Hydrodynamical wind on a magnetized ADAF with thermal conduction

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ABSTRACT

We examine the effects of a hydrodynamical wind on advection-dominated accretion flows (ADAFs) with thermal conduction in the presence of a toroidal magnetic field under a self-similar treatment. The disc gas is assumed to be isothermal. For a steady state structure of such accretion flows a set of self-similar solutions are presented. The mass-accretion rate $\dot{M}$ decreases with radius $r$ as $\dot{M} \propto r^{s+1/2}$, where $s$ is an arbitrary constant. We show that existence of wind will lead to an increase of the accretion velocity. Cooling effects of outflows or winds are noticeable and should be taken into account for calculating luminosity and effective temperature of optically thin and thick ADAFs. Increasing the effect of wind decreases the disc’s temperature because of energy flux which is taken away by winds. We will see that for a given set of input parameters, the solution reaches a non-rotating limit at a specific value of $\phi_s$. The values of this limitation on $\phi_s$ will increase by adding $s$, the wind parameter. In fact, the higher values of $s$ will allow the disc to have a physical rotating solution for larger $\phi_s$.

Key words: accretion, accretion discs – conduction – stars: winds, outflows.

1 INTRODUCTION

Accretions on to compact objects are the most energetic processes in the Universe. It is believed that many astrophysical objects are powered by mass accretion on to black holes. The standard geometrically thin, optically thick accretion disc model can successfully explain most of the observational features in active galactic nuclei (AGN) and X-ray binaries (Shakura & Sunyaev 1973).

Accretion discs are an important ingredient in our current understanding of many astrophysical systems on all scales. Examples of presumed disc acertors include young stars, compact objects in close binary systems, AGN and quasars (QSOs). There is also evidence that the process of mass accretion via a disc is often and perhaps always associated with mass-loss from the disc in the form of a wind or a jet. Mass-loss appears to be a common phenomenon among astrophysical accretion disc systems. These mass-loss mechanisms are observed in microquasars, young stellar objects and even in brown dwarfs (Ferrari 1998; Whelan et al. 2005; Bally, Reipurth & Davis 2007). An outflow emanating from an accretion disc can act as a sink for mass, angular momentum and energy, and can therefore alter the dissipation rates and effective temperature across the disc (Knigge 1999). The accretion flows lose their mass by the winds as they flow on to the central object. As a result of mass-loss, the accretion rate, $\dot{M}$, is no longer constant in radius $r$. It is often expressed as $\dot{M} \propto r^s$ with $s$ being a constant of the order of unity (Blandford & Begelman 1999).

Astrophysical jets and outflows emanating from accretion discs have been extensively investigated by many researchers. Various driving sources are proposed, including thermal, radiative and magnetic ones. Traditionally the name of the wind depends on its driving force. In this paper we will follow the hydrodynamical (thermal) wind which has been discussed by many authors (e.g. Meier 1979, 1982; Fukue 1989; Takahara, Rosner & Kusnose 1989).

Accretion disc models have been extensively studied during the past three decades (see Kato et al. 2008 for a review). Besides introducing the traditional standard disc by Shakura & Sunyaev (1973), there are new-type discs, such as advection-dominated accretion flows (ADAFs) or radiatively inefficient accretion flows (RIAFs) for very small mass-accretion rates (Narayan & Yi 1994), and supercritical accretion discs or so-called slim discs, for very large mass-accretion rates (Abramowicz et al. 1988). ADAFs with winds or outflows have been studied extensively during recent years, but thermal conduction has been neglected in all these ADAF solutions with winds and toroidal magnetic field. Since the advection-dominated discs have high temperature, the internal energy per particle is high. This is one of the reasons why advective cooling overcomes radiative cooling. For the same reason, turbulent-heat transport by conduction in the radial direction is non-negligible in the heat balance equation. Shadmehi (2008) and Tanaka & Menou (2006) have studied the effect of hot accretion flow with thermal...
conduction. Shadmehri (2008) has shown that thermal conduction opposes the rotational velocity, but increases the temperature. In advection-dominated inflow–outflow solutions (ADIOS) it is assumed that the mass-flow rate has a power-law dependence on radius with the power-law index, $s$, treated as a parameter (e.g. Blandford & Begelman 1999). Kitabatake, Fukue & Matsumoto (2002) studied supercritical accretion disc with winds, though angular momentum loss of the discs, because of the winds, has been neglected. In this paper, we present self-similar solutions for ADAFs with thermal conduction, wind and a toroidal magnetic field, on the basis of solutions presented by Akizuki & Fukue (2006). In Section 2, we present assumptions and basic equations. Self-similar solutions are investigated in Section 3. The aim of this paper is to consider the possibility that winds in the presence of thermal conduction and a toroidal magnetic field, which has been largely neglected before, could affect the global properties of hot accretion flows substantially. In Section 4 we show the results and give discussions of the results.

2 THE BASIC EQUATIONS

We investigate the effect of mass outflow by the wind and mass-accretion rate by viscosity simultaneously. We consider an accretion disc that is stationary and axisymmetric ($\frac{\partial \Phi}{\partial r} = 0$) and geometrically thin $H/R < 1$. In cylindrical coordinates $(r, \varphi, z)$, we vertically integrate the flow equations; also we suppose that all flow variables are only a function of $r$. We ignore the relativistic effect and we use Newtonian gravity. We adopt the $\alpha$-prescription for viscosity of rotating gas. For the magnetic field we consider a toroidal configuration.

The equation of continuity gives

$$\frac{\partial}{\partial r}(r \Sigma v_r) + \frac{1}{2\pi} \frac{\partial M_w}{\partial r} = 0,$$

where $v_r$ is the accretion velocity ($v_r < 0$) and $\Sigma = 2\rho H$ is the surface density at a cylindrical radius $r$. The mass-loss rate by outflow/wind is represented by $M_w$. So

$$M_w(r) = \int 4\pi r' m_w(r') r' dr',$$

where $m_w(r)$ is mass-loss rate per unit area from each disc face. On the other hand, we can rewrite the continuity equation:

$$\frac{1}{r} \frac{\partial}{\partial r}(r \Sigma v_r) = 2\rho H,$$

where $\rho$ is the mass-loss rate per unit volume and $H$ is the disc half-thickness.

The equation of motion in the radial direction is

$$v_{\varphi} = \frac{v_r^2}{r} \frac{GM_c}{r^2} - \frac{1}{\Omega} \frac{\partial}{\partial r}(\Sigma C_s^2) - \frac{C_A^2}{r} - \frac{1}{2\Omega} \frac{d}{dr} \left(\Sigma C_\lambda^2\right),$$

where $v_r$, $C_s$ and $C_\lambda$ are the rotational velocity of the gas disc, the sound and Alfvén velocities of the fluid, respectively. Sound speed is defined as $C_s^2 = \frac{\gamma}{\gamma-1} \frac{\dot{\rho}}{\rho}$, $p_g$ being the gas pressure, and Alfvén velocity is defined as $C_\lambda^2 = \frac{\frac{\dot{B}_\perp^2}{4\pi\rho}}{\frac{\dot{B}_z^2}{4\pi\rho}} = \frac{2p_m}{\rho}$, where $p_m$ is the magnetic pressure.

The integrated angular momentum equation over $z$ gives (e.g. Shadmehri 2008)

$$r \Sigma v_z \frac{d}{dr}(r v_r) = \frac{d}{dr} \left(\frac{r^3 v_z}{2\pi} \frac{d\Omega}{dr}\right) - \frac{\Omega(r)^2}{2\pi} \frac{dM_w}{dr},$$

where the last term on the right-hand side represents angular momentum carried by the outflowing material. Here, $l = 0$ corresponds to a non-rotating wind and $l = 1$ to outflowing material that carries away the specific angular momentum (Knigge 1999). Also $\nu$ is the kinematic viscosity coefficient and we assume

$$\nu = \alpha C_s H,$$

where $\alpha$ is a constant less than unity. By integrating over $z$ of the hydrostatic balance, we have

$$\frac{GM}{r^2} H^2 = C_s^2 \left[ 1 + \frac{1}{2} \left(\frac{C_A}{C_s}\right)^2 \right] = (1 + \beta) C_s^2,$$

where $\beta = \frac{p_m}{p_g} = (1/2)\left(C_A/C_s\right)^2$ which shows the importance of magnetic field pressure compared to gas pressure. We will show the dynamical properties of the disc for different values of $\beta$. Now we can write the energy equation considering cooling and heating processes in an ADAF. We assume the generated energy due to viscous dissipation and the heat conduction into the volume are balanced by the advection cooling and energy loss of outflow. Thus,

$$\frac{\Sigma v_r}{r} \frac{dC_s^2}{dr} - 2H v_r C_s^2 \frac{d\rho}{dr} = \frac{f}{\Omega} \frac{\Sigma C_s^2}{r^2} \frac{\dot{\rho}}{\Omega} \frac{d\Omega}{dr} - \frac{2H}{r} \frac{d}{dr} (r F_i) - \frac{1}{2} \frac{\eta m_w}{r^4} v_r^2(r),$$

where the second term on the right-hand side represents energy transfer due to the thermal conduction and $F_i = 5\Phi \rho C_i^2$ is the saturated conduction flux (Cowie & Mckee 1977). Dimensionless coefficient $\Phi_i$ is less than unity. Also, the last term on the right-hand side of the energy equation is the energy loss due to wind or outflow (Knigge 1999). Depending on the energy-loss mechanisms, the dimensionless parameter $\eta$ may change. In our case we consider it as a free parameter in our models so that larger $\eta$ corresponds to more energy extraction from the disc because of outflows (Knigge 1999). Finally, since we consider the toroidal component for the global magnetic field of central stars, the induction equation with field escape can be written as

$$\frac{d}{dr} (V_i B_\varphi) = B_\varphi,$$

where $B_\varphi$ is the field escaping/creating rate due to magnetic instability or the dynamo effect.

3 SELF-SIMILAR SOLUTIONS

Self-similar solutions cannot describe the global behaviour of the solutions, because in this method there are no boundary conditions that have been taken into account. Although we are not interested in the solutions near the boundaries, such solutions describe correctly the true and useful asymptotic behaviour of the flow in the intermediate areas.

We assume that the physical properties are self-similar in the radial direction. In the self-similar model the velocities are assumed to be expressed as follows:

$$v_i(r) = -C_1 \alpha v_r(r),$$

$$v_\varphi(r) = C_2 v_r(r),$$

$$C_s(r) = C_3 v_r^2(r),$$

$$C_\lambda^2(r) = \frac{B_z^2(r)}{4\pi\rho(r)} = 2\beta C_3 v_r^2(r).$$
where
\[ v_k(r) = \sqrt{\frac{GM}{r}} \]  
and constants \( C_1, C_2 \) and \( C_3 \) will be determined later. From the hydrostatic equation, we obtain the disc half-thickness \( H \) as
\[ H = \sqrt{C_3(1 + \beta)} = \tan \sigma. \]  
Hence, a hot accretion flow with winds also has a conical surface, whose opening (half-thickness) angle is \( \sigma \).

We assume the surface density \( \Sigma \) is in the form of
\[ \Sigma = \Sigma_0 r^s. \]  
Note that the value of \( s \) should be determined iteratively for consistency. Then we assume that the power-law index of the density \( \rho \) in the radial direction is constant regardless of \( z \). Hence we set \( \rho \propto r^{s-1} \),
\[ \dot{\rho} = \rho_0 r^{-5/2}, \]  
\[ \dot{B}_e = B_0 r^{-(s-1)/2}, \]  
\[ M_w = M_0 r^{s+1/2} \]  
and
\[ \dot{m}_w = m_0 r^{-3/2}. \]  
It should be noted that, for a self-similar disc without any wind mass-loss, the suffix \( s \) is \( s = -1/2 \).

By substituting the above self-similar solutions into the dynamical equations of the system, we obtain the following dimensionless equations, to be solved for \( C_1, C_2 \) and \( C_3 \):
\[ \dot{\rho}_0 = -\left( s + \frac{1}{2} \right) \frac{(C_1 \alpha \Sigma_0)}{2} \sqrt{\frac{GM_\alpha}{(1 + \beta)C_3}}, \]  
\[ H = \sqrt{(1 + \beta)C_3 r}, \]  
\[ -\frac{1}{2} C_2^2 s^2 = C_2 - 1 - [s - 1 + \beta(s + 1)] C_3, \]  
\[ C_1 = 3(s + 1)C_3 + \left( s + \frac{1}{2} \right) I^2 \dot{m}, \]  
\[ \left( \frac{1}{\gamma - 1} \right) C_1 C_3 = \frac{9}{4} f C_1 C_2^2 - \frac{5 \Phi_\alpha}{\alpha} \left( s - \frac{1}{2} \right) C_3^{3/2} = \frac{3}{8} \eta \dot{m}, \]  
and
\[ \dot{m} = 2C_1, \]  
where \( \dot{m} = \frac{M_0}{\pi \Sigma_0 \sqrt{G M}} \) is the non-dimensional mass-accretion rate.

After algebraic manipulations, we obtain a fourth-order algebraic equation for \( C_1 \):
\[ D^2 C_1^4 + 2DBC_1^2 + \left( B^2 + 2D(E - 1) \right) C_1^2 + \left[ 2(BE - 1) - A^2 C_1 + (E - 1)^2 \right] = 0, \]  
where
\[ D = \frac{1}{2} s^2 \]

\[ B = \frac{4}{9f} \left( \frac{1}{\gamma - 1} - \frac{1}{2} \right) - [s - 1 + \beta(s + 1)] \left[ \frac{1 - 2(s + 1/2)^2}{3(s + 1)} \right] \]  
\[ A = \frac{20 \Phi_\alpha}{9f \alpha} \left( s - \frac{1}{2} \right) \left[ \frac{1 - 2(s + 1/2)^2}{3(s + 1)} \right]^{1/2}, \]  
\[ E = \eta \left( s + 1 \right) \]  
\[ \frac{1}{3} \left[ 1 - 2(s + 1/2)^2 \right]. \]  
Abbassi, Ghanbari & Najjar (2008) have solved these equations when \( s = -1/2 \) because they did not have any wind or mass-loss in their model. This algebraic equation shows that the variable \( C_1 \) which determines the behaviour of radial velocity depends only on \( \alpha, \Phi_\alpha, \beta, f, s \) and \( \eta \). Other flow quantities such as \( C_2 \) and \( C_3 \) can be obtained easily from \( C_1 \):
\[ C_2^2 = \frac{4C_1}{9f} \left[ \frac{1}{\gamma - 1} - \frac{1}{2} \right] + \frac{20 \Phi_\alpha}{f \alpha} \left( s - \frac{1}{2} \right) \times \left[ \frac{1 - 2(s + 1/2)^2}{3(s + 1)} \right]^{1/2} C_1^{1/2} + \frac{\eta}{3} \left[ 1 - 2(s + 1/2)^2 \right] \]  
\[ C_3 = C_1 \left[ 1 - 2(s + 1/2)^2 \right] \frac{3(s + 1)}{2}. \]  
We can solve these simple equations numerically, and clearly just physical solutions can be interpreted. Without wind, thermal conduction and magnetic field, \( s = l = \gamma = \phi = \beta = 0 \), the equations and their similarity solutions reduce to the standard ADAF solution (Narayan & Yi 1994). They also reduce to the results of Abbassi et al. (2008) without wind. Now we can analyse the behaviour of solutions.

### 4 RESULTS

Now we can analyse the behaviour of solutions in the presence of outflow, thermal conduction and magnetic field. Our primary goal is to investigate these effects via parameters \( s, l, \eta, \phi, \alpha \) and \( \beta \).

This algebraic equation shows that the variable \( C_1 \) which determines the behaviour of radial velocity depends only on \( \alpha, \Phi_\alpha, \beta, f, s \) and \( \eta \). Other flow quantities such as \( C_2 \) and \( C_3 \) can be obtained easily from \( C_1 \) via the above equations. The parameters of the model are the ratio of specific heats \( \gamma \), the standard viscosity parameter \( \alpha \), the energy-advection parameter \( f \), the degree of magnetic pressure to gas pressure \( \beta \) and \( s \) which determines the outflow from the discs.

Fig. 1 shows the coefficients \( C_i \) in terms of advection parameter \( f \) for different values of \( \beta \). In the upper panel we show \( C_1 \) as a function of advection parameter. It represents the behaviour of radial flows of accretion materials. Although in ADAFs the radial velocity is generally sub-Keplerian, it becomes larger by increasing \( f \). By adding \( \beta \), which indicates the role of magnetic field on the dynamics of accretion discs, we will see that the radial flow will increase. On the other hand the radial flow increases when the toroidal magnetic field becomes large which agrees with the results of Abbassi et al. (2008). This is because the magnetic tension term dominates the magnetic pressure term in the radial momentum equation, which assists the radial velocity of accretion flows. The middle panel shows the coefficient \( C_2 \) which is the ratio of the rotation velocity to the Keplerian velocity as a function of advection parameter for different values of \( \beta \). We can see that by adding the magnetic field influences (adding \( \beta \)), the rotation velocity will decrease. This
Figure 1. Numerical coefficient $C_t$ as a function of advection parameter $f$ for several values of $\beta$, magnetic field strength. All these figures were set up for $s = -0.3$, $\alpha = 0.01$, $\phi = 0.001$ and $l = \eta = 1$.

Figure 2. Numerical coefficient $C_t$ as a function of advection parameter $f$ for several values of $\phi$, thermal conduction parameter. All these figures were set up for $s = -0.3$, $\alpha = 0.01$, $\beta = 0.1$ and $l = \eta = 1$.

is because the disc should rotate faster than the case without the magnetic field which results in the magnetic tension. In the lower panel, we have plotted $C_{t2}$, the coefficient of squared sound speed as a function of $f$. As the advection parameters become large, the sound speed and therefore the disc’s thickness become large. Sound speed also depends on the magnetic field influences. By adding magnetic field influence we will see that sound speed increases, as well as the vertical thickness of accretion flows.

In Fig. 2 we investigate the role of saturated thermal conduction in radial, toroidal and sound speed of accretion flows. Increasing the thermal conduction coefficient $\phi_3$ will decrease the radial velocity. It will have a large effect on the rotational velocity of accretion flows (middle panel). It will decrease the sound speed and therefore vertical thickness of the discs too (lower panel).

To show the behaviour of the solution with respect to the wind influences, we have plotted the disc’s physical quantities for different values of $s$ and $l$ in Figs 3 and 4. The value of $s$ measures the strength of outflows, and larger values of $s$ denote strong outflows. Each curve is labelled with its corresponding $s$. We can see that ADAFs with wind rotate more quickly than those without wind; radial flows of the accretion materials also increase for larger $s$. Strong-wind models have a lower vertical thickness compared to the weak-wind model because by adding $s$ we can see that $C_{t3}$ decreases, which means that the sound speed and vertical thickness decrease as well.

There are some limitations for having hot discs with sufficiently low $f$. The middle panel in Fig. 3 shows that there is no physical solution for ADAFs with $f < 0.4$ for the case of no wind. But by adding $s$ we can see that this limitation can be cancelled. The outflow and wind act as cooling agents, while thermal conduction provides an extra heating source so these two factors affect the dynamical behaviour of the flows. While effects of wind and thermal conduction on the profiles of sound speed and vertical thickness of the discs are similar, their effects on the profiles of radial and rotational velocities oppose each other. Both of them lead to a decrease in the sound speed and therefore the vertical thickness of the discs decreases. They lead to enhanced radial and rotational velocities.
\[ \eta = s \phi_0 \cdot C_{0.3} f \]

because equation (31) gives a

\[ 0 = - \ldots \]

\[ B_{10} = \leq \rightarrow 0 \quad \text{and} \quad \beta, f \]

will account for

as a function of thermal conduction

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Clearly thermal conduction can signif-

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\[ l \geq 0.3 \]

\[ \eta = 0.3 \]

\[ s \]

\[ \phi \]

\[ 0 = \phi \]

\[ f = 0.7 \]

\[ l = \eta = 1. \]

\[ s = -0.3 \]

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bathed in a toroidal magnetic field. In this section we will discuss the general properties of accretion flow and we will investigate possible effects of wind, thermal conduction and magnetic field on the physical quantities of accretion flows.

Using the self-similar solution we will estimate the mass-accretion rate as

\[ M = -2\pi r^3 v_\Sigma = 2\pi \Sigma \Omega c \sqrt{GM} \rho^{-1/2} \]

\[ = M_{\text{out}} \left( \frac{r}{r_{\text{out}}} \right)^{s+1/2}, \]

(33)

where \( r_{\text{in}} \) is the disc’s outer radius and \( M_{\text{out}} \) is the mass-accretion rate there. In the case of an accretion disc with no wind, \( s = -1/2 \), the accretion rate is independent of the radius, while for those with wind, \( s \approx -1/2 \), the accretion rate decreases with radius as we expect. Because winds start from various radii, the mass-loss rate is not constant but depends on the radius. As a result, some parts of accretion materials are not concentrated at the centre, but are diluted over a wide space. According to wind mass-loss, the accretion rate decreases with radius as we expected.

With a simple calculation we will see that a significant amount of accretion materials are flung away via the wind and mass-loss. Only 1–10 per cent of \( M_{\text{out}} \) is ultimately accreted on to the central accretor. Using the above expression for accretion rate, we have

\[ \frac{M_{\text{in}}}{M_{\text{out}}} \sim \left( \frac{r_{\text{in}}}{r_{\text{out}}} \right)^{s+1/2}. \]

If we estimate \( r_{\text{in}} \sim 10^{-3} r_{\text{out}} \), we finally have for \( s = 1/2 \)

\[ \frac{M_{\text{in}}}{M_{\text{out}}} \sim 10^{-3}. \]

We can show the radial behaviour of temperature of the present self-similar ADAF discs with toroidal magnetic field and outflow. These ADAFs occur in two regimes depending on their mass-accretion rate and optical depth. Optical depths of accretion flows are highly dependent on their accretion rate. In a high mass-accretion rate, the optical depth becomes very high and the radiation generated by accretion flows can be trapped within the disc. This type of accretion discs is known as optically thick or slim discs. In the limit of low mass-accretion rates, the disc becomes optically thin. In this case, the cooling time of accretion flows is longer than the accretion time-scale. The energy generated by accretion flows therefore remains mostly in the discs and the discs cannot radiate their energy efficiently. We will examine the influence of the wind and thermal conduction on the radial appearances of the temperature in these two cases. In the optically thin case, where the gas pressure dominates, we can adopt the ideal gas law to estimate the effective temperature as (Akizuki & Fukue 2006):

\[ \frac{R}{\mu T} = c_s^2 = \frac{GM}{r}, \]

(34)

where \( T \) is the gas temperature, \( R \) the gas constant and \( \mu \) the mean molecular weight. If we use \( GM = r_c e \), where \( r_c \) and \( e \) are the Schwarzschild radius and light speed, respectively, the temperature gradient is expected as

\[ T = C_1 \frac{c^2}{2} \frac{r_e}{R/\mu r}. \]

(35)

which means that \( T \propto C_1/r \). This has similar form (radial dependency) to the non-magnetic case, but coefficient \( C_1 \) implicitly depends on magnetic field \( \beta \), outflow effect \( s \), advection parameter \( f \) and the effect of thermal conduction \( \phi \). For the case of strong wind \( C_1 \) decreases (Fig. 3) so the surface temperature and vertical thickness of the disc will decrease. Thermal conduction has the same effect (Fig. 2), while the magnetic field will increase the surface temperature and vertical thickness of the disc.

It should be emphasized that the radiative appearance of optically thin discs, such as \( L_{\text{in}} \), cannot be calculated easily. In this case it should demonstrate the importance of advective cooling in optically thin discs. In order to clarify this problem we should show how cooling and heating change with \( r \), \( \Sigma \) and \( M \). Radiative cooling generally has very complicated parameter dependencies. In the optically thin cases, emission from the gas is not a blackbody continuum. Bremsstrahlung cooling by non-relativistic or relativistic, synchrotron and Compton cooling may have possible roles to reproduce emission spectra.

In the optically thick case, where the radiation pressure dominates, sound speed is related to radiation pressure. We can write the average flux \( F \) as

\[ F = \sigma T^4 = \frac{3c}{8H} \Pi_{\text{gas}} = \frac{3}{8} \Sigma \sigma \frac{C_3}{1 + \beta} G M r^{-2}, \]

(36)

where \( \Pi_{\text{gas}} = \Sigma c^2 \) is the height-integrated gas pressure, \( T \) the disc central temperature and \( \sigma \) the Stefan–Boltzmann constant. In the moderately to the strongly magnetized \( \beta \geq 1 \) cases the magnetic pressure is comparable with gas pressure and it should take into account:

\[ \Pi = \Pi_{\text{gas}} + \Pi_{\text{mag}} = (1 + \beta) \Pi_{\text{gas}}, \]

so the calculation should be modified by multiplying with \( 1 + \beta \). For the optically thick case the optical thickness of the disc in the vertical direction is

\[ \tau = \frac{1}{2} \kappa \Sigma = \frac{1}{2} \kappa \Sigma_0 r', \]

where \( \kappa \) is the electron-scattering opacity. So we can calculate the effective flux and effective temperature of the disc surface as

\[ \sigma T_{\text{eff}}^4 = \frac{\sigma T^4}{\tau} = \frac{3c}{4k} \left( C_3 \sqrt{\frac{G M}{1 + \beta}} \right)^4 \]

\[ = \frac{3}{4} \left( \frac{C_3}{1 + \beta} \right) \frac{L_E}{4\pi r^2}, \]

(37)

\[ \frac{T_{\text{eff}}}{L_{\text{disc}} = \frac{3}{4} \left( \frac{C_3}{1 + \beta} \right) L_k \ln \frac{r_{\text{out}}}{r_{\text{in}}} \]

(38)

where \( L_E = 4\pi c G M \) is the Eddington luminosity of the central object. If we integrate these equations radially we have the disc luminosity as

\[ L_{\text{disc}} = \frac{3}{4} \left( \frac{C_3}{1 + \beta} \right) L_E \ln \frac{r_{\text{out}}}{r_{\text{in}}} \]

(39)

As this equation shows, the optically thick disc’s luminosity is affected by magnetic field explicitly, \( \beta \), but it would be affected by outflow, thermal conduction and viscosity through \( C_1 \) implicitly.

It should be emphasized that the luminosity and effective temperature of the disc, \( L_{\text{disc}} \) and \( T_{\text{eff}} \), are not affected by the mass-loss through outflow (there are no \( s \) dependencies). But wind would affect the radiative appearance of the disc through the \( C_1 \) in these formulae, implicitly. The average flux decreases all over the disc when we have mass-loss outflow compared with the case of no mass-loss. The surface density and therefore optical depth decrease for the mass-loss case. So we can see the deep inside of the disc.
6 SUMMARY AND CONCLUSION

In this paper we have studied accretion discs around a black hole in an advection-dominated regime in the presence of a toroidal magnetic field. Thermal conduction and wind as energy sources and angular momentum transport were adopted. We have presented the results of self-similar solutions to show the effects of various physics behaviours on the dynamical quantities of accretion flows. We adopt the solution presented by Fukue (2004), Akizuki & Fukue (2006) and Abbassi et al. (2008) to present the dynamical behaviour of the ADAFs. Some approximations were made in order to simplify the main equations. We assume an axially symmetric, static disc with the α-prescription of viscosity. We ignored the relativistic effects and the self-gravity of the discs. Considering the weakly collisional nature of hot accretion flow (Tanaka & Menou 2006; Abbassi et al. 2008), a saturated form of thermal conduction was adopted as a possible mechanism for energy transportation. We have accounted for this possibility by allowing the saturated thermal conduction constant, $\phi_s$, to vary in our solutions.

Theoretical arguments and observations suggest the mass-loss via wind in RIAFs. By assuming that the flow has self-similarity structure in the radial direction, a power-law wind is adopted in our model. Using some assumptions, we made a simplified toy model which included the effect of thermal conduction, wind and $B$-field. In this toy model, we can easily investigate its possible combined effects on the dynamical quantities of the fluid.

We have shown that strong wind could have a lower temperature which is consistent with the results presented by Kawabata & Mineshige (2009), which could make significant differences in the observed flux compared to standard ADAFs. The most important finding of our simple self-similar solutions is that the accreting flow is strongly affected not only by mass-loss but also by energy loss by the wind.

There are some limitations in our solutions. One of them is that the magnetic field also has an important role in producing wind (magnetically driven wind). To have X-wind, pure toroidal $B$-field is not enough and the disc should have a $z$-component $B$-field. So our case is not a good model for magnetically driven discs. The other limitation of our solutions is the anisotropic character of conduction in the presence of magnetic field. Balbus (2001) has argued that the dynamical structure of the hot flows could be affected by the anisotropic character of thermal conduction in the presence of $B$-field.

Although our preliminary self-similar solutions are too simplified, they clearly improve our understanding of the physics of ADAFs around a black hole. To have a realistic picture of accretion flow a global solution is needed rather than the self-similar solution.

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