Are the superstructures in the two-degree field galaxy redshift survey a problem for hierarchical models?

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ABSTRACT

We introduce an objective method to assess the probability of finding extreme events in the distribution of cold dark matter such as voids, overdensities or very high mass haloes. Our approach uses an ensemble of \( N \)-body simulations of the hierarchical clustering of dark matter to find extreme structures. The frequency of extreme events, in our case the cell or smoothing volume with the highest count of cluster-mass dark matter haloes, is well described by a Gumbel distribution. This distribution can then be used to forecast the probability of finding even more extreme events, which would otherwise require a much larger ensemble of simulations to quantify. We use our technique to assess the chance of finding concentrations of massive clusters or superclusters, like the two found in the two-degree field galaxy redshift survey (2dFGRS), using a counts-in-cells analysis. The Gumbel distribution gives an excellent description of the distribution of extreme cell counts across two large ensembles of simulations covering different cosmologies, and also when measuring the clustering in both real and redshift space. We find examples of structures like those found in the 2dFGRS in the simulations. The chance of finding such structures in a volume equal to that of the 2dFGRS is around 2 per cent.

Key words: methods: numerical – methods: statistical – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

The discovery of extreme objects, such as large voids or highly overdense regions called superclusters, in which many galaxy clusters are found close together, has often been presented as a challenge to the hierarchical structure formation paradigm. However, the main drawback of using the presence of unusual structures to rule out a particular model is that it is not always clear how to assess the probability of finding such objects. In this paper we introduce a new methodology to address this problem in which we use \( N \)-body simulations and extreme value theory to provide a quantitative assessment of the probability of finding rare structures in a given cosmology.

Claims of rare structures are common in the literature. Frith et al. (2003) argued that the shape of the local galaxy counts implies an underdense volume in the southern sky \( \sim 300 \, h^{-1} \) Mpc across (see also Busswell et al. 2004; Frith, Shanks & Ourtam 2005; Frith, Metcalfe & Shanks 2006). Cruz et al. (2005) found a cold spot in the cosmic microwave background radiation that is much bigger than expected in a Gaussian distribution. Rudnick, Brown & Williams (2007) suggested that this cold spot is a secondary anisotropy, coinciding with the angular position of a void in a survey of radio galaxies. Swinbank et al. (2007) found a large association of galaxy clusters, a supercluster of galaxies, at \( z \sim 0.9 \) in the UK Infrared Deep Sky Survey. Sylos Labini, Vasilyev & Baryshev (2009a,b) argue that large-scale density fluctuations are present in the local galaxy surveys which cannot be explained in current structure formation models. There are two common problems with the interpretation of such results. First, what is the selection function of these objects, which would allow us to fix the frequency of finding such structures? Secondly, what exactly are we looking at? For example, in the case of an overdensity of galaxies have we seen one massive cluster or are we looking at a projection of smaller structures along the line of sight? How should we compare the observations to theoretical predictions?

In this paper, we assess how common the superclusters found in the two-degree Field Galaxy Redshift Survey (2dFGRS; Colless et al. 2001, 2003) are in the cold dark matter (CDM) cosmology. These structures were identified as ‘hotspots’ in the distribution of galaxy counts-in-cells (Baugh et al. 2004; Croton et al. 2004). One structure is in the NGP part of the 2dFGRS at a redshift of \( z = 0.08 \).
and a right ascension of 13 h, and the other is in the SGP region at z = 0.11 at a right ascension of 0.5 h. The higher order moments of the counts are strongly influenced by the presence of these structures (Croton et al. 2004; Nichol et al. 2006). A subsequent analysis of galaxy groups in the 2dFGRS revealed that these regions contain a surprisingly large fraction of all the massive clusters in the survey (Eke et al. 2004a). Of the 94 groups in the full flux limited 2dFGRS out to z ~ 0.15 with nine or more members and estimated masses above $5 \times 10^{14} h^{-1} \text{M}_\odot$, 20 per cent reside in these superclusters (Padilla et al. 2004). The supercluster in the NGP region of the 2dFGRS is part of the ‘Sloan Great Wall’ (Gott et al. 2005).

The 2dFGRS superclusters are by no means the largest superclusters in the local universe (for a list of superclusters, see Einasto et al. 2001). The Shapley supercluster, for example, contains more Abell clusters than either of the 2dFGRS structures (Raychaudhury et al. 1991; Proust et al. 2006; Munoz & Loeb 2008). However, not all of the clusters contained within Shapley and similar mass concentrations have measured redshifts. Many of the member clusters have been identified in projection, and so their actual size is open to debate (Sutherland & Efstathiou 1991). The advantage of focusing on the 2dFGRS structures is that they have been identified from an unbiased redshift survey which was designed to map a particular volume of the Universe, rather than by targeting known structures. The volume of space in which the superclusters are found is therefore well defined. Furthermore, through the construction of the 2dFGRS Percolation Inferred Galaxy Group (2PIGG) catalogue (Eke et al. 2004a), there is a clear, objective way to connect the observed properties of the galaxy groups which make up the superstructures to dark matter haloes in N-body simulations.

In this paper, we use extreme value theory to assess the probability of finding structures like the 2dFGRS superclusters in the CDM cosmology. Previous attempts to address the probability of finding such structures have used small numbers of simulations, and so have not been able to make definitive statements. For example, Croton et al. (2004) measured the moments of the galaxy cell count distribution in the 22 mock catalogues whose construction was described by Norberg et al. (2002). None of these mocks displayed higher order moments that looked like those measured in the 2dFGRS, giving a probability of less than 5 per cent that such a structure could arise in a CDM model. One possible way around this problem is to generate estimates of the error on a measurement from the data itself (see Norberg et al. 2009). Around 50 such estimates are required to get an accurate estimate of the variance on a measurement in the case of Gaussian statistics, and this method is clearly not applicable to a structure which appears once or twice in the data set. The method we describe in this paper is calibrated against N-body simulations and can be extrapolated to very low probabilities, without requiring any assumption about the detailed form of the underlying distribution, just its asymptotic behaviour.

The layout of this paper is as follows. In Section 2 we first give a very brief overview of extreme value theory, before discussing how we connect dark matter haloes from an N-body simulation to galaxy groups in the 2PIGG catalogue. Finally, in Section 2 we describe the ensembles of N-body simulations that we use to measure the counts-in-cells distribution of massive haloes. The results of the paper are presented in Section 3, in which we show the impact of mass errors on the distribution of halo counts-in-cells and demonstrate how well extreme value theory describes the probability of finding a ‘hot’ cell. Our conclusions are given in Section 4.

## 2 METHOD

In this section we first give a brief introduction to extreme value theory (Section 2.1), before describing how we will look for superclusters like those in the 2dFGRS in N-body simulations (Section 2.2). The simulations we use are outlined in Section 2.3.

### 2.1 Extreme value theory

Extreme value theory is used extensively by weather forecasters, seismologists and actuaries, but less so by cosmologists. However, there are some recent examples of applications to features in the cosmic microwave background (Mikelsens, Silk & Zuntz 2009) and the fluctuations in the density around galaxies (Antal et al. 2009). The main result of interest for this paper is the probability distribution of extreme events. In our application the extreme event is a high count of galaxy clusters within a volume of space covered by a cell. There is no need to specify the form of the underlying probability distribution of cluster counts. The only requirement for the extreme value distribution to be applicable to the extrema of this process is that the underlying distribution is well behaved, which means that it is continuous and the cumulative distribution has an inverse. There are three types of extreme value distribution, which are distinguished by the shape of the tail of the underlying distribution. The Gumbel distribution is a type one extreme value distribution, in which case the ‘shape parameter’ of the distribution tends to zero (e.g. as is the case for a Gaussian or exponential distribution).

Returning to the application of extreme value theory in this paper, imagine a population of galaxy redshift surveys. For each survey, the distribution of the counts in cells of galaxy clusters is measured for a particular cell size. The maximum count obtained within a cell is recorded for each survey. The cumulative distribution of the maximum count across the population of surveys is given by the Gumbel distribution:

$$F(x; \mu, \beta) = \exp \left( - \exp \left( \frac{x - \mu}{\beta} \right) \right),$$

where the mean of the distribution is $\mu + \gamma \beta$, where $\gamma = 0.577 216$ is the Euler–Mascheroni constant. The standard deviation is given by $\beta \sqrt{\gamma}$. The value of $\beta$, a parameter in the Gumbel distribution, is obtained from the standard deviation of maximum cell counts values across different realizations of the density field. Likewise, the other parameter $\mu$ is derived from the mean of the maximum cell counts.

### 2.2 The extreme value data: 2dFGRS superclusters

As mentioned in the previous section, the extreme value data we will assess are the two ‘hot’ cells in the 2dFGRS identified by Baugh et al. (2004) and Croton et al. (2004). These hot cells were initially identified in a volume-limited sample of $L_v$ galaxies. On leaving out the galaxies within the two hot cells, the higher-order moments of the count distribution had the expected form on large scales. The cell radius used by Croton et al. was equivalent to a cube of side $40.6 h^{-1}$ Mpc. On cross-matching the hot cells in the galaxy distribution with the 2PIGG catalogue, there are 10 groups in each cell with an estimated mass in excess of $5 \times 10^{14} h^{-1} \text{M}_\odot$ (Padilla et al. 2004). Rather than construct mock galaxy catalogues to compare with the 2dFGRS, we shall consider instead the counts of dark matter haloes. This removes a layer of theoretical uncertainty, as there is no longer a need to include a model of galaxy formation, which may in
any case have produced a group catalogue with different properties from those of the 2PIGG sample. Furthermore, there is an objective prescription which relates the mass of a galaxy group in the 2PIGG catalogue to the mass of a dark matter halo in an N-body simulation (Eke et al. 2004b). Careful tests using simulations show that there is a scatter and a small systematic bias between the true mass of the halo in the N-body simulation $m_{\text{true}}$ and the estimated mass inferred from the galaxy groups found using a percolation algorithm (Eke et al. 2004b):

$$m_{\text{estimated}} = m_{\text{true}} \times 10^{0.1+0.3\sigma}, \quad (2)$$

where the systematic bias is 0.1 dex and $\sigma$ is a Gaussian deviate with zero mean and unit variance. Hence, given the true halo mass obtained from the simulation, we can generate an estimated mass using equation (2), to mimic the mass that would have been recovered for this halo by the 2PIGG algorithm.

2.3 The N-body simulations

We use two ensembles of 50 large volume N-body simulations called L-BASICC (Angulo et al. 2008). The ensembles correspond to different choices of the values of the cosmological parameters within a flat $\Lambda$CDM cosmology (Sánchez et al. 2006, 2009). The L-BASICC ensemble uses the same cosmological parameters as the Millennium Simulation of Springel et al. (2005): a matter density parameter, $\Omega_m = 0.25$, an energy density parameter for the cosmological constant, $\Omega_{\Lambda} = 0.75$, a normalization of density fluctuations, $\sigma_8 = 0.9$, a Hubble constant of $h = 0.73$, a scalar spectral index $n_s = 1$ and a baryon density parameter of $\Omega_b = 0.045$. The L-BASICC II ensemble uses a parameter set which is in somewhat better agreement with the latest observations of the cosmic microwave background and the local large-scale structure of the galaxy distribution (Sánchez et al. 2006): $\Omega_m = 0.237$, $\Omega_b = 0.041$, $n_s = 0.954$, $\sigma_8 = 0.77$ and $h = 0.735$. Each of the L-BASICC and L-BASICC II simulations covers a comoving cubical region of side $1340 h^{-1}$ Mpc using $448^3$ particles. This gives a particle mass comparable to that employed in the Hubble Volume simulation (Evrard et al. 2002). The equivalent Plummer softening length in the gravitational force is $\epsilon = 200 h^{-1}$ kpc. The volume of each computational box, $2.41 h^{-3}$ Gpc$^3$, is almost 20 times that of the Millennium Simulation, and more than three times the volume of the luminous red galaxy sample from the Sloan Digital Sky Survey (SDSS) used to make the first detection of the acoustic peak by Eisenstein et al. (2005). The computational box is 300 times the volume of the region covered by the volume-limited 2dFGRS sample of $L_*$ galaxies. The total volume of the ensemble is $120 h^{-3}$ Gpc$^3$, more than four times that of the Hubble Volume simulation. We use the friends-of-friends halo catalogue extracted from the $z = 0$ simulation output in our analysis, retaining objects with 10 or more particles (corresponding to a mass limit of $1.75 \times 10^{13} h^{-1} M_\odot$) to analyse the likelihood of finding rare massive structures.

3 RESULTS

In this section we present the results for the distribution of the counts in cells of dark matter haloes in the L-BASICC and L-BASICC II simulation ensembles.

We first examine the impact on the halo mass function of including the bias and error expected if the 2PIGG group finding algorithm was to be applied to a mock volume made from the N-body simulations. Fig. 1 shows the abundance of haloes at $z = 0$, both without and with incorporating the error implied by the distribution given by equation (2). There is a considerable spread in the halo mass function between the individual realizations of the ensembles for the most massive haloes. There is a significant change in the abundance of haloes at the high-mass end of the mass function upon including the mass error. The abundance of haloes of mass $\log_{10}(M_{\text{halo}}/h^{-1} M_\odot) \sim 15.5$ increases by an order of
magnitude when the mass errors are included. This is readily understood in terms of the exponential shape of the mass function for haloes of these masses. There are many more haloes of low mass than high mass. Hence, on applying a perturbation in halo mass corresponding to a symmetrical error distribution, there is a net transfer of haloes from lower mass bins to higher mass bins. This effect is compounded by the small systematic overestimate of halo mass resulting from the 2PIGG algorithm. The analytic mass functions give a reasonable match to the unperturbed mass function, except in the case of the Press & Schechter theory, which predicts too few massive haloes. This discrepancy has been noted before (see e.g. Efstathiou et al. 1988). Finally, we note that there is an appreciable reduction in the abundance of haloes of a given mass in the L-BASCC II cosmology compared with the L-BASICC cosmology, as is clear by comparing the simulation results with the analytical models in Fig. 1, which are the same in each panel.

We now measure the distribution of the counts in cells of dark matter haloes in the simulations. We consider haloes with mass in excess of $5 \times 10^{14} h^{-1} M_\odot$, using both the true mass estimated directly from the simulation by a friends-of-friends halo finder using the dark matter particles and the perturbed mass, which attempts to mimic the mass the halo would have been assigned by the 2PIGG algorithm. We use cubical cells of side $40.6 h^{-1}$ Mpc. To find cells of a similar halo overdensity to those corresponding to the 2dFGRS superclusters, we need to find cells which contain 10 or more haloes of the above mass. The halo distribution is oversampled by throwing down many more cells than would fit independently into the simulation volume. This is important because the halo count within a cell could change significantly with a small change in the location of the cell. This oversampling is taken into account when plotting the counts-in-cells probability distribution. We oversample the density field 1000 times; in practice the count probability distribution converges after a few tens of regriddings.

We show some images of selected regions in the L-BASICC simulations in Fig. 2. These plots show the projected density of dark matter in slices of $40 h^{-1}$ Mpc. The volume in these slices is 50 per cent of the volume of the 2dFGRS $L_\star$ sample. The top panels are centred on ‘hot’ cells which contain 10 or more haloes more massive than $5 \times 10^{14} h^{-1} M_\odot$ and the lower panels show randomly selected regions.

Figure 2. The projected density of dark matter in selected simulations. The colour scale is the same in each panel and covers the range from 1 (black) to 16 (red) dark matter particles per pixel, with $256 \times 256$ pixels per image. Red corresponds to a projected dark matter density of $1.5 \times 10^{13} h^{-1} M_\odot/(h^{-1} \text{Mpc})^2$. The thickness of the slices is $40 h^{-1}$ Mpc. The top two panels are centred on ‘hot’ cells in the distribution of massive haloes, and the lower panels show randomly selected regions.
We first measure the cell counts using the true positions of the haloes, without taking into account the impact of peculiar velocities. The cell counts for the two ensembles of simulations are plotted in Fig. 3. In all 100 realizations covering both cosmologies, we did not find any cells with the required occupancy of massive haloes to match the 2dFGRS superclusters, when using the true halo masses. The conclusion is very different if we consider the estimated halo masses rather than the true halo masses. Now there are many cells with 10 or more haloes of the required mass. In the L-BASICC cosmology, in the majority of realizations the hottest cells contain in excess of 10 massive haloes. Around a quarter of the realizations in the L-BASICC II ensemble contain cells with counts above this threshold. This difference in the tail of the count distribution is driven primarily by the difference in the value of $\sigma_8$ between L-BASICC and L-BASICC II.

The impact of peculiar velocities on the appearance of largescale structure in the halo distribution is taken into account using the distant observer approximation. One of the Cartesian axes of the simulation cube is taken to be the line of sight. The peculiar motion of the halo along this axis is added to its position, after applying a suitable scaling to convert the velocity into an equivalent displacement in Mpc. The change in the cell count probability is quite dramatic, as can be seen from Fig. 3. There is a shift in the maximum cell count and an increase in the spread of the maximum cell count across the realizations of the simulation ensembles. The impact of redshift-space distortions on the cell counts is akin to changing cosmology from that used in the L-BASICC to L-BASICC II simulations (which is largely due to the change in the value of $\sigma_8$).

In order to quantify these changes more clearly, we plot in Fig. 4 the distribution of the maximum cell count over the different realizations. The histograms show the distribution of hot cell counts extracted from the simulations. The curves show the Gumbel distribution of equation (1), plotted using the corresponding mean and variance of the hot cell distributions as measured from the simulations. In real space, the mean and variance of the hot cells for the L-BASICC ensemble is 11.1 and 0.995, respectively, which correspond to $\mu = 10.65$ and $\beta = 0.78$; in the case of L-BASICC II these values become a mean of 8.92 and variance of 1.562, which become $\mu = 8.21$ and $\beta = 1.22$. The agreement between the Gumbel distributions and the simulation results is impressive, being equally good in real and redshift space and with and without mass errors.

Our results are sensitive to the assumed error on the halo mass returned by the group finding algorithm applied to the 2dFGRS. In Fig. 5, we show the impact on the cell count probability of a modest increase in the size of the scatter in halo mass, using 0.4 dex instead of 0.3 dex. This 25 per cent increase in the mass error produces a substantial shift in the tail of the count distribution and, correspondingly, on the distribution of maximum cell counts, shown in the lower panel of Fig. 5. Eke et al. (2004b) made a careful assessment of the error on the recovered mass of 2dFGRS groups, using N-body simulations combined with a galaxy formation model. However, it is not inconceivable that the parameters used in the group finding algorithm could have some dependence on the galaxy formation model used in the calibration, as could the error on the recovered masses, particularly at the 25 per cent level.

So far, we have assessed the chance of finding a hot cell within the full simulation volume which is many times larger than the volume of the 2dFGRS $L_v$ volume-limited sample. We have found that cells with as many massive haloes as found in the 2dFGRS are not uncommon in the current best-fitting CDM models, particularly when the clustering is measured in redshift space and the effects of mass errors introduced by the group finding procedure are included. The volume of the $L_v$ sample is equivalent to around 120 of the cubic cells of side 40.6 $h^{-1}$ Mpc. We have measured the probability of finding a hot cell in groups of 120 cells drawn from the simulations. The cells are contiguous in the $x$ and $y$ planes within the simulation cube, but do not represent an attempt to extract a region with the same geometry as the 2dFGRS itself, just the same volume. In the L-BASICC II ensemble in redshift space, we find that in around 2 per cent of cases, a hot cell is found. Hence, the chance of finding a hot cell like the ones in the 2dFGRS is low but not negligible.

Figure 3. The count probability distribution for haloes with mass in excess of $5 \times 10^{14} h^{-1} M_\odot$ in cubical cells of side 40.6 $h^{-1}$ Mpc. In the left-hand panels, the cell counts are measured using the real-space positions of the haloes and in the right-hand panels using the redshift-space positions. The upper row shows the measurement in the L-BASICC ensemble and the lower row in the L-BASICC II simulations. Each curve shows the count distribution in one of the realizations. The different coloured lines in each panel show the measurements without and with mass errors as indicated by the key.
4 CONCLUSIONS

We have introduced a new objective methodology for assessing the likelihood of finding extreme structures in hierarchical structure formation models. Quite often the probability of finding an unusual structure such as a large void or an overdensity is estimated using a Gaussian distribution, because the smoothing scale in question is large. This is a good approximation for events which represent small departures from the mean density. However, for extreme events this is a poor assumption. The probability distribution of the density contrast on a particular smoothing scale, although assumed to be initially Gaussian in most models, rapidly evolves away from this form due to gravitational instability (e.g. Gaztañaga, Fosalba & Elizalde 2000). Assuming a Gaussian probability distribution rather than the true distribution would lead to a misestimation of the probability of finding a cell with an extreme density by many orders of magnitude.

The advantage of our approach is that we do not need to specify the actual form of the probability distribution of cell counts. We have shown that the distribution of extreme cell counts is well described by a Gumbel distribution in a range of different situations: real space, redshift space and both with and without errors in halo mass. The simulations give the mean and variance of the Gumbel distribution. The analytic form can then be extrapolated into the tails to give the probability of events which would require hundreds or even thousands of realizations of $N$-body simulations to find. By using $N$-body simulations, we can assess the probability of events...
which cannot be calculated analytically, such as the largest Einstein ring expected in a CDM model (e.g. Oguri & Blandford 2009).

Cells with the number of massive haloes seen in the 2dFGRS can be found within our simulations, provided that the clustering of these haloes is measured in redshift space and the mass errors introduced by the group finding algorithm are taken into account. However, if we consider a volume of the size of the 2dFGRS $L_s$ sample, which is 300 times smaller than our simulation volume, then we expect to find such an overdensity of cluster-mass haloes in approximately two out of a hundred cases.

Norberg et al. (2010) have carried out a related analysis using the final release of the SDSS. These authors applied a different technique to identify overdense regions. They split the galaxy distribution into zones, as would be done to carry out a jackknife estimation of the error on clustering statistics (Norberg et al. 2009). By comparing the distribution of two- and three-point correlation functions measured from the jackknife resamplings, a zone whose omission produced an outlying estimate of the clustering was found. However, when applying the same analysis to the same ensemble of $N$-body simulations used in this paper, Norberg et al. (2010) found that such outliers were quite common. There are some key differences between our analyses. The SDSS volume-limited sample is an order of magnitude larger than the 2dFGRS sample considered in this paper. Norberg et al. found one ‘unusual’ structure in the SDSS volume-limited sample for $L_s$ galaxies. Also, the method for quantifying unusual structures is different from the one we use, and will pick up a very different type of structure. The zones used by Norberg et al. sample conical volumes of space, covering a large baseline in radial distance. The superstructure in their case could be a projection of independent structures along the line of sight. In our case, we use compact cells. Norberg et al. (2010) conclude that in the larger volume of the SDSS $L_s$ sample, the structures found by their correlation function study are consistent with those found in CDM. Our results do not contradict this; we have applied a different test to search for overdense regions with a different structure.

ACKNOWLEDGMENTS

YY acknowledges a studentship from CONICET. This work was supported by the European Commission through its funding of the LENAC training project under the ALFA-II programme and by the Science and Technology Facilities Council. We acknowledge Stephane Colombi, Peter Coles and John Barrow for stimulating discussions on extreme value theory and the referee for a helpful report.

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