The protomagnetar model for gamma-ray bursts

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ABSTRACT

Long duration gamma-ray bursts (GRBs) originate from the core collapse of massive stars, but the identity of the central engine remains elusive. Previous work has shown that rapidly spinning, strongly magnetized protons (millisecond protomagnetars) produce outflows with energies, time-scales and magnetizations \( \sigma_0 \) (maximum Lorentz factor) that are consistent with those required to produce long duration GRBs. Here we extend this work in order to construct a self-consistent model that directly connects the properties of the central engine to the observed prompt emission. Just after the launch of the supernova shock, a wind heated by neutrinos is driven from the protomagnetar. The outflow is collimated into a bipolar jet by its interaction with the progenitor star. As the magnetar cools, the wind becomes ultrarelativistic and Poynting flux dominated \( \sigma_0 \gg 1 \) on a time-scale comparable to that required for the jet to clear a cavity through the star. Although the site and mechanism of the prompt emission are debated, we calculate the emission predicted by two models: magnetic dissipation and shocks.

Our results favour magnetic dissipation as the prompt emission mechanism, in part because it predicts a relatively constant ‘Band’ spectral peak energy \( E_{\text{peak}} \) with time during the GRB. The baryon loading of the jet decreases abruptly when the neutron star becomes transparent to neutrinos at \( t = t_{\nu-\text{thin}} \sim 10–100 \) s. Jets with ultrahigh magnetization cannot effectively accelerate and dissipate their energy, which suggests this transition ends the prompt emission. This correspondence may explain both the typical durations of long GRBs and the steep decay phase that follows. Residual rotational or magnetic energy may continue to power late time flaring or afterglow emission, such as the X-ray plateau. We quantify the emission predicted from protomagnetars with a wide range of physical properties (initial rotation period, surface dipole field strength and magnetic obliquity) and assess a variety of phenomena potentially related to magnetar birth, including low-luminosity GRBs, very luminous GRBs, thermal-rich GRBs/X-ray flashes, very luminous supernovae and short-duration GRBs with extended emission.

Key words: MHD – gamma-ray burst: general – stars: neutron – stars: winds, outflows.

1 INTRODUCTION

Soon following the discovery of gamma-ray bursts (GRBs; Klebesadel, Strong & Olson 1973), there were possibly more theories for their origin than theorists (Ruderman 1975). However, once GRBs were confirmed to originate from cosmological distances (e.g. Metzger et al. 1997), the joint requirements of supernova-scale (SN-scale) energies, short (millisecond) time-scales and relativistic speeds significantly narrowed the list of plausible central engines. It is now generally accepted that GRBs result from the formation or catastrophic rearrangement of stellar-mass black holes (BHs) or neutron stars (NSs). This conclusion has only been strengthened in recent years due to the much richer picture of the prompt and afterglow emission provided by the Swift and Fermi missions.
However, despite a wealth of new data, the identity of the central engine remains elusive.

At least some long-duration GRBs originate from the death of very massive stars (Woosley & Bloom 2006), as confirmed by their observed association with energetic core-collapse SNe (e.g. Galama et al. 1998; Bloom et al. 1999; Stanek et al. 2003; Chornock et al. 2010; Starling et al. 2010). It nevertheless remains unsettled whether the central engine is a rapidly accreting BH (Woosley 1993; MacFadyen & Woosley 1999; Nagataki et al. 2007; Barkov & Komissarov 2008; Lindner et al. 2010) or a rapidly spinning, strongly magnetized NS (a ‘millisecond magnetar’; Usov 1992; Thompson 1994; Blackman & Yi 1998; Wheeler et al. 2000; Zhang & Mészáros 2001; Thompson, Chang & Quataert 2004; Bucciantini et al. 2007, 2008, 2009; Metzger, Thompson & Quataert 2007). Although much less is known about the origin of short duration GRBs, the properties of their host galaxies and their notable lack of an accompanying SN are consistent with an origin associated with the merger of NS–NS and NS–BH binaries (Berger et al. 2005; Hjorth et al. 2005; Bloom et al. 2006; see e.g. Berger 2010 for a recent review). However, the unexpected discovery that many short GRBs are followed by an energetic X-ray tail lasting ∼100 s has challenged basic predictions of the merger model (e.g. Gal-Yam et al. 2006; Gehrels et al. 2006; Perley et al. 2009) and may hint at an alternative origin for some events, such as magnetar formation via the accretion-induced collapse (AIC) of a white dwarf (WD) (Metzger, Quataert & Thompson 2008a).

The large range in length-scales and the complexity of the physics involved in producing a GRB have thus far prevented all steps in the phenomena from being studied in a single work. Any attempt to construct a ‘first principles’ model is hindered by uncertain intermediate steps relating the physics of the central engine to the properties of the relativistic jet and the gamma-ray emission mechanism. Nevertheless, in this paper, we argue that the magnetar model is uniquely predictive. This allows us to construct a self-consistent model which can in principle be compared directly with observations. Although we focus on magnetars formed via the core collapse of massive stars, we also apply our results to AIC (Section 6.7). Our primary conclusion is that a remarkable fraction of GRB properties find natural explanations within the protomagnetar model.

1.1 Black hole versus magnetar

In the original collapsar model, Woosley (1993) envisioned a ‘failed SN’, in which the energy released by core collapse is insufficient to unbind the majority of the star, such that a BH necessarily forms. If the collapsing envelope has sufficient angular momentum, it accretes through a centrifugally supported disc. Energy released by accretion, or via the accretion-mediated extraction of the BH’s spin (Blandford & Znajek 1977), then powers a relativistic jet, which burrows through the star and ultimately powers the GRB at larger radii (MacFadyen & Woosley 1999; Matzner 2003; Proga et al. 2003; Morsony, Lazzati & Begelman 2007).

The discovery that long GRBs are accompanied by hyperenergetic (∼10^{52} erg) SNe propelled the collapsar model to the theoretical forefront. However, it also proved, somewhat ironically, that GRB SNe are far from the complete ‘failures’ envisioned by Woosley (1993). Indeed, if the collapsar scenario is correct, then either (1) the BH forms promptly following stellar collapse and the explosion mechanism associated with GRB SNe is fundamentally different from that associated with the death of normal (slower rotating) stars, which are instead powered by NS formation; or (2) a BH forms only after several seconds delay, due to the ‘fall-back’ of material that remains gravitationally bound despite a successful and energetic SN (e.g. Chevalier 1993; Fryer 1999; Zhang, Woosley & Heger 2008; Moriya et al. 2010).

Modern core-collapse simulations find that the shock produced at core bounce initially stalls due to neutrino and photodissociation losses (e.g. Rampp & Janka 2000; Liebendörfer et al. 2001; Thompson, Burrows & Pinto 2003). It has long been thought that neutrino heating from the proto-NS may revive the shock, resulting in a successful explosion (Bethe & Wilson 1985). Recent simulations suggest that the neutrino mechanism may work for low-mass progenitors (e.g. Scheck et al. 2006), but higher mass stars appear more difficult to explode. Although multidimensional effects not captured by present simulations may be a crucial missing ingredient (e.g. Nordhaus et al. 2010), neutrinos alone may well prove incapable of powering ∼10^{52} erg explosions.

GRB progenitors are, however, far from typical. Essentially all central engine models require rapid rotation and a strong, large-scale magnetic field (≥10^{15} G; e.g. McKinney 2006). These ingredients may go hand-in-hand in core collapse because differential rotation provides a source of free energy to power field growth, via, for example, an α–Ω dynamo in the convective proto-NS (Duncan & Thompson 1992) or the magnetorotational instability (MRI; e.g. Akiyama et al. 2003; Thompson, Quataert & Burrows 2005). The crucial question then arises: Do SNe indeed fail and lead to BH formation if the progenitor core is rapidly rotating? or stated more directly: Are the requisite initial conditions for the collapsar model self-consistent?

An additional energy reservoir (rotation) and means for extracting it (magnetic fields) make magnetorotational effects a more promising way to produce hypernovae than neutrinos alone (e.g. LeBlanc & Wilson 1970; Symbalisty 1984; Ardeljan, Bisnovatyi-Kogan & Moiseenko 2005). Only recently, however, have simulations begun to capture the combined effects of magnetohydrodynamic (MHD) and neutrino heating (e.g. Burrows et al. 2007b).

Dessart et al. (2008) (hereinafter D08) calculate the collapse of a rotating 35-M⊙ zero-age main sequence collapsar progenitor of Woosley & Heger (2006), which they endow with a pre-collapse magnetic field that results in a ∼10^{15} G field strength when compressed to NS densities. This reproduces the field strength, if not the field topology, expected from the saturated state of the MRI. Soon after core bounce, a bipolar MHD-powered outflow develops from the proto-NS. Although the explosion is not initially successful over all solid angles, matter continues to accrete through an equatorial disc. By accreting angular momentum, the NS remains rapidly spinning, which in turn enhances the mass-loss from higher latitudes due to magnetocentrifugal slinging (e.g. Thompson et al. 2004; Metzger et al. 2007; see equation A12). Importantly, in the strongly magnetized model of D08, the wind mass-loss rate eventually exceeds the accretion rate, such that for t ≥ 300 ms the NS mass begins to decrease. Although D08 cannot address the possibility of later fall-back, and a different progenitor angular momentum profile could change the conclusion, their result is none the less suggestive: a core self-consistently endowed with the properties required to produce a GRB may not leave a BH at all. The results of D08 highlight the fact that the BH versus NS formation may not be a function of the progenitor mass and metallicity alone. Delineating this dichotomy more definitively will, however, require addressing challenging theoretical issues, such as the precise mechanism responsible for amplifying the magnetic field (see Spruit 2008 for a discussion).

Fig. 1 is a schematic diagram of the possible effects of rapid rotation and strong magnetic fields on the regimes of the NS versus
BH formation as a function of the main-sequence stellar mass, $M_\star$, and the initial NS rotation period $P_0$. The collapse of slowly rotating, low-mass stars may result in a normal SN with kinetic energy $\sim 10^{51}$ erg powered by neutrinos. For higher mass stars, however, neutrino-powered explosions are less likely (or are accompanied by significant ‘fall-back’ accretion) due to more massive, compact iron cores and higher envelope-binding energies $E_{\text{bind}}$. For these reasons, it has been argued that stars with $M_\star \gtrsim 25 M_\odot$ leave BH remnants at the subsolar metallicities that appear to characterize GRB progenitors (e.g. Fryer 1999; Heger et al. 2003; O’Connor & Ott 2010).

Above the dashed line in Fig. 1, however, the rotational energy, $E_{\text{rot}}$, of the proto-NS (equation 1) exceeds the binding energy of the stellar envelope, where

$$E_{\text{rot}} \simeq (1/2)I \Omega^2$$

$$\approx 3 \times 10^{52} \text{ erg} \left( \frac{M_{\text{ms}}}{1.4 M_\odot} \right) \left( \frac{R_{\text{ms}}}{12 \text{ km}} \right)^2 \left( \frac{P}{\text{ ms}} \right)^{-2},$$

(1)

with $I = (2/5)M_{\text{ms}}R_{\text{ms}}^2$, $M_{\text{ms}}$, $R_{\text{ms}}$ and $\Omega = 2\pi/P$ are the NS moment of inertia, mass, radius and rotation rate, respectively. We have defined $E_{\text{bind}}$ exterior to $1.8 M_\odot$, as calculated by Dessart, Livne & Waldman (2010) from the stellar profiles of Woosley, Heger & Weaver (2002). Although the efficiency with which $E_{\text{rot}}$ couples to the SN shock depends on uncertain details during the first few hundred milliseconds after core bounce, if $E_{\text{rot}} > E_{\text{bind}}$, then a NS remnant could in principle result, even for very massive stars. The hypothetical boundary between NS and BH formation based on the above discussion is shown with a solid line in Fig. 1. We note that there is indeed evidence that some Galactic magnetars may have stellar progenitors with masses $\gtrsim 40 M_\odot$ (Muno et al. 2006), although (consistent with Fig. 1) this does not exclusively appear to be the case (Davies et al. 2009).

If an MHD-powered SN does not leave a BH, then a rapidly spinning, strongly magnetized NS (a ‘protomagnetar’) may instead remain behind the outgoing SN shock. The rotational energy $E_{\text{rot}} > 10^{52}$ erg of a magnetar with $P_0 \sim 1$ ms is more than sufficient to power most long GRBs. However, not all of this energy is available to produce high-energy emission; a fraction of $E_{\text{rot}}$, for instance, is expended as the jet emerges from the star or is used to power an accompanying hypernova (dashed line; Fig. 1). The right-hand axis in Fig. 1 shows the magnetic field strength, $B_{\text{eq}}$, that would be generated if the magnetic energy in the dipole field is $\sim 0.1$ per cent of $E_{\text{rot}}$ (equation 4). A dot–dashed line shows the minimum rotation rate required to produce a classical GRB from a magnetar with a field strength, $B_{\text{dip}}$, based on the model presented in Section 4. The conditions for a hypernova and a GRB from a protomagnetar are thus remarkably similar.

1.2 Summary of the magnetar model and this paper

In this section, we summarize the organization of this paper and orient the reader with a brief description of the model time-line (more details and references are provided in subsequent sections).
In Section 2, we present calculations of the time-dependent properties of protomagnetar winds and quantify the stages of the protomagnetar model. The basic picture is summarized by Fig. 2, which shows the wind power $\dot{E}$ and magnetization $\sigma_0$ (maximum Lorentz factor) as a function of time following core bounce, calculated for a protomagnetar with a surface dipole magnetic field strength $B_{\text{dip}} = 2 \times 10^{15}$ G, initial spin period $P_0 = 1.5$ ms and magnetic obliquity $\chi = \pi/2$. Changes in the wind properties with time are driven largely by the increase in $\sigma_0(t)$ as the proto-NS cools.

Within the first few hundred milliseconds following core bounce, a successful SN shock is launched by neutrino heating or MHD forces (Stage I). Soon after, a wind heated by neutrinos expands freely from the NS surface into the cavity evacuated by the outgoing shock. The wind is initially non-relativistic ($\sigma_0 \lesssim 1$) because the neutrino-driven mass-loss rate is high (Stage II). However, as the proto-NS cools, $\sigma_0$ increases to $\gtrsim 1$ and the wind becomes relativistic (Stage III). The wind is collimated by its interaction with the star into a bipolar jet, which breaches the stellar surface after $\sim 10$ s. After jet break-out, the relativistic magnetar wind is directed through a relatively clear channel out of the star and the GRB commences (Stage IV; Section 4). Averaging over variability imposed by, for example, interaction with the jet walls (Section 4.2), the time-evolution of the power and mass-loading of the jet match those set by the magnetar wind at much smaller radii. In Section 3, we provide a more quantitative description of the individual model stages described above using an extensive parameter study of wind models.

Although the site and mechanism of prompt GRB emission remain uncertain, in Section 4, we calculate the light curves and spectra within two emission models. Depending on the means and efficacy of the jet's acceleration (Section 4.1), GRB emission may be powered by the dissipation of the jet's Poynting flux directly ('magnetic dissipation'; Section 4.3) near or above the photosphere and/or via 'internal shocks' within the jet at larger radii (Section 4.4). As Fig. 2 makes clear, self-interaction in the jet is inevitable because $\sigma_0$ – and hence the jet speed – increases monotonically as the proto-NS cools.

After $t \sim 30–100$ s, $\sigma_0$ increases even more rapidly as the proto-NS becomes transparent to neutrino emission. Because magnetic dissipation and jet acceleration become ineffective when $\sigma_0$ is very large, this abrupt transition likely ends the prompt GRB. In Section 5, we address the possibility that residual rotational or magnetic energy may continue to power late-time flaring or afterglow emission, such as the X-ray plateau. In Section 6, we discuss the implications of our results for the diversity of GRB-related phenomena, including low-luminosity GRBs (Section 6.3), very...
luminous GRBs (VLGRBs) (Section 6.2), X-ray flashes (XRFs) (Section 6.4), Galactic magnetars (Section 6.6) and magnetar formation via AIC (Section 6.7). We summarize our conclusions in Section 7.

2 PROTON MAGNETAR WINDS

In this section, we present calculations of the time-dependent properties of magnetized proto-NS winds (Thompson et al. 2004; Metzger et al. 2007). In Section 2.1, we summarize the model, which is similar to that presented in Metzger et al. (2007) but includes additional details not addressed in previous work. Our results are presented in Section 2.2.

2.1 Evolutionary wind model

2.1.1 Model description

The two most important properties of the proton magnetar wind are the mass-loss rate $\dot{M}$ and the energy-loss rate, or wind power, $E$. The wind power contains kinetic and magnetic (Poynting flux) components: $E = E_{\text{kin}} + E_{\text{mag}}$. A related quantity, determined from $M$ and $E_{\text{mag}}$, is the wind magnetization

$$\sigma_0 \equiv \frac{\phi_0^2 \Omega^2}{M c^2},$$

where $\Omega$ is the NS rotational rate, $\phi_0 \equiv B_0 r_0^2$ is the magnetic flux threading the open magnetosphere divided by 4\pi sr (Michel 1969) and $B_0 \sim r_0$ the poloidal field strength. As shown in Appendix A, $\phi$ is directly related to the Poynting flux $E_{\text{mag}}$ (equation A3). The magnetization is important because it delineates non-relativistic (\(\sigma_0 \lesssim 1\)) from relativistic (\(\sigma_0 \gtrsim 1\)) outflows and affects the asymptotic partition between the kinetic and magnetic energy in the wind. In particular, in relativistic outflows most of the wind power resides in Poynting flux ($E_{\text{mag}} \gg E_{\text{kin}}$) at the fast magnetosonic surface. The value of $\sigma_0$ in this case crucially affects the efficiency with which the jet may accelerate and dissipate its energy (Section 4.1) and is approximately equal to the outflow’s maximum achievable Lorentz factor $\Gamma_{\text{max}} \approx E/Mc^2 \approx \sigma_0$.

In Appendix A, we describe in detail how $E$, $M$ and $\sigma_0$ are determined in magnetized proto-NS winds. To briefly summarize, mass-loss during the first $t \sim 30–100$ s is caused by neutrino heating in the proto-NS atmosphere. As a result, $M \propto \dot{M}_c^{5/3} \epsilon_v^{10/3}$ depends sensitively on the neutrino luminosity, $\dot{L}_c$, and the mean neutrino energy, $\epsilon_v$, during the Kelvin–Helmholtz cooling phase (equation A8). In most cases, we take $L_c(t)$ and $\epsilon_v(t)$ from the proto-NS cooling calculations of Pons et al. (1999) (see Fig. A1), but modified by a ‘stretch factor’ $\eta_t$ (defined in equation A11) that qualitatively accounts for the effects of rotation on the cooling evolution.

We assume that mass-loss from the proto-NS occurs only from portions of the surface threaded by the open magnetic flux. We assume a dipolar magnetosphere bounded by the bundle of ‘last-closed’ field lines which intersect the ‘Y’ point radius in the magnetic equator (Fig. 3 is an illustration of the relevant geometry). We determine the dependence of the Y-point radius on the wind properties using results from the axisymmetric MHD simulations of Bucciantini et al. (2006), which span the $\sigma_0 \sim 1$ to $\sigma_0 \sim 3$ transition. Using numerical results from Metzger, Thompson & Quataert (2008b), we further account for the enhancement in $M$ that occurs due to magnetocentrifugal forces in the heating region. This effect is most important when the NS is rotating very rapidly ($P \lesssim 2$ ms) and the magnetic obliquity is large, such that the polar caps samples regions near the rotational equator. After $t \approx t_{\text{spin}} \sim 30–100$ s, the proto-NS becomes transparent to neutrinos, which causes $L_c$ and $\epsilon_v$ to decrease sharply (Fig. A1). Once neutrino heating decreases sufficiently, other processes (e.g. $\gamma-B$ or $\gamma-\gamma$ pair production) likely take over as the dominant source of mass-loading (Hibschman & Arons 2001; Thompson 2008) and the wind composition may change from baryon- to pair-dominated. Lacking a predictive model for $M$ at late times, we assume that $M$ scales with the Goldreich & Julian (1969) flux for a fixed value of the pair multiplicity $\mu_{\gamma\gamma} = 10^6$. Our conclusions are fortunately insensitive to this choice (see Section 5). The full expression for $M$ is given in equation (A15).

Protomagnetar winds are magnetically driven throughout most of their evolution. When the wind is non-relativistic, its speed at the fast surface is $v_{\text{sw}} \approx \sigma_0^{1/3} c$, the wind power is $E \propto \sigma_0^{2/3} M \propto M^{1/3}$ and $E_{\text{mag}} = 2 E_{\text{kin}}$ (Lamers & Cassinelli 1999). For relativistic winds, $E \propto \sigma_0 M$ is approximately independent of $M$, and $E_{\text{mag}} \gg E_{\text{kin}}$ at the fast point. Indeed, in the limit that $\sigma_0 \gg 1$ we assume that $E$ is the force-free spin-down rate (Spitkovsky 2006), which depends only on $\sigma$ and $E$. Even for relatively large (but finite) values of $\sigma_0$, however, spin-down occurs more rapidly than in the force-free case because the ‘Y’ point radius $R_Y$ resides inside the light cylinder (Fig. 3). The full expression for $E$ is given in equation (A5).

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1 Note that this definition may differ from that used elsewhere in the literature. In particular, what we define as $\sigma_0$ is sometimes referred to as the ‘baryon loading’ parameter (e.g. Drenkhahn & Spruit 2002).

Figure 3. Geometry of magnetized proto-NS winds. The NS radius, $R_{ns}$, is initially large (>20 km) following the launch of the SN shock, but decreases to its final value $R_{ns} \approx 12$ km in a few seconds (Fig. A1). The NS rotates at an angular velocity $\Omega = 2\pi/P$ about the vertical axis, where $P$ is the rotational period; the light cylinder radius is $R_L = c/\Omega \approx 50(P/\text{ms})$ km. The magnetic dipole moment $|\mu| = B_{\text{dip}} R_{ns}^3$ makes an angle $\chi$ with respect to the rotation axis. The angle $\theta_{open}$ defines the size of the open magnetosphere on the NS surface. The magnetosphere is closed at angles $\theta > \theta_{open}/2$ from the magnetic pole, while field lines with $\theta < \theta_{open}/2$ form an ‘open’ or ‘wind’ zone along which matter may escape to infinity. The size of the open zone affects both the spin-down rate and the mass-loss rate from magnetized proto-NS winds. The bundle of last closed field lines intersects the magnetic equator at the ‘Y’ point radius $R_Y$. Ultrarelativistic, force-free winds ($\sigma_0 \gg 1$) have $R_Y \approx R_L$, while less-magnetized winds in general have $R_Y < R_L$ (see Section A2 and Fig. 4).
2.1.2 Spin-down evolution and initial conditions

Protomagnetar winds are magnetorotationally powered throughout most of their evolution. The NS thus loses angular momentum \( J = I \dot{\Omega} \) to the wind at the rate \( J = -E/\Omega \). Neglecting mass-loss (a good approximation), the rotation rate \( \dot{\Omega} \) evolves according to

\[
\dot{\Omega} = -\frac{2 R_{\text{ns}}}{R_{\text{m}}} \frac{2 E}{E_{\text{rot}}},
\]

where \( E_{\text{rot}} \) is the NS rotational energy (equation 1). In equation (3), we neglect angular momentum losses due to gravitational waves, which become important if the NS is sufficiently aspherically distorted by its strong interior magnetic field (e.g. Cutler 2002; Arons 2003; Stella et al. 2005; Dall’Osso, Shore & Stella 2009). This is a good approximation, provided that either the magnetic obliquity is small or the interior magnetic field is less than \( \sim 100 \) times stronger than the outer dipole field. We also neglect gravitational wave emission due to non-axisymmetric waves or instabilities (e.g. \( \tau \) modes; Andersson 1998), although these are implicitly taken into account through the maximum initial NS rotation rate that we consider (see below). We also neglect the possibility of late-time accretion on to the protomagnetar (e.g. Metzger et al. 2008a; Zhang & Dai 2009), which could affect the spin-down evolution both through accretion torques and through altering the geometry of the magnetosphere.

Given \( E \) and \( M \) as a function of \( \Omega \) and time, we solve equation (3) to obtain \( \Omega(t) \), \( M(t) \), \( E(t) \) and \( \sigma(t) \). A wind solution is thus fully specified by just four parameters: the NS mass \( M_{\text{ns}} \); the ‘initial’ angular rotation rate \( \Omega_0 = 2 \pi / P_0 \); the surface dipole magnetic field strength \( B_{\text{dip}} \); and the inclination angle \( \chi \) (‘obliquity’) between the magnetic and rotational axes (see Fig. 3). Since the proto-NS is still contracting for several seconds following core bounce, \( \Omega_0 \) and \( B_{\text{dip}} \) are more precisely defined as the maximum values that would be achieved were the NS to contract at constant angular momentum \( J \propto R_{\text{ns}}^2 M_{\text{ns}} \Omega \) and magnetic flux \( \Phi \propto B_{\text{dip}} R_{\text{ns}}^2 \).

If the magnetic field is amplified on a time-scale comparable to the duration of the NS cooling epoch (e.g. via linear field winding), the assumption of a fixed dipole flux may be a poor approximation. On the other hand, if field growth occurs more rapidly via a convection-driven dynamo (Duncan & Thompson 1992) or the dynamical-time-scale MRI (e.g. Akiyama et al. 2003; Thompson et al. 2005), then the field is probably established and finds a stable configuration (Braithwaite & Spruit 2006) – in less than a few seconds (Spruit 2008). In this case, the assumption that \( \Phi \) is fixed may be reasonable.

Given the uncertainty in the origin of magnetar fields, in general, we allow both \( P_0 \) and \( B_{\text{dip}} \) to vary independently within their respective physical ranges (\( P_0 \gtrsim 1 \) ms, \( B_{\text{dip}} \lesssim 3 \times 10^{16} \) G; see below). However, if the magnetic field is in fact generated from the free energy available in differential rotation, then a relationship between \( B_{\text{dip}} \) and \( P_0 \) of the form

\[
B_{\text{dip}} = 10^{16} \text{G} \left( \frac{P_0}{10^{-3}} \right)^{1/2} \left( \frac{R_{\text{ns}}}{12 \text{ km}} \right)^{-1/2} \left( \frac{P_0}{\text{ms}} \right)^{-1}
\]

could result, where we have assumed that the magnetic energy in the dipole field (\( \propto B_{\text{dip}}^2 R_{\text{ns}}^2 \)) is a fraction \( \epsilon_B \) of the rotational energy \( E_{\text{rot}} \propto R_{\text{ns}}^2 P_0^{-1} \) (equation 1) and that the energy in differential rotation scales with \( E_{\text{rot}} \). In our models, we require that \( P_0 \gtrsim 1 \) ms because this is the allowed range of stable proto-NS rotational periods (e.g. Strobel, Schaab & Weigel 1999). This maximum rotation rate may be enforced in practice by the efficient loss of angular momentum incurred by very rapidly spinning NSs to MRI-generated turbulence or waves radiated by non-axisymmetric instabilities (e.g. Ott et al. 2005; Thompson et al. 2005; Wheeler & Akiyama 2007).

2 Note the distinction between the conserved dipole flux through the stellar interior \( \Phi \) defined here and the open flux through the magnetosphere \( \phi \) (equation 2), which evolves in time.

2.2 Results

The results of our calculations are summarized in Figs 2–5 and Table 1. As already discussed, Fig. 2 shows the wind magnetization

![Figure 4. Time-evolution of the light cylinder radius \( R_L \) (solid line), Alvén radius \( R_A \) (dot–dashed line; equation A2), ‘Y’ point radius \( R_Y \) (dashed line), sonic radius \( R_s \) (double dot–dashed line) and NS radius \( R_{\text{ns}} \) (see Fig. A1) for the solution shown in Fig. 2.](https://academic.oup.com/mnras/article-abstract/413/3/2031/967037/)

![Figure 5. Same as Fig. 2, but calculated for different protomagnetar properties. The first two models are for \( P_0 = 1 \) ms and \( B_{\text{dip}} = 10^{16} \) G, and assume values of the magnetic obliquity \( \chi = \pi/2 \) (dotted line) and \( \chi = 0 \) (solid line), respectively. The dashed line shows a lower spin-down case, calculated for \( P_0 = 2 \) ms, \( B_{\text{dip}} = 10^{15} \) G and \( \chi = \pi/2 \).](https://academic.oup.com/mnras/article-abstract/413/3/2031/967037/)
Table 1. Properties of protomagnetar winds.

| $B_{\text{dp}}^{(0)}$ (G) | $P_0/P_{\text{bo}}$ | $M_{\text{NS}}$ (M$_{\odot}$) | $\zeta_0$ (rad) | $\sigma_0$ (G) | $\sigma_{\text{avg}}$ (G) | $\Gamma_{\text{avg}}$ | $T_{\text{end}}^{(0)}$ (s) | $t_{\text{end}}^{(0)}$ (s) | $E_{\text{E}}^{(0)}$ (10$^{50}$ erg) | $E_{\text{B}}^{(0)}$ (10$^{50}$ erg) | $E_{\text{B}}^{(0)}$ (10$^{60}$ erg s$^{-1}$) | $\tau_{\text{dip}}^{(0)}$ (s) |
|---------------------------|--------------------------|--------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $2 \times 10^{14}$       | 1.5 [4,3]                | $1.4 \pi/2$               | 3              | 22             | 570            | 64             | 47             | 50             | 1.46           | 24             | 0.25            | 270             | 20 |
| $2 \times 10^{15}$       | 1.5 [4,3]                | $1.2 \pi/2$               | 3              | 45             | 220            | 30             | 46             | 50             | 1.83           | 35             | 0.24            | 400             | 20 |
| $2 \times 10^{15}$       | 1.5 [4,3]                | $1.4 \pi/2$               | 3              | 50             | 250            | 35             | 50             | 70             | 1.23           | 19             | 0.19            | 430             | 20 |
| $2 \times 10^{15}$       | 1.2 [4,3]                | $1.2 \pi/2$               | 3              | 60             | 150            | 40             | 60             | 80             | 1.95           | 35             | 0.18            | 380             | 20 |
| $2 \times 10^{15}$       | 1.5 [4,3]                | $1.2 \pi/2$               | 3              | 64             | 260            | 45             | 66             | 85             | 1.46           | 14             | 0.19            | 390             | 20 |
| $2 \times 10^{15}$       | 1.5 [4,3]                | $1.4 \pi/2$               | 1              | 60             | 360            | 90             | 12             | 110            | 1.43           | 14             | 0.17            | 660             | 20 |
| $2 \times 10^{15}$       | 1.2 [4,3]                | $1.4 \pi/2$               | 1              | 70             | 500            | 100            | 11             | 180            | 1.43           | 14             | 0.17            | 710             | 20 |
| $2 \times 10^{15}$       | 1.5 [4,3]                | $2 \pi/2$                 | 3              | 80             | 600            | 200            | 12             | 200            | 1.43           | 14             | 0.17            | 810             | 20 |
| $2 \times 10^{15}$       | 1.5 [4,3]                | $1.4 \pi/2$               | 1              | 90             | 400            | 110            | 12             | 220            | 1.43           | 14             | 0.17            | 910             | 20 |
| $2 \times 10^{15}$       | 1.4 [4,3]                | $1.2 \pi/2$               | 1              | 100            | 500            | 120            | 12             | 300            | 1.43           | 14             | 0.17            | 1000            | 20 |

Notes:
- $^a$Surface dipole magnetic field strength following NS contraction.
- $^b$Spin period if the NS were to contract to the final radius with a fixed angular momentum (actual initial spin period at $t = 0$).
- $^c$Magnetic obliquity (see Fig. 3).
- $^d$Stretch' correction applied to neutrino luminosities and energies to account for the effects of rotation (see equation A11).
- $^e$Magnetic field at the jet breaks out of the stellar surface at $t = t_{\text{bo}} = 10^5$ s (equation 5; Fig. 6).
- $^f$Energy-weighted average magnetization between the jet break-out and the end of the prompt emission. $t_{\text{bo}} < t < t_{\text{end}}$ (Fig. 11).
- $^g$Energy-weighted Lorentz factor of the bulk shell produced by internal shocks (Fig. 12).
- $^h$Duration of the prompt GRB emission $T_{\text{end}} = t_{\text{end}} - t_{\text{min,NS}}$ (Fig. 13).
- $^j$Time after core bounce when the prompt GRB emission ends, defined as the point when the ‘saturation radius’, $r_{\text{mag}}$ (equation 10), exceeds the internal shock radius, $r_{\text{ch}}$ (equation A3). This transition generally occurs simultaneously with the transition of the proto-NS to neutrino transparency (see Fig. 2).
- $^k$Maximum ‘thermal’ energy produced by the jet (equation 8).
- $^l$Maximum GRB energy, defined as the rotational energy released in the time-interval $[t_{\text{bo}}, t_{\text{min,NS}}] < t < t_{\text{end}}$ (see Fig. 10).
- $^m$Wind power at $t = t_{\text{end}}$.
- $^n$Dipole spin-down time-scale at $t = t_{\text{end}}$.
- $^o$Calculated using the neutrino cooling calculations of Hudepohl et al. (2010).

σ₀(t) and power $\dot{E}(t)$ as a function of time since core bounce, calculated for $M_{\text{NS}} = 1.4 M_{\odot}$, $P_0 = 1.5$ ms, $B_{\text{dp}} = 2 \times 10^{15}$ G and $\chi = \pi/2$. Fig. 4 shows the time-evolution of several critical radii associated with this wind solution.

During the first few seconds, $\dot{E}$ rises because $\Omega$ and $B_{\text{dp}}$ increase by angular momentum and magnetic flux conservation, respectively, as the proto-NS contracts to its final radius. On longer time-scales, $\dot{E}$ reaches a maximum and then decreases once the NS begins to spin-down and the open magnetosphere shrinks. The latter results because both the spin-down and the larger wind magnetization cause $R_K$ to increase (see Figs 3 and 4). Fig. 2 also shows that $\sigma_0$ increases rapidly for the first $\sim 100$ s as the NS cools and the neutrino-driven mass-loss rate decreases. This results in several distinct stages in the wind evolution, which we denote by Roman numerals in Fig. 2 and are discussed individually in the next section. At late times, $\sigma_0$ plateaus and then begins to decrease once the wind mass-loss rate reaches its minimum value proportional to the Goldreich–Julian flux (equation A14). Once $\sigma_0 \gg 1$ force-free spin-down obtains, such that $\dot{E}$ asymptotes at late times to the standard force-free decay $\dot{E} \propto t^{-2}$.

Fig. 5 shows three additional wind models, calculated for different values of $B_{\text{dp}}$, $P_0$ and $\chi$. The models shown with the solid and dotted lines correspond, respectively, to high spin-down cases with $B_{\text{dp}} = 10^{26}$ G, $P_0 = 1$ ms, calculated for different values of the magnetic obliquity $\chi = 0$ and $\pi/2$. The third model shown with a dashed line is a lower spin-down case with $B_{\text{dp}} = 10^{24}$ G, $P_0 = 2$ ms and $\chi = \pi/2$. Although the evolution of $\dot{E}(t)$ and $\sigma_0(t)$ is qualitatively similar to the fiducial model in Fig. 2, differences are apparent. Note that the higher(lower) spin-down models achieve

Note, however, that the measured braking indices of Galactic pulsars generally differ from the force-free prediction (e.g. Livingstone et al. 2007; see Section 5).
larger (smaller) values of $E$ and $\sigma_0$, except at late times. Also note that at fixed $B_{\text{dip}}$ and $P_0$, $\sigma_0$ is larger for the aligned rotator ($\chi = 0$) than in the oblique case ($\chi = \pi/2$) due to the enhanced mass-loss in the latter case caused by centrifugal ‘slinging’ (see equation A12 and surrounding discussion).

Table 1 summarizes the results of several additional calculations, which explore the sensitivity of our results to variations in the protomagnetar properties and in the adopted NS cooling model. Our primary conclusion is that key observables are most sensitive to the dipole field $B_{\text{dip}}$, rotation rate $P_0$ and obliquity $\chi$. Plausible variations in the NS mass, $M_{\text{ns}}$, stretch parameter, $\eta_s$, and the cooling model, on the other hand, generally result in at most of the order of unity differences. For this reason, we fix $M_{\text{ns}} = 1.4 M_\odot$ and $\eta_s = 3$ in the sections to follow and confine our analysis to the 3D parameter space ($B_{\text{dip}}$, $P_0$, $\chi$).

3 STAGES OF THE PROTOMAGNETAR MODEL

In this section, we describe the stages of the protomagnetar wind evolution and quantify their relationship to the GRB phenomenology. Our discussion is guided closely by Figs 2–5.

I. Pre-SN/thermally driven wind

($\sigma_0 \lesssim 10^{-3}$; $t \lesssim$ few × 100 ms)

Simulations of core collapse fail to produce a prompt explosion, suggesting that the proto-NS continues to accrete for several hundred milliseconds before a delayed explosion occurs. The proto-NS forms hot and its initial radius exceeds ~30 km. Since magnetic forces are unlikely to be dynamically important yet, an explosion at this stage would be neutrino-driven (Bethe & Wilson 1985). If this ‘standard’ scenario applies, thermal pressure is initially responsible for accelerating the neutrino-heated wind into the cavity behind the proto-NS. But if the resulting explosion occurs, the wind is thus neutrino-driven from its onset. The division between thermally and magnetically driven winds occurs at a critical magnetization $\sigma_0 \sim 10^{-3}$, because above this value the asymptotic speed of a magnetically driven wind, $v_{\infty} = \sigma_0^{1/3} c$, exceeds the speed $v_{\infty} \sim 0.1 c$ obtained via thermal acceleration alone (Metzger et al. 2007).

II. Magnetically driven, non-relativistic wind

($10^{-3} \lesssim \sigma_0 \lesssim 1$; few × 100 ms $t \lesssim$ few s)

Regardless of whether the SN itself is powered by thermal or magnetic forces, the neutrino wind becomes magnetically driven ($\sigma_0 \gtrsim 10^{-3}$) less than a second later. Because the neutrino luminosity $L_{\nu}$ is still large at these early times (Fig. A1), the wind mass-loss rate, $\dot{M}$, remains high. Though powerful at this stage, the outflow is thus still non-relativistic ($\sigma_0 \lesssim 1$). Non-relativistic magnetized winds are efficiently self-collimated by hoop stresses (e.g. Sakurai 1985). The protomagnetar wind thus forms a bipolar jet, which catches up to the slower SN shock and begins boring a collimated cavity into the unshocked star.

III. Magnetically driven, relativistic wind (pre-break-out)

($1 \lesssim \sigma_0 \lesssim 10–100$; few $s \lesssim t \lesssim t_{\text{bo}}$)

As the NS continues to cool, $\sigma_0$ exceeds unity within a few seconds and the wind becomes relativistic. Self–collimation fails in ultrarelativistic outflows (e.g. Bucciantini et al. 2006). The wind power thus becomes concentrated at low latitudes, where it collides with the slowly expanding SN ejecta and forms a hot ‘protomagnetar nebula’ (Bucciantini et al. 2007). As toroidal flux accumulates in the nebula, magnetic forces – and the anisotropic thermal pressure they induce – redirect the equatorial outflow towards the poles (Begelman & Li 1992; Königl & Granot 2002; Bucciantini et al. 2007, 2008, 2009; Komissarov & Barkov 2007; Uzdensky & MacFadyen 2007). Stellar confinement thus produces a mildly relativistic jet, which continues to drill a bipolar cavity where the earlier non-relativistic outflow left off.

The jet propagates through the star at a significant fraction $\beta$ of the speed of light (e.g. MacFadyen & Woosley 1999; Aloy et al. 2000; Ramirez-Ruiz, Celotti & Rees 2002b; Zhang, Woosley & MacFadyen 2003; Morsony et al. 2007; Bucciantini et al. 2008), such that it ‘breaks out’ of the stellar surface of radius $R_s$ on a time-scale

$$t_{\text{bo}} \approx \frac{R_s}{\beta c} \sim 7 \left(\frac{R_s}{10^{11}\text{ cm}}\right) \left(\frac{\beta}{0.5}\right)^{-1} \text{s.} \quad (5)$$

Although the precise value of $t_{\text{bo}}$ will in general depend on both the properties of the jet and the properties of the star, in what follows we assume a fixed value $t_{\text{bo}} = 10$ s. Although this is a reasonable estimate for moderately powerful jets, weaker jets could require significantly longer to reach the surface. Below a critical jet power $E < E_{\text{min}} \sim 10^{48}$ erg s$^{-1}$, both hydrodynamic (e.g. Woosley & Zhang 2007) and MHD outflows (Bucciantini et al. 2009) may fail to produce stable clean jets (e.g. Matzner 2003) which may instead be ‘choked’ inside the star, resulting in little direct electromagnetic radiation (see Section 6.5).

IV. Magnetically driven, relativistic wind (GRBs)

($10–100 \lesssim \sigma_0 \lesssim 10^4$; $t_{\text{bo}} \lesssim t \lesssim t_{\text{end}}$)

After the jet breaches the stellar surface, a relatively clean opening is soon established through the star (e.g. Morsony et al. 2007). Simulations suggest that after this point the power and mass loading of the jet reflect, in a time- and angle-averaged sense, the values of $E(t)$ and $\dot{M}(t)$ set by the protomagnetar wind at much smaller radii (e.g. Bucciantini et al. 2009; Morsony, Lazzati & Begelman 2010).

Fig. 6 shows contours of the wind magnetization $\sigma_0$ at break-out ($t = t_{\text{bo}} = 10$ s), as calculated using a grid of wind models spanning the physical range of magnetar parameters $B_{\text{dip}}$ and $P_0$ for two values of the magnetic obliquity $\chi = 0$, $\pi/2$. Note that high spin-down magnetars (upper left-hand corner) produce outflows that are ultrarelativistic at break-out, that is, $\sigma_0|_{t_{\text{bo}}} > \pi x s(10–100)$.

Over the next tens of seconds, $\sigma_0$ increases from $\sigma_0|_{t_{\text{bo}}} \gtrsim 10^4$ (Figs 2 and 5), resulting in ideal conditions for high-energy emission. Assuming that the wind is collimated into a jet with a
half-opening angle \( \theta_j \), the ‘isotropic’ jet luminosity \( E_{\text{iso}} \) is larger than the wind power \( \dot{E} \) by a factor \( f_b^{-1} \), where \( f_b \approx \frac{\theta_j^2}{2} \) is the beaming fraction (Rhoads 1999). Using axisymmetric MHD simulations, Bucciantini et al. (2009) found \( \theta_j \approx 5 \times 10^{-5} \) for a magnetar with \( B_{\text{dp}} \approx 3 \times 10^{15} \) G and \( P_0 \approx 1 \) ms, values consistent with the typical opening angles inferred from GRB afterglow modelling (e.g. Frail et al. 2001; Bloom, Frail & Kulkarni 2003a).

Although the more general dependence of \( \theta_{\text{jet}} \) on the properties of the magnetar and stellar progenitor has not yet been determined, some insight is provided directly from observations. By combining the well-known correlation between the peak energy of the prompt emission spectrum, \( E_{\text{peak}} \), and the isotropic energy, \( E_{\text{iso}} \), \( E_{\text{iso}} \propto E_{\text{peak}} E_{\text{iso}}^{0.4} \) (Amati et al. 2002), with the correlation \( E_{\text{peak}} \propto E_{\gamma}^{0.7} \), between \( E_{\text{peak}} \) and the beaming-corrected energy \( E_{\gamma} = f_b E_{\text{iso}} \) (Ghirlanda, Ghisellini & Lazzati 2004), we obtain the empirical relationship (cf. Nava et al. 2006):

\[
f_b \approx 2 \times 10^{-3} \left( \frac{E_{\gamma}}{10^{51} \text{ erg}} \right)^{-3/4} \quad ; \quad \theta_j \approx 3.3 \left( \frac{E_{\gamma}}{10^{51} \text{ erg}} \right)^{-3/8}.
\]

In what follows, we assume for simplicity a fixed beaming fraction \( f_b = 2 \times 10^{-3} \), but we return to an implication of the correlation \( f_b \propto E_{\gamma}^{-3/4} \) in Section 4.3.

To produce high-energy emission, the jet must both accelerate to a high Lorentz factor \( \Gamma_j \approx \sigma_0 \gg 1 \) and dissipate much of its bulk energy internally. Both the emission models that we consider in Section 4, magnetic dissipation and internal shocks, predict a characteristic emission radius where most dissipation occurs, \( R_{\gamma} = R_{\text{mag}} \) and \( R_{\text{in}} \), respectively, that increases with time. Here \( R_{\text{mag}} \) and \( R_{\text{in}} \) are the radii at which magnetic dissipation peaks and internal shocks occur, respectively (see below). Whether photons escape the emission region at a given epoch depends on the location of \( R_{\gamma} \) with respect to the radius of the Thomson photosphere of the jet (e.g. Giannios 2006):

\[
R_{\gamma} \approx \frac{E_{\text{iso}} \kappa_{\gamma} \sigma_0}{8\pi c^2 \sigma_0},
\]

where \( \kappa_{\gamma} \) is the Thomson opacity and we have assumed an efficient acceleration, that is, \( \Gamma_j \approx \sigma_0 \gg 1 \) (Section 4.1).

Figure 6. Contours of the wind magnetization at jet break-out \( t = t_{\text{bo}} = 10 \) s, as a function of the magnetic field strength, \( B_{\text{dp}} \), and initial rotation period, \( P_0 \), of the magnetar. The solid and dotted lines show calculations, assuming magnetic obliquities \( \chi = 0 \) and \( \pi/2 \), respectively.

Figure 7. Photosphere radius, \( R_{\gamma} \) (solid line; equation 7), internal shock radius, \( R_{\text{sh}} \) (dashed line; equation B3) and the ‘saturation’ radius at which magnetic dissipation peaks, \( R_{\text{mag}} \) (dotted line; equation 10) in the protomagnetar jet as a function of time since core bounce, calculated for the model shown in Fig. 2. The jet breaks out of the star at the time \( t = t_{\text{bo}} = 10 \) s. At times \( t_{\text{bo}} \lesssim t \lesssim t_{\text{thin, mag}}(t_{\text{thin, mag}}) \), magnetic dissipation (internal shocks) occurs below the photosphere and the resulting emission will be thermalized (Stage IVA). By contrast, at times \( t \gtrsim t_{\text{thin, mag}}(t_{\text{thin, is}}) \), emission occurs in an optically-thin environment and may be non-thermal (Stage IVb). The end of the GRB is defined as when \( R_{\text{mag}} = R_{\gamma} \) (Stage V).

### IVa. Quasi-thermal, photospheric emission

\( (t_{\text{bo}} \lesssim t \lesssim t_{\text{thin}}; \ R_{\gamma} < R_{\text{sh}}) \)

Fig. 7 shows the time-evolution of the photosphere radius, \( R_{\gamma} \), and the radii at which internal shocks occur (\( R_{\text{sh}} \); equation B3) and magnetic dissipation (\( R_{\text{mag}} \); equation 10) occur, calculated for the fiducial model shown in Fig. 2. Just after break-out, the magnetic dissipation and internal shocks occur below the photosphere, that is, \( R_{\gamma} = \{R_{\text{mag}}, R_{\text{sh}}\} \ll R_{\gamma} \), such that high-energy emission will be partially thermalized and suppressed due to adiabatic losses. At later times, jet dissipation occurs in an optically-thin environment (\( R_{\gamma} > R_{\gamma} \)), such that brighter non-thermal emission is more likely.

Fig. 8 shows contours of the time after core bounce at which \( R_{\text{sh}} = R_{\text{sh}} \) (dotted line) and \( R_{\text{sh}} = R_{\gamma} \) (solid line) as a function of \( B_{\text{dp}} \) and \( P_0 \) for \( \chi = \pi/2 \). Low-field magnetars (lower diagram) produce jets that are optically thick at break-out (i.e. \( t_{\text{thin}} > t_{\text{bo}} \approx 10 \) s) and thus experience a phase of quasi-thermal photospheric emission, as in the fiducial model described above (Stage IVA). In fact, if \( t_{\text{thin}} \) becomes comparable to the GRB duration itself (cf. Fig. 13, shown later), a thermally-rich subluminous GRB or XRF may result instead of a classical GRB (Section 6.4). By contrast, jets from strongly magnetized magnetars (upper diagram) dissipate their energy in an optically-thin environment just after jet break-out, thereby skipping Stage IVA entirely.

Fig. 9 shows contours of the fraction of the energy released in thermal emission during the GRB phase \( f_{\text{th}} = E_{\text{th}}/(E_{\gamma} + E_{\text{sh}}) \). Here \( E_{\text{th}} \) is the total non-thermal emission during the GRB (quantified in the next section) and \( E_{\text{sh}} \) is the maximum thermal energy, which we

\[5\] Throughout this paper, we define ‘non-thermal’ emission as a non-blackbody spectrum. This does not necessarily imply that the radiating electrons have a non-thermal energy distribution.
estimate as (e.g. Mészáros & Rees 2000):

$$E_{th} = \int_{t_{bo}}^{t_{th}} \epsilon_{r} E \left( \frac{R_{th}}{R_{th}} \right)^{-2/3} dt,$$

(8)

where the factor $(R_{th}/R_{th})^{-2/3}$ accounts for adiabatic losses and we have (optimistically) assumed a radiative efficiency $\epsilon_{r} = 0.5$ (equation B7). High-field magnetars (upper diagram) produce little thermal emission, $E_{th} \approx 0$, because the jet is already optically thin at break-out (i.e. $t_{th} < t_{bo}$; cf. Fig. 8). By contrast, somewhat lower field magnetars (middle left-hand diagram) have $t_{th} \gtrsim 0.1 \left( E_{th} \sim 10^{38} \text{ to } 10^{40} \text{ erg} \right)$, consistent with measurements or upper limits on the quasi-thermal photospheric emission from GRBs (e.g. Mészáros & Rees 2000; Ramirez-Ruiz, MacFadyen & Lazzati 2002a; Ryde 2005; Guiriec et al. 2011).

**IVb. Main GRB emission**

$(t_{th} \lesssim t \lesssim t_{end}; R_{\gamma} > R_{ph})$

From the time $t = \min \{t_{th}, t_{bo} \}$ until the GRB ends at $t = t_{end}$ (which we define more precisely below), shocks or reconnection occurs above the photosphere and non-thermal emission is likely. Fig. 10 shows contours of the total energy released by the magnetar wind $E_{\gamma} \equiv \int E \sigma_{r} dt$ integrated over the duration of the GRB. High spin-down magnetars (upper left-hand diagram) achieve values $\sigma_{avg} \approx \Gamma_{max} \sim 10^{2} \text{ to } 10^{4}$ which are consistent with observational constraints on the GRB Lorentz factors (i.e. $\Gamma \gtrsim 100$–1000; e.g. Lithwick & Sari 2001; Zou & Piran 2010; Zou, Fan & Piran 2011). We caution, however, that although $\sigma_{avg}$ approximately equals the jet’s maximum instantaneous Lorentz factor, for internal shocks the Lorentz factor of the emitting material, $\Gamma_{e}$, is generally lower than $\sigma_{avg}$ because the faster jet interacts with slower material released at earlier times (Section 4.4). In Fig. 12, we show contours of the (energy-weighted) mean Lorentz factor, $\Gamma_{avg}$, of the bulk shell, from behind which internal shock emission originates. Note that in general $\Gamma_{avg}$ is a factor of a few times lower than $\sigma_{avg}$. A comparison of Fig. 10 with Figs 11 and 12 reveals a positive correlation between $E_{\gamma}$ and the mean magnetization/Lorentz factor. We discuss this correlation and its implications further in Section 4.3.

**Figure 10.** Contours of the maximum non-thermal gamma-ray emission, $E_{\gamma}$, in erg as a function of $B_{ph}$ and $P_{0}$ for $\chi = 0$ (solid line) and $\chi = \pi/2$ (dotted line). We calculate $E_{\gamma}$ as the total energy released by the magnetar in the time-interval $\max \{t_{th}, t_{bo} \} \lesssim t \lesssim t_{end}$ times a factor $\epsilon_{r} = 0.5$ to account for the maximum radiative efficiency. Here $t_{bo}$ is the time required for the jet to propagate through the star. $t_{th}$ is the time after which the outflow is optically thin at the internal shock radius (Fig. 8) and $t_{end}$ is the end of the GRB, defined as when $R_{mag} = R_{b}$ (see Figs 7 and 13).
σ is the jet break-out time, 30–100 s (Fig. A1), after and P is the time after internal shocks occur.

The true predicted (rest-frame) T ≲ ∼ 10–100 s, and accounts for why the prompt high-energy emission ends the prompt emission (i.e. when R_{mag} = R_{ns}; see Fig. 7).

Fig. 13 shows contours of the rest-frame GRB duration, T_{90}, defined as the time-interval within max[t_{bo}, t_{thin, is}] ≤ t ≤ t_{end} during which 90 per cent of the wind energy is released. Note that high spin-down (‘GRB-capable’) magnetars have T_{90} ~ 40–50 s, similar to the average observed rest-frame duration of long GRBs. A qualitatively similar, though somewhat shorter, T_{90} distribution results if we assume that emission begins at t_{thin, mag} (magnetic dissipation) rather than t_{thin, is} (internal shocks). The true predicted (rest-frame) GRB duration distribution will of course be broader than suggested by Fig. 13 because we have not taken into account variations in the time-scale for jet break-out, t_{bo} (equation 5), and neutrino transparency, t_{thin}, the latter of which depends on the NS mass and rotation rate. Realistic variations in t_{bo}, t_{ns}, M_{ns} will undoubtedly broaden the rest-frame T_{90} distribution by factors of a few as observed (see Table 1 for examples).

Except in the case of VLGRBs (Section 6.2), most of the magnetar’s initial rotational energy remains when the prompt emission ends. Though not released as gamma-rays, this residual energy may be dissipated at later times or larger radii and hence may contribute, for instance, to the GRB X-ray afterglow. In Section 5, we discuss the emission during the late-time high-σ phase.

4 GAMMA-RAY BURST EMISSION

In this section, we calculate the emission during the prompt phase (t_{bo} ≤ t ≤ t_{end}; Stage IV). We begin with a discussion of the mechanisms for jet acceleration (Section 4.1) and variability (Section 4.2) and then present calculations of the gamma-ray emission produced by magnetic dissipation (Section 4.3) and internal shocks (Section 4.4).

4.1 Acceleration

The energy carried by the relativistic wind is primarily in the magnetic field near the light cylinder radius, R_{lc} ~ 10^7 cm. Because GRBs originate from ultrarelativistic outflows (e.g. Lithwick & Sari 2001), this magnetic energy must be transferred to kinetic energy.

As σ continues to increase, the jet becomes less and less effective at accelerating and dissipating its ordered energy (Section 4.1). A particularly abrupt jump in σ occurs once the NS becomes transparent to neutrinos at t = t_{thin} ~ 30–100 s (Fig. A1), after which σ rises to very large values ≥10^6. This transition likely ends the prompt high-energy emission. Although the argument for why t_{end} ~ t_{thin} is quite general, we can be concrete by defining t_{end} as the time after which the magnetic dissipation radius, R_{mag} (equation 10), exceeds the internal shock radius, R_{ns}. For t ≥ t_{end}, the jet magnetization at the shock radius exceeds the critical value σ ~ 0.1 above which strong shocks are suppressed (Kennel & Coroniti 1984). The association of t_{end} with t_{thin} both explains the typical duration of long GRBs, T_{90} ~ 10–100 s, and accounts for why the prompt ~MeV emission declines more rapidly at later times ≥T_{90} (αr^3) than the jet luminosity predicted by most central engine models (e.g. Tagliaferri et al. 2005; Barniol Duran & Kumar 2009).

Here t_{bo} is the jet break-out time, t_{thin} is the time after internal shocks occur above the photosphere (Fig. 8) and t_{end} is the end of the prompt emission (i.e. when R_{mag} = R_{ns}, see Fig. 7).
prior to the radii $\sim 10^{12}$–$10^{16}$ cm at which the high-energy emission occurs. Unconfined, time-stationary Poynting-flux-dominated outflows do not accelerate efficiently in ideal MHD (Goldreich & Julian 1970; Beskin, Kuznetsova & Rafikov 1998; Bogovalov & Tsynganos 1999). The Lorentz factor achieved when acceleration slows near the fast magnetosonic surface $\Gamma_{\infty} \sim \sigma_0^{1/3}$ (Goldreich & Julian 1970) is much less than the maximum possible value $\Gamma_{\text{max}} \approx \sigma_0$. Full acceleration to $\Gamma_{\infty} \sim \Gamma_{\text{max}}$ therefore appears to require a combination of a differentially collimated (non-monopolar) geometry, time-variability or violations of ideal MHD (see Komissarov 2010 for a recent review).

At small radii the wind is concentrated in the rotational equator. On larger scales, the outflow is redirected into a bipolar jet by its interaction with the star (Komissarov & Barkov 2007; Bucciantini et al. 2009). Analytic (e.g. Vlahakis & Königl 2001; Narayan, McKinney & Farmer 2007) and numerical (Komissarov et al. 2007, 2009; Tchekhovskoy, McKinney & Narayan 2009; Tchekhovskoy, Narayan & McKinney 2010) calculations show that if the jet is confined to a parabolic shape, additional acceleration is possible due to ‘equilibrium collimation’. However, the maximum Lorentz factor that can be achieved in this manner is $\Gamma_{\infty} \sim 1/\theta_\ast \sim 10$ (equation 6) because only while $\Gamma \theta_\ast \lesssim 1$ does the jet remain in lateral causal contact. Although an additional boost of acceleration (by a factor of $\lesssim 10$) may occur as the jet emerges from the stellar surface (Komissarov et al. 2009; Tchekhovskoy et al. 2009), reaching $\Gamma_{\infty} \gtrsim 10^2$, and simultaneously achieving high conversion efficiency of magnetic to kinetic energy appears difficult via collimation alone.

A time-dependent flow can also produce acceleration. In the so-called ‘astrophysical plasma gun’ or ‘magnetic rocket’ mechanism (Contopoulos 1995; Lyutikov & Lister 2010; Granot, Komissarov & Spitkovsky 2011; Lyutikov 2011), a high-$\sigma_0$ magnetic pulse of finite width expands into a lower density medium (‘vacuum’; see, however, Levinson 2010). As the shell propagates, it ‘self-accelerates’ via magnetic pressure gradients which develop as a rarefaction wave passes through the shell. In this case, $\Gamma$ increases $\propto r^{1/3}$ (e.g. Granot et al. 2011), similar to the magnetic dissipation model described below. Faster acceleration $\propto \sqrt{r}$ is possible in standard (high-entropy) GRB fireball models (e.g. Goodman 1986), but it remains unclear how the necessary thermalization would occur inside the star.\(^6\) Note that no ideal MHD model for jet acceleration accounts for the dissipation of energy responsible for powering the GRB, which must instead occur at larger radii after acceleration is complete.

An alternative possibility for jet acceleration is magnetic dissipation, that is, a break-down of ideal MHD (Spruit, Daigne & Drenkhahn 2001; Drenkhahn 2002; Drenkhahn & Spruit 2002). One way this can occur is if the rotation and magnetic axes of the NS are misaligned ($\chi > 0$), such that the outflow develops an alternating or ‘striped’ magnetic field geometry (Coroniti 1990) on the scale of the light cylinder radius. If this non-axisymmetric pattern is preserved when the flow is redirected along the polar jet, the resulting geometry is conducive to magnetic reconnection. Magnetic dissipation occurs gradually from small radii up to the ‘saturation’ radius $R_{\text{mag}}$, beyond which reconnection is complete and the flow achieves its terminal Lorentz factor. During this process, approximately half the Poynting flux is directly converted into kinetic energy (producing acceleration) and the other half is deposited into the internal (thermal) energy of the flow. Acceleration and emission thus both result from the same physical mechanism.

Drenkhahn (2002) shows that the Lorentz factor of the jet as a function of radius is given by

$$\Gamma = \begin{cases} \sigma_0 (r/R_{\text{mag}})^{1/3}, & r < R_{\text{mag}} \\ \sigma_0, & r > R_{\text{mag}} \end{cases}$$

where the saturation radius is

$$R_{\text{mag}} = \frac{\pi c \sigma_0^2}{3 \epsilon / \Omega} = 5 \times 10^{22} \text{ cm} \left( \frac{\sigma_0}{10^2} \right)^2 \left( \frac{P}{10^{-8} \text{ ms}^{-1}} \right) \left( \frac{\epsilon}{0.01} \right)^{-1}$$

and $\epsilon \lesssim 1$ parametrizes the reconnection speed $v_r = \epsilon v_A$, where $v_A \simeq 0$ is the Alfve´n speed. In our calculations, we assume that $\epsilon = 0.01$, independent of the radius or jet properties. This value is motivated by recent work finding a reconnection rate of this order due to secondary tearing instabilities in the current sheets (e.g. Uzdensky, Louie & Schekochihin 2010), even in highly collisional environments, that characterizes the jet close to the central engine. On the other hand, at larger radii (yet still well below the nominal saturation radius), reconnection may occur in the collisionless regime, such that faster reconnection is also likely (see e.g. Arons 2008; McKinney & Uzdensky 2010 for specific physical dissipation mechanisms).

4.2 Variability

Although GRBs are variable on time-scales down to fractions of a millisecond (Schafer & Walker 1999; Walker, Schaefer & Fenimore 2000), most Fourier power is concentrated on a characteristic time-scale $\sim 1$ s (Beloborodov, Stern & Svensson 1998, 2000). GRB variability may be related to the emission mechanism itself or it may reflect real variations in the power and mass loading of the jet (e.g. MacFadyen & Woosley 1999; Aloy et al. 2000; Mizuta & Aloy 2009; Morsony et al. 2010).

There are several potential sources of variability in protomagnetar outflows. Sporadic changes to the magnetosphere could modulate the magnetar wind properties on short ($\lesssim$millisecond) time-scales due to the reconnection near the light cylinder (Bucciantini et al. 2006) or on longer time-scales due to the neutrino heating in the closed zone (Thompson 2003). Longer time-scale variability could also be imposed on the outflow as it propagates to the stellar surface, due to instabilities associated with the termination shock(s) in the protomagnetar nebula (Bucciantini et al. 2009; Camus et al. 2009) or at larger distances as the jet propagates through the stellar envelope (Morsony et al. 2007, 2010). The latter possibility is particularly promising because the sound-crossing time across the jet near the stellar radius is in fact $\sim 1$ s (e.g. Morsony et al. 2010; Lazatti et al. 2011) and might not evolve appreciably throughout the burst, a fact consistent with observations (Ramirez-Ruiz & Fenimore 1999).

The time-averaged wind properties calculated in Section 2.1 ($\dot{E}$, $M$ and $\sigma_0$) do not account for any of the variability discussed above. In fact, given the stochastic nature of the GRB emission, it seems unlikely that any model will be capable of predicting the detailed light curve of individual bursts. In our calculations below, we instead focus on predicting the time-averaged high-energy emission over time-scales of seconds or longer, which may be usefully compared with integrated GRB light curves and spectra (e.g. McBreen et al. 2002). We nevertheless emphasize that variability affects the observed emission differently depending on the emission model. Magnetic dissipation, for instance, occurs at relatively small radii, such that variability is directly encoded in the emitted

\(^6\) One possibility is if instabilities act within the jet to randomize the magnetic field, such that it behaves as a $y = 4/3$ relativistic gas (Heinz & Begelman 2000; Giannios & Spruit 2006).
radiation. Variability from internal shocks instead manifests indirectly through the effects of subsequent collisions at larger radii.

4.3 Emission from magnetic dissipation

Drenkhahn & Spruit (2002) make specific predictions for the rate at which the magnetic energy is dissipated with the radius. However, reconnection can lead to plasma heating and acceleration in localized regions\(^7\) (e.g. current layers). Alternatively, reconnection may drive bulk motions in the jet that excite Alfvénic turbulence (e.g. Thompson 1994), which cascades to small scales and heats larger volumes in the plasma. We follow the model of Giannios (2006, 2008) who assumes that the dissipated energy heats the plasma smoothly throughout the flow (slow heating model; see Ghisellini & Celotti 1999; Stern & Poutanen 2004). Similar qualitative conclusions would, however, result from any model that invokes localized modest particle acceleration close to the photosphere (e.g. Lazzati & Begelman 2010).

\(^7\) This is the approach adopted by Lyutikov & Blandford (2003). However, because the mechanisms responsible for particle acceleration in magnetic reconnection are uncertain, it is difficult to make concrete predictions for the resulting GRB emission in this case.

Giannios (2008) shows that the energy dissipated at large Thomson optical depths is thermalized, such that a portion emerges through the photosphere with a peak at \(\sim\)MeV energies (cf. Goodman 1986; Mészáros & Rees 2000; Ramirez-Ruiz 2005; Giannios 2006; Pe'er, Mészáros & Rees 2006; Beloborodov 2010). At times when \(R_{\text{mag}} \gtrsim R_{\text{ph}}\), most of the Poynting flux is dissipated near or above the photosphere and the equilibrium temperature of the electrons exceeds the radiation temperature. Inverse-Compton scattering of the photons advected outwards with the flow then results in power-law emission with a flat spectral slope, \(E_L \propto E^0\), above the thermal peak. Larger radii in the flow are heated to yet higher temperatures, resulting in an additional component of synchrotron and synchrotron-self-Compton emission at lower frequencies [i.e. optical, ultraviolet (UV) and X-ray bands]. This softens the spectrum below the MeV peak close to the observed \(E_L \propto E^2\) value.

Fig. 14 shows the bolometric (isotropic) luminosity due to magnetic dissipation, \(L_{\text{mag}}\), calculated for the fiducial model shown in Fig. 2. At late times \(t \gtrsim t_{\text{thin,mag}}\approx 20\) s, magnetic dissipation occurs above the photosphere (\(R_{\text{mag}} \gtrsim R_{\text{ph}}\)) and \(L_{\text{mag}} = E_{\text{iso}}/2\) (Drenkhahn & Spruit 2002). At early times \(t \lesssim t_{\text{thin,mag}}\) when \(R_{\text{mag}} \lesssim R_{\text{ph}}\), by contrast, \(L_{\text{mag}}\) is suppressed below \(E_{\text{iso}}\) by an additional factor \(\sim 0.4(R_{\text{mag}}/R_{\text{ph}})^{2/3}\) due to adiabatic losses incurred between the dissipation radii and the photosphere.

Fig. 15 shows snapshots of the high-energy spectrum, calculated at the times \(t = 15, 20, 25\) and \(30\) s. The spectrum at \(t = 15\) s
corresponds to an early epoch when $R_{\text{ph}} \gg R_{\text{mag}}$ and the dissipated energy is thermalized; the spectrum in this case is approximately Planckian with temperature $T \approx 2 \text{ keV}$. Due to its low luminosity and X-ray peak, such a component of early thermal emission may be challenging to detect in an actual GRB (Fig. 9). At later times $t \gtrsim t_{\text{th,mag}} \approx 20 \text{ s}$, by contrast, the dissipation peaks near or above the photosphere. This results in a spectral peak at energy $E_{\text{peak}} \sim 10^7 \text{ keV}$ (described below) and a non-thermal Comptonized tail that extends to increasingly higher energies as $\sigma_0$ rises and the outflow becomes cleaner with time. Also note the component of the synchrotron emission at softer X-ray/UV wavelengths, which increases in relative importance to the Comptonized gamma-rays as the jet magnetization increases and dissipation peaks at larger radii.

The magnetic dissipation model predicts a spectral energy peak, $E_{\text{peak}}$ (or break), similar to the observed Band spectrum peak $\sim$ few hundred keV and which is relatively insensitive to the jet properties. Giannios & Spruit (2007) show that to good approximation

$$E_{\text{peak,mag}} \simeq 270 \text{ keV} \left( \frac{E_{\text{iso}}}{10^{52} \text{ erg s}^{-1}} \right)^{0.11} \left( \frac{\epsilon \Omega}{10^5} \right)^{0.33} \left( \frac{\sigma_0}{10^2} \right)^{0.2} \text{ (11)}$$

for $R_{\text{mag}} \gtrsim R_{\text{ph}}$.

Fig. 16 shows $E_{\text{peak}}$ as a function of time for the fiducial model shown in Figs 14 and 15. Note that although $E_{\text{peak}}$ rises rapidly at times $t \lesssim t_{\text{th,mag}}$ (when the luminosity is highly suppressed), $E_{\text{peak}}$ is relatively constant during the GRB itself, increasing from $\sim$200 to $\sim$400 keV between $t = t_{\text{th,mag}}$ and $t_{\text{end}}$. This evolution results from the weak dependence of $E_{\text{peak}}$ on $E_{\text{iso}}(t)$ and $\sigma_0(t)$ in equation (11). The prediction that $E_{\text{peak}}$ should rise with time seems in conflict with the observation that GRBs generally spectrally ‘soften’ throughout their duration. This behaviour may, however, still be consistent with the spectral evolution predicted by magnetic dissipation if the synchrotron emission at lower frequencies begins to contaminate the soft X-ray bands at late times ($E_{\text{peak,mag}}$ refers to the spectral peak of the inverse-Compton emission; see Fig. 15).

We now consider the implications of equation (11) for the population of magnetar-powered GRBs as a whole. Fig. 17 shows a scatter plot of the average magnetization, $\sigma_{\text{avg}}$, during the GRB (Fig. 11) as a function of the average GRB luminosity, $L_{\gamma} \equiv E_{\gamma}/T_{90}$ (Figs 10 and 13), where we have included data points from all models within the range of magnetar parameters explored previously (1 $\lesssim P_0 \lesssim 5 \text{ ms}$; $3 \times 10^{14} \lesssim B_{\text{mag}} \lesssim 3 \times 10^{16} \text{ G}$; $\chi = 0$ and $\pi/2$). Magnetars lying on the one-parameter family $B_{\text{mag}} \propto P_0^{-1}$ defined by equation (4) for $\epsilon_0 = 10^{-3}$ are connected with a solid line.

Fig. 17 shows that the magnetar model predicts, with large scatter, a positive correlation between $\sigma_{\text{avg}}$ and $L_{\gamma}$. In particular, for the one-parameter family of solutions we find that $\sigma_{\text{avg}} \propto L_{\gamma}^\alpha$, where $\alpha \approx 0.5$–1, depending on $L_{\gamma}$ and $\chi$. Assuming that the GRB duration, radiative efficiency, $\epsilon_r$, and beaming fraction, $\eta$, are similar from burst to burst, this correlation implies that $E_{\gamma} \propto \sigma_{\text{avg}}^{1/\alpha}$. From equation (11), this in turn leads to the prediction that $E_{\text{peak}} \propto E_{\gamma}^{0.2-0.4}$ for $\epsilon \Omega \sim$ constant. Note that this is close to the Amati et al. (2002) relationship $E_{\text{peak}} \propto E_{\gamma}^{0.3}$. If one furthermore drops the assumption that $f_{\text{pl}}$ is constant and instead assumes $f_{\text{pl}} \propto E_{\gamma}^{-0.75}$, as motivated by the combined Amati and Ghirlanda relations (equation 6), one finds $E_{\text{peak}} \propto E_{\gamma}^{0.3-0.5}$, resulting in even better agreement with observations. A qualitatively similar correlation is predicted between $E_{\gamma}$ and the peak jet power, consistent with the related ‘Yonetoku’ relation (Yonetoku et al. 2004). We emphasize that both the normalization and the slope of the Amati–Yonetoku correlations are reproduced if we assume a reconnection rate $\epsilon = 10^{-2}$ favoured by recent work (Uzdensky et al. 2010).

### 4.4 Emission from internal shocks

If the acceleration of the jet is efficient (Section 4.1), then a significant fraction of the Poynting flux is converted into kinetic energy. The kinetic luminosity and Lorentz factor of the outflow in this case are given by $L_j(t) \simeq (1 - \epsilon_{\text{mag}})E(t)$ and $\Gamma(t) \simeq (1 - \epsilon_{\text{mag}})\sigma_0(t)$, respectively, where $\epsilon_{\text{mag}} \lesssim 0.5$ is the fraction of the power radiated during the acceleration phase, due to magnetic dissipation (Section 4.3). In what follows we assume $\epsilon_{\text{mag}} = 0.5$, although $\epsilon_{\text{mag}} = 0$ would be appropriate if the magnetic energy that is dissipated is not radiated away or if acceleration is achieved by another mechanism.

Because $\sigma_0 \sim \Gamma$ increases monotonically during the GRB (Fig. 2), slower material is released prior to faster material. Strong shocks will occur once the faster material catches up, provided that the residual magnetization of the jet is $\lesssim 0.1$ (Kennel & Coroniti 1984; Mimica, Giannios & Aloy 2009). This scenario is similar to the standard internal shock model for GRB emission (e.g. ...
Rees & Meszaros 1994; Kobayashi, Piran & Sari 1997; Daigne & Mochkovitch 1998) with a few key differences to be discussed below.

Immediately after the jet breaks out of the star, the fast and slow ejecta have comparable energies and speeds. With time, however, the slow material released early accumulates into a common ‘bulk’ shell, which we characterize by its total rest mass \( M_s = \int_0^t M_i \, dt \), energy \( E_s \), mean velocity \( \beta_s \), and mean Lorentz factor \( \Gamma_s \equiv E_s / M_i c^2 \), where \( M_i = L_s / \Gamma_s c^2 \) (see Fig. 12). At most times the jet’s self-interaction is well described as a collision between the fast, variable jet and a slower (yet still ultrarelativistic) shell. We model this interaction using a 1D kinematic model, as described in Appendix B. Although this approach neglects the effects of pressure forces and the true multidimensional geometry (e.g. Zhang & MacFadyen 2009), it provides a reasonable first approximation to the full hydrodynamic problem (Daigne & Mochkovitch 1998, 2000, 2003).

Fig. 14 shows the (average) bolometric luminosity \( L_{\gamma} = \epsilon_{\gamma} L_{\gamma} \) from shocks as a function of the observer time \( t_{\text{obs}}(1 + z) \), calculated assuming that the fraction of the shock’s energy imparted to electrons is \( \epsilon_{\gamma} \approx 1 \). As in the case of magnetic dissipation, at times \( t \lesssim t_{\text{thick}} \), we suppress \( L_{\gamma} \) by an additional factor \( (R_{\text{sh}} / R_i)^{-2/3} \) to account for adiabatic losses when shocks occur below the photosphere, where \( R_i \) is the internal shock radius given in equation (B3). Note that although \( L_{s} \) decreases by a factor of \( \approx 6 \) throughout the burst, \( L_{\gamma} \) changes by only a factor of a few. Indeed, both magnetic dissipation and shock models predict that the bolometric luminosity should be relatively constant in time, a result in agreement with the approximately linear slope of cumulative GRB light curves (e.g. McBreen et al. 2002).

In Appendix B, we calculate the peak energy, \( E_{\text{peak, obs}} \), of the synchrotron spectrum as a function of the jet and shell properties, assuming that a fraction \( \xi_e \) of electrons are accelerated and that a fraction \( \xi_g \) of the shock energy goes into generating the magnetic field (see equation B13 and surrounding discussion).

Fig. 16 shows the evolution of \( E_{\text{peak, obs}} \) during the GRB for the fiducial model, calculated assuming \( \xi_e \approx 1, \xi_g = 0.1 \) and \( \xi_e = 0.3 \). These microphysical parameters are chosen ad hoc such that \( E_{\text{peak}} \) attains a value \( \sim 10^3 \text{keV} \) at the peak luminosity characteristic of observed GRB spectra. Even after this fine-tuning, however, two problems remain for the internal shock model. First, Fig. 16 shows that \( E_{\text{peak, obs}} \) increases by over three orders of magnitude during the burst, in contradiction with the relatively constant (or decreasing) peak energy measured during actual bursts. Although both \( \Gamma_s \) and \( t_i \) increase with time, \( E_{\text{peak, obs}} \propto t_i^{-1/2} \Gamma_s^{1/2} \) increases because the jet Lorentz factor, \( \Gamma_i \), increases even more rapidly (see equation B13). Although a slowly evolving peak energy could in principle be recovered by invoking, for example, time-dependent microphysical parameters, fine-tuning appears unavoidable (e.g. Zhang & Meszaros 2002). A second problem is that the variability time-scale produced by subsequent internal collisions, \( \delta t_{\text{var}} \propto R_{\text{sh}} / 2 \Gamma_s c \), is predicted to increase linearly with time, again contrary to observations suggesting that \( \delta t_{\text{var}} \) evolves weakly during the burst (Ramirez-Ruiz & Fenimore 1999). We conclude that the synchrotron emission from internal shocks appears disfavoured as the source of prompt emission from protomagnetars.

5 LATE-TIME EMISSION

When \( \sigma_0 \) becomes very large at late times (\( \gtrsim 10^5 \); Fig. 2), \( R_{\text{mag}} \) becomes so large that even if reconnection occurs at the speed of light in the comoving frame (\( \epsilon \sim 1 \)), no efficient acceleration or dissipation occurs before the outflow begins to interact with itself or the external interstellar medium. Without acceleration, shocks cannot occur and without efficient reconnection, there can be no dissipation-powered emission. As we argued in Section 3, this transition ends the phase of prompt internal emission. Similar physics occur in the wind from the Crab Pulsar, for which the very high initial magnetization may prevent internal dissipation prior to the wind termination shock at \( R \sim 10^{15} \text{cm} \) (Kennel & Coroniti 1984; Lyubarsky & Kirk 2001). The emission from the nebula may be the result of the forced reconnection at the termination shock itself (Lyubarsky 2003, 2005) or from dissipation in a striped wind

Fig. 17. Average magnetization, \( \sigma_{\text{avg}} \), during the GRB versus the average GRB luminosity \( L_s \equiv E_s / T_90 \). Each point represents a model calculated within the range of initial spin periods \( 1 \lesssim P_0 \lesssim 5 \text{ ms} \) and surface dipole fields \( 3 \times 10^{14} \lesssim B_{\text{mag}} \lesssim 3 \times 10^{16} \text{ G} \). The left-hand and right-hand panels show calculations performed assuming the magnetic obliquity \( \chi = 0 \) and \( \pi/2 \), respectively. A solid line connects solutions lying along the one-parameter family, \( B_{\text{mag}} \propto P_0^{-1} \), defined by equation (4) assuming \( \epsilon_B = 10^{-3} \).
When the prompt emission ends, a significant fraction of the magnetar’s initial rotational energy remains to be released in other forms. Since the beginning of the *Swift* mission, evidence has been accumulated that GRB central engines are indeed active at late times, from minutes to \( t > t_{\text{end}} \) following the burst. The X-ray afterglow in particular shows a complex evolution, including a ‘plateau’ phase in the light curve which is not predicted by the standard forward shock model (Nousek 2006; Zhang et al. 2006; Willingale et al. 2007), superimposed on the smoother afterglow are large-amplitude X-ray flares (Burrows et al. 2005, 2007a; Piro et al. 2005; Chincarini et al. 2007, 2010), which share many properties with the prompt GRB emission (Margutti et al. 2010) and also appear to result from late-time central engine activity (Lazzati & Perna 2007; Margutti et al. 2011).

Although the magnetic dissipation or internal shocks responsible for the prompt emission become ineffective when \( \sigma_0 \) is very large, spin-down luminosity can, in principle, power late-time emission in other ways. Indeed, a spin-down origin for the X-ray plateau is suggested by the ‘plateau-like’ evolution of the late-time wind power, \( E(t) \), illustrated in Figs 2 and 5. Spin-down can in principle power X-ray emission either indirectly by refreshing the forward shock (e.g. Granot & Kumar 2006; Dall’Osso et al. 2011) or directly (‘internally’) by, for example, forced reconnection at the forward shell (e.g. Lyubarsky 2003, 2005; Thompson 2006; Zhang & Yan 2011; Zou et al. 2011) or by upscattering forward shock photons (Pannaitescu 2008). Internal emission appears favoured in at least some cases due to the very steep decay observed in the X-ray flux following the plateau (e.g. GRB 070110; Troja et al. 2007; L10; Rowlinson et al. 2010).

Fig. 18 shows a scatter plot of the wind power evaluated at the beginning of the plateau-like high-\( \sigma_0 \) phase \( E_{\text{plateau}} \equiv E_{\text{end}} \) as a function of the spin-down time-scale \( \tau_{\text{s, end}} \), calculated for models spanning the usual range of magnetar parameters (1 \( \lesssim P_0 \lesssim 5 \) ms and surface dipole fields \( 3 \times 10^{14} \lesssim B_{\text{dip}} \lesssim 3 \times 10^{16} \) G; results are shown for both magnetic obliquities \( \chi = 0 \) and \( \pi/2 \). We also show for comparison the luminosities and end-times, \( t_{\text{end}, X} \), of the sample of plateaus from Lyons et al. (2010) (hereinafter L10), which show a steep decline in flux at times \( t > t_{\text{end}, X} \). The triangles and diamonds show the luminosities calculated assuming that the ratio between (observed) isotropic X-ray luminosity and wind power is equal to or is a factor of 10 larger than, respectively, the gamma-ray beaming fraction (equation 6).

(Coroniti 1990) if the pair multiplicity is higher than is commonly assumed (e.g. Arons 2008). Although internal dissipation in protomagnetar winds is unlikely at \( t > t_{\text{end}} \), forced reconnection at large radii is a potential source of late-time emission in this case as well (see below).
we overplot $t_{\text{end},X}$ and the luminosity from $L_{10} = L_{X,\text{iso}} / \eta_X^{-1}$ corrected by a factor $\eta_X = f_{\text{beam},X} \epsilon_{\text{X}}$ that accounts for both the X-ray beaming fraction, $f_{\text{beam},X}$, and the efficiency that spin-down power is converted into X-ray luminosity, $\epsilon_{\text{X}}$. We show two cases, in which $\eta_X$ equals to, or is a factor of $\geq 10$ times larger than, the gamma-ray beaming fraction $f_{\gamma}$ (which we estimate using equation (6) and the measured isotropic GRB energies). Note that because $t_{\text{end},X}$ is a lower limit on $\tau_{\gamma}$, Fig. 18 shows that all of the plateaus measured by $L_{10}$ are consistent with being powered by magnetar spin-down for $\eta_X \lesssim 10 f_{\gamma}$. The spin-down power escaping through the jet and radiated in X-rays is $\ll 10^{-5} f_{\gamma}$. Our results indicate that either (1) the jet opening angle during the plateau phase is a few times larger than during the GRB itself, that is, $f_{\text{beam},X} \gg f_{\gamma}$; and/or (2) the fraction of the spin-down power escaping through the jet is very low at late times, too low of an efficiency may be inconsistent with afterglow energetics. It is also possible that a fraction of the late-time spin-down energy is instead transferred to the SN shock, although numerical simulations of the interaction of the wind with the star suggest this need not be the case during the GRB itself (Bucciantini et al. 2009).

Late-time magnetar activity could also produce X-ray flaring. Margutti et al. (2011) find that the average flare luminosity decreases as $L_{\text{flare}} \propto \tau^{-\alpha}$, where $\alpha = 2.7$ (cf. Lazzati, Perna & Begelman 2008; Margutti et al. 2011). Although standard force-free spin-down predicts $\alpha = 2$ at times $\tau > 10^2$, steeper decays are inferred from the measured braking indices of some pulsars [e.g. $\alpha = 4/(n-1) \approx 2.42$ for PSR J1846–0258 with $n = 2.65$; Livingstone et al. 2007]. If the prompt emission is indeed suppressed at late times by the high magnetization of the jet, periodic enhancements in the jet’s mass-loading could temporarily ‘revive’ prompt-like internal emission, resulting in flaring. Temporarily enhanced mass-loss could result, for instance, from currents driven by a sudden rearrangement of the magnetosphere, analogous to Galactic magnetar flares (Thompson & Beloborodov 2005). Indeed, X-ray flares could also be powered by the release of magnetic energy itself, which is $\gtrsim 10^{48}–10^{50}$ erg for typical values of the interior field strength $B \sim 10^{16}–10^{17}$ G. Giannios (2010) recently proposed searching for such ‘superflares’ in nearby galaxies, which could in principle be observed even long after the GRB, possibly in coincidence with a relic radio afterglow.

6 DISCUSSION – A DIVERSITY OF PHENOMENA

Magnetars may form with a variety of properties (and under a variety of conditions) which, in turn, manifest as a diversity of high-energy phenomena. Fig. 19 summarizes the possible observable signatures of the magnetar birth as a function of the dipole field strength, $B_{\text{dip}}$, and birth period $P_0$. Although the plot shown is for an aligned rotator ($\chi = 0$), qualitatively similar results apply to the oblique case as well.

6.1 Classical GRBs

Magnetars in the upper-left quadrant of Fig. 19 produce ‘classical GRBs’ because (1) above the dotted lines the high-energy emission is almost exclusively non-thermal because the relativistic jet dissipates its energy – through reconnection or shocks – above the photosphere beginning just after stellar break-out; (2) magnetars to the left-hand side of the dot–dashed line produce GRBs with energies $E_\gamma \gtrsim 10^{50}$ erg (see Fig. 10); and (3) magnetars in this regime produce outflow with the average magnetization $\sigma_{\text{avg}} \sim 10^{-2}–10^{-3}$, consistent with the inferred Lorentz factors of long GRBs (Figs 11 and 12). Note that the initial rotational energies of magnetars in this parameter regime are $\gtrsim 3 \times 10^{51}$ erg ($P_0 \lesssim 3$ ms), implying that the requirements for a classical GRB and a hyperenergetic SN are remarkably similar (Fig. 1).

6.2 VLGRBs

Magnetars in the extreme upper left-hand corner of Fig. 19 produce classical GRBs with energies $E_\gamma \sim 10^{52}$ erg which are comparable to the total rotational energy available (equation 1). Evidence has recently grown for a class of ‘VLGRBs’ (e.g. Cenko et al. 2010a,b), which includes several Fermi bursts, such as GRB 080916C, with an isotropic energy $E_{\gamma,\text{iso}} \approx 8 \times 10^{51}$ erg (e.g. Abdo et al. 2009). The observation that energetic Fermi bursts appear to be distinguished by larger inferred Lorentz factors ($\Gamma \gtrsim 30$) than is estimated for more typical GRBs is consistent with the correlation predicted by the magnetar model between the average GRB luminosity and jet magnetization $\sigma_{\text{avg}}$ (maximum Lorentz factor), as shown in Fig. 17. Many extremely energetic GRBs, such as GRB 990123 (e.g. Kulkarni et al. 1999) and 080319B (Bloom et al. 2009), are also distinguished by the bright optical emission coincident with the GRB. The synchrotron emission predicted by the magnetic dissipation model at optical–UV wavelengths contributes an especially large fraction of the total radiated energy in bursts with large magnetization $\sigma_{\text{avg}}$ (see the late-time spectra in Fig. 15).

At present, the properties of VLGRBs appear consistent with resulting from magnetars with extreme, but physically reasonable, properties. However, measurements of the total energy in relativistic ejecta, $E_{\text{tot}} = E_\gamma + E_k$ (where $E_k$ is the kinetic energy), could constrain – or even rule out – the magnetar model as the central engine if $E_{\text{tot}}$ were found to exceed the maximum rotational energy $\sim k\sigma_{\text{avg}} (P_0 \approx 1 \text{ ms}) \times 3 \times 10^{52}$ erg. Although efforts are presently under-way to determine $E_{\text{tot}}$ for a sample of well-studied bursts (Cenko et al. 2010a,b), the results of these studies are hindered at present by simplifying assumptions in the afterglow modelling and jet structure, which may lead to systematic overestimates in $E_{\text{tot}}$ (e.g. Zhang & MacFadyen 2009; van Eerten, Zhang & MacFadyen 2010). Nevertheless, VLGRBs provide important probes of the most-extreme central engine properties.

6.3 Low-luminosity GRBs

Magnetars to the right-hand side of the dot–dashed line in Fig. 19 produce GRBs with energies $\lesssim 10^{50}$ erg which may contribute to the class of the so-called ‘low-luminosity GRBs’ (LLGRBs; e.g. Soderberg et al. 2004b; Cobb et al. 2006; Kaneko et al. 2007; Liang et al. 2007). LLGRBs are distinguished from classical GRBs by their lower energies, simple gamma-ray light curves (generally a single pulse), longer durations and higher local rates (e.g. Le & Dermer 2007; Liang et al. 2007). Because large angular momentum is probably rare in core-collapse SNe, LLGRB-producing magnetars with

10 As would be the case if spin-down triggers an abrupt end to the emission due to, for example, the delayed formation of a BH from a rotationally supported hypermassive NS (e.g. Baumgarte, Shapiro & Shibata 2000).

11 Note, however, that the lower limit constraints on $\Gamma$ derived for Fermi bursts become weaker if the ~GeV and ~MeV photons originate from different radii (e.g. Zou et al. 2011).
weaker fields and/or slower rotation may indeed be formed more commonly than the magnetars responsible for classical GRBs.

6.4 Thermal-rich GRBs and X-ray flashes

Magnetars below the dotted lines in Fig. 19 produce jets that dissipate a significant fraction of their energy under optically thick conditions after breaking through the star (i.e. they pass through Stage IVa described in Section 3) and produce jets with lower Lorentz factors than classical GRBs, that is, $\sigma_{\text{avg}} \lesssim 10^2$. We speculate that protomagnetars in this regime may produce X-ray-rich GRBs or XRFs (Heise et al. 2001; Mazzali et al. 2006) because they are accompanied by lower frequency, quasi-thermal emission with an energy comparable to, or somewhat lower than, the non-thermal GRB emission itself (Fig. 9). Although XRFs share many properties with long GRBs, such as an association with massive star formation (e.g. Bloom et al. 2003b; Soderberg et al. 2004a; Soderberg et al. 2007), they may be distinguished from GRBs by their ability to couple a significant energy to highly relativistic material (e.g. Soderberg et al. 2004a). This is consistent with the fact that magnetars in the lower portions of Fig. 19 indeed radiate a smaller fraction of their total energy during the GRB (as compared to the radiatively-inefficient high-$\sigma_0$ phase; Section 5) than magnetars in the classical GRB regime.

6.5 Choked jets and very luminous SNe

Magnetars in the lower right-hand corner of Fig. 19 produce jets with peak isotropic luminosities $\lesssim 10^{48}$ erg s$^{-1}$. Low-power jets may be unstable (Bucciantini et al. 2009) or take longer to propagate through the star than the duration of the GRB (Waxman & Meszaros 2003). Magnetars in this regime may thus produce ‘choked’ jets with little direct electromagnetic radiation (although they could still be a source of high-energy cosmic rays or neutrinos; e.g. Vietri 1995; Waxman 1995; Ando & Beacom 2005; Murase, Meszaros & Zhang 2009).

A number of core-collapse SNe have been recently discovered that are unusually bright and/or optically energetic (e.g. Gal-Yam et al. 2007; Ofek et al. 2007; Quimby et al. 2007', Quimby et al. 2009; Smith et al. 2007, Smith et al. 2008; Rest et al. 2009). Proposed explanations for these events, collectively known as very luminous SNe (VLSNe), include pair-instability SNe (Barkat, Rakavy & Sack 1967); interaction of the SN shock with dense circumstellar material (e.g. Gal-Yam et al. 2007; Smith et al. 2007, 2008; Metzger 2010); and the injection of late-time rotational energy from a rapidly spinning magnetar (Kasen & Bildsten 2010; Woosley 2010). In order to energize the SN ejecta on the ~days–weeks time-scales relevant for powering VLSNe, Kasen & Bildsten (2010) conclude that a magnetar with $B_{\text{dip}}$ ~ $5 \times 10^{14}$ G must possess an initial rotation period $P_0$ ~ 2–20 ms. This nominally places VLSNe-producing
magnetars in the ‘choked jet’ regime. We note, however, that in order to explain VLSNe, the initially Poynting-flux-dominated magnetar wind must thermalize its energy behind the SN shock, instead of escaping in a jet (Bucciantini et al. 2009), which might still be able to propagate through the star on the longer time-scales of relevance for VLSNe.

6.6 Galactic magnetars

If known Galactic magnetars were born with magnetic fields similar to their current observed strengths, $B_{\text{int}} \sim 10^{15} - 10^{18}$ G (e.g. Kouveliotou et al. 1998), and as fast rotators, then Fig. 19 suggests that their formation was accompanied by a thermal-rich GRB/XRF or choked jet, depending on their initial rotational period. Slower rotation, corresponding to a choked jet, may be likely in the majority of cases because Galactic magnetars are formed in ~10 per cent of core-collapse SNe (Woods & Thompson 2006), yet only a small fraction of envelope-stripped SNe are accompanied by relativistic ejecta (Soderberg et al. 2006). Furthermore, the SN remnants of Galactic magnetars do not show evidence for hyperenergetic SN explosions (e.g. Vink & Kuiper 2006; see, however, Horvath & Allen 2010).

6.7 Accretion-induced collapse

This paper has focused on the core collapse of massive stars, but magnetars may also form via the AIC of WDs (e.g. Nomoto et al. 1979; Usov 1992). Although AIC is probably intrinsically rarer than the standard core collapse (e.g. Fryer et al. 1999), millisecond magnetars may be a more common by-product of AIC because the WD is spun up considerably as it accretes up to the Chandrasekhar mass.

A distinguishing characteristic of AIC is the lack of a massive overlying stellar envelope. However, AIC does not produce a vacuum around the magnetar. A small quantity of mass, $\sim 10^{-3} - 10^{-1} M_\odot$, is ejected during the SN explosion itself (Dessart et al. 2006) and in the early, mildly-relativistic phase of the neutrino wind (Stage II). If the collapsing WD furthermore has sufficient angular momentum, an accretion disc forms around the NS (Michel 1987; Dessart et al. 2006). As this disc accretes on to the NS on a time-scale $\lesssim 1$ s, outflows from the disc powered by nuclear recombination eject $\gtrsim 10^{-2} M_\odot$ in nickel-rich material (Metzger, Piro & Quataert 2009).12

Because the protomagnetar is surrounded by a modest ‘sheath’ of material, its relativistic wind from the magnetar may be collimated into a bipolar jet, analogous to the standard core-collapse case. Because of the lower inertia of this surrounding mass, however, collimation may be less effective and the opening angle of any ‘jet’-like structure may be considerably larger. If this speculation is correct, then it would imply a larger beaming fraction, $\beta_{\text{b}}$, lower isotropic luminosity and softer spectral peak (e.g. equation 11) than in the standard core-collapse case. Perhaps equally important, the fact that the jet is no longer required to escape the star in order to produce high-energy emission may ‘select’ for magnetars with lower fields and/or slower rotation (and hence lower spin-down luminosities, lower Lorentz factors and softer spectra) than in the standard core-collapse case.

One of the biggest mysteries associated with short-duration GRBs is that $\lesssim 1/4$ are followed by a ‘tail’ of emission (usually soft X-rays) starting $\sim 10$ s after the GRB and lasting for $\sim 30-100$ s (Gal-Yam et al. 2006; Gehrels et al. 2006; Norris & Bonnell 2006; Perley et al. 2009; Norris, Gehrels & Scargle 2010). Although the large inferred energies and durations of the tails are difficult to explain in NS merger models (e.g. Metzger et al. 2010), their properties are similar to the prompt emission expected from magnetar birth via AIC. Metzger et al. (2008a) proposed an AIC model for ‘short GRBs with extended emission’, in which the short GRB is powered by the accretion of the disc on to the NS as described above and the subsequent ‘tail’ is powered by the (wider angle) protomagnetar wind. This model is consistent with the host galaxy demographics, and the lack of a bright SN, associated with short GRBs (e.g. Berger et al. 2005; Hjorth et al. 2005; Bloom et al. 2006; Perley et al. 2009; Berger 2010; Fong, Berger & Fox 2010). Such a possibility is supported by the recent discovery of a $\approx 2 M_\odot$ pulsar by Demorest et al. (2010), which demonstrates that the high-density equation of state is relatively stiff.

7 CONCLUSIONS

In this paper, we take the first steps towards developing the millisecond protomagnetar model into a quantitative theory for GRBs. Using detailed evolutionary models of magnetar spin-down, we explore a wide range of magnetar properties and calculate the prompt emission predicted by magnetic dissipation and internal shock models. Although the picture we construct may not be accurate in all details, it serves as a ‘proof of principle’ that the basic concepts can be constructed into a self-consistent model. Our work also provides a baseline for future improvements, as will be necessitated in particular by advances in our understanding of the origin of the prompt GRB emission.

Several theoretical uncertainties remain that should be addressed with future work. These include a more detailed understanding of the effects of rotation and convection on the cooling evolution of the proto-NS, and the effects of strong magnetic fields on the neutrino-driven mass-loss rate. Although most of our results are at least qualitatively robust to these uncertainties, predictions for the GRB duration (and how it correlates with other observables) are in particular sensitive to the time of neutrino transparency. The mass-loss rate from the proto-NS (and hence the wind magnetization) also depends on the fraction of the magnetosphere open to outflows, which depends on the poorly understood sources of dissipation near the Y-point. Future studies would also be aided by a more detailed understanding of the dependence of the jet properties (e.g. break-out time and opening angle) on the properties of the protomagnetar and the stellar progenitor. The source of the rapid rotation and strong magnetic fields required to produce millisecond magnetars also remains a major uncertainty. However, we note that BH models place similar, if not more extreme, constraints on the progenitor rotation and the large-scale magnetic field of the central engine (e.g. McKinney 2006).

Our primary conclusion is that a surprisingly large fraction of GRB properties can be explained by the magnetar model. These include the following:

(i) Energy. Magnetars with properties in the ‘classical GRB’ regime in Fig. 19 radiate $E_{\gamma} \sim 10^{50} - 10^{52}$ erg during the GRB phase, consistent with the beaming-corrected gamma-ray energies inferred from afterglow modelling. Magnetars with stronger(weaker) magnetic fields and/or shorter(longer) initial periods may produce very luminous (low-luminosity) GRBs.

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12 This implies that although AIC is not accompanied by a bright SN, it may produce a dimmer, ∼day-long transient (Fryer et al. 2009; Metzger et al. 2009; Darbha et al. 2010).

(ii) Lorentz factor. Magnetars in the ‘classical GRB’ regime produce jets with average and instantaneous magnetizations, $\sigma_0 \gtrsim 10^7$–$10^9$ (Fig. 11), which are remarkably similar to the typical Lorentz factors inferred from GRB observations (cf. Fig. 12). The baryon loading of the jet is not fine-tuned or put in by hand, but instead results naturally from the physics of neutrino heating above the protomagnetar surface. This is in contrast to BH models, for which current predictions for $\Gamma$ depend on the uncertain rate at which baryons diffuse into an otherwise clean jet (e.g. Levinson & Eichler 2003; McKinney 2005). The magnetar model predicts that $\sigma_0$ (and probably $\Gamma$) increases monotonically with time during the burst. Among other things, this implies that any thermal emission present will be strongest at early times and will decrease in relative strength as the outflow becomes cleaner with time (Fig. 15).

(iii) Duration. The GRB begins once the jet breaks out of the star and becomes optically thin at the internal shock or magnetic dissipation radius. The GRB ends once the jet magnetization increases sufficiently that jet acceleration and dissipation become ineffective. Because the latter generally occurs when the NS becomes transparent to neutrinos at $t = t_{\text{trans}}$, $\sim 10$–$100$ s (Fig. A1), the magnetar model naturally explains the typical durations of long GRBs.

(iv) Steep-decay phase. The abrupt onset of the high-$\sigma$ transition at $t \approx t_{\text{trans}}$ (Fig. 2) explains why GRB prompt emission decreases rapidly after the prompt emission ends (the ‘steep-decay’ phase; e.g. Tagliaferri et al. 2005).

(v) Association with supernovae. It is natural to associate energetic, MHD-powered SNe with magnetar birth. If the magnetar model is correct, all long GRBs formed from the core collapse of massive stars should be accompanied by an energetic (and possibly hyperenergetic) SN. Magnetars formed via AIC, by contrast, may produce long GRBs not accompanied by a bright SN. This is a promising explanation for the $\sim 100$ s X-ray tails observed following some short GRBs (Section 6.7) and explains why they resemble long GRBs in many of their properties.

(vi) High Lorentz factors $\leftrightarrow$ energetic bursts. The magnetar model predicts a positive correlation (with significant scatter) between the (energy-weighted) average magnetization, $\sigma_{\text{avg}}$, of the jet and the (beaming-corrected) GRB luminosity/energy (Fig. 17). This is consistent with the fact that energetic Fermi bursts appear to have the largest Lorentz factors.

(vii) High radiative efficiency. Both magnetic dissipation and internal shocks may occur in protomagnetar winds, resulting in the prompt high-energy emission. Both models predict maximum radiative efficiencies, $\epsilon_r \sim 30$–$50$ per cent, consistent with the high values of $\epsilon_r$ inferred from afterglow modelling (e.g. Panaitescu & Kumar 2001; Zhang et al. 2007).

(viii) Amati–Yonetoku relation. Our spectral modelling favours magnetic dissipation over internal shocks as the prompt emission mechanism, in part because magnetic dissipation predicts a relatively constant spectral energy peak, $E_{\text{peak}}$, as a function of time (Fig. 16). Strong internal shocks may be suppressed by the residual magnetization of the ejecta or if the toroidal field geometry is not conducive to particle acceleration (e.g. Sironi & Spitkovsky 2011).

In combination with the predicted $\sigma_{\text{avg}} - L_{\nu}$ correlation (Fig. 17), the magnetic dissipation model reproduces both the slope and the normalization of the observed Amati–Yonetoku correlations.

(ix) Late-time emission. Although we expect that prompt internal emission becomes ineffective when $\sigma_\nu$ becomes very large at late times, the plateau X-ray afterglow phase may also be powered by magnetar spin-down, as proposed by previous authors and suggested by Fig. 2. The predicted correlation between the plateau luminosity and duration (Fig. 18) is consistent with the sample of ‘internal’ plateaus studied by L10. Late-time X-ray flaring may be powered by residual rotational or magnetic energy.

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APPENDIX A: PROTOMAGNETAR WIND PROPERTIES

In this appendix, we describe how to calculate the power $E$ (Section A1) and mass-loss rate $\dot{M}$ (Section A2) of protomagnetar winds.

A1 Energy-loss rate $\dot{E}$

The winds from millisecond protomagnetars are accelerated primarily by magnetic forces rather than by thermal pressure. At large radii, the wind power can be divided into components of kinetic energy and magnetic Poynting flux, viz. $\dot{E} = \dot{E}_{\text{kin}} + \dot{E}_{\text{mag}}$.

The kinetic luminosity of the wind is $\dot{E}_{\text{kin}} = (\Gamma_{\infty} - 1)\dot{M}c^2$, where $\Gamma_{\infty} \equiv (1 - v_{\infty}^2/c^2)^{-1/2}$ and $v_{\infty}$ are the asymptotic Lorentz factor and velocity of the outflow, respectively. Non-relativistic outflows have magnetization $\sigma_0 \lesssim 1$ and reach an asymptotic speed $v_{\infty} \approx \sigma_0^{1/3}$, resulting in a kinetic luminosity $\dot{E}_{\text{kin}} \propto (1/2)\dot{M}v_{\infty}^2$ (Lamers & Cassinelli 1999). Relativistic outflows have $\sigma_0 \gtrsim 1$ and achieve $\Gamma_{\infty} \approx \sigma_0^{1/3}$ near the fast magnetosonic surface, beyond which acceleration effectively ceases (Goldreich & Julian 1970). This weak 1/3 power embodies the classical problem that (unconfined, time-stationary) high-$\sigma_0$ winds accelerate inefficiently in ideal MHD (Kennef & Corbini 1984; see Section 4.1 for further discussion).

The magnetosphere is completely open outside the Alfven radius $R_A$. The Poynting flux $\dot{E}_{\text{mag}}$ is thus related to the open magnetic flux $\phi$ and hence to the magnetization $\sigma_0$ (equation 2), via the relationship

$$\dot{E}_{\text{mag}} \simeq \left( \frac{c}{4\pi} |\mathbf{E} \times \mathbf{B}| \right) n \approx 2\left( \frac{\dot{q} B_0^2 r^2}{3} \right) \frac{2\Omega^2}{\Omega^2} \approx 2\left( \frac{\dot{M} c^2 \sigma_0^{2/3}}{3} \right), \quad \sigma_0 \ll 1 \quad \sigma_0 \gg 1, \quad (A1)$$

where $\mathbf{E} = -(\mathbf{v}/c) \times \mathbf{B}$ is the electric field and the factor 2/3 accounts for the angular integration. The equalities in the second line follow because (1) the outflow corotates with the star, such that $v_r \sim \Omega r$ out to radii $\sim R_A$, where $v_r$ is the toroidal velocity; and (2) near $R_A$, the poloidal field becomes a fraction of the fluid’s inertia. The toroidal magnetic field strength $B_0$ thus becomes comparable to the poloidal field $B_0$ at $\mathbf{r} \sim R_A$, such that $\phi \equiv B_0 r^2 \sigma_0 \approx B_0 R_A^2$ (the equality is exact in the case of force-free winds). In writing the third and fourth lines, we have used that

$$R_A = \left\{ \begin{array}{ll} v_{\infty}/\Omega & \sim R_0 \sigma_0^{1/3}, \quad \sigma_0 \ll 1 \\
L = c/\Omega & \sigma_0 \gg 1. \end{array} \right. \quad (A2)$$

The open magnetic flux $\phi$ of a rotating dipole with the surface magnetic field strength $B_{\text{dip}}$ is given by

$$\phi \simeq f_{\text{open}} B_{\text{dip}} R_\text{in}^2, \quad (A3)$$

where

$$f_{\text{open}} \simeq (1/2\pi) \int_0^{2\pi} \! \mathbf{d} \phi \int_0^{\theta_{\text{open}}/2} \! \sin \theta \, d\theta$$

$$\approx 1 - \cos \left( \frac{\theta_{\text{open}}}{2} \right) \frac{\sigma_0^{1/3}}{R_0 \gg R_A} \approx (1 + \sin^2 \chi)^{1/2} \frac{R_{\text{in}}}{2R_Y}, \quad (A4)$$

is the fraction of the NS surface threaded by an open field. Here $\theta_{\text{open}} \simeq 2\sin^{-1}(R_{\text{in}}/R_Y)^{1/2}$ is the opening angle of the polar cap, corrected by a factor $(1 + \sin^2 \chi)^{1/2}$ to account for the larger cap size of an oblique rotator (e.g. Cheng, Ruderman & Zhang 2000; Bai & Spitkovsky 2010); $R_{\text{in}}$ is the NS radius; $R_Y$ is the ‘Y’ point radius where the close zone ends in the magnetic equatorial plane; and the second equality holds in the small-cap limit $\theta_{\text{open}} \ll 1$ ($R_Y \gg R_{\text{in}}$) (see Fig. 3 for an illustration of the relevant geometry).

Just after core bounce, thermal pressure may dominate above the NS surface and the entire magnetosphere may open into a ‘split-monopole’ configuration with $f_{\text{open}} \sim 1$. As the NS contracts, cools and spins up, its magnetic field, however, is amplified and magnetic pressure eventually comes to dominate. This produces a ‘closed’ or ‘dead’ zone at low magnetic latitudes from which a steady-state wind cannot escape (i.e. $R_Y > R_{\text{in}}$). In the limit of a force-free wind ($\sigma_0 \gg 1$), the radius of the Y-point likely extends close to the radius of the light cylinder, but in general $R_Y \ll R_{\text{in}}$ for less-magnetized (finite-$\sigma_0$) winds. Following Metzger et al. (2007), we assume that $R_Y/R_{\text{in}} = \min[0.3\sigma_0^{1/3}, 1]$ for $R_Y > R_{\text{in}}$, based on an empirical fit to the axisymmetric relativistic MHD simulations of Bucciantini et al. (2006), which span the non-relativistic to relativistic transition. The values of $R_Y$ that we adopt are similar to those we estimate by applying the toy model of Mesetel & Spruit (1987) to the protomagnetar context. Determining the detailed time-dependence of $R_Y$ will, however, ultimately require incorporating a self-consistent, physical model for the resistivity in the magnetosphere and equatorial current sheet.

Combining our results, the total wind power is given by

$$\dot{E} = \dot{E}_{\text{mag}} + \dot{E}_{\text{kin}} \simeq \begin{cases} \dot{M} c^2 \sigma_0^{2/3}, & \sigma_0 \ll 1 \\
(2/3)\dot{M} c^2 \sigma_0, & \sigma_0 \gg 1 \end{cases}, \quad (A5)$$

where the magnetization (equation 2) can now be written as

$$\sigma_0 \simeq \frac{B_{\text{dip}} R_\text{in}^2 \Omega^2 f_{\text{open}}^2}{\dot{M} c^3} \simeq \frac{B_{\text{dip}}^2 R_\text{in}^4 \Omega^4 (1 + \sin^2 \chi)}{4 \dot{M} c^3} \left( \frac{R_Y}{R_{\text{in}}} \right)^{-2}. \quad (A6)$$

Note that in the non-relativistic case, the kinetic and magnetic contributions to the total power are similar ($\dot{E}_{\text{mag}} \approx 2\dot{E}_{\text{kin}}$), while in the relativistic case, the outflow is Poynting dominated since $\dot{E}_{\text{mag}}/\dot{E}_{\text{kin}} \sim \sigma_0^{2/3} \gg 1$.

A2 Mass-loss rate $\dot{M}$

Mass-loss from the proto-NS results from neutrino heating in the atmosphere just above the NS surface. The dominant heating and cooling processes are the charged-current reactions:

$$\nu_e + n \leftrightarrow e^- + p \quad \text{and} \quad \bar{\nu}_e + p \leftrightarrow e^+ + n. \quad (A7)$$

For unmagnetized winds, the mass-loss rate is well approximated by the analytic expression (Qian & Woosley 1996):

$$\dot{M}_v = 5 \times 10^{-5} M_\odot s^{-1} \left( \frac{L_v}{10^{52} \text{erg s}^{-1}} \right)^{5/3}$$

$$\times \left( \frac{\epsilon_v}{10 \text{MeV}} \right)^{10/3} \left( \frac{M_\odot}{1.4 M_\odot} \right)^{-2} \left( \frac{R_{\text{in}}}{10 \text{km}} \right)^{5/3} (1 + \epsilon_{\text{vol}})^{5/3}. \quad (A8)$$

Although both electron neutrinos and antineutrinos contribute to the heating, for simplicity, we combine their contributions into a single product of the neutrino luminosity $L_v$ and mean energy $\epsilon_v$ defined by

$$L_v \epsilon_v \equiv L_{\nu_e} |\nu_e|^2 + L_{\bar{\nu}_e} |\bar{\nu}_e|^2, \quad (A9)$$
where the [...] represent an appropriate average over the neutrino absorption cross-sections. The normalization adopted in equation (A8) includes both this averaging and a general relativistic correction (Thompson, Burrows & Meyer 2001). The parameter

\[
\epsilon_{\alpha} = 0.2 \left( \frac{M_{\text{in}}}{1.4 M_\odot} \right) \left( \frac{R_{\text{in}}}{10 \text{ km}} \right)^{-1} \left( \frac{\epsilon_v}{10 \text{ MeV}} \right)^{-1}
\]

is a correction \( \leq 1 \) for the additional heating due to inelastic electron scattering (see Qian & Woosley 1996, equation 50).

In most calculations, we use \( L_\nu(t), \epsilon_v(t) \) and \( R_{\text{in}}(t) \) from Pons et al. (1999) (hereinafter P99) who calculate the deleptonization and cooling evolution of non-rotating proto-NSs (cf. Burrows & Lattimer 1986). Examples of \( L_\nu(t), \epsilon_v(t) \) and \( R_{\text{in}}(t) \) are shown in Fig. A1 for different NS masses. Note that for \( t \gtrsim 1 \text{ s} \), \( L_\nu \) and \( \epsilon_v \) decrease relatively gradually as a power law until a time \( t_{\text{in-\text{thin}} \sim 10-60 \text{ s}} \), after which \( L_\nu \) and \( \epsilon_v \) plummet as the proto-NS becomes transparent to neutrinos. As we have shown in Section 3, \( t_{\text{in-\text{thin}}} \) determines the GRB duration in the protomagnetar model.

Since \( M_\text{in} \) depends sensitively on \( L_\nu \) and \( \epsilon_v \), we briefly discuss the limitations and the uncertainties in the calculations of P99. First, although portions of the proto-NS are convectively unstable during its early cooling evolution (Burrows & Fryxell 1993; Keil, Janka & Mueller 1996), convective transport is not accounted for by P99. The primary effect of convection is to increase the cooling rate and hence to speed up the temporal evolution of the neutrino luminosity (L. Roberts, private communication). P99 find that the rate at which \( L_\nu \) and \( \epsilon_v \) decrease at late times, and hence the precise value of \( t_{\text{in-\text{thin}}} \), also depends sensitively on the high-density equation of state, which is uncertain. In order to explore the sensitivity of our results to uncertainties in \( L_\nu \) and \( \epsilon_v \), we also calculate models using neutrino luminosities and energies from the recent proto-NS cooling calculations of Hudepohl et al. (2010) (hereinafter H10) (L. Roberts, private communication), which follow a successful electron-capture SN (Kitaura, Janka & Hillebrandt 2006). This calculation, which includes improvements in the neutrino opacities over previous work, is shown for comparison in Fig. A1. The primary difference between the cooling curves of P99 and H10 is the significantly faster late-time evolution found by H10.

Finally, neither P99 or H10 include the effects of magnetic fields or rotation, yet this paper focuses on protomagnetars rotating at a significant fraction of their break-up speed. Rapid rotation decreases the interior temperature of the NS, which slows its cooling evolution. Using one-dimensional rotating core-collapse calculations, Thompson et al. (2005) find that \( L_\nu \) and \( \epsilon_v \) are reduced by factors of \( \sim 0.5 \) and \( \sim 0.8 \), respectively, in their fastest rotating model at \( t \approx 600 \text{ ms} \) following core bounce, compared to an otherwise equivalent non-rotating case. Ideally, the effects of rotation on \( L_\nu \) and \( \epsilon_v \) should be calculated self-consistently. Lacking such a model, however, we account for rotational effects qualitatively by introducing a ‘stretch’ parameter \( \eta_1 \), which modifies the cooling evolution from the non-rotating case \( (\Omega = 0) \) as follows:

\[
L_\nu \rightarrow L_\nu |\eta_1|\epsilon_1^{1/4}, \quad t \rightarrow t|\eta_1|\epsilon_1^{1/4}, \quad \epsilon_v \rightarrow \epsilon_v|\eta_1|\epsilon_1^{1/4},
\]

where a value \( \eta_1 \sim \frac{\epsilon_1}{\epsilon_\nu} \) is motivated by the calculations of Thompson et al. (2005) for millisecond rotators. Note that this simple parametrization preserves the total energy radiated in electron neutrinos and increases the time of neutrino transparency \( t_{\text{in\text{-}thin}} \propto \eta_1 \). Although we expect \( \eta_1 \) to be an increasing function of \( \Omega \), in our calculations, we fix \( \eta_1 = 3 \) for the lack of a predictive model. We also neglect differences in the neutrino radiation field with latitude caused by rapid rotation (e.g. Brandt et al. 2011), which if properly included would impart the total wind mass-loss rate with an additional dependence on the magnetic obliquity \( \chi \).

A strong magnetic field modifies \( M \) from the standard expression in equation (A8) in three ways. First, \( M \) is reduced by a factor of \( f_{\text{open}} \) (equation A4) since only the open fraction of the surface contributes to the outflow. Secondly, \( M \) is enhanced by a factor of \( f_{\text{cent}} \) due to centrifugal ‘slinging.’ This occurs when rotation is sufficiently rapid and the magnetic field is sufficiently strong that centrifugal forces increase the scaleheight in the heating region (Thompson et al. 2004). By fitting the numerical results of Metzger et al. (2008b), we find that the maximum centrifugal enhancement to \( M \) (obtained in the strong-field limit of strict corotation described below) is well approximated by the functional form (for \( P \gtrsim 1 \text{ ms} \)):

\[
f_{\text{cent, max}} \approx \exp[(P_*/P)^\beta]
\]

for a value \( \beta \approx 1.5, \) where

\[
P_* \simeq 2.1 \sin \alpha \left( \frac{R_{\text{in}}}{10 \text{ km}} \right)^{3/2} \left( \frac{M_{\text{in}}}{1.4 M_\odot} \right)^{-1/2} \text{ ms},
\]

where \( \alpha \approx \max[\beta_{\text{open}}/2, \chi] \) is a typical angle from the rotational axis sampled by the open zone. The normalization we adopt for
$P_c$ is determined by fitting the numerical results of Metzger et al. (2008b), which were calculated for equatorial field lines ($\alpha = \pi/2$). The scaling of $P_c$ with mass, radius and $\alpha$, however, are chosen based on the theoretical expectation that $P_c \propto R^5_c / c_s$ (Thompson et al. 2004), where $R_c \sim R_\text{in} \sin \alpha$ and $c_s$ are the centrifugal ‘lever arm’ and sound speed in the gain region, respectively. The latter is proportional to the NS escape speed $\propto (M/R_{\text{in}})^{1/2}$ (see Qian & Woosley 1996, equation 45).

Although $f_{\text{cent, max}}$ is the maximum enhancement of $M$, it obtains only if the magnetic field is sufficiently strong that the outflow corotates with the star to a location outside the sonic radius $R_\text{s}$. This requires $R_\text{s} \gtrsim R_\text{in} = (GM^2 / \Omega^4)^{1/3}$ (Lamers & Cassinelli 1999).

Using the numerical results of Metzger et al. (2008b), we find that a satisfactory interpolation of the mass-loss enhancement between the centrifugal ($R_\text{s} \gg R_\text{in}$) and non-centrifugal ($R_\text{s} \ll R_\text{in}$) regimes is given by $f_{\text{cent}} = f_{\text{cent, max}} (1 - \exp [-R_\text{s} / R_\text{in}]) + \exp [-R_\text{s} / R_\text{in}]$. Note that relativistic outflows are necessarily in the centrifugal regime because for $\sigma \equiv 1$, $R_\text{s} \sim R_\text{in} = c / \Omega > R_\text{in}$ (equation A2).

Finally, a strong magnetic field changes $M_c$ (equation A8) by altering the neutrino heating and cooling rates in the proto-NS atmosphere (e.g. Duan & Qian 2004; Riquelme et al. 2005). The most important effect is that the electron and positron participating in the charged-particle reactions (equation A7) are restricted into discrete Landau levels (Duan & Qian 2004). In this paper, we neglect these effects because we estimate that the corrections to $M_c$ are relatively minor for surface field strengths $B_{\text{surf}} \lesssim 3 \times 10^{16}$ G. However, more detailed future work should address the dependence of $M_c$ on $B_{\text{surf}}$.

Once the NS becomes transparent to neutrinos at late times, $L_\nu$ and $\epsilon_\nu$ decrease rapidly (Fig. A1) and the mass-loss rate decreases abruptly. Neutrino heating only determines $\dot{M}$ so long as the magnetosphere is sufficiently dense that vacuum electric fields do not develop. This assumption breaks down, however, once $\dot{M}$ decreases to near the critical Goldreich & Julian (1969) (hereinafter GJ69) mass-loss rate. However, because in actual pulsar winds $\dot{M}$ exceeds the GJ69 value, we instead assume that the minimum mass-loss rate in the pair-dominated regime is given by a multiple of the GJ69 rate, namely

$$M_{\text{GJ}} \equiv \mu_{-+} m_e \frac{1}{e} \sim 3 \times 10^{-15} M_\odot \; s^{-1} \times \left( \frac{H_{\text{surf}}}{10^7} \right) \left( \frac{B_{\text{surf}}}{10^{15} \text{G}} \right) \left( \frac{P}{\text{ms}} \right)^{-2} \left( \frac{R_\text{s}}{10 \text{km}} \right)^3,$$

where $I \equiv 4\pi R_\text{GJ}^2 \rho_\text{GJ} / c$, $m_e$ and $e$ are the electron mass and charge, and $\rho_\text{GJ} \approx (\Omega B/2\pi c)$ is the GJ69 charge density, evaluated at the light cylinder. The multiplicity $\mu_{-+}$ of positrons/electrons produced by magnetospheric acceleration is uncertain, especially in the case of magnetars (Thompson 2008). Lacking a predictive model, in our calculations, we fix the multiplicity at a value $\mu_{-+} = 10^8$ which is consistent with estimates based on detailed synchrotron emission models of pulsar wind nebulae (e.g. Bucciantini, Arons & Amato 2011). Although the late-time wind magnetization depends sensitively on the multiplicity ($\sigma_{\mu} |_{\mu_{-+}} \propto 1 / \mu_{-+}$), most of our conclusions regarding late-time emission (Section 5) are insensitive to this choice.

13 Note that $\dot{M}_c$ depends most sensitively on the heating and cooling rates in the gain region, which is typically located a few kilometres above the NS surface, where the magnetic field strength $B_{\text{surf}} \sim 10^{16}$ G is a factor of a few weaker than the surface field strength.
We now consider how the jet and shell interact in greater detail in order to evaluate the peak energy of the resulting synchrotron emission. In the frame of the bulk shell (hereinafter denoted by a tilde), the jet velocity and Lorentz factor are, respectively, given by

$$\tilde{\beta}_j \approx 1 - \Gamma_j^2 / \tilde{\Gamma}_j^2$$

and

$$\tilde{\Gamma}_j \approx (1 - \tilde{\beta}_j^2)^{-1/2} \Gamma_j / \tilde{\Gamma}_s.$$  (B8)

Note that unlike in the standard internal shock scenario, the relative Lorentz factor between the shells $\tilde{\Gamma}_j$ is $\gg 1$ at late times when $\Gamma_j \gg \Gamma_s$. This allows for high radiative efficiency.

If the shocked gas is relativistically hot, the (rest-frame) post-shock number and energy densities are, respectively, given by (Blandford & McKee 1976):

$$n_{sh} = (4\tilde{\Gamma}_j + 3)n_j \quad \text{and} \quad \epsilon_{sh} = (\tilde{\Gamma}_j - 1)n_{sh}m_pc^2,$$  (B9)

where

$$n_j = \frac{L_{j,iso}}{4\pi \tilde{\Gamma}_j^2 R_{sh}^2 m_pc^3}$$

(B10)

is the pre-shock density and $L_{j,iso} \equiv L_j f_b^{-1}$.

If a fraction $\epsilon_e$ and $\epsilon_b$ of the post-shock energy is partitioned into electron kinetic and magnetic energy, respectively, the resulting (electron) random Lorentz factor and magnetic field strength are, respectively, given by

$$\tilde{\gamma}_e = \frac{\epsilon_em_p(\tilde{\Gamma}_j - 1)}{\zeta_em_c} + 1 \approx \frac{\epsilon_em_p\Gamma_j}{2\zeta_em_c\Gamma_s}$$

and

$$\tilde{B} = \frac{(8\pi\epsilon_b\tilde{\epsilon}_b)^{1/2}}{(8\pi\epsilon_b\tilde{\epsilon}_b)^{1/2}} \approx \frac{2\epsilon_bL_{j,iso}}{\Gamma_j^2 R_{sh}^2 c} \frac{\epsilon_b^{1/2} L_{j,iso}^{1/2}}{\Gamma_j^{3/2} \Gamma_s^{1/2} \epsilon_c^{3/2} \tilde{\Gamma}_s}.$$  (B11)

$$B = \frac{(8\pi\epsilon_b\tilde{\epsilon}_b)^{1/2}}{(8\pi\epsilon_b\tilde{\epsilon}_b)^{1/2}} \approx \frac{2\epsilon_bL_{j,iso}}{\Gamma_j^2 R_{sh}^2 c} \frac{\epsilon_b^{1/2} L_{j,iso}^{1/2}}{\Gamma_j^{3/2} \Gamma_s^{1/2} \epsilon_c^{3/2} \tilde{\Gamma}_s}.$$  (B12)

where $\zeta_e$ is the fraction of electrons accelerated.

The peak synchrotron photon energy as seen by the observer is then

$$E_{\text{peak,iso}} \approx \frac{eB\hbar\tilde{\gamma}_e^2 \tilde{\Gamma}_s}{m_ec} \approx \frac{4.7\text{ MeV} \epsilon_e^{1/2} \epsilon_b^{-1}}{10^5 \text{ erg s}^{-1}} \times \left( \frac{L_{j,iso}}{10^{51} \text{ erg s}^{-1}} \right)^{1/2} \left( \frac{t_j}{10^5 \text{ s}} \right)^{-1} \Gamma_j^2 \Gamma_s^{-4}.$$  (B13)

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