Extended X-ray emission from radio galaxy cocoons

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ABSTRACT
We study the emission of X-rays from lobes of Fanaroff–Riley type II (FRII) radio galaxies by inverse-Compton scattering of microwave background photons. Using a simple model that takes into account injection of relativistic electrons, their energy losses through adiabatic expansion, synchrotron and inverse-Compton emission, and also the stopping of the jet after a certain time, we study the evolution of the total X-ray power, the surface brightness, angular size of the X-ray bright region and the X-ray photon index, as functions of time and cocoon size, and compare the predictions with observations. We find that the radio power drops rapidly after the stopping of the jet, with a shorter time-scale than the X-ray power. The X-ray spectrum initially hardens until the jet stops because the steepening of electron spectrum is mitigated by the injection of fresh particles, for electrons with $\gamma \geq 10^3$. This happens because of the concurrence of two times scales, that of the typical jet lifetimes and cooling due to inverse-Compton scattering ($\sim 10^{7-8}$ yr), of electrons responsible for scattering cosmic microwave background photons into keV range photons (with $\gamma \sim \sqrt{T_{\text{keV}}/A_{\text{CMB}}}$). Another finding is that the ratio of the X-ray to radio power is a robust parameter that varies mostly with redshift and ambient density, but is weakly dependent on other parameters. We also determine the time-averaged ratio of X-ray to radio luminosities (at 1 keV and 151 MHz) and find that it scales with redshift as $\propto (1 + z)^{3.8}$, for typical values of parameters. We then estimate the X-ray luminosity function for FRII radio galaxies and estimate the number of these diffuse X-ray bright objects above a flux limit of $\sim 3 \times 10^{-18}$ erg cm$^{-2}$ s$^{-1}$ to be $\sim 25$ deg$^{-2}$.

Key words: galaxies: active – intergalactic medium – X-rays: galaxies.

1 INTRODUCTION
A number of radio galaxies have been observed to emit diffuse X-rays in the region between the nucleus and radio hot spots in recent years. This emission has been interpreted as inverse-Compton (IC) scattering of the cosmic microwave background (CMB) photons by relativistic electrons in the radio lobes that have lost most of their energy, and their Lorentz factor have come down to $\gamma \sim 10^3$ (Fabian et al. 2003; Croston et al. 2005; Blundell et al. 2006; Johnson et al. 2007; Erlund, Fabian & Blundell 2008; Fabian et al. 2009; Itoh et al. 2009). That relativistic electrons in radio galaxy cocoons could upscatter CMB photons to X-rays has long been anticipated since the discovery of CMB (e.g. Felter & Rees 1967).

Since the increase in the CMB energy density with redshift $(\propto (1 + z)^4)$ compensates for the cosmological surface brightness dimming, this extended emission has been seen at both low and high redshift, and has been used as a probe of the relativistic plasma in the lobe. Combined with radio observations, this emission provides constraints on the magnetic field and on the energy distribution among the relativistic particles, or both. While electrons with $\gamma \geq 10^4$ emit GHz synchrotron radiation (for typical magnetic fields of a few $\mu$G), it requires $\gamma \geq 10^3$ to upscatter the CMB photons to the observed 1–10 keV range in X-rays.

The simultaneous radiation in two different wavelengths caused by the same parent electron population provides an excellent opportunity to probe the physical nature of the radiating object. Comparing the X-ray and radio emission from lobes, Blundell et al. (2006) showed that the injected electron energy distribution has a low-energy turn-over at $\gamma_{\text{min}} \geq 10^3$, since the X-ray bright regions did not coincide with the radio hot spots. The comparison between X-ray and radio studies also shed light on the magnetic field. Croston et al. (2005) argued that the inferred magnetic field strength is close to the equipartition value (lying between 0.3 and 1.3 times the equipartition value).

In this paper, we use a variant of a model of emission from Fanaroff–Riley type II (FRII) radio galaxies advocated by Kaiser & Alexander (1997) (hereafter KA97) and Kaiser, Dennett-Thorpe & Alexander (1997) (hereafter KDA97) to study the evolution of X-ray power and surface brightness with time, and as functions of the radio lobe size, ambient density, redshift and the jet power. We do not assume self-similarity of the evolution of the radio lobe.
assumed by KA97 and KDA97, in order to study the effect of the cessation of jet activity on the X-ray surface brightness.

The evolution of radio lobes after the stopping of jets has been discussed by Komissarov & Gubanov (1994), although they did not address the soft X-ray emission by old electrons. Kaiser & Cotter (2002) have also discussed the evolution of radio galaxies in connection with the observations of radio relics, and Reynolds, Heinz & Begelman (2002) have numerically studied the expansion of ‘dead’ radio galaxies.

We first discuss the KA97 and KDA97 model and the changes introduced in the model in the present paper, and then discuss the results of our calculations.

2 EVOLUTION OF RADIO GALAXY LOBES FOR FRII SOURCES

2.1 The KDA97 model

The KDA97 model of the dynamical expansion of an FRII-type radio source assumes a self-similar expansion, driven by twin jets emerging from the nucleus in opposite directions, pushing the surrounding environment. The jets produce strong shocks where the jet particles are accelerated and these particles made the cocoon expand. The density distribution in the ambient medium is assumed to be a power-law, \( \rho = \rho_0 (r/a_0)^{-\beta} \), where \( \rho_0 \) is the density at a core radius \( a_0 \). If the jet power is denoted by \( Q_j \), then the combination \( Q_j / \rho_0 a_0^3 \) has the dimension of length, and, in a self-similar model, it is proportional to the length of the jet, \( L_j(t) \) (Falle 1991). Half of the cocoon volume is approximated by a cylinder of length \( L_j \) and a base radius. The ratio \( R \) between the length and half-width of this cylinder is referred to as its axial ratio.

The dynamics of the cocoon is determined by the cocoon pressure, \( p_c = (\Gamma_c - 1)(u_e + u_b) \), where the contributions from different components have been added: (a) relativistic electrons with energy density \( u_e \) and adiabatic index \( \Gamma_c \) and (b) a tangled magnetic field with energy density \( u_b \). (This implicitly assumes that \( \Gamma_c = 4/3 \), the adiabatic index appropriate for magnetic field pressure.) Suppose the cocoon volume scales as \( n^\alpha \). Then the cocoon pressure evolves with time as \( p_c \propto t^{-\alpha+1} \) (see KDA97). The cocoon is assumed to be divided into small volume segments filled with magnetized plasma and particles that are injected into the cocoon at the jet terminal shock.

The initial electron energy distribution is assumed to be a power law in energy,

\[
n(\gamma, t_0) d\gamma = n_0 \gamma^{-\beta} d\gamma,
\]

where the spectrum ranges between \( \gamma_{\text{min}} \) and \( \gamma_{\text{max}} \). The electrons lose energy through adiabatic expansion, synchrotron and IC losses. KDA97 found a closed-form solution for the energy distribution at a later time \( n(\gamma, t) \) in the case of a self-similar expansion of the cocoon.

The ratio of the energy density in particles to that in magnetic field is assumed to be \( r \), so that the magnetic energy density \( u_b \) at time \( t \) is given by

\[
u_b(t) = \frac{r p_e(t)}{(\Gamma_c - 1)(r + 1)}.
\]

Assuming that for synchrotron radiation, electrons emit only at their critical frequency \( \nu_c = \nu_c \gamma_0 \), where \( \nu_c \) is the Larmor frequency, the total radio power emitted by electrons in a volume element \( dV \) of the cocoon (radio power per unit frequency and solid angle) is given by

\[
\delta P = \frac{1}{6\pi} c u_b \frac{\gamma^2}{\nu} n(\gamma) dV.
\]

Here, the volume element at time \( t \) is related to the initial time interval over which the particles residing in it were injected, as

\[
\delta V(t) = \frac{(\Gamma_c - 1)Q_j}{\rho_c(t)} (4R_j^2)^{1/(\Gamma_c-1)} \left( \frac{L_j}{R_j} \right)^{\alpha} \delta t_0.
\]

The total radio power is then calculated by integrating over all time,

\[
P_c = \int_0^t \delta P_c,
\]

using equations (3) and (4).

2.2 Evolution without self-similarity

In this paper, we would like to study the evolution of radio and X-ray power on a time-scale that is longer than the typical jet lifetime, and study the effect of the stopping of jets on the X-ray emission. As we will explain later in the paper, the stopping of the jet has important effects on X-ray power of the cocoon, and the results are markedly different than in the case where the jet continues to be active. To do this, we cannot use the self-similar evolution assumed in KDA97. Instead, we calculate the size evolution the radio lobe in the following way that captures the basic physical processes in a simple manner.

Following the standard evolutionary picture (Scheuer 1974; Begelman & Cioffi 1989; Reynolds & Begelman 1997), radio jets inject relativistic particles into a cocoon, which is overpressured (Nath 1995) and drive a strong shock into the ambient medium. The speed with which this shocked shell expands is determined by the ram pressure of the ambient gas that enters the shock. Assuming that the pressure inside the cocoon at any given time is uniform (see KA97), and a value \( p_c(t) \), and assuming that radiation loss is small, we have for the expansion of the cocoon,

\[
Q_j(t) = \frac{1}{\Gamma_c - 1} \left( V_c p_c + \Gamma_c p_v V_v \right)
\]

\[
\frac{dL_j}{dt} = \sqrt{\frac{p_c}{p_0}},
\]

where \( V_c \) is the volume of the cocoon and \( \rho_0 \) is the ambient density. For simplicity, we assume a constant axial ratio \( R \), and the volume is assumed to be \( V_c = \frac{4}{3\pi} L_j^3 \) (KA97).

For the jet luminosity we specify a jet lifetime \( t_j \), so that \( Q_j \) is a constant for \( t \leq t_j \) and \( Q_j = 0 \) for \( t > t_j \), where \( t_j \sim 10^{-8} \) yr. Numerical simulations of ‘dead’ radio galaxies after the cessation of jet activity have shown that the overpressured cocoon continues to expand until a pressure equilibrium is established with the ambient medium (Reynolds et al. 2002).

We assume a constant axial ratio \( R \sim 2 \), which is an average value (Leahy & Williams 1984), and, following Wang & Kaiser (2008), we use a profile of the ambient density as given by,

\[
p_\Lambda = \rho_0 (r/a_0)^{-\beta} = \Lambda r^{-\beta},
\]

where \( \Lambda = \rho_0 a_0^3 \). For \( \beta = 2 \), \( \Lambda \) has the units of \( \text{g cm}^{-1} \), and one can infer its value from the observations of environments of radio galaxies. Typically, for \( \beta = 2 \) and \( a_0 \sim 200 \text{ kpc} \), an electron density \( n_e = \rho_0/\mu m_p \sim 10^{-4} \text{ cm}^{-3} \) would imply \( \Lambda \sim 10^{10} \text{ g cm}^{-1} \), a value we assume for most of our models in this paper. Observations of individual galaxies show \( \Lambda \) to be in the range of \( 10^{10-20} \text{ g cm}^{-1} \) (Fukazawa, Makishima & Ohashi 2004), whereas Jetha et al. (2007)
inferred $\Lambda \sim 10^{39} - 10^{40} \text{g cm}^{-1}$ in group environments, although these determinations assume $\beta \sim 1.5$. For simplicity, because of the ability of combining two free parameters ($p_0$ and $a_0$) into one ($\Lambda$), we assume a value of $\beta = 2$ for our calculations.

Solving equations (6), we can therefore determine the cocoon pressure $p_c(t)$ at any instant, from which we determine an equipartition value of magnetic field, using equation (2). Assuming (as in KDA97) a ratio $r$ between particle and magnetic field energy density. The corresponding electron energy density is then given by $u_e(t) = u_B(t)/r$, assuming that there is no thermal particles in the cocoon (i.e. in the language of KDA97, $u_T = 0$).

In the KDA97 model, particles are assumed to be injected at the termination point of the jet. In our case, after the jet stops, the injection of particles also stops, and so the integration in the equation (5), for the determination of radio power at a given time $t$, has an upper limit of min ($t_i$, $t_j$). To perform this integration, we rewrite the equation for volume segments (equation 4) which in the KDA97 model used the exponent $a_I$ for self-similar evolution of cocoons. For $t_i \leq t_j$, any small volume segment of the cocoon at a given instant $t$ can be related to the pressure at the time ($t_j$) when the electrons in this segment were injected into the cocoon. One can rewrite equation (4) in the KDA97 model as (for $t < t_j$)

$$\delta V(t) = \left( \frac{r_c}{r_i} \right)^{a_I} \frac{1}{4R^2} \frac{dV}{dV_i} \frac{p_i(t)}{p_i(t_j)}^{1/r_c} \delta t_i.$$  \hspace{1cm} (8)

The evolution of the Lorentz factor is explicitly solved using the loss equation, 

$$\frac{d\gamma}{d\tau} = \frac{1}{3} \frac{1}{V} \frac{dV}{d\tau} - \frac{4}{3} \frac{\sigma_T c}{m_e} \gamma^2 (u_B + u_e),$$  \hspace{1cm} (9)

where the first term denotes adiabatic energy loss and is computed using the results of equations (6). The second term combines radiation loss in synchrotron and IC scattering. Here, $u_e = aT_{\text{CMB}}$ is the CMB photon energy density, $\sigma_T$ is the Thomson cross-section, $m_e$ is the electron mass and $c$ is the speed of light.

Thus, beginning with an initial energy distribution law with a power-law index $p$, one can solve for the energy distribution at any given time, $n(\gamma, t)$, given the initial distribution, $n(\gamma, t_j)$. Note that this was analytically done by KDA97 for a self-similar evolution of the cocoon, and we explicitly solve it in order to go beyond self-similarity.

Using this knowledge of $n(\gamma, t)$, we can then use equation (3) in conjunction with equation (8), and integrate over time to calculate the radio power at a given frequency and at a given instant.

### 2.3 Inverse-Compton radiation

We extend this formalism further to determine the IC emission as a function of time. First, we note that the Lorentz factor $\gamma$ boosts a CMB photon of frequency $\nu_{\text{CMB}}$ into an energy $h\nu_{\gamma}^2 \nu_{\text{CMB}}$. The precise calculation of the IC power for this photon would require one to consider the total spectrum of CMB photons. But we can simplify it for our purpose here by assuming all CMB photons to have a single frequency. We assume that the CMB photon distribution function is given by $\nu(\epsilon) = \nu_0 \delta(\epsilon - k_B T_{\text{CMB}})$, where the normalizing factor $\nu_0$ can be calculated by requiring that total energy density $\int \nu(\epsilon) d\epsilon = aT_{\text{CMB}}^4$, and one has $\nu_0 = aT_{\text{CMB}}^4/k_B$. The total scattered power depends on the integral [see equation (7.29a) in Rybicki & Lightman 1979]

$$I' = \int \nu(\epsilon) \frac{d\nu}{d\epsilon} = \frac{aT_{\text{CMB}}^4}{k_B} (k_B T_{\text{CMB}})^{(p-1)/2}.$$  \hspace{1cm} (10)

If one had used the blackbody distribution function $\nu(\epsilon) = (8\pi^2/\hbar^3 c^3) \exp(\epsilon/k_B T_{\text{CMB}}) - 1)^{-1}$, then the corresponding integral would have yielded (see equation 7.31 in Rybicki & Lightman 1979)

$$I = \int \nu(\epsilon) e^{\nu(\epsilon)/\nu_{\gamma}}$$  

$$= \frac{8\pi}{h^c c^3} (k_B T_{\text{CMB}})^{2/3} \frac{\Gamma(p+5/2)}{\Gamma(p+5/2)} \zeta \left( \frac{p+5}{2} \right),$$  \hspace{1cm} (11)

where the symbols have the standard meanings. Then the ratio, $I'/I = \frac{\pi^4}{15} \left( \frac{\nu_{\gamma}^2}{\nu} \right) \zeta \left( \frac{p+5}{2} \right)$, shows the error one incurs in assuming all CMB photons to have the peak frequency in estimating the IC power. For $p = 2.2$, one finds $I'/I \sim 1.6$. Therefore, the total scattered power calculated using this assumption is correct within an accuracy of 60 per cent, and we adopt it for simplicity in our calculation.

We therefore calculate the IC power at a given frequency, in the manner of previous equation (3), as

$$P_{\text{IC,\nu}} = \int_0^{\ell(t)} \frac{1}{6\pi} \sigma_T c u_\gamma^{3/2} \nu n(\gamma) d\nu, $$  \hspace{1cm} (13)

using equation (8) for $\delta V$, and the value of $n(\gamma, t)$ found from solving equation (9).

Not all volume elements in the cocoon would contribute to the IC radiated power through. Electrons in some volume element that was injected at an earlier time $t_i$ may lose energy to the extent that they fail to boost CMB photons to the observed X-ray band. These volume elements would not contribute to the X-ray power.

We calculate the total projected area of the cocoons that contribute to X-ray power, assuming the cocoon to be in the plane of the sky, and an axial ratio $R$. This allows us to calculate the X-ray surface brightness. We also calculate the fraction $f_\gamma$ of the total surface area of the cocoon that contributes to X-ray emission.

### 3 RESULTS

To begin with, as a fiducial case, we consider $Q_1 = 10^{46} \text{erg s}^{-1}$, $\Lambda = 10^{40} \text{g cm}^{-1}$ at $z = 0.1$, and a constant axial ratio $R = 2$. Following KDA97, we also adopt $r = 0.785$ and $p = 2.14$, $\gamma_{\text{max}} = 10^8$ and $\gamma_{\text{min}} = 1$. We refer to this set of parameters as Case I.

The set of thick lines in Fig. 1 shows the results for this fiducial case. We plot the time evolution of cocoon size in the top-left panel, radio power at 178 MHz in the top-right panel and IC power at 1 keV in the bottom-left panel. The corresponding evolution of the ratio of synchrotron to IC power with cocoon size is shown in the bottom-right panel. The solid lines show the case for $t_j = 10^7$ yr, and dotted and dashed lines show the cases $t_j = 5 \times 10^7$ and $10^8$ yr, respectively.

In the same figure, we also show the results of another case, with a higher jet power and one located at a higher redshift than in the fiducial case. The thin set of lines in Fig. 1 shows the case with $Q_1 = 10^{46} \text{erg s}^{-1}$ at $z = 1$ (Case II), keeping the values of other parameters the same.

The evolution of the size of the cocoon is self-similar, as expected, till $t \sim t_j$ (top-left panel of Fig. 1), after which the cocoon size expands slower than before. The radio luminosity as a function of time (top-right panel) is comparable to the results of KDA97 for the case of $t_j \sim 10^7$ yr, with sources at higher redshift dimming faster than their low-redshift counterparts because of IC loss. For
The time evolution of radio and X-ray powers are shown in this plot. Thick lines show the result for Case I \(Q_j = 10^{45} \text{ erg s}^{-1}, \Lambda = 10^{19} \text{ cm}^{-1}\) at \(z = 0.1\), with \(\gamma_{\text{min}} = 1\), whereas thin lines show the results for Case II \(Q_j = 10^{46} \text{ erg s}^{-1} \text{ at } z = 1\), with the same ambient density as in Case I. Dotted, solid and dashed lines show the cases for \(t_j = 5 \times 10^7\), \(10^8\) and \(10^9\) yr, respectively.

The X-ray luminosity of the cocoon at 1 keV (bottom-left panel) is shown in the top-left panel, radio power at 178 MHz (in W/Hz sr) in the top-right and the IC power at 1 keV in the same units is shown in the bottom-left panel. The bottom-right panel shows the corresponding ratio of IC to radio power as a function of cocoon size for the three cases. Two horizontal dot–dashed lines indicate the range of observed ratios (see text).

Figure 1. Time evolution of radio and X-ray powers are shown in this plot. Thick lines show the result for Case I \(Q_j = 10^{45} \text{ erg s}^{-1}, \Lambda = 10^{19} \text{ cm}^{-1}\) at \(z = 0.1\), with \(\gamma_{\text{min}} = 1\), whereas thin lines show the results for Case II \(Q_j = 10^{46} \text{ erg s}^{-1} \text{ at } z = 1\), with the same ambient density as in Case I. Dotted, solid and dashed lines show the cases for \(t_j = 5 \times 10^7\), \(10^8\) and \(10^9\) yr, respectively. Time evolution of the cocoon size \(L_c\) is shown in the top-left panel, radio power at 178 MHz (in W/Hz sr) in the top-right and the IC power at 1 keV in the same units is shown in the bottom-left panel. The bottom-right panel shows the corresponding ratio of IC to radio power as a function of cocoon size for the three cases. Two horizontal dot–dashed lines indicate the range of observed ratios (see text).

The ratio of X-ray to radio power (bottom-right panel), therefore, shows a mild increase till \(t_j\), after which it increases rapidly, because of the drop in radio luminosities. We note that, typically, the value of the ratio at \(t \sim t_j\) is of order \(P_\gamma/P_r \sim 10^{-10} - 10^{-9}\). For a single power-law electron energy distribution with index \(p = 2\alpha + 1\), where \(\alpha\) is the synchrotron (and inverse Compton) radiation spectral index, the ratio of relative luminosities can be written in terms of the strength of the magnetic field and the CMB temperature. For \(p = 2.5\), we have (e.g. Tucker 1977)

\[
P_\gamma/P_r \approx 4.7 \times 10^{-11} \left(\frac{T_{\text{CMB}}}{2.73 \text{ K}}\right)^{3\alpha} \left(\frac{B}{10^{-12} \text{ G}}\right)^{-(\alpha+1)}
\]

where the coefficient is \(\sim 2.7 \times 10^{-11}\) for \(p = 2\), and \(8.3 \times 10^{-12}\) for \(p = 3\). These values are somewhat lower than predicted from our detailed calculations, which take into account the electron energy distribution in different parts of the cocoon as they evolve. Although the magnetic field strength throughout the cocoon is assumed to be constant, the evolution of electron energy spectrum in different parts of the cocoon can give rise to a value of the ratio \(P_\gamma/P_r\) that is different from the above simple estimate. Admittedly, the flux ratio is mainly driven by magnetic field, which is what the above simple estimate indicates, but our calculations show the important role played by the evolution of electron spectrum prior to the observed epoch for an object.

The observed ratios of fluxes at 1 keV and 178 MHz fall in the range of \(P_\gamma/P_r \sim 10^{-10} - 10^{-9}\), for objects with a variety of linear sizes and at different redshifts [e.g. 3C 215 (\(z = 0.41\)), 3C 334 (\(z = 0.56\)), 3C 9 (\(z = 2.0\)) (Bridle et al. 1994), 3C 219 (\(z = 0.17\)) (Perley et al. 1980), 3C 265 (\(z = 0.81\)) (Bondi et al. 2004), 3C 109 (\(z = 0.31\)) (Giovannini et al. 1994), 3C 179 (\(z = 0.85\)) (Samburua et al. 2002)]. We indicate this range by two dot–dashed lines in the figure. Since the evolution of this ratio depends on many parameters (as listed in Table 1, for example), our goal in this paper is not to reproduce all the observed parameters, but to prove whether or not the model yields representative values consistent with observations.

It is apparent from the figure that the observed range of ratios is mostly consistent with model predictions for different parameters. It also appears that the observed range is not related to the rapidly rising part of the evolutionary curves. In other words, the systems observed in X-ray (near 1 keV) should also be radio bright at low frequencies, of the order of \(\sim 150\) MHz, that we have used for our calculations (\(\sim 178\) MHz). Low-frequency radio observations therefore should be an important probe of X-ray bright cocoons, as has been pointed out by previous authors.

Another aspect is that cocoons spend a relatively short time in the rapidly rising parts of the curves for ratio between \(P_\gamma/P_r\), because of the precipitous drop in radio luminosity after \(t_j\). In other words, radio galaxies that have faded from the window of frequencies of the order of a few hundred MHz are unlikely to be X-ray bright, since the X-ray power would also drop (see bottom-left panel of Fig. 1).

Next, we plot in Fig. 2 the corresponding flux in the 1–5 keV band (top-left panel) for the cases shown in Fig. 1. Our results show that for the parameters used here the cocoons typically emit at a flux of \(\sim 10^{-14}\) erg s\(^{-1}\) cm\(^{-2}\) per keV (or a few micro-Crab) before

<table>
<thead>
<tr>
<th>Model</th>
<th>(Q_j) (erg s(^{-1}))</th>
<th>(z)</th>
<th>(t_j) (yr)</th>
<th>(\Lambda) (cm(^{-1}))</th>
<th>(\gamma_{\text{min}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>(10^{45})</td>
<td>0.1</td>
<td>(10^8)</td>
<td>(10^{39})</td>
<td>1</td>
</tr>
<tr>
<td>Case II</td>
<td>(10^{46})</td>
<td>0.1</td>
<td>(10^8)</td>
<td>(10^{39})</td>
<td>1</td>
</tr>
<tr>
<td>Case III</td>
<td>(10^{46})</td>
<td>0.2</td>
<td>(10^8)</td>
<td>(10^{39})</td>
<td>1</td>
</tr>
</tbody>
</table>
determination. We have estimated the cocoon angular size from visual inspection from the observations cited in the caption, assuming the cocoons to be projected in the plane of the sky, and estimated the physical size assuming a Λ cold dark matter (ΛCDM) cosmology with $h = 0.7$, $Ω_m = 0.3$, $Ω_Λ = 0.7$. The error bars on the data points are large, but there appears to be a trend of decreasing photon index with increasing cocoon size: larger cocoons show flatter X-ray spectrum.

Our model predictions show that initially the X-ray spectrum gradually gets flatter (smaller value of Γ) till $t \sim t_f$. This is because of the fact that for electrons $γ \geq 10^3$, the injection of fresh particles mitigates the radiative steepening of electron spectrum. The injection of electrons from the jet compensates for synchrotron energy losses for electrons with $γ \sim 10^5$, where two relevant time-scales become comparable: (a) the jet lifetime ($10^7$–$10^8$ yr) and (b) IC cooling time-scale of electrons with $γ \sim 10^6$ ($3 \times 10^8$ yr at present cosmological epoch). This energy scale (corresponding to $γ \sim 3$) Characteristic energy of the CMB photons in keV range) also happens to be the one required to upscatter CMB photons to keV range. The concurrence of these two time-scales produces a temporary hardening of the soft X-ray spectrum.

Then, after the jet stops, the number of electrons at this energy scale rapidly decreases because of IC and adiabatic loss, and the spectrum becomes soft again. This softening occurs rapidly at high redshift, as expected from increasing IC loss. With regard to some of the data points with large cocoons and Γ $\sim 1$, we found that they can be explained with models using a jet power of the order of $10^{46}$ erg s$^{-1}$ at low redshifts.

### 3.1 Variations with parameters

Although we have shown the results for cocoons for a few cases, varying parameters such as the jet power, redshift and jet lifetime, there are other free parameters in this model whose effects must be understood. The X-ray properties of cocoons also depend on the ambient density (here parametrized as $\Lambda$) and the lower cut-off in the electron energy distribution ($\gamma_{\text{min}}$). To study the effect of these parameters, we first plot in Fig. 3 (with solid lines) the results for a fiducial case (Case III): $Q_j = 10^{46}$ erg s$^{-1}$, $z = 0.2$, $t_f = 10^6$ yr and $Λ = 10^{10}$ g cm$^{-3}$, $\gamma_{\text{min}} = 1$. In this section we will study the effect of changing the values of $L_j$, $Λ$, $\gamma_{\text{min}}$ separately, and compare with the results for our fiducial case. We will not vary the redshift and the jet lifetime, as the discussion in the previous section has already considered the effects of changing these parameters. Also, we keep $\gamma_{\text{min}}$ fixed at $10^6$, and we do not change the values of $p$ and $r$.

In Fig. 3, we have plotted the X-ray flux (in 1–5 keV band; top-left panel), ratio between X-ray and radio power (top-right panel), X-ray surface brightness (in 1–5 keV; bottom-left panel) and the photon index (Γ) in the bottom-right panel. All these parameters are plotted against the cocoon size.

First, the results for the fiducial case mentioned above are shown with solid lines in all panels for easy comparison. Then, with dotted line, we show the results of changing jet power to $10^{47}$ erg s$^{-1}$. As expected this cocoon expands to occupy a large volume, and is more X-ray bright than the fiducial case. The ratio of X-ray to radio power does not change, however, but the surface brightness increases. The photon index continues to decrease and the X-ray spectrum becomes harder with time (and size) for $t \sim t_f$ after which it becomes softer.

Next, we change the value of $\gamma_{\text{min}}$ to $10^4$ (keeping all other values same as in the fiducial case), and the results are shown with short-dashed lines. Increasing the lower cut-off results in increasing the
Curves in the top-right panel of Fig. 3 show that the ratio of X-ray to radio power is a robust parameter, in the sense that it does not vary with changes in $\gamma_{\text{min}}$ and jet power at fixed times. The curves are plotted against $D$, but the kink in the curves due to the stopping of jet occurs for all curves at the same time, namely $t_f$, and a comparison of the solid, short dashed (which happens to be superposed on the solid curve) and the dotted curves shows that the ratio is a robust parameter. It does vary with redshift (as we have seen in Fig. 1) and with ambient density (long dashed line in Fig. 3). This robustness allows us to use this parameter to characterize the connection between radio and X-ray properties of cocoons, and we will discuss this issue further in the next section.

3.2 Time-averaged ratio of X-ray to radio power

Our results show that many of the X-ray properties of FRII radio galaxy cocoons vary substantially with time, even when all other parameters such as jet power, ambient density and others are kept constant. It is therefore not easy to predict the X-ray properties of these sources that can be tested with observations, because it is difficult to determine the age of these sources from radio or other observations.

One can, however, use the fact that the evolutionary time-scale of radio galaxies, of the order of a few hundred Myr, is much shorter than the Hubble time, even at redshifts when the radio galaxy population peaked in number density ($z \sim 2$). This implies that one can use time-averaged quantities related to X-ray emission, and speculate upon their average properties that could be observed and tested.

In particular, we would like to determine the average property of the ratio of luminosities in X-ray and radio frequencies, $v_x P_x/v_r P_r$ of the sources under consideration, averaged over time until their X-ray or radio emission drops rapidly: $\int \frac{v_x P_x}{v_r P_r} dt / \int dt$. Our results (Fig. 3, top-right panel) show that this ratio of luminosities is a robust parameter, and it varies negligibly with the variations in jet power, lower cut-off in $\gamma$, although it varies strongly with changes in ambient density. This ratio is however likely to increase with redshift, and we wish to determine the scaling with redshift.

We therefore compute the X-ray luminosity $v_x P_x$ in the 1–5 keV band, and we choose a radio frequency of $v_r = 151$ MHz to compute $v_r P_r$. We compute a time-average of this ratio, summing over the duration in which the sources are both X-ray and radio bright in these frequencies (in the source frame).

Celotti & Fabian (2004) have discussed the significance of this ratio of X-ray and radio luminosities in the case of radiation from electrons with a single power-law energy distribution. In this case, this ratio can be written as

$$\frac{v_x P_x}{v_r P_r} = \frac{U_{\text{CMB}}}{U_B} \left( \frac{v_x v_B}{v_{r,x}} \right)^{1-\alpha} (1+z)^{3+\alpha}, \quad (15)$$

where $\alpha$ is the radio spectral index, $U_{\text{CMB}}$ and $U_B$ are the energy density in CMB photons (at $z = 0$) and magnetic field, respectively. Also, $v_B$ and $v_{r,x}$ are the gyrofrequency and the peak frequency of CMB photons at present epoch. For a field strength of $B = 0.1–10 \mu G$, the ratio between the luminosities at 1 keV and 151 MHz (in the observer frame) can take values 0.08–300 at $z = 0$, for $p = 2.6$, and 0.08–50 for $p = 2$. The ratio increases with redshift, although the increase can be mitigated by the possibility that magnetic field may change with redshift. Celotti & Fabian chose a conservative value of unity for this ratio, at all redshifts, and discussed the possible implications.
to the two different cosmological models. The luminosities in two different models are related as,
\[ P_1 D_1^{-2} = P_2 D_2^{-2}, \tag{18} \]
where
\[ D(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1 + z')^3 + \Omega_k}} \tag{19} \]
is the comoving distance in a flat cosmological model. Also, the comoving volume in equation (17) is given by
\[ dV = 4\pi D(z)^2 dD(z). \]
Having done this, we have associated a diffuse X-ray luminosity with each FRII sources with the scaling result from the last section, and determined the X-ray luminosity function of FRII galaxies.

Fig. 5 shows the computed X-ray luminosity function, in the units of number per unit (comoving) Mpc$^3$, per 10$^{44}$ erg s$^{-1}$, for three redshifts: $z = 0$ (solid), $z = 1$ (dotted) and $z = 2$ (short-dashed line). The peak of the X-ray luminosity function shifts to increasing luminosity because of the strong redshift evolution of the X-ray to radio power ratio. We note that Celotti & Fabian (2004) assumed a constant ratio of unity and so their X-ray luminosity function peaked at the same luminosity at different redshifts (their fig. 3).

We compare the computed luminosity function with that of clusters in X-ray in the 0.5–2 keV band (shown here with long-dashed line) as determined from the ROSAT-ESO Flux Limited X-ray (REFLEX) sample by Böhringer et al. (2002) (for a similar cosmology but with $h = 0.5$). Recently, Mullis et al. (2004) found that the cluster X-ray luminosity function decreases with increasing redshift at the high luminosity end, but does not evolve significantly below $10^{44}$ erg s$^{-1}$. The comparison with the luminosity function expected from FRII galaxies shows that the number density of these sources become comparable at the high luminosities at $z \sim 2$.

### 3.3 X-ray luminosity function of FRII radio galaxies

We are now in a position to estimate the number density of X-ray bright radio galaxy cocoons using the radio luminosity function of these sources, and using the above results for the relation between X-ray and radio emission. We use the radio luminosity function of FRII galaxies, as determined by Willott et al. (2001) from 3CRR, 6C and 7CRS samples. This luminosity function was determined assuming a cosmological model with $\Omega_0 = 0 = \Omega_\Lambda$, $\Omega_k = 1$, $h = 0.5$. We have converted it for the $\Lambda$CDM cosmology with $\Omega_m = 0.7$, $\Omega_\Lambda = 0.3$, $h = 0.7$ using the relation (Peacock 1985)
\[ \rho_1(P_1, z) \frac{dV_1}{dz} = \rho_2(P_2, z) \frac{dV_2}{dz} \tag{17} \]
where $P$ is the luminosity derived in a specific cosmological model for a measured flux and at a given redshift $z$, and the indices refer to the two different cosmological models.
We can also estimate the number of diffuse X-ray emitting cocoons in a given area of the sky above a given X-ray flux limit. Integrating the above luminosity function to $z \sim 2$, we find that the number of sources above a flux limit of $\sim 3 \times 10^{-16} \text{erg cm}^{-2} \text{s}^{-1}$ is of the order of $\sim 27$ per deg$^2$. Recently, Finoguenov et al. (2010) deleted $\sim 6$ X-ray emitting radio lobes in 1.3 deg$^2$ above $2 \times 10^{-15}$ erg cm$^{-2}$ s$^{-1}$, and given the uncertainties, our estimate is consistent with it.

4 DISCUSSION

The expected number of X-ray bright radio galaxy cocoons as estimated above can be compared with the observed values. Bauer et al. (2002) found six extended sources in the Chandra Deep Field North survey, within an area of $\sim 130$ arcmin$^2$, implying a surface density of $\sim 167^{+70}_{-60}$ deg$^{-2}$, at a limiting soft X-ray flux of $\sim 3 \times 10^{-16} \text{erg cm}^{-2} \text{s}^{-1}$. Our estimate shows that a small fraction of the order of $\sim 0.1$–$0.3$ of such extended soft-X-ray sources in the sky could be due to FRII radio galaxies.

The X-ray luminosity function in the soft band can also be used to estimate mirror effect of X-ray emission, namely the Sunyaev–Zeldovich (SZ) effect on the CMB. To some extent, it would underestimate the effect because of the fact the soft-band X-ray power is smaller than the total X-ray power. Keeping this in mind, we can determine the ratio of the total radiation energy density in the soft-X-ray band that is emitted by these sources and that is present in the CMB, by integrating the luminosity function: $\Delta \epsilon_x \sim \int \frac{d\epsilon_x}{\epsilon_x dN(x)} \int L_x \frac{d\epsilon_x}{\epsilon_x} dL_x$. We find that $\Delta \epsilon_x/\epsilon_x \sim 2 \times 10^{-7}$, integrating up to a redshift $z \sim 2$, although strictly speaking this is a lower limit. This implies a Compton $y$-parameter of the order of $y \sim (1/4)\Delta \epsilon_x \geq 10^{-7}$. Yamada, Sugiyama & Silk (1999) considered the distortion of the CMB from the population of radio galaxy cocoons, using Press–Schechter mass function and assuming that haloes above a certain mass limit produce radio galaxies, and estimated that $y \sim 6 \times 10^{-5}$. Enßlin & Kaiser (2000) however estimated from radio galaxy luminosity functions a total optical depth of the relativistic electrons to be $\tau \sim 10^{-7}$. For relativistic electrons with $\gamma_{\text{min}} \geq 1$, one has $\gamma_{\text{min}} \geq \tau$, and our estimate is consistent with that of Enßlin & Kaiser (2000).

It is not only the cocoons of radio galaxies that have been observed in X-rays, but the jets have also been recently detected by Chandra (e.g. Schwartz 2002). Although some uncertainties remain in identifying the emission mechanism behind the X-ray radiation, IC scattering of CMB photons is thought to be one of the possibilities (Stawarz et al. 2004). It is possible that the X-ray radiation from the jets may contaminate the emission from the cocoons and make the study of cocoons in X-rays difficult, especially at high redshift where angular resolution may become an issue.

5 CONCLUSION

We have used an evolving model of FRII radio galaxy cocoons that takes into account the stopping of the jet after a jet lifetime of $t_j$, and we have studied the X-ray property of cocoons from IC scattering of CMB photons. We have shown that the X-ray power (in the 1–5 keV band) and the X-ray surface brightness decrease rapidly after the stopping of the jet, as well as the radio power. At the same time, the X-ray photon index is found to first decrease to some extent while the jet is active. This occurs because at the important energy scale for IC scattering of CMB photons to keV range ($\gamma \sim 10^7$), two relevant time-scales become comparable: the jet lifetime and that of cooling due to IC ($10^{7-8}$ yr). After the stopping of the jet, the photon index increases, making the spectrum softer. We find that typical radio galaxy parameters produce an X-ray flux (1–5 keV band) of the order of 1-10 micro-Crab, being emitted from regions with sizes of the order of tens of arcsec.

We have also shown that the ratio of X-ray to radio power changes little with jet power and lower cut-off in the electron energy distribution, and it changes mostly with redshift and ambient density. We have then determined the time-averaged ratio of X-ray to radio power and studied its scaling with redshift, which we found to scale as $\alpha (1 + z)^3$, almost independent of the injected electron energy spectrum, contrary to the expectations from a population of electrons with a single power-law distribution of energy.

Using these scaling relations, we estimated the X-ray luminosity function of FRII radio galaxies at various redshifts, and estimated the number of such diffuse X-ray sources to be $\sim 25$ deg$^{-2}$ above an X-ray flux limit of $\sim 3 \times 10^{-16} \text{erg cm}^{-2} \text{s}^{-1}$ (compared to $\sim 167$ deg$^{-2}$ diffuse soft X-ray sources observed in the Chandra Deep Field North for a similar flux limit).

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