Candidate subdwarfs and white dwarfs from the 2MASS, Tycho-2, XPM and UCAC3 catalogues

George A. Gontcharov,1⋆ Anisa T. Bajkova,1⋆ Peter N. Fedorov2⋆ and Vladimir S. Akhmetov2⋆

1Central Astronomical observatory at Pulkovo of RAS, Pulkovskoye chaussee 65/1, 196140 Saint Petersburg, Russia
2Institute of Astronomy of Kharkiv National University, Sums’ka 35, 61022 Kharkiv, Ukraine

Accepted 2010 December 17. Received 2010 November 4

ABSTRACT
Photometry from the Two-Micron All-Sky Survey (2MASS), United States Naval Observatory CCD Astrograph Catalog (UCAC3) and SuperCosmos catalogues, together with proper motions from the Tycho-2, Kharkiv Proper Motions (XPM) and UCAC3 catalogues, is used to select all-sky samples of 28 candidate white dwarfs, 1826 evolved and 7641 unevolved subdwarfs for $R$ from 9–17 mag. The samples are separated from main-sequence stars with an admixture of less than 10 per cent, owing to an analysis of the distribution of the stars in colour index versus reduced proper-motion diagrams for various latitudes using related Monte Carlo simulations. It is shown that the XPM and UCAC3 catalogues have the same level of proper-motion accuracy. Most of the selected stars have at least six-band photometry. This allows us to eliminate some admixtures and reveal some binaries. Empirical calibrations of absolute magnitude versus colour index and reduced proper motion for Hipparcos stars give us distances and a three-dimensional (3D) distribution for all the selected stars. It is shown that the subdwarf samples are almost complete for the Tycho-2 stars, i.e. to 11 mag or 150 pc from the Sun. For fainter stars from the XPM and UCAC3 catalogues, the subdwarf samples are complete only to 20–60 per cent because of the selection method and incompleteness of the catalogues. Some conclusions can be made, however, especially for Tycho-2 stars with known radial velocities and metallicities. The subdwarfs show some concentration in the Galactic Centre hemisphere, with voids due to extinction in the Gould belt and the Galactic plane. Some as yet unexplained overdensities of evolved subdwarfs are seen in several parts of the sky. For 176 stars with radial velocities, the 3D motion and Galactic orbits are calculated. For 57 stars with Fe/H we find relations of the metallicity with colour index, asymmetric drift velocity and orbital eccentricity. All the data are consistent with the suggestion that most unevolved subdwarfs belong to the low-metallicity halo with large asymmetric drift, whereas evolved subdwarfs have various metallicities and velocities and include both disc and halo stars. The lower limit of the local mass density of unevolved subdwarfs, estimated as $2 \times 10^{-5} M_\odot$ pc$^{-3}$, appears twice as high as traditional estimates. The selected stars are listed in the new catalogue of candidate subdwarfs and white dwarfs from the 2MASS, Tycho-2, XPM and UCAC3 catalogues (hereafter SDWD catalogue) for future spectroscopic confirmation of the subluminous status of these stars, because the majority of them are now classified for the first time.

Key words: stars: Population II – subdwarfs – white dwarfs – Galaxy: stellar content.

1 INTRODUCTION
The spatial distribution, kinematics and age–velocity–metallicity relations of subluminous stars such as subdwarfs (SDs) and white dwarfs (WDs) are important yet poorly known data with which to study the structure, formation and history of the Galaxy. Traditionally, three ways have been used to separate subluminous stars from the rest. First, if the distance is known, a subluminous
star can be selected in the Hertzsprung–Russell (HR) diagram as one hotter than solar-metallicity zero-age main sequence (ZAMS) or solar-metallicity zero-age horizontal branch (ZAHB) stars of the same luminosity or absolute magnitude and fainter than ZAMS or ZAHB stars of the same temperature or colour index. Some stellar evolutionary tracks have been calculated as a theoretical basis for the existence of such stars, as discussed later. Secondly, selection by spectroscopy has been based on some intrinsic spectral features established for subluminous stars with respect to other stars of the same spectral class, such as weak lines for metal-poor subdwarfs. Thirdly, many subluminous stars were recognized as Galactic Population II members and so selection by their fast motion with respect to the Sun is possible.

Unfortunately, all three approaches have strong selection effects. As a result, the distribution of subluminous stars in the HR diagram as well as their age, metallicity and velocity is not clear.

The limitations of the above approaches have the result that only few dozen subluminous stars with precise parallaxes have been selected in the HR diagram, only a few thousands have been found after star-by-star investigation of their spectra and only hundreds with precise proper motions and/or radial velocities have been identified until recently.

New all-sky astrometric and photometric surveys of millions of stars, such as Tycho-2 (Høg et al. 2000), the Two-Micron All-Sky Survey (2MASS: Skrutskie et al. 2006), XPM (Fedorov, Myznikov & Akhmetov 2009) and United States Naval Observatory CCD Astrograph Catalog (UCAC3: Zacharias et al. 2010), provide a good source with which to select large samples of candidate subluminous stars to be confirmed by spectroscopy. This spectroscopy-independent outlook on the stars is possible by the use of multi-colour photometry and its combination with proper motions known as reduced proper motions (RPM). The RPM is defined for the 2MASS $K_s$ photometric band (and similarly to others) as

$$ M_{K_s} \equiv K_s + 5 + 5 \log(\mu) + A_{K_s}, $$

where $\mu$ is total proper motion in arcsec yr$^{-1}$ and $A_{K_s}$ is the interstellar extinction. The latter is negligible for the nearest part of the Galaxy: $A_{K_s} \approx 0.1 A_V$. This is an advantage of the use of the 2MASS catalogue, which is the only current all-sky source of precise infrared photometry.

As shown by Monte Carlo simulations (Gontcharov 2009a, hereafter GG2009), $M_{K_s}$ can be used instead of absolute magnitude $M_K$ to select some classes of stars and calculate their photoastrometric distances via empirical calibration

$$ M_{K_s} \text{ versus } M_K, $$

which is comparable to their photometric distances via calibration

$$ (J - K_s) \text{ versus } M_{K_s}. $$

Then one can consider stellar 3D distribution and kinematics. GG2009 proves that the introduced biases can be taken into the account by Monte Carlo simulations. This has been realized for clump red giants by Gontcharov (2008b). Both studies show that purity, completeness in some region of space and symmetry without a bias in favour of slow or fast stars cannot be combined in the same sample at the current level of photometric and astrometric errors.

The first investigations of subluminous stars using RPM made about 50 yr ago were discussed by Greenstein (1965). A notable investigation of SDs from the SDSS survey (Finlator et al. 2000) in a part of the sky making use of the RPM and colour index has been carried out by Smith et al. (2009).

The selection of an all-sky sample of SDs from Tycho-2 by RPM and colour index was proposed by GG2009. The current paper is a realization of that proposition expanded to fainter stars from the XPM and UCAC3 catalogues. We select SDs and WDs as all the stars in some domains in the colour–RPM ‘$(J - K_s)$–$M_{K_s}$’ diagram. Then we consider the distribution of the samples in space, colours, velocities, metallicities, elements of their Galactic orbits and Galactic populations.

The limitations of the catalogues used give us only a few WDs, far from a complete sample. Therefore, we place more attention on SDs.

We do not deal with the classic spectroscopic classification of the SDs into sdO, sdB, etc. The selected samples can be considered as candidate SDs and WDs for spectroscopy.

2 SUBDWARFS

The theoretical position of some SDs in the effective temperature–luminosity ($\log(T_{eff})–\log(L)$) diagram is shown in Fig. 1. The solar-metallicity ZAMS ($Z = 0.019$) is shown as a thick line, while those for $Z = 0.008$ and $Z = 0.0004$ are shown as dashed and dotted lines respectively and marked together as ‘ZAMS’ (Girardi et al. 2000). A classic stellar evolutionary track for the planetary nebula nucleus (PNN) stage and WD sequence of a star with initial main-sequence mass $0.9 M_\odot$ and core mass $0.6 M_\odot$ and $Z = 0.004$ is shown as a thick line at high $T_{eff}$ marked as ‘PNN–WD’ (Vassiliadis & Wood 1994). The subdwarfs should be somewhere between the thick lines. Indeed, many theoretical evolutionary tracks have been proposed to enter this HR diagram domain. However, the statistics of real samples are so poor and theoretical conclusions so contradictory that one should not calculate the probability of these tracks. For example, classical evolutionary tracks (without convective overshooting) never draw a ZAHB bluer than the solar ZAMS, but convective overshooting tracks can.

As an example, three evolutionary tracks are shown in Fig. 1 as coloured lines (with jumps from the helium flash to ZAHB shown as dashed lines):

(i) $Z = 0.0004, Y = 0.23, M = 0.9 M_\odot$ before and $0.5 M_\odot$ after the helium flash (Girardi et al. 2000) (blue);

(ii) $Z = 0.008, Y = 0.25$ and the same masses (Girardi et al. 2000) (green);

(iii) $Z = 0.04, Y = 0.46$ and the same masses (Bertelli et al. 2008) (red).

Some models allow such considerable mass loss from $0.9 M_\odot$ to $0.5 M_\odot$ on the red giant branch and its tip at the helium flash (Cassisi et al. 2003), but even a star with lower mass loss, from $0.7 M_\odot$ to $0.5 M_\odot$, can be a SD (Girardi et al. 2000).

One should note that the recently calculated red track shows that some stars with a metallicity higher than solar could be in the SD domain of the HR diagram. Another example is a quite unusual but realistic low-mass high-metallicity PNN–WD track from Fogotto et al. (1994) for $M = 0.5 M_\odot, Z = 0.05, Y = 0.35$, shown in Fig. 1 as the purple line. Such stars should be considered as SDs by definition from their positions in the HR diagram. The existence of such high-metallicity evolved SDs may be confirmed by observations.

Besides the quite exotic evolutionary tracks, evolution in some close binaries is considered as a way for a star to become an SD in the grey ellipse shown in Fig. 1 (Han et al. 2003). It was even the main explanation for the existence of SDs before convective overshooting was engaged.

© 2011 The Authors, MNRAS 413, 1581–1599

Finally, theoretically two SD domains in the $\log T_{\text{eff}}$–$\log L$ plane between the solar ZAMS and the classic PNN–WD track should be the most populated:

(i) *evolved SDs, hereafter ESDs*, hot SDs shown in Fig. 1 as the light blue domain with $4.05 < \log T_{\text{eff}} < 4.8$ (50000 < $T_{\text{eff}}$ < 60000 K, spectral classes sdO and sdB, $(J - Ks) < 0.0$, $0 < \log L < 3.2$, $M_{Ks} < 8$);

(ii) *unevolved SDs, hereafter USDs*, cool SDs shown in Fig. 1 as the light red domain with $\log T_{\text{eff}} < 3.85$ ($T_{\text{eff}} < 7000$ K, spectral classes sdF and later, $(J - Ks) > 0.1$, $\log L < 0.5$, $M_{Ks} > 2$).

The gap between these domains should not be populated by USDs because they would have low metallicity but high mass, corresponding to an age lower than 7 Gyr. It suggests too slow evolution of a rather massive star. However, some binaries of an ESD and a redder dwarf must have $(J - Ks) > 0$ and fill the gap. Real data should be used to test this.

The USDs look quite a homogeneous population: low-metallicity Population II stars with little or no Galactic rotation. By definition they are on a low-metallicity ZAMS. Therefore, their distribution with mass and age is determined by their as yet poorly known birthrate. For example, if all low-metallicity stars were born more than 7 Gyr ago then all USDs would have masses $M < 0.9 M_\odot$ (more massive SDs have left the ZAMS) and hence $T_{\text{eff}}$, spectra and $(J - Ks)$ are as pointed out earlier. However, if some low-metallicity stars were born within the last 7 Gyr in a satellite merged with the Galaxy then one might find more massive, hot and bright USDs, still fast with respect to the Sun. Therefore, kinematically selected samples of SDs could be useful to study the evolution of the Galaxy.

In contrast to USDs, the ESDs is a heterogeneous population. The only common feature for ESDs is the mass, nearly 0.4–0.6 $M_\odot$, because there is no theoretical track of a more massive star entering the ESD domain, while less massive ones are not so evolved. Various scenarios of the mass loss at the red giant branch and helium flash at its tip, as well as the evolution of close binaries are most important here because future ESD has to be quite massive before the helium flash to rich it for a reasonable time yet much less massive to enter the ESD domain. Many known spectroscopically selected ESDs are helium-rich and many are helium-core-burning stars with extremely thin hydrogen envelopes (extended horizontal branch stars). The tracks in Fig. 1 show that ESDs could have various metallicities: very low, nearly solar, very high and even exotic with low $Z$. 

Figure 1. $\log T_{\text{eff}}$ versus $\log L$ for ZAMS with $Z = 0.019, 0.008$ and 0.0004 – thick, dashed and dotted lines marked as ZAMS; PNN–WD track for 0.9 $M_\odot$ and $Z = 0.004$ – thick line on the left marked as PNN–WD; PNN–WD track from Fagotto et al. (1994) for 0.5 $M_\odot$, $Z = 0.05$ and $Y = 0.35$ – purple curve; three evolutionary tracks for stars of 0.9 $M_\odot$ before and 0.5 $M_\odot$ after the helium flash for $Z = 0.0004$ with $Y = 0.23$ (blue curve), $Z = 0.008$ with $Y = 0.25$ (green) and $Z = 0.04$ with $Y = 0.46$ (red), with jumps from the helium flash to ZAHB shown as dashed parts. The grey ellipse shows the domain of subdwarfs generated after evolution in close binaries. Finally, the theoretical domains of evolved and unevolved subdwarfs are coloured light blue and light red respectively.
and high Y. Also, the ESDs could have various ages and velocities belonging to different Galactic populations. The reason for this diversity of ESDs is that the usual evolutionary sequence from the red giant branch through horizontal branch, asymptotic giant branch, PN to WD can be deviated or interrupted to place a star in the ESD domain. Briefly, a star becomes an ESD coming from the red giant branch tip (Catelan 2007), extended horizontal branch (Fagotto et al. 1994), early asymptotic giant branch and asymptotic giant branch (Fagotto et al. 1994; Catelan 2007), as a WD (hot-flash scenario with helium-flash delay: Miller Bertolami et al. 2008), pair of merging WDs or the evolution of binary (Han et al. 2003), and so on.

To be rather numerous in large surveys, ESDs must (1) stay in the domain for a long time: several Gyr for USDs and about 100 Myr for ESDs, (2) deviate considerably from the solar ZAMS (the data used must be quite precise).

3 THE DATA

The Tycho-2 $B_T$, $V_T$ photometry and proper motions are used when the precision is better than 0.2 mag and 7 mas yr$^{-1}$ respectively.

The XPM catalogue was made in Kharkov National University, Ukraine. It combines positions from the 2MASS and USNO-A2.0 (Monet 1998) catalogues in order to derive the absolute proper motions of about 280 million stars distributed all over the sky, excluding a small region near the Galactic Centre, in the magnitude range 12 mag < $B$ < 19 mag. The mean epoch difference of the positions used is about of 45 yr for the Northern hemisphere and 17 yr for the Southern one. The zero-point of the absolute proper-motion frame (the ‘absolute calibration’) was specified with the use of about 1.45 million galaxies from 2MASS. Most of the systematical errors inherent in the USNO-A2.0 catalogue were eliminated before the calculation of proper motions. The mean formal error of absolute calibration is less than 1 mas yr$^{-1}$ (Fedorov et al. 2009).

The third US Naval Observatory CCD Astrograph Catalog, UCAC3, is a compiled all-sky star catalogue covering mainly the 8–16 mag range in a single bandpass between $V$ and $R$. We use its proper motions and photometry. The latter consists of UCAC3 own-band photometry (hereafter $R_{\text{UCAC3}}$), three bands of 2MASS ($J,H,K_s$) and three bands from the SuperCosmos project (Hambly, Irwin & MacGillivray 2001; hereafter $R_{\text{SC}}, R_{\text{SC}}, I_{\text{SC}}$). The proper motions of bright stars are based on about 140 catalogues, including Hipparcos (European Space Agency 1997; van Leeuwen 2007), Tycho and all catalogues used for the Tycho-2 proper motion calculations. The UCAC3 and SuperCosmos photometry covers all of the key data for our study. Therefore, we consider only stars with $B_{\text{UCAC3}} < 13.5$ should therefore be used with caution. (Zacharias et al. 2010). Unpublished plate measurement data from several astrometric catalogues have considerably contributed to improving proper motions for stars, mainly in the 10–14 mag range (the interval that is most interesting for us); however, these data do not cover the whole sky, as pointed out by Zacharias et al. (2010).

As pointed out earlier, precise infrared photometry makes up the key data for our study. Therefore, we consider only stars with $6 < K_s < 14$ mag, following the error budget of the 2MASS photometry. The UCAC3 and SuperCosmos photometry covers all of this photometric interval, but not for all stars. The $R_{\text{UCAC3}}$ accuracy is believed to be at the level of 0.1–0.2 mag, whereas SuperCosmos typical photometric accuracy is about 0.3 mag. However, as declared by Hambly et al. (2001), the $B_{\text{SC}}–R_{\text{SC}}$ and $R_{\text{SC}}–I_{\text{SC}}$ colours are accurate to 0.07 mag.

There are about 47 million stars with $K_s < 14$ common to 2MASS, XPM and UCAC3. The mean difference of their proper motions in the sense ‘UCAC3 minus XPM’ is quite small: $\Delta \mu_h \cos(\delta) = -2.8$ mas yr$^{-1}$, $\Delta \mu_d = 4.0$ mas yr$^{-1}$. The standard deviations of their proper motion differences are $\sigma(\Delta \mu_h \cos(\delta)) = 12$ mas yr$^{-1}$, $\sigma(\Delta \mu_d) = 12$ mas yr$^{-1}$. Taking into account that majority of common stars has $K_s \approx 14$, the accuracy of the UCAC3 proper-motion components is near 6–8 mas yr$^{-1}$, as declared for faint stars by Zacharias et al. (2010). Consequently, we conclude that the XPM proper-motion accuracy has nearly the same level. A detailed comparison of XPM and UCAC3 as well as analysis of 2MASS proper-motion errors is presented elsewhere (Fedorov et al. 2010).

4 SELECTION OF THE STARS

4.1 General principle

The usual means of selection of candidate subluminous stars in a ‘colour index versus RPM’ plane simply selects all the stars in the region of a plane defined by some linear functions of the colour index (Smith et al. 2009). In contrast to this, we try to combine three independent sources of proper motions (Tycho-2, XPM and UCAC3) with different errors, magnitude ranges and sky coverage. Therefore we have to analyse and compare the distribution of the stars in the $(J–K_s)–M_{K_s}$ plane for various latitudes for all three catalogues. Then we determine the cut lines (a priori different for the catalogues) between main-sequence (MS) stars, ESDs and WDs, giving minimal admixtures to the SD and WD samples.

A more detailed study of the distribution of the stars in the $(J–K_s)–M_{K_s}$ plane is now in progress to determine the supergiant–giant and giant–MS cut lines as well as to explain all the main features of the distribution. It will be presented elsewhere. A fairly simple structure of the HG diagram at the fainter edge of the MS and in the domain of subluminous stars allows us to restrict our current consideration to the determination of the MS–SD and SD–WD cut lines as polynomial functions of $(J–K_s)$.

4.2 Simulations

The contents of the 2MASS, XPM and UCAC3 catalogues for MS, ESD, USD and WD stars are reproduced by Monte Carlo simulations similar to Gontcharov (2010) to analyse their distribution in the $(J–K_s)–M_{K_s}$ plane and estimate the cut lines. These simulations are largely based on the Besänçon model of the Galaxy (BMG) by Robin et al. (2003) as applied to the solar neighbourhood with a radius of 2.5 kpc (due to the above 2MASS limitation $K_s < 14$ mag and the subluminous stars limitation $M_{K_s} > 2$ mag). Namely, the age, metallicity (Fe/H) mean and its dispersion around the mean, radial metallicity gradient (dex kpc$^{-1}$), initial mass function and star formation rate of the stellar components are taken from table I of Robin et al. (2003). We consider thin and thick discs and the stellar halo but not the bulge of the BMG, because there are too few bulge stars within the 2.5-kpc radius. We do not reproduce supergiants, giants, subgiants and brown dwarfs because GG2009 has shown that they are absent in the considered SD and WD ranges of the RPM and colour index.

We use normal and uniform distributions realized with the Microsoft Excel 2007 random number generator, a general description.
of which was given by Wichman & Hill (1982). Two million model stars are generated.

Let us specify the initial mass function from $0.4\,M_\odot - 2\,M_\odot$ according to the BMG (assuming that more massive stars cannot be SDs): we calculate the stellar mass (in solar masses) as $M = (M')^{-1/\alpha}$, where $M$ is a random variable distributed uniformly over the interval 0.1–6 and $\alpha$ is a parameter that depends on the stellar mass and population. In accordance with the BMG, we take $\alpha = 2$ for thick-disc and halo stars whereas $\alpha = 1.6$ for thin-disc stars with a mass $< 1\,M_\odot$ and $\alpha = 3$ for more massive thin-disc stars.

According to BMG, we assume the star formation in the thin disc to be constant over the last 10 Gyr whereas the only burst of stellar formation in the thick disc occurred at 11 Gyr and the only one in the halo at 14 Gyr. This determines the distribution of stars in age and metallicity and, consequently, in the $(J - Ks)$ versus $M_{Ks}$ plane following Girardi et al. (2000), Girardi et al. (2005) and Bertelli et al. (2008) (see their YZVAR data base of evolutionary tracks and isochrones at http://stev.oapd.inaf.it/YZVAR).

Since the statistics of the evolutionary tracks entering the ESD domain is poor, the durations of the pre-ESD and ESD stages are taken into account assuming that every star of an initial mass from $0.4\,M_\odot - 1\,M_\odot$ becomes an ESD at a particular stage of its evolution determined by the tracks from Bertelli et al. (2008) and (Fagotto et al. 1994). Because of the poor statistics, we have to accept a normal distribution of the ESDs in $(J - Ks)$ with a mean of $-0.15$ mag and standard deviation of $0.07$ mag and an independent normal distribution of them in $M_{Ks}$ with mean of 5 mag and standard deviation of 1 mag.

WDs are taken into account separately but self-consistently according to BMG (section 2.5 of Robin et al. 2003).

The local mass density of the stellar components, density laws and associated parameters of the stellar components are assumed according to BMG (tables 2 and 3 of Robin et al. (2003)). Within a 2.5-kpc radius from the Sun it means an almost uniform distribution of all stars in rectangular Galactic coordinates $X$ and $Y$ with a slight increase of the density in the Galactic Centre direction. The distribution of the halo stars in coordinate $Z$ is almost uniform, with a slight decrease with $|Z|$.

The distributions of the stars in standard rectangular velocity components $U$, $V$ and $W$ are calculated from the velocity dispersions and asymmetric drift taken from BMG (table 4 of Robin et al. 2003).

The interstellar extinction $A_K$ is calculated as $0.11A_V$, where $A_V$ is calculated following a new model of extinction by Gontcharov (2009b) taking into the account the extinction in both the Galactic plane and the Gould belt.

The errors of photometry for $Ks < 14$ mag are accepted as 0.05 mag, whereas the errors of the proper motions are varied widely in the simulations to reproduce the level of the errors of real catalogues.

For every model star we calculate

(i) true distance $R = (X^2 + Y^2 + Z^2)^{1/2}$,
(ii) longitude $l$ and latitude $b$: $\tan(l) = Y/X$, $\tan(b) = Z/(X^2 + Y^2)^{1/2}$,
(iii) proper-motion components $\mu_l = [U\sin(l) - V\cos(l)]/(4.74R)$, $\mu_b = [U\cos(l) \sin(b) + V\cos(l) \sin(b) - W\cos(b)]/(4.74R)$,
(iv) total proper motion $\mu = (\mu_l^2 + \mu_b^2)^{1/2}$,
(v) $Ks = M_{Ks} - 5 + 5 \log(R) + A_K$,
(vi) truncation of the sample at $Ks = 14$ mag,
(vii) $M_{Ks}$ from $Ks$, $\mu$ and $A_K$, following equation (1).

---

Figure 2. The simulated distribution of the MS, SD and WD stars in plane (a) $(J - Ks)$ versus $M_{Ks}$, (b) $(J - Ks)$ versus $M_{Ks}$ with proper-motion errors $\sigma(\mu) = 1$ mas yr$^{-1}$, (c) the same but $\sigma(\mu) = 10$ mas yr$^{-1}$, (d) the same but with photometric errors of 0.05 mag, (e) the same but $\sigma(\mu) = 20$ mas yr$^{-1}$ without photometric errors. The MS domain is shown as the light grey spot, the SD ones darker spots and the intersection of the MS and SD domains black spots whereas WDs are shown as discrete black points.
Table 1. Coefficients of equation (4) approximating the distribution of XPM, UCAC3 and Tycho-2 MS and SD stars with $-0.34 < (J - Ks) < 0.35$ along $M'_{Ks}$. The Tycho-2 SD distribution is not fitted because there are no WDs in the Tycho-2 sample and all its stars fainter than the MS–SD cut point are sampled as SDs.

| Catalogue | $|b|$ | MS $k_1$ | MS $k_2$ | MS $k_3$ | MS $k_4$ | SD $k_1$ | SD $k_2$ | SD $k_3$ | SD $k_4$ |
|-----------|------|----------|----------|----------|----------|----------|----------|----------|----------|
| TYC2      | <10  | 11000    | 5.9      | 2.2      | -0.08    |           |           |           |           |
| TYC2      | >50  | 7500     | 7.2      | 2.15     | -0.08    |           |           |           |           |
| XPM       | <10  | 280000   | 7.7      | 2.4      | -0.06    | 9000     | 11.0     | 0.95     | 0.02     |
| XPM       | >50  | 31000    | 8.1      | 2.3      | -0.09    | 600      | 11.5     | 0.8      | 0.01     |
| UCAC3     | <10  | 170000   | 7.4      | 2.2      | -0.065   | 5000     | 11.0     | 0.85     | 0.02     |
| UCAC3     | >50  | 38000    | 8.0      | 2.2      | -0.08    | 300      | 11.5     | 0.8      | 0.01     |

Some examples of results of the simulations are presented in Fig. 2. The simulations yield about 4000 SDs and 10 WDs among 2 million model stars; the rest are MS stars. Therefore, for better representation of the simulation results the regions with the highest density of SDs, containing 90 per cent of them, are shown as two grey spots of middle intensity, one for ESDs and another for USDs; a region with an MS star density higher than the density of SDs at the edge of their spots is shown as a light grey spot, whereas the WDs are shown as black discrete points. The contamination of the samples can be estimated at the intersection of the MS and SD spots, shown as black domains, as well as at that of WD points over SD spots.

The distribution of the stars in the ($J - Ks$) versus $M'_{Ks}$ diagram is shown in Fig. 2(a). The MS, SD and WD stars are completely separated in this diagram. There are no SDs and WDs for ($J - Ks$) > 0.7 because of the magnitude limits of the catalogues.

The distribution of the stars in the ($J - Ks$) versus $M'_{Ks}$ diagram is shown in Fig. 2: in (b) the proper-motion error is 1 mas yr$^{-1}$, no errors of photometry, in (c) the proper-motion error is 10 mas yr$^{-1}$, no errors of photometry, in (d) the proper-motion error is 10 mas yr$^{-1}$, photometry error is 0.05 mag and in (e) the proper-motion error is 20 mas yr$^{-1}$, no errors of photometry. One can see that the photometric errors at the level of 0.05 mag have little influence on the contamination of the MS and SD samples. In contrast, the proper-motion errors are very important for the separation of the samples because the $M'_{Ks}$ dispersion increases with an increase in proper-motion errors. Almost pure and complete SD and WD samples for ($J - Ks$) < 0.7 can be obtained with proper-motion errors at the level of 1 mas yr$^{-1}$. In contrast, practically no SD or WD sample can be obtained with errors at the level of 20 mas yr$^{-1}$, the level of typical SD and WD proper motion itself.

Fig. 2(d) shows that the actual level of accuracy of the XPM and UCAC3 catalogues, about 10 mas yr$^{-1}$, together with the actual level of photometric accuracy of the 2MASS catalogue, are acceptable for the selection of quite clear SD and WD samples with less than 10 per cent contamination by each other and by MS stars. However, in this way we lose the slowest ESDs and especially USDs. Consequently, the samples obtained in this way have strong biases. In this case the MS–SD cut line can be drawn as quite a simple polynomial function of ($J - Ks$).

The SD–WD cut line can be determined only in the range $-0.1 < (J - Ks) < 0.4$. For $-0.1 < (J - Ks) < -0.04$, the ESDs and WDs cannot be separated because of their proximity in the HR diagram; for ($J - Ks$) > 0.4 no WD must be in the samples because of the magnitude limits of the catalogues.

The reddening applied to the stars has little effect on the selection of USDs, WDs and the faintest ESDs but quite a noticeable one on the selection of the brightest ESDs: shifted to redder colours, they are lost among the MS stars.

4.3 Selection in detail

To determine the cut-line polynomials we consider the one-dimensional distributions of stars for sections of ($J - Ks$) from −0.3 to 0.7 mag with a variable width from 0.01–0.1 mag and for empirically determined sections of latitude ($|b| > 50^\circ, 30^\circ < |b| < 50^\circ, 10^\circ < |b| < 30^\circ$ and $|b| < 10^\circ$) accepted as having enough stars in every section.

We suspect that the distributions of MS and SD stars in every section can be approximated by two asymmetric Gaussians of the form

$$k_1 \exp \left[ -\left( M'_{Ks} - k_2 \right)^2 / 2 \left( k_3 + k_4 M'_{Ks} \right)^2 \right] / \left( k_3 + k_4 M'_{Ks} \right).$$

where $k_1$ is the amplitude, $k_2$ is the mean and $k_3 + k_4 M'_{Ks}$ is the standard deviation of the Gaussian. The asymmetry of the distribution can be explained. Even though the initial luminosity function

![Image](https://academic.oup.com/mnras/article-abstract/413/3/1581/963737/1586)
Candidate subdwarfs and white dwarfs

The stars selected by use of equation (5): (a) Tycho-2 stars in the diagram \((J - Ks)\) versus \((B_T - Ks)\), (b) XPM–UCAC3 ones in the diagram \((J - Ks)\) versus \((BSC - Ks)\). The empirical ZAMS are shown by solid lines. The dashed lines show the theoretical reddening slope and separate stars with normal colour relations (below the dashed lines) from those with colour discrepancies (above the dashed lines).

The stars selected by use of equation (5): (a) Tycho-2 stars in the diagram \((B_T - V_T)\)–\(M_V\) (the Hipparcos stars with parallax relative error less than 0.3 are shown as the grey bulk for comparison), (b) XPM–UCAC3 stars in the diagram \((BSC - ISC)\)–\(M_{BSC}\). The lines separate the suspected SDs and WDs (below) from the remaining stars (above).

Therefore, all Tycho-2 stars fainter than the MS–SD cut point are sampled as SDs and the Tycho-2 SD distribution is not fitted.

The distributions of the stars along \(M_{Ks}\) and their approximations for the section \(0.34 < (J - Ks) < 0.35\) are shown in Fig. 3 as black lines for XPM, red lines for UCAC3 and blue lines for the Tycho-2 catalogue for different latitudes: (a) \(|b| < 10^\circ\) and (b) \(|b| > 50^\circ\). Solid, dashed and dotted lines are the distribution, MS Gaussian and SD Gaussian respectively. The coefficients of the Gaussians are taken from Table 1. For high latitudes the sum of the Gaussians perfectly fits the fainter end of the distribution (the black and red dotted lines coincide with the solid ones in panel b). No SD Gaussian is fitted for Tycho-2 (no blue dotted line is drawn).

The XPM and UCAC3 distributions are very similar for high latitudes and slightly different near the equator. The reason for the latter is that the XPM contains more equatorial stars in some overcrowded zones. They were processed with lower accuracy, giving a larger standard deviation of the MS Gaussian. The worse proper motions of these stars influences the overall error budget of the XPM. In contrast, the UCAC team simply eliminates such stars from the processing. We do not consider these stars, taking into the account only the ones common to XPM and UCAC3. Otherwise very similar distributions of XPM and UCAC3 stars on \(M_{Ks}\) mean that these catalogues have the same level of proper-motion accuracy. Together with the fact that SDs at different latitudes cover approximately the same interval of \(M_{Ks}\) (as seen in Fig. 3), this allows us to accept the unified cut points for all catalogues and all latitudes: for example, for the section \(0.34 < (J - Ks) < 0.35\) \(M_{Ks} = 12.5\) mag for the MS–SD cut point and \(M_{Ks} = 15\) mag for the SD–WD cut point as marked in Fig. 3.
Finally, we accept the MS–SD cut line
\[ M'_{Ks} = 11.8(J - Ks)^3 - 16.8(J - Ks)^2 + 12.4(J - Ks) + 9.5, \]
as well as the SD–WD cut line, applicable for \(-0.1 < (J - Ks) < 0.4\) and only at low latitudes:
\[ M'_{Ks} = -13(J - Ks)^2 + 11.6(J - Ks) + 12.6. \]

4.4 Additional cleaning of the samples

The stars selected by use of equation (5) are shown in Fig. 4: (a) Tycho-2 stars in the diagram \((J - Ks)\) versus \((B_T - Ks)\) and (b) the remaining stars in the diagram \((J - Ks)\) versus \((B_{SC} - Ks)\). The empirical ZAMS \((B_T - Ks) = 5.6(J - Ks) - 0.1\) and the line \((B_T - Ks) = 7.5(J - Ks) + 1.4\) with theoretical reddening slope, both taken from Gontcharov (2008a), are shown in Fig. 4(a) for Tycho-2 stars by thick and dashed lines respectively. Similar lines shifted due to \(B_T - B_{SC}\) differences are shown in Fig. 4(b) for the remaining stars: \((B_{SC} - Ks) = 5.6(J - Ks) - 0.4\) and \((B_{SC} - Ks) = 7.5(J - Ks) + 2.1\). The spread of the dots near the ZAMS is due to the photometric errors and reddening spread. The outliers lower than the ZAMS prove to be binaries with composite spectra (Stark & Wade 2003). The outliers higher than the reddening lines show extraordinary discrepancies between their colours. They can be explained by stellar duplicity, known in most cases or hidden. It gives visual/infrared identification mistakes and/or unusual energy distributions in the common spectrum. However, one should not

Table 2. Number of selected stars by band, magnitude range, median magnitude and median photometric error.

<table>
<thead>
<tr>
<th>Band</th>
<th>Stars</th>
<th>Mag. range</th>
<th>Median mag.</th>
<th>Median error</th>
</tr>
</thead>
<tbody>
<tr>
<td>BT</td>
<td>615</td>
<td>7.6–13.1</td>
<td>11.9</td>
<td>0.10</td>
</tr>
<tr>
<td>VT</td>
<td>538</td>
<td>7.3–12.8</td>
<td>11.7</td>
<td>0.13</td>
</tr>
<tr>
<td>BSC</td>
<td>8825</td>
<td>8.9–18.4</td>
<td>15.3</td>
<td>≈0.2</td>
</tr>
<tr>
<td>RSC</td>
<td>8972</td>
<td>9.0–16.9</td>
<td>14.6</td>
<td>≈0.2</td>
</tr>
<tr>
<td>IS</td>
<td>8972</td>
<td>9.3–16.5</td>
<td>14.1</td>
<td>≈0.2</td>
</tr>
<tr>
<td>RUCAC3</td>
<td>7316</td>
<td>9.4–17.3</td>
<td>15.0</td>
<td>0.13</td>
</tr>
<tr>
<td>J</td>
<td>9495</td>
<td>6.7–14.7</td>
<td>13.8</td>
<td>0.03</td>
</tr>
<tr>
<td>Ks</td>
<td>9495</td>
<td>6.5–14.0</td>
<td>13.5</td>
<td>0.03</td>
</tr>
</tbody>
</table>
simply eliminate these stars because some of them could contain SD or WD components, specifically taking into account that the binary fraction in ESD stars seems to be much higher than that in MS stars (Østensen 2009).

The pairs with subluminous components can be separated from the rest of the binaries in the diagrams \((B_T - V_T) - M'_V\) and \((B_{SC} - I_{SC}) - M'_{BSC}\) after the main selection by use of equation (5). These two diagrams for stars selected by use of equation (5) are shown in Figs 5 (a) and (b) respectively. In fact, the former diagram contains Tycho-2 stars and the latter the remaining stars. The \(Hipparcos\) stars with parallax relative error less than 0.3 are shown in the former diagram as a grey ‘cloud’ of points. This could not be provided for the latter diagram because no precise \(B_{SC}\) and \(I_{SC}\) for the \(Hipparcos\) stars are available. After star-by-star inspection, all stars higher than the lines \(M'_V = 10(B_T - V_T) + 6\) and \(M'_{BSC} = 4.5(B_{SC} - I_{SC}) + 5\) shown in Fig. 5 appear to be extraneous and are eliminated.

5 STATISTICS OF THE SELECTED STARS

The final sample of 9495 candidate SDs and WDs is shown in Fig. 6 in the diagram \((J - Ks) - M'_Ks\), together with the \(Hipparcos\) stars with parallax relative error less than 0.3 (the grey ‘cloud’ of points). The 863 stars selected from Tycho-2 are shown in panel (a) and 8632 from XPM and UCAC3 in panel (b) (358 Tycho-2 stars are also XPM and UCAC3 ones, but their Tycho-2 proper motions are preferred). The accepted MS–SD cut line is shown as the solid black curve at the higher border of the black bulk of the points.

The SD–WD cut line from equation (6) is shown as a dashed line. For \((J - Ks) < -0.1\) we accept an ESD–WD cut line that has all known WDs lower than it and is also at a drop of the distribution of the selected stars in the \((J - Ks) - M'_Ks\) diagram. We suspect that this drop is due to a difference between the local densities of ESDs and WDs. We accept the ESD–WD cut line

\[
M'_Ks = 10.3 - 10(J - Ks),
\]

shown in Fig. 6 as the solid black line at lower left border of the black bulk of the points.

The gap between the grey and black bulks in Fig. 6 means that we have to apply rather a strong cut to obtain very pure samples. Therefore, we expect to lose many slow SDs and even some known ones selected by spectroscopy. The percentage of completeness will be estimated later in this paper. However, the completeness and purity of the sample should be proven by spectroscopic studies.

Two bulks, the ESDs and USDs, are evident in Fig. 6, with highest density at \((J - Ks) \approx -0.1 - 0\) and \(\approx 0.3 - 0.4\) respectively. As expected, USDs are much more numerous than ESDs. Most ESDs are selected from Tycho-2 whereas most USDs are from

Figure 9. Some colour–colour diagrams to separate USDs (grey points) from the remaining stars (black points).

© 2011 The Authors, MNRAS 413, 1581–1599
XPM and UCAC3. Tycho-2 contains too few USDs because of the magnitude limitation.

The ESD and USD bulks are evident in the distribution of the Tycho-2 (open diamonds) and XPM–UCAC3 (black points) selected stars in the diagram of $(J - K_S)$ versus total proper motion shown in Fig. 7. The proper motions are in mas yr$^{-1}$. The ESD and USD bulks fall into the same $(J - K_S)$ colours for bright (Tycho-2) and faint (XPM–UCAC3) stars. This means that the gap between the bulks is not catalogue- or magnitude-dependent.

It is important that the lower edge of the bulks rises with colour. The ESD sample is more complete than the USD one: the former contains quite slow stars with $\mu \approx 10$ mas yr$^{-1}$.

The number of selected stars by band, magnitude range, median magnitude and median photometric error is presented in Table 2.

The distribution of the selected stars in $(J - K_S)$ is shown in Fig. 8. The ESD and USD Gausians are evident here, with some asymmetries due to binaries (particularly at $-0.1 < (J - K_S) < +0.1$) and reddening (particularly at $(J - K_S) > 0.4$).

The USDs can be separated from the remaining stars in some colour–colour diagrams due to differences in their spectra. We accept the set of the following empirical conditions to select USDs: $$(J - K_S) > 0.06 \ (0.15 \text{ for Tycho-2}),$$ $$(B_{SC} - I_{SC}) > 0.75,$$ $$(B_{SC} - R_{SC}) > 0.42,$$ $$(R_{UCAC3} - K_S) > 1.25,$$ $$(B_{SC} - K_S) > 1.8 - (R_{UCAC3} - R_{SC}).$$

Some examples of colour–colour diagrams with USDs selected according to the above conditions as grey points and the remaining stars as black ones are presented in Fig. 9. One can see two overdensities in the bulk of the non-USD stars: at $(J - K_S) \approx -0.2$ and $(J - K_S) \approx 0$. The latter contains many binaries of an ESD and a red dwarf, making for a redder total colour index. It is hard, however, to separate single ESDs and binaries by such colour–colour diagrams.

Equations (8) and (9) are used to separate three samples among the 9495 selected stars: 7641 USDs, 1826 ESDs and 28 WDs.

The distribution of the selected stars in $J$ magnitude is shown in Fig. 10(a): a dash–dotted line for WDs, black solid line for ESDs and grey solid line for USDs. The step changes at $J \approx 8$, $J \approx 11$ and $J \approx 14$ mag, corresponding to the magnitude limits of the Hipparcos, Tycho-2 and 2MASS catalogues respectively. We suspect that the samples are complete for the brightest stars. The expected numbers of stars are calculated assuming their spatial distribution according to the BMG (for example, almost uniform for USDs, as for halo stars). The expected numbers are shown as black and grey dashed lines for ESDs and USDs respectively. The relation of the number of the selected stars to the expected number of stars should be considered as the percentage of the completeness of the sample. Its distribution in $J$ magnitude for ESDs (black curve) and USDs (grey) is shown in Fig. 10(b).

It is evident that the selection criteria are stronger for fainter stars. For Tycho-2 stars, or to $J \approx 11$ mag or the distance $R \approx 150$ pc (taking $M_J < 5$ mag), the ESD sample is almost complete whereas the USD one is complete to the level of 60 per cent. This is a reason to consider the distributions of the selected Tycho-2 stars separately in further figures in order to understand the importance of sample completeness.

Hipparcos contains 139 selected stars. There is a spectral classification for 129 stars from the Tycho-2 Spectral Types catalogue (TST) by Wright et al. (2003) or other sources. In many cases the classification is marked as doubtful. Only 27 stars are classified as SDs and 10 as WDs.

Fig. 11 shows the diagram $(J - K_S) \text{ versus } M_{K_S}$, where $M_{K_S}$ is calculated from the Hipparcos parallaxes for

(i) all Hipparcos stars with parallax relative error less than 0.3, shown as a grey ‘cloud’ of points,

(ii) two selected ESDs previously classified as such by the TST and other sources, shown as blue open circles,

(iii) 10 selected ESDs without previous classification, shown as blue filled circles,

(iv) one previously classified ESD not selected by us, shown as a brown filled circle,

(v) nine selected WDs previously classified, shown as green open diamonds,

(vi) three selected WDs without previous classification, shown as green filled diamonds,

(vii) three previously classified WDs not selected by us, shown as red diamonds,

(viii) 10 selected USDs previously classified, shown as purple open squares,

(ix) 23 selected USDs without previous classification (including two outliers discussed later), shown as purple filled squares,

(x) 23 previously classified USDs not selected by us, shown as orange squares.

It is evident from the absolute magnitudes that the ESDs and WDs that are classified but not selected (red and brown symbols) have the wrong spectral classification. Perhaps, the same is true of some of the classified but not selected USDs (orange squares); however, it seems that most of them are lost due to our strong cut. Thus, mistakes in the spectral classification of subluminous stars are common. Our method helps to verify the status of the stars.
A preliminary comparison of non-Hipparcos stars of our samples with known lists of classified subluminous stars reveals six common WDs and 152 common ESDs using the BG survey (Green 2008), but no common ones using the 1717 stars from Smith et al. (2009). The reasons are the different magnitude and colour-index ranges, different sky coverage and very different principles of selection. A detailed comparison of our samples with lists of known subluminous stars will be provided elsewhere.

Two outliers in Fig. 11 at $0.2 < (J - K_s) < 0.3$ and $M_{K_S} \approx 7$ are interesting examples of the influence of duplicity. They are HIP 10529 and 3446. HIP 10529 is a known pair with HIP 10531, probably optical. Its Hipparcos parallax may be wrong. The colours are agreed; therefore it must be a USD. HIP 3446 has a large error of parallax (3 mas) and a discrepancy of colours. This must mean duplicity with a component that is a USD. The arrows in Fig. 11 show the suspected relocation of these outliers to more convenient positions in the plane.

The lines in Fig. 11 are some $(J - K_S)$ versus $M_{K_S}$ calibrations. The dotted line is the accepted empirical calibration for WDs:

$$M_{K_S} = -23.364(J - K_S)^4 + 33.36(J - K_S)^3$$
$$- 14.05(J - K_S)^2 + 3.35(J - K_S) + 12.53. \quad (10)$$

It approximates the theoretical calibrations from the BMG and from the TRILEGAL code by Girardi et al. (2005). The empirical intrinsic spread of the WDs about the line is 0.2 mag.

The solid line is the accepted empirical calibration for SDs:

$$M_{K_S} = 18.28(J - K_S)^4 - 42.968(J - K_S)^3$$
$$+ 30.36(J - K_S)^2 - 2.13(J - K_S) + 2.38. \quad (11)$$

Its intrinsic spread is about 0.3 mag for USDs and about 1 mag for ESDs. In fact it reflects the homo-/heterogeneous nature of these stars. As expected, for USDs this calibration follows the general direction of the isochrones. The dash–dotted and dashed lines show the isochrones for metallicities $Z = 0.001$ and $Z = 0.019$. The accepted calibrations are shown for SDs (solid line) and WDs (dotted line). The arrows show the suspected replacement of two outliers.

It has been shown that the $(J - K_S)$ for some binaries can be erroneous. Therefore, for all selected stars with colour-index discrepancies the empirical calibration is used:

$$M_{K_S} = 0.544M'_{K_S} - 0.88. \quad (12)$$

The accuracy of this calibration is estimated from the spread of Hipparcos stars with well-known parallaxes as about 1 mag.

Photoastrometric (in cases of colour-index discrepancies) and photometric (in other cases) distances as well as related rectangular coordinates XYZ are calculated for the selected stars by use of the calibrated $M_{K_S}$ from

$$\log(R_{ph}) = (K_S - M_{K_S} + 5 - A_{K_S})/5 \quad \text{(for the sake of simplicity all obtained distances are referred to as } R_{ph} \text{ hereafter).}$$

The interstellar extinction $A_{K_S}$ is generally no more than...
Some overdensities of ESDs are seen both among Tycho-2 and UCAC3-XPM stars at

(i) $l \approx 218, b \approx +5$,
(ii) $l \approx 278, b \approx -32$, in front of the Large Magellanic Cloud,
(iii) $l \approx 287, b \approx -2$, probably near the $\eta$ Car region,
(iv) $l \approx 314, b \approx +15$, probably near the Sco–Cen association,
(v) $l \approx 318, b \approx -12$,
(vi) $l \approx 332, b \approx -2$,
(vii) $l \approx 5, b \approx -42$,
(viii) $l \approx 22, b \approx -32$,
(ix) $l \approx 33, b \approx -41$,
(x) $l \approx 94, b \approx -2$,
(xi) $l \approx 137, b \approx -22$.

The existence of the overdensities among the almost complete sample of Tycho-2 ESDs proves that they are not a selection effect. A detailed investigation of individual stars in the overdensities shows that they are groups of ESDs of similar magnitudes and, consequently, similar distances. Thus, these overdensities are real concentrations of ESDs in space. Moreover, they show similar proper motions within the overdensities. Unfortunately, no radial velocity or metallicity is known for these stars. In all cases except three, we have found no known stellar clusters, moving groups or other features in these regions. The nature of the overdensities is to be revealed. One may suspect them as some remnants of dwarf galaxies or globular clusters. Alternatively, since some of the overdensities are near star-formation regions, one might suspect intense mass loss for stars near the regions making so many ESDs.

6 3D DISTRIBUTION AND MOTION OF THE STARS

The spatial distributions of the selected ESDs are shown in Fig. 14: all ESDs projected into the (a) $XY$, (b) $XZ$ and (c) $YZ$ planes as well as Tycho-2 ESDs projected into the same planes in panels (d), (e) and (f) (distances in kpc). The almost complete sample of Tycho-2 ESDs shows the same features as the very incomplete one of fainter stars. The same data for USDs are shown in Fig. 15. The USDs from Tycho-2 show no interesting features containing nearby stars only. The distribution of the selected WDs is not shown, because all of them are within 50 pc. The voids due to extinction in the Gould belt are evident in both Figs 14 and 15 for all samples reaching at least 500 pc. These voids are marked by arrows in the $XZ$ plots. Higher stellar density in the Galactic Centre hemisphere according to BMG is evident in the $XY$ and $XZ$ plots for ESDs and fainter USDs.

The Pulkovo Compilation of Radial Velocities (PCRV) catalogue (Gontcharov 2006) contains radial velocities for 72 selected stars with precision better than 5 km s$^{-1}$. Less precise radial velocities are collected from other sources for 104 more selected stars. Precise
radial velocities are the crucial data for modern kinematic analysis of any stellar sample. Some advances in radial velocity data are expected in near future. This preliminary study with poor statistics based either on the 72 best radial velocities or all 176 is only a test of the feasibility of our data and results.

These radial velocities are used together with $\alpha$, $\delta$, $\mu$ and $\pi$ or $R_{\text{ph}}$ to calculate rectangular components $U$, $V$, $W$ of the stellar space motion with respect to the Sun. No correction for Galactic rotation and solar motion to the apex is applied.

The Hipparcos distances highly correlate with $R_{\text{ph}}$ for the 72 best stars. Therefore, the main features of the stellar distribution in the six-dimensional space $XYZUVW$ is the same for both sets, based on Hipparcos distances or $R_{\text{ph}}$. Here we consider only $XYZUVW$ based on $R_{\text{ph}}$.

The means and dispersions for $U$, $V$ and $W$ velocity components for 11 ESDs and 60 USDs are presented in Table 3 (in km s$^{-1}$). One should remember that these samples are kinematically biased and far from complete, so it is only a test of the feasibility. Asymmetric drift (considerable negative $V$) is evident for both ESDs and USDs. For USDs it is similar to the BMG value for a halo: $-226$ km s$^{-1}$ (Robin et al. 2003). For ESDs it is between the Besançon halo and thick-disc ($-53$ km s$^{-1}$) values, supporting a hypothesis that the ESDs comprise a heterogeneous sample, with disc and halo stars.

The distributions of 11 ESDs (circles) and 60 USDs (open squares) with precise radial velocities in the projection on the $UV$, $UW$ and $VW$ planes (velocities in km s$^{-1}$) are shown in Fig. 16. A strong bias due to the selection in favour of higher velocities with respect to the Sun is evident. However, one can conclude that the dispersion of the velocities and asymmetric drift in the $V$ component do not contradict the hypothesis that the ESDs under consideration contain both disc and halo stars whereas all or almost all USDs belong to the halo. Some of the USDs have retrograde motion.

Metallicities Fe/H are collected for 57 stars from various sources. The relation of Fe/H and velocity component $V$ is shown in Fig. 17 for ESDs (circles) and USDs (open squares) (again only as a test of the feasibility of our results). The error bars show an estimation of the accuracy of $V$ from $\mu$, radial velocity and $R_{\text{ph}}$ as well as the error of about 0.3 dex accepted for Fe/H. The solar position is marked. The vertical lines show the separation of thin/thick disc at
about Fe/H ≈ −0.3 (Z ≈ 0.01) and thick disc/halo at about Fe/H ≈ −1.3 (Z ≈ 0.001), similarly to Robin et al. (2003). The ESDs are found both in halo and disc (one ESD with halo metallicity, HIP 103755, is a known RR Lyr variable (European Space Agency 1997)). The thin–thick disc separation is not evident in these data. The USDs belong to the halo and probably the thick disc (taking into account that the errors for the most metal-rich USDs are at the halo/disc boundary). The velocity–metallicity trend is evident for the USDs but should be taken with caution because of poor and biased statistics.

Our data show a relation between Fe/H of SDs and their position in the HR diagram, which should be taken with caution because of poor and biased statistics. The ESDs (circles) and USDs (open squares) are shown in Fig. 18 for (a) $M_K$ versus Fe/H and (b) $(J-K_S)$ versus Fe/H. In fact, the relation for ESDs is due to the two most luminous stars with high velocity and low metallicity and, consequently, may be an artefact. The relations for the USDs can be explained by colour–age and colour–metallicity relations (Girardi et al. 2000): the turn-off point becomes redder with an increase of age and metallicity. Therefore, one can see that the redder part of the USD sample has a low dispersion of Fe/H, containing only old stars with moderate metallicity, whereas the bluer part has a larger dispersion of Fe/H because of the admixture of younger stars with higher Fe/H and older stars with lower Fe/H.

As a test of the feasibility of our results, the Galactic orbits are calculated for 176 selected stars with $XYZUVW$ accepting a solar galactocentric distance of 8.5 kpc. We used the following fixed Galactic potential by Fellhauer et al. (2006) and Helmi et al. (2006):

$$\Phi = \Phi_{\text{halo}} + \Phi_{\text{disc}} + \Phi_{\text{bulge}}.$$  \hspace{1cm} (13)

The halo is represented by a logarithmic potential of the form

$$\Phi_{\text{halo}}(R, Z) = v_0^2 \ln(1 + R^2/d^2 + Z^2/d^2),$$  \hspace{1cm} (14)

with $v_0 = 134$ km s$^{-1}$ and $d = 12$ kpc. The disc is represented by a Miyamoto–Nagai potential

$$\Phi_{\text{disc}}(R, Z) = -G M_d \left[ \frac{R^2 + [b + (Z^2 + c)^{1/2}]^2}{R^2 + b^2} \right]^{1/2} \hspace{1cm} (15)$$

<table>
<thead>
<tr>
<th>$U$</th>
<th>$V$</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESD</td>
<td>30/92</td>
<td>−124/113</td>
</tr>
<tr>
<td>USD</td>
<td>−23/189</td>
<td>−267/94</td>
</tr>
</tbody>
</table>

Table 3. Mean/dispersion for $U$, $V$ and $W$ velocity components for ESDs and USDs (in km s$^{-1}$).
Candidate subdwarfs and white dwarfs

Figure 16. The distributions of ESDs (circles) and USDs (open squares) with precise radial velocities in a projection in the $UV$, $UW$ and $VW$ planes (velocities in $\text{km s}^{-1}$).

Figure 17. The relation of Fe/H and velocity component $V$ for ESDs (circles) and USDs (open squares).

Figure 18. The relation of Fe/H and (a) $M_{\text{Ks}}$, (b) $(J - Ks)$ for ESDs (circles) and USDs (open squares).

with $M_0 = 9.3 \times 10^{10} \text{M}_\odot$, $b = 6.5 \text{kpc}$, $c = 0.26 \text{kpc}$. The bulge is modelled as a Hernquist potential

$$\Phi_{\text{bulge}}(R) = -GM_b/(R + a)$$

with $M_b = 3.4 \times 10^{10} \text{M}_\odot$, $a = 0.7 \text{kpc}$. The circular speed at the solar radius is accepted as 220 km s$^{-1}$. The solar velocity with respect to the local standard of rest is taken from Dehnen & Binney (1998). The orbits were calculated over a total of 1.1 Gyr backwards.

The eccentricities for three WDs are 0.03 for the star 2MASS PSC 1098632336, 0.11 for HIP 14754 and 0.31 for HIP 101516. The projections of the WD orbits in the $XY$, $XZ$ and $YZ$ planes are shown in Fig. 19(a) by solid, dotted and dashed curves for different stars (distances in kpc). All of these orbits better fit the thin disc.
The projections of the ESD and USD orbits in the $XY$, $XZ$ and $YZ$ planes are shown in Fig. 19(b) and (c) respectively (distances in kpc). Note the different scales in the different panels.

The distributions of ESDs (dashed line) and USDs (solid line) with orbital eccentricity are shown in Fig. 20. The vertical lines show the suspected separation into thin disc, thick disc and halo. The mean eccentricity for the orbits of 40 ESDs is $0.4 \pm 0.3$ and that for 133 USDs is $0.8 \pm 0.2$. Because of poor and biased statistics, these values should be considered only as a test of the feasibility of our method, giving no unexpected orbit. However, we can conclude that ESDs belong to both disc and halo, while probably not all USDs belong to the halo.

In Fig. 21 we show the average spatial distribution of ESDs (dashed line) and USDs (solid line) as a function of $Z$ distance (in kpc) based on their orbits. It is given by the statistics of orbital points calculated with equal time-steps of 1 Myr. It is similar to the analysis of Galactic orbits of the sdB stellar sample by de Boer et al. (1997). In contrast to that work, our results expand far beyond $|Z| = 4$ kpc and our distributions do not have local minima at $Z = 0$. The reason is that their choice of stars was determined.
solely by the quite poor availability of spectroscopic data, yielding even more selection effects than in our case.

The exponential distribution of the USDs with $Z$ distance allows us to calculate the lower limit of their local mass density assuming a mean stellar mass of $0.5 \, M_\odot$: $\rho_0 = 2 \times 10^{-5} \, M_\odot \, pc^{-3}$. It is nearly twice the local mass density of all halo stars accepted for the BMG by Robin et al. (2003), although the BMG value is initial-mass-function dependent.

The relations of Fe/H versus eccentricity, perigalactic distance $R_{\text{min}}$ (in kpc) and apogalactic distance $R_{\text{max}}$ (in kpc) are shown in Fig. 22 (a), (b) and (c) respectively for ESDs (circles) and USDs (open squares). Despite the poor and biased statistics, one can conclude that few if any USDs belong to the disc whereas the ESDs belong to both halo and disc.

7 CATALOGUE FORMAT

The properties of the 9495 selected stars are presented as the catalogue ‘Candidate subdwarfs and white dwarfs from the 2MASS, Tycho-2, XPM and UCAC3 catalogues’ (hereafter the SDWD catalogue) in Table 4 (the full table is presented in electronic form – see Supporting Information). Table 5 gives its format.

8 CONCLUSIONS

New all-sky astrometric and photometric surveys (2MASS, XPM, UCAC3, SuperCosmos, Tycho-2) appear suitable for selecting the largest sample of candidate subluminous stars: 7641 un-evolved subdwarfs, 1826 evolved subdwarfs and 28 white dwarfs. The key feature of this selection is the detailed analysis of the
Table 4. The candidate subdwarfs and white dwarfs from the 2MASS, Tycho-2, XPM and UCAC3 catalogues (data for the first three stars; the full table is presented in electronic form – see Supporting Information).

| 2MASS IAU2000 number | Tycho-2 number | Stellar Class (WD = 1, ESD = 2, USD = 3) | Obtained $M_X$ | $\mu_1$ cos($b$) in mas yr$^{-1}$ | $\mu_2$ in mas yr$^{-1}$ | $Hipparcos$ parallax in mas | parallax precision in mas | $B_1$ mag from Tycho-2 | $B_1$ precision from Tycho-2 | $V_1$ mag from Tycho-2 | $V_1$ precision from Tycho-2 | $J$ mag from 2MASS | $Ks$ mag from 2MASS | $R_{UCAC3}$ mag from UCAC3 | $R_{UCAC3}$ precision from UCAC3 | $B_{SC}$ mag from SuperCosmos | $R_{SC}$ mag from SuperCosmos | $I_{SC}$ mag from SuperCosmos | $B_{SC}$ quality flag from UCAC3 | $R_{SC}$ quality flag from UCAC3 | $I_{SC}$ quality flag from UCAC3 | Obtained photometric distance in pc | Obtained $X$ distance in pc | Obtained $Y$ distance in pc | Obtained $Z$ distance in pc | Radial velocity in km s$^{-1}$ | Radial velocity precision in km s$^{-1}$ | Fe/H |
|----------------------|----------------|---------------------------------|----------------|------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 00001260–55454517    | 8464 01239     | 3                               | 3              | $-273.6$                     | $-14.9$        | 1.33           | 2.5            | 12.04          | 0.08           | 12.07          | 0.13           | 10.72          | 14.19          | 13.45          | 15.43          | 14.78          | 0.18           | 0.07           | 16.13          | 15.22          | 14.95          | 14.13          | 1                | 1              | 1              | 0              | 271            | 866            | 616            | 98             | 237            | 137            | $-89$          | $-101$         | $-326$         | $-827$         | $-600$         |
| 00002663–4054014      |                |                                 |                |                             |               |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |
| 00003667–3358357      |                |                                 |                |                             |               |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |                |

The distributions of all stars from these catalogues in the ‘colour index versus reduced proper motion’ plane, as well as Monte Carlo simulations of the distributions. It is found that only surveys with proper-motion accuracy better than 10 mas yr$^{-1}$ provide an acceptable separation of subluminous stars from the main sequence, albeit with a 10 per cent admixture and considerable selection biases in favour of faster stars. It is proven that most of the Tycho-2, XPM and UCAC3 stars fit this criterion of proper-motion accuracy. It is pointed out that future surveys with proper-motion accuracy better than 1 mas yr$^{-1}$ can provide perfect separation without an admixture and bias. It is pointed out that the current level of photometric accuracy is acceptable for such tasks. Multi-colour photometry also appears useful for subluminous star selection and the separation of evolved and unevolved subdwarfs, as well as single and binary stars.

The calibrations of absolute magnitude versus colour index and reduced proper motion made with the best Hipparcos stars allow us to calculate photometric and photoastrometric distances for all selected stars and consider their 3D distribution, which is almost complete only within about 150 pc from the Sun. The use of radial velocities for 176 stars and Fe/H metallicities for 57 stars allows us to consider their 3D motion and metallicity–velocity relation, while the statistics is poor and biased.

This investigation tests some theories related to subluminous stars. Namely, our results are consistent with the statement about different evolutionary status, kinematics and metallicity of unevolved and evolved subdwarfs. All or almost all unevolved subdwarfs are Population II low-metallicity high-asymmetric-drift stars from the halo. Their local mass density lower limit is estimated as $2 \times 10^{-5} M_{\odot}$ pc$^{-3}$, which is twice the traditional estimation. The evolved subdwarfs are a heterogeneous group containing stars from disc and halo with metallicities from low to solar and with halo to thin-disc kinematics. The evolved subdwarfs appear the most interesting group. They show some spatial overdensities of as yet unknown nature, a large fraction of binaries and a vast diversity of properties.

All important parameters of the selected stars are compiled into the SDWD catalogue in order to continue investigations, especially to prove their status by spectroscopy.

**ACKNOWLEDGMENTS**

We thank Dr. Elizabeth M. Green from Steward Observatory of the University of Arizona for providing a comparison of her list of subdwarfs with ours. We are grateful to Professor Martin C. Smith from Kavli Institute for Astronomy and Astrophysics of the Peking University for providing his list of subdwarfs. We are grateful to Professor Martin C. Smith from Kavli Institute for Astronomy and Astrophysics of the Peking University for providing his list of subdwarfs. We are thankful to the referee, Dr. Wyn Evans, for many reasonable suggestions.

This study was supported by the Fundamental Researches State Fund of Ukraine (project No. FRSF-28/238) and the Russian Foundation for Basic Research (projects No. 08-02-00400 and No. 09-02-90443-Ukr-f), and in part by the ‘Origin and Evolution of Stars and Galaxies – Program of the Presidium of the Russian Academy of Sciences and the Program for State Support of Leading Scientific Schools of Russia’ (NSh-6110.2008.2).
Table 5. Format of the catalogue of candidate subdwarfs and white dwarfs from 2MASS, Tycho-2, XPM and UCAC3.

<table>
<thead>
<tr>
<th>Field</th>
<th>Format</th>
<th>Positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2MASS IAU2000 number</td>
<td>I16</td>
<td>1–16</td>
</tr>
<tr>
<td>Tycho-2 number</td>
<td>A10</td>
<td>18–27</td>
</tr>
<tr>
<td>Hipparcos number</td>
<td>I6</td>
<td>29–34</td>
</tr>
<tr>
<td>Stellar class (WD = 1, ESD = 2, USD = 3)</td>
<td>I1</td>
<td>36–36</td>
</tr>
<tr>
<td>Obtained $M_K$</td>
<td>F5.2</td>
<td>37–41</td>
</tr>
<tr>
<td>$\mu_\alpha$ (in mas yr$^{-1}$)</td>
<td>F7.1</td>
<td>42–48</td>
</tr>
<tr>
<td>$\mu_\delta$ (in mas yr$^{-1}$)</td>
<td>F7.1</td>
<td>49–55</td>
</tr>
<tr>
<td>Hipparcos parallax in mas</td>
<td>F6.2</td>
<td>56–61</td>
</tr>
<tr>
<td>Parallax precision in mas</td>
<td>F4.1</td>
<td>62–65</td>
</tr>
<tr>
<td>$B_V$ mag from Tycho-2</td>
<td>F5.2</td>
<td>66–70</td>
</tr>
<tr>
<td>$B_V$ precision from Tycho-2</td>
<td>F4.2</td>
<td>71–74</td>
</tr>
<tr>
<td>$V_B$ mag from Tycho-2</td>
<td>F5.2</td>
<td>75–79</td>
</tr>
<tr>
<td>$V_B$ precision from Tycho-2</td>
<td>F4.2</td>
<td>80–83</td>
</tr>
<tr>
<td>$J$ mag from 2MASS</td>
<td>F6.2</td>
<td>84–89</td>
</tr>
<tr>
<td>$K_s$ mag from 2MASS</td>
<td>F6.2</td>
<td>90–95</td>
</tr>
<tr>
<td>$R_{UCAC3}$ mag from UCAC3</td>
<td>F6.2</td>
<td>96–101</td>
</tr>
<tr>
<td>$R_{UCAC3}$ precision from UCAC3</td>
<td>F4.2</td>
<td>102–106</td>
</tr>
<tr>
<td>$B_{SC}$ mag from SuperCosmos</td>
<td>F5.2</td>
<td>107–112</td>
</tr>
<tr>
<td>$R_{SC}$ mag from SuperCosmos</td>
<td>F5.2</td>
<td>113–118</td>
</tr>
<tr>
<td>$I_{SC}$ mag from SuperCosmos</td>
<td>F5.2</td>
<td>119–124</td>
</tr>
<tr>
<td>$B_{SC}$ quality flag from UCAC3</td>
<td>I1</td>
<td>126</td>
</tr>
<tr>
<td>$R_{SC}$ quality flag from UCAC3</td>
<td>I1</td>
<td>127</td>
</tr>
<tr>
<td>$I_{SC}$ quality flag from UCAC3</td>
<td>I1</td>
<td>128</td>
</tr>
<tr>
<td>Obtained photometric distance in pc</td>
<td>I5</td>
<td>129–133</td>
</tr>
<tr>
<td>Obtained X distance in pc</td>
<td>I5</td>
<td>134–138</td>
</tr>
<tr>
<td>Obtained Y distance in pc</td>
<td>I5</td>
<td>139–143</td>
</tr>
<tr>
<td>Obtained Z distance in pc</td>
<td>I5</td>
<td>144–148</td>
</tr>
<tr>
<td>Radial velocity in km s$^{-1}$</td>
<td>F6.1</td>
<td>149–154</td>
</tr>
<tr>
<td>Radial velocity precision in km s$^{-1}$</td>
<td>I2</td>
<td>155–156</td>
</tr>
<tr>
<td>Fe/H</td>
<td>F4.1</td>
<td>157–161</td>
</tr>
</tbody>
</table>

REFERENCES

European Space Agency (ESA), 1997, Hipparcos and Tycho catalogues.
ESA Publication Division, Noordwijk
Monet D., 1998, BAAS, 30, 1427
Zacharias N. et al., 2010, AJ, 139, 2184

SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article:

Table 4. The candidate subdwarfs and white dwarfs from the 2MASS, Tycho-2, XPM and UCAC3 catalogues.

Please note: Wiley–Blackwell are not responsible for the content or functionality of any supporting materials supplied by the authors. Any queries (other than missing material) should be directed to the corresponding author for the article:

This paper has been typeset from a TeX/LATEX file prepared by the author.