all the physical effects discussed in Section 2 is still not available, there is a good agreement in the community about the list of relevant physical processes that have to be included. Moreover, all those effects have been already discussed in the literature by at least one group, so we have estimates of the final impact of these corrections on the recombination history (see Fig. 1).

Based on these estimates, we have quantified the impact of future corrections to the recombination history using the approximation described in equation (3), where $f(z)$ is taken from Fig. 1, as described in equation (4). Using this function, we have obtained the posterior distributions for the same case discussed above (nominal Planck satellite sensitivities and a six-parameter analysis). The basic results are shown in Table 2 and Fig. 6. The biases in the different parameters increase very significantly, as one would expect from the inspection of Fig. 1, specially for $n_S (-2.3\sigma)$, $\Omega_b h^2 (-1.65\sigma)$ and $\log(10^{10} A_S) (-1\sigma)$. Therefore, as pointed out by several authors (see e.g. Chluba & Sunyaev 2008b; Hirata 2008), the detailed treatment of the hydrogen Ly$\alpha$ radiative transfer problem constitutes the most significant correction to our present understanding of the recombinational problem. If this effect is not taken into account when analysing Planck data, the final constraints could be significantly biased.

4.1 Extended cosmological models

In this section, we describe to what extent the full set of corrections to the recombination history may affect the cosmological constraints on some extensions to the (minimal) six-parameter model which was used in this work. Throughout this section, we compare the complete recombination history (which includes the additional corrections shown in Fig. 1) with the constraints that would be inferred using RECFAST v1.4.2.

4.1.1 Tensor perturbations

We first consider the case of including tensor perturbations in addition to the previous model. For these computations, we consider...
an eight-parameter model, by including the spectral index of the primordial tensor perturbation ($n_T$) and the tensor-to-scalar ratio $r$, in addition to the parameters in equation (2). As pivot scale, we are using $k_0 = 0.05$ Mpc$^{-1}$.

Fig. 7 shows the impact of recombination uncertainties on the $r$–$n_S$ plane. The constraints on $r$ are determined by large-scale information, and therefore the modifications of the recombination history do not bias this parameter. However, due to the important bias on $n_S$, the 2D contours on this plane are significantly shifted.

4.1.2 Scale dependence of spectral index

The running of the spectral index is a possible extension to the simple $\Lambda$ cold dark matter model which is under debate in the literature in light of WMAP observations (see e.g. Spergel et al. 2007, which gives $n_{\text{run}} = dn_S/d\ln k = -0.055 \pm 0.030$). Fig. 8 illustrates the impact of the recombination uncertainties on the determination of the running of the spectral index. For this computation, we considered a seven-parameter model by adding $n_{\text{run}} = dn_S/d\ln k$, which again is computed at the same pivot scale of $k_0 = 0.05$ Mpc$^{-1}$. When studying the 1D posterior distributions for all parameters, we find that the inclusion of $n_{\text{run}}$ does not affect the shape of the rest of the posteriors. The bias on $n_{\text{run}}$ due to recombination uncertainties is not very significant (changes from $n_{\text{run}} = -0.0012 \pm 0.0050$ to $n_{\text{run}} = -0.0034 \pm 0.0050$, i.e. 0.4σ), but has to be taken into account.

For completeness, we have run a nine-parameter case, in which we allow to vary simultaneously $r$, $n_T$ and $n_{\text{run}}$ in addition to the other six parameters. In this case, we have checked that the contours shown in Figs 7 and 8 are not affected by the inclusion of the other parameter.

4.1.3 Curvature

Another possible extension of the minimal six-parameter model is to constrain simultaneously the spatial curvature, $\Omega_K$. The inclusion of this additional parameter introduces a practical complication, since in our new parametrization $p_{\text{new}}$ (equation 2), the $H_0$ parameter is highly correlated with $\Omega_K$. Fig. 9 presents the posterior distributions for this case, in which the degeneration between $H_0$ and $\Omega_K$ is clearly visible. There are two things to note. First, there is no significant bias to $\Omega_K$ due to the inclusion of recombination uncertainties, as one would expect since this parameter is mainly constrained by information at angular scales around the first Doppler peak. Secondly, the shape of the remaining posteriors and the biases to the parameters are not significantly affected by the inclusion of this additional parameter.

4.1.4 Residual Sunyaev–Zeldovich clusters/point source contributions

One common extension of the minimal model is the inclusion of some parameters describing the residual contribution of Sunyaev–Zeldovich (SZ) clusters or point sources which are left in the maps after the component separation processes. For the case of the Planck satellite, these are known to be the major contaminants at small angular scales (see e.g. Leach et al. 2008).

For illustration, here we will consider the case of the residual SZ cluster contribution. One of the simplest parametrizations of the SZ contribution to the angular power spectrum is to use a fixed template from numerical simulations, and fit for the relative amplitude by
Table 2. Estimate of the impact of remaining recombination uncertainties on the confidence limits of the cosmological parameters for the Planck satellite. Confidence intervals are derived as in Table 1 so the confidence interval encompasses 68 per cent of the probability.

<table>
<thead>
<tr>
<th></th>
<th>With $f(z)$ from Fig. 1</th>
<th>With RECFAST (v1.4.2)</th>
<th>Bias (in $\sigma$)</th>
<th>Bias (in per cent)</th>
<th>Fiducial model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$ ($\times 10^2$)</td>
<td>$2.274^{+0.014}_{-0.015}$</td>
<td>$2.250^{+0.015}_{-0.013}$</td>
<td>$-1.65$</td>
<td>$-1.01$</td>
<td>$2.273$</td>
</tr>
<tr>
<td>$\Omega_{dm} h^2$</td>
<td>$0.1098^{+0.0013}_{-0.0012}$</td>
<td>$0.1098^{+0.0013}_{-0.0012}$</td>
<td>$0.02$</td>
<td>$-0.09$</td>
<td>$0.1099$</td>
</tr>
<tr>
<td>$H_0$</td>
<td>$71.9 \pm 0.6$</td>
<td>$71.7 \pm 0.6$</td>
<td>$-0.41$</td>
<td>$-0.28$</td>
<td>$71.9$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.087 \pm 0.006$</td>
<td>$0.086 \pm 0.006$</td>
<td>$-0.18$</td>
<td>$-1.15$</td>
<td>$0.087$</td>
</tr>
<tr>
<td>$n_S$</td>
<td>$0.963^{+0.0037}_{-0.0026}$</td>
<td>$0.955^{+0.0035}_{-0.0038}$</td>
<td>$-2.27$</td>
<td>$-0.83$</td>
<td>$0.963$</td>
</tr>
<tr>
<td>$\log(10^{10} A_S)$</td>
<td>$3.064 \pm 0.009$</td>
<td>$3.055 \pm 0.009$</td>
<td>$-0.99$</td>
<td>$-0.26$</td>
<td>$3.063$</td>
</tr>
</tbody>
</table>

Figure 6. Posterior distributions obtained with the remaining recombination uncertainties (see text for details). The solid line corresponds to the posteriors using the correct description of the recombination history, while the dashed lines represent the case in which the current description (RECFAST v1.4.2) is used.

using an additional parameter, $A_{SZ}$. This approach is similar to the one used in Spergel et al. (2007), where they parametrize the SZ contribution by the model of Komatsu & Seljak (2002) but allowing for a different normalization through the $A_{SZ}$ parameter.

When the parameter constraints are inferred with the inclusion of this additional parameter, we find that (neglecting the intrinsic bias due to the degeneracy between $n_S$ and $A_{SZ}$) the relative bias is practically not affected. In particular, we obtain a bias of $-1.42$, $-0.44$, $-2.09$ and $-1.03\sigma$ for $\Omega_b h^2$, $H_0$, $n_S$ and $\log(10^{10} A_S)$, respectively.

Those numbers are comparable to the net biases which have been obtained for the minimal model in Table 2.

5 IMPACT ON PARAMETER ESTIMATES USING PRESENT-DAY CMB EXPERIMENTS

One would expect that the order of magnitude of the corrections to the recombination history discussed in previous sections (at the level of 1–2 per cent) would have a negligible impact on the
parameter constraints that we would infer from present-day CMB experiments. As shown in FCRW09, the changes introduced in the power spectra (both temperature and polarization) are significant at high multipoles, in that they are larger than the benchmark level estimated as $\pm 3/\ell$ (see Seljak et al. 2003).

To quantify this fact, we have obtained the posterior distributions for the case of the minimal model with six free parameters, combining the CMB information from WMAP5 (Hinshaw et al. 2009), ACBAR (Kuo et al. 2007), CBI (Sievers et al. 2007) and Boomerang (Jones et al. 2006; Montroy et al. 2006), together with measurements on the linear matter power spectrum based on luminous red galaxies from Sloan Digital Sky Survey Data Release 4 (SDSS-DR4) (Tegmark et al. 2006). Fig. 10 presents the results for the case of using the standard RECFAST recombination history, together with the case of using our most complete description of the recombination history, as presented in the previous section. As expected, the modifications on the shape of the posteriors are very small and no biases are seen in the parameters except for $n_S$ and log(10$^{10}$A$_S$), which are slightly biased. Our analysis including the full description of the recombination history gives $n_S = 0.970 \pm 0.013$ and log(10$^{10}$A$_S$) = 3.075 $\pm$ 0.038, while the result using RECFAST v1.4.2 gives $n_S = 0.967^{+0.013}_{-0.012}$ and log(10$^{10}$A$_S$) = 3.066$^{+0.038}_{-0.036}$. In other words, this is a $\sim -0.25$ and $\sim -0.22\sigma$ bias on $n_S$ and log(10$^{10}$A$_S$), respectively. For completeness, we have run also the MCMC for the case of using WMAP5 data alone. In that case, the bias decreases to $\lesssim -0.15\sigma$ for those two parameters.

6 DISCUSSION

In this section, we now discuss the results presented in this paper focusing in three particular aspects. First, we discuss the robustness of our results against possible modifications of the physical description of the recombination process. Secondly, we also consider the dependence of the obtained biases if additional parameters are included in the MCMC analysis. Finally, we discuss the possible impact of recombination uncertainties on the results obtained from other cosmological probes different from CMB anisotropies.

6.1 Dependence of the results on the description of the recombination process

As discussed above (Section 2), there is a wide agreement in the community about the list of physical processes which should be included in the description of the cosmological recombination process. In many cases, these physical processes have been treated separately by at least two separate groups, and the agreement on the signs and amplitudes of the corrections is excellent in most of the cases (e.g. see the compilation of uncertainties in the physics of recombination in table 2.1 of Wong 2008). Although an agreement at the level of $\lesssim 0.1$ per cent is still not reached, we are almost there, as the remaining uncertainties seem to be at the level of 0.1–0.3 per cent between the different groups. In this sense, one would expect that a code which includes self-consistently all those processes should obtain essentially the same biases that have been described in Section 4, and have been reported in Table 2.

However, apart from the processes described in Section 2.3, there are still some possible uncertainties which might lead to measurable biases on the cosmological parameters. Below, we now briefly address them.

6.1.1 Hydrogen recombination

One effect which might lead to additional biases on the cosmological parameters is the inclusion of very high-$n$ states in the cosmological hydrogen recombination. The computations in this paper are based on a training set which uses $n = 75$ shells to describe the hydrogen atom. To explore the dependence of higher number of shells, we have repeated the standard six-parameter computation for the mock Planck data presented in Sections 3.1 and 4, but taking as a reference model the one computed using $n = 110$ hydrogen shells, and trying to recover it with rico (which uses $n = 75$ hydrogen shells). The recovered posteriors using the rico code in this case do not show any appreciable bias in any of six parameters of the minimal model. This is illustrated in Fig. 11, where we present the posterior distribution for the $n_S$ parameter, which is the one having the largest bias.

We would like to stress that this conclusion has been obtained for the case of using Planck data alone. Thus, it should be revised if we incorporate in the analysis additional constraints from other cosmological data sets [e.g. reionization constraints from 21 cm data, baryon acoustic oscillations (BAOs), supernova data, etc.] which might help in reducing the error bars and in breaking some degeneracies from CMB data alone. Here, in particular, the degeneracies in connection with reionization [e.g. between $\tau$, $n_S$ and log(10$^{10}$A$_S$)] and the ‘relatively large’ error on $\tau$ from the Planck alone ($\sim 7$ per cent) are important. Since a detailed treatment of the high-$n$ states leads to a slight increase of the residual electron fraction at low redshifts, a part of this effect is currently buried in the uncertainties due to reionization. In this context also the possible
annihilation of dark matter will have to be considered carefully (see Section 6.2).

6.1.2 Helium recombination

As mentioned in Section 2.3, and discussed in FCRW09, we do not expect major changes in our current understanding of the helium recombination, at a level which might be relevant for the computation of CMB anisotropies. This statement assumes that the uncertainties in the modelling of the helium atom are well controlled, but, as described in Rubiño-Martín et al. (2008), we still lack an accurate description of the photoionization cross-sections, energies or transition rates for the helium atom, which might lead to small changes in these results. One may also wonder whether the small differences found for the correction caused by helium feedback processes (see Section 2.3.2) could matter.

By far, the most relevant correction to the helium recombination history is caused by the absorption of HeI photons by neutral hydrogen, although the inclusion of the $^2\text{P}_1 - ^1\text{S}_0$ intercombination line also gives some contribution. These two effects are already included in the codes, both in RICO (FCRW09) and in RECFAST v1.4.2 (Wong et al. 2008). For completeness, we have also quantified for this paper the impact that these two corrections to the helium recombination would have on the recovered cosmological parameters. Our computations show that, if these corrections are not included (which in practice corresponds to setting RECFAST\_Heswitch = 0 in RECFAST v1.4.2, or equivalently, using RECFAST v1.3), then the resulting biases on the parameters are found to be $-3.2$, $-2.0$, $-1.2$ and $-0.7\sigma$ for $n_S$, $\Omega_b h^2$, $\log(10^{10} A_S)$ and $H_0$, respectively. For illustration, we show in Fig. 12 a comparison of the posterior distribution obtained for the $n_S$ parameter when using the RECFAST code with (RECFAST\_Heswitch = 6) and without (RECFAST\_Heswitch = 0) all the corrections to the helium recombination.

This simple computation shows that additional corrections to the helium recombination history at the $\sim 0.1$ per cent level will not matter much to the analysis of future Planck data. Therefore, the physics of helium recombination already seems to be captured at a sufficient level of precision, when including the acceleration caused by the hydrogen continuum opacity and the $^2\text{P}_1 - ^1\text{S}_0$ intercombination line, which together lead to a $\sim -3$ per cent correction to $X_e(z)$ at $z \sim 1800$ (FCRW09).
Figure 10. Impact on parameter estimates for present-day data (WMAP5+ACBAR+CB1+B03), together with SDSS LRGs. For this minimal six-parameter case, there are no obvious biases on the parameters, although there is a small difference in the shape of the posterior for $n_S$. As in Fig. 6, solid lines represent the posterior distributions obtained when our most complete description of the recombination process is used, while dashed lines use the current description (RECFAST v1.4.2).

6.2 Recombination uncertainties and other extended cosmological models

In addition to those extended models described in Section 4.1, there are a number of possible non-standard models for which the inclusion of refined recombination physics might be of importance. For example, when using current CMB data to constrain the presence of hypothetical sources of Ly$\alpha$ resonance radiation or ionizing photons at high redshifts (e.g. Peebles, Seager & Hu 2000; Bean, Melchiorri & Silk 2003, 2007); or to probe dark matter models with large annihilation cross-section (e.g. Padmanabhan & Finkbeiner 2005; Galli et al. 2009a; Huetsi, Hektor & Raidal 2009); or energy release by long-lived unstable particles (Chen & Kamionkowski 2004; Zhang et al. 2007); or when exploring the variation of fundamental constants with time [see e.g. Galli et al. (2009b) for Newton’s gravitational constant or Landau et al. (2008) for the fine structure constant and the Higgs vacuum expectation value], it is obvious that neglecting physically well understood additions to the recombination model, as described in Section 2, could lead to spurious detections or confusion, in particular, when the possible effects are already known to be rather small (e.g. see Galli et al. 2009a, in the case of dark matter annihilations).

More generally speaking, given that the largest recombination uncertainties are obtained for $n_S$, $\Omega_b h^2$ and $\log(10^{10} A_s)$, one can say that any additional parameter showing a strong correlation with those three might be biased if an incomplete description of the recombination physics is used. In this sense, neglecting the refinements to the recombination model could be as important as not taking into account, for instance, uncertainties in the beam shapes, which also have been shown to compromise our ability to measure $n_S$ (Colombo et al. 2009) or the combined effect of beam and calibration uncertainties, which introduce significant biases to $n_S$ and $\Omega_b h^2$ (Bridle et al. 2002), although other parameters (like $\Omega_k$) are essentially not affected because these are basically constrained by the position of the peaks, and not by their amplitudes.

6.3 Recombination modelling and other cosmological probes

The combination of CMB data with other data sets usually helps to improve the parameter constraints, in some cases by breaking internal degeneracies which are inherent to the CMB data alone. One of the commonly used external data sets is BAOs. de Bernardis et al. (2009) have recently shown that a possible delay of recombination (Peebles et al. 2000) could lead to spurious detections or confusion, in particular, when the possible effects are already known to be rather small (e.g. see Galli et al. 2009a, in the case of dark matter annihilations).
photon dates to the inclusion of an additional number of shells in the description of the hydrogen atom. The black solid line has been obtained from a fiducial model with \( n = 110 \) hydrogen shells. The dashed line corresponds to the posterior distribution recovered using the \texttt{RICO} code only, with a training set based on \( n = 75 \) shells. The dotted line corresponds to the posterior distribution recovered using the current \texttt{RECFAST} code. Note that we did not include for this computation the corrections due to Ly\( \alpha \) radiative transfer and Raman scattering.

![Figure 11. Bias on the \( n_s \) parameter due to the inclusion of an additional number of shells in the description of the hydrogen atom. The black solid line corresponds to the posterior distribution recovered using the \texttt{RICO} code only, with a training set based on \( n = 75 \) shells. The dotted line corresponds to the posterior distribution recovered using the current \texttt{RECFAST} code. Note that we did not include for this computation the corrections due to Ly\( \alpha \) radiative transfer and Raman scattering.](https://academic.oup.com/mnras/article-abstract/403/1/439/1019597)

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![Figure 12. Bias on the \( n_s \) parameter for different modelings of the helium recombination history. The black solid line corresponds to the posterior distribution recovered using the \texttt{RICO} code and the additional corrections used in Section 4. The dotted line corresponds to the posterior distribution recovered using the current \texttt{RECFAST} code (v1.4.2), while the dashed line uses the same version of the code but without any correction to the helium recombination history.](https://academic.oup.com/mnras/article-abstract/403/1/439/1019597)

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In addition, we would like to point out that in principle one could reduce the uncertainty in our knowledge of the recombination epoch and its possible non-standard extensions [e.g. due to annihilating dark matter (Padmanabhan & Finkbeiner 2005)] in two ways. On the one hand, one could search for the imprint of the cosmological hydrogen recombination lines on the CMB angular power spectrum (Rubíñ--Martín et al. 2005; Hernández-Monteagudo, Rubíñ--Martín & Sunyaev 2007), which arises due to the resonant scattering of CMB photons by hydrogen atoms at each epoch (Basu, Hernández-Monteagudo & Sunyaev 2004). On the other hand, one could also try to directly observe the photons that are emitted during the recombination epoch. Today, these photons should still be visible as small distortions of the CMB energy spectrum in the mm, cm and dm spectral bands (e.g. see Dubrovich 1975; Dubrovich & Stolyarov 1997; Chluba & Sunyaev 2006a; Rubíñ--Martín et al. 2006; Chluba et al. 2007; Rubíñ--Martín et al. 2008; Chluba & Sunyaev 2009b). These observations would not only open another possibility to determine some of the key cosmological parameters, such as the primordial helium abundance, the number density of baryons and the CMB monopole temperature at recombination (e.g. Chluba & Sunyaev 2008a), but they would also allow us to directly check our understanding of the recombination process and possible non-standard aspects (e.g. see Sunyaev & Chluba 2009, for an overview), for example, in connection with early energy release (Chluba & Sunyaev 2009d) or dark matter annihilations (Chluba 2009).

7 CONCLUSIONS

In this paper, we have performed a MCMC analysis of the expected biases on the cosmological constraints to be derived from the upcoming Planck data, in the light of recent developments in the description of the standard cosmological recombination process. Our main conclusions are as follows.

(i) An incomplete description of the cosmological recombination process leads to significant biases (of several \( \sigma \)) in some of the basic parameters to be constrained by the Planck satellite (see Table 2), and in general by any future CMB experiment. However, these corrections have a major impact for present-day CMB experiments: for instance, using WMAP5 data plus other cosmological data sets, we find a \( \approx -0.25 \) and \( \approx -0.22 \sigma \) bias on \( n_s \) and \( \log(10^{10} A_s) \), respectively, while the rest of the parameters remain unchanged.

(ii) Today, it seems that our understanding of cosmological recombination has reached the sub-per cent level in \( X_e \) at redshifts 500 \( \lesssim z \lesssim 1600 \). However, it will be important to cross-validate all of the considered corrections in a detailed code comparison, which currently is under discussion among the different groups.

(iii) Given the range of variation of the relevant cosmological parameters, it is possible to incorporate all the new recombination corrections by using (cosmology independent) fudge functions. Here, we described one possibility which uses a simple correction factor to the results obtained with \texttt{RECFAST} (see Section 3.2). We provide the function \( f(z) \) on the \texttt{RICO} webpage.6

(iv) The physics of helium recombination already seems to be captured at a sufficient level of precision, when including the acceleration caused by the hydrogen continuum opacity and the \( 2\text{P}_1 - 1\text{S}_0 \) intercombination line. The biases caused by neglecting only these corrections are \(-0.8 \) and \(-0.4 \sigma \), for \( n_s \) and \( \Omega_b h^2 \), respectively.

(v) When allowing for more non-standard additions to the recombination model (e.g. related to annihilating dark matter), the biases introduced by an inaccurate recombination model could lead to spurious detections or additional confusion (see Section 6.2).

We note that most of the constraints and biases on the different parameters obtained in this paper and described in these conclusions have been produced for the case of mock Planck data alone. Therefore, our conclusions should be updated if additional data sets either from the CMB side (e.g. small scale measurements or future polarization measurements) or from other cosmological probes

6 http://cosmos.astro.uiuc.edu/rico