Simulated evolution of the dark matter large-scale structure of the Universe

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ABSTRACT
We analyse the evolution of the basic properties of the simulated elements of the large-scale structure of the Universe (LSS) formed by dark matter (DM), and we confront it with the observed evolution of the Lyman α forest. In three high-resolution simulations, we have selected samples of compact DM clouds of moderate overdensity. Clouds are selected at redshifts 0 \( \leq z \leq 3 \) with the minimal spanning tree technique. The main properties of the clouds selected in this way are analysed in three-dimensional space and with the core-sampling approach. This allows us to compare estimates of the DM LSS evolution obtained with two different techniques, and to clarify some important aspects of the LSS evolution. In both cases, we find that regular redshift variations of the mean characteristics of the DM LSS are accompanied only by small variations of their probability distribution functions (PDFs), which indicates the self-similar characteristic of the DM LSS evolution. A high degree of relaxation of DM particles, compressed within the LSS, is found along the shortest principal axis of the clouds. We see that the internal structure of the selected clouds depends upon the mass resolution and scale of the perturbations achieved in the simulations. It is found that the low-mass tail of the PDFs of the LSS characteristics depends upon the procedure of cloud selection.

Key words: methods: statistical – quasars: absorption lines – quasars: general – large-scale structure of Universe.

1 INTRODUCTION
At \( z \leq 1.5 \), the large-scale structure of the Universe (LSS) is observed in many galaxy catalogues, such as 2dFGRS, SDSS, DEEP2 and VIRMOS (Percival et al. 2001; Abazajian et al. 2003; Davis et al. 2003; Verde et al. 2003; Le Fevre et al. 2005). It is manifested as a strong galaxy concentration within rich walls and filaments surrounding regions with a low density of galaxies (voids). At redshifts \( z \geq 2 \), the LSS is observed as the Lyman α (Lyα) forest and rare metal systems in high-resolution spectra of the farthest quasars. The observed properties of these populations of the LSS elements are different, but now it is commonly believed that both represent different manifestations of the same structure formed by dark matter (DM) and baryonic components. This suggests that Lyα absorbers can be associated with low-mass structure elements formed by non-luminous baryonic and DM components, while galactic walls and filaments represent a fraction of the richer LSS elements. This means that the DM structure traced by the Lyα absorbers is qualitatively similar to the rescaled structure observed in the spatial distribution of galaxies. In particular, this inference is consistent with observations of absorbers in the vicinity of galactic filaments, and even galaxies within voids (Morris et al. 1993a,b; Penton, Shull & Stocke 2000; McLin et al. 2002; Penton, Stocke & Shull 2002; Williger et al. 2010).

The facilities of both observational approaches are limited, but they are complementary to each other. Thus, the analysis of the galaxy surveys is focused on the richer LSS elements in a limited range of redshifts \( z \leq 1–1.5 \). This allows us to determine the main characteristics of such elements (see, for example, Doroshkevich et al. 2004), but we cannot trace the LSS evolution for large redshift intervals. This analysis is concentrated on the investigations of galactic properties and their environmental dependence. For example, it allows us to test the star formation histories (see, for example, Panter et al. 2007; Skibba et al. 2009) and correlations between orientations of the LSS elements and the angular momentum of galaxies (see, for example, Trujillo, Carretero & Patiri 2006; Aragon-Calvo et al. 2007; Paz, Stasyszyn & Padilla 2008; Slosar et al. 2009; Jimenez et al. 2010).

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However, observations of the Lyα forest at $z \sim 2$–4 allow us to trace some characteristics of the low-mass LSS elements and their evolution. However, these rely on the observed evolution of neutral hydrogen, which can be caused by many factors. Thus, in addition to the actual evolution of the DM components of the Lyα absorbers, the observed fraction of neutral hydrogen depends upon poorly known variations of the ultraviolet (UV) background. In spite of this, it is clear that there is a close link between observed Lyα absorbers and DM clouds formed at high redshifts.

It seems that the close link between the Lyα forest and the galaxy LSS can be established by numerical simulations of the structure evolution. However, it is not yet possible to simulate the LSS evolution in a wide range of scales. Thus, simulations performed within large boxes with a moderate spatial and mass resolution (for reviews, see Frenk 2002; Springel, Frenk & White 2006) reproduce reliably only the formation of the richer DM LSS elements similar to those observed in the galaxy distribution. However, high-resolution simulations performed with small box sizes can reproduce some characteristics of the Lyα forest (see, for example, Weinberg et al. 1999; Zhang et al. 1998; Davé et al. 1999; Theuns et al. 1999; Theuns, Schaye & Haehnelt 2000; Schaye 2001; Meiksin, Bryan & Machacek 2001), but obviously they cannot simulate the formation of the richer LSS elements. Perhaps more progress can be achieved with simulations performed with variable resolution (see, for example, Springel et al. 2008; Diemand et al. 2008).

The technical limitations of the box size and resolution lead to small- and large-scale cut-offs of the simulated power spectrum. These cut-offs restrict the potential of such simulations and do not allow us to reproduce reasonably well the observed characteristics of the Lyα forest (see, for example, Meiksin et al. 2001; Gnedin & Hamilton 2002; Demiański & Doroshkevich 2003; Manning 2003a,b; Seljak, McDonald & Makarov 2003; Tegmark & Zaldarriaga 2003). Now, such simulations are used mostly for the surprisingly stable reconstruction of the small-scale initial power spectrum from characteristics of the forest (see, for example, Croft et al. 2001, 2002; Viel et al. 2004a; Viel, Haehnelt & Springel 2004b; McDonald et al. 2005; Seljak et al. 2005). An alternative approach is to focus mainly on the regular trends in the evolution of the simulated LSS, and to compare these with observations of the forest properties in a wide range of redshifts. Such trends can be revealed with special methods applied to representative high-resolution simulations. The successful investigation of the process of halo formation (see, for example, Navarro, Frenk & White 1995, 1996, 1997; Bullock et al. 2001; Tahtisomi et al. 2004) demonstrates the high potential of this approach.

A statistical analysis of a large sample of the Lyα absorbers (Demiański, Doroshkevich & Turchaninov 2006; Demiański & Doroshkevich 2010) reveals some unexpected features of the forest evolution. The most important are the regular redshift variations of the mean observed characteristics of the Lyα absorbers and the surprisingly weak variations of their probability distribution functions (PDFs). These results demonstrate the self-similar characteristic of the evolution of the observed LSS, which must be tested with numerical simulations.

Other important problems, still open to debate, are connected with the internal structure of the Lyα absorbers and its evolution with time. Also included are an estimate of the degree of relaxation of matter accumulated by absorbers with a variety of richness, and the link between the thermal and large-scale bulk motions of the gas and the observed Doppler parameters of the forest. At the present time, a reliable observational discrimination between contributions of these factors is problematic. We can also note the surprisingly weak redshift evolution of the observed mean Doppler parameter and a complex shape of its PDF, indicating the existence of a rich sample of DM clouds with small Doppler parameter $b \lesssim (b)$.

A comparison of the observed evolution of the forest with the simulated evolution of the DM clouds allows us to clarify how they are related. For this purpose, in this paper, we investigate the evolution of the DM LSS in high-resolution simulations, using the minimal spanning tree (MST) and core-sampling approaches (see, for example, Doroshkevich et al. 2004). The space and force resolutions achieved in these simulations allow us to characterize the evolution of the compact DM LSS elements with moderate richness, which is itself an important problem. Thus, we reveal significant differences in the properties and evolution rates of the LSS elements selected in simulations with different mass and space resolutions. These results demonstrate the strong influence of small-scale perturbations on the internal structure of the LSS elements.

All selected DM elements are significantly larger than those observed as the Lyα forest, which does not allow us to perform a direct quantitative comparison of the observed and simulated clouds. However, the basic properties of the selected sample of DM clouds are found to be similar to the properties of the observed Lyα forest. In particular, we confirm the self-similar characteristic of the evolution of the DM LSS, we reproduce the redshift dependence and the PDF of the Doppler parameter, and we estimate the cloud rotation, the contribution of macroscopic turbulent motions and the degree of relaxation of compressed matter measured along the principal axes of the clouds. The properties of the LSS elements are found to be qualitatively consistent with the predictions of the Zel'dovich approximation (Zel'dovich 1970; Shandarin & Zel’dovich 1989; Demiański & Doroshkevich 1999, 2004).

This paper is organized as follows. In Section 2, we briefly present the simulations that we use and the methods used for their analysis. In Section 3, we describe the evolution of the basic characteristics of the DM LSS. The results of the core-sampling analysis are given in Section 4. A discussion of obtained results and conclusions can be found in Section 5.

2 NUMERICAL SIMULATIONS

In this paper, we investigate the process of formation and evolution of the DM LSS within the concordance cosmological model:

$$\Omega_\Lambda = 0.7, \quad \Omega_m = 0.3, \quad h = 0.7, \quad \sigma_8 = 0.9, \quad n = 1.$$ 

Here, $\Omega_\Lambda$ and $\Omega_m$ denote the dimensionless densities of dark energy and matter, $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$ is the dimensionless Hubble parameter, $\sigma_8$ is the amplitude of density perturbations and $n$ is the power index of the spectrum of perturbations.

One simulation (hereafter referred to as $S_{\text{iso}}$) was performed with an mpi version of the adaptive refinement tree code (Kravtsov et al. 1997) within a box of $L_{\text{box}} = 150 h^{-1}$ Mpc with 256$^3$ particles, a cell size $\approx 0.6 h^{-1}$ Mpc, and mass and force resolutions of $1.7 \times 10^{10}$ $h^{-1}$ M$_\odot$ and $18 h^{-1}$ kpc, respectively (see Wojtak et al. 2005 for more details). We have analysed the simulated DM distribution at four redshifts (i.e. at $z = 0, 1, 2$ and 3), which roughly covers the interval of observed redshifts.

For comparison, we have also used the DM distribution obtained in the MareNostrum universe (Gottlöber & Yepes 2007). This non-radiative smoothed particle hydrodynamics (SPH) simulation was performed with the GADGET2 code (Springel 2005) using the concordance model. This simulation (hereafter referred to as $S_{\text{iso}}$) consists of 1024$^3$ dark and 1024$^3$ gas particles in a box of $L_{\text{box}} = 500 h^{-1}$ Mpc on a side. Both the mass and force resolutions ($0.75 \times 10^{10}$ $h^{-1}$ M$_\odot$...
and $15 \, h^{-1}$ comoving kpc) are comparable to those achieved in the previous simulation. Such a comparison allows us to check the impact of the baryonic component, the code used and the initial realization on the properties of the simulated LSS. As a rule, the properties of clouds selected in this simulation are close to those obtained for the simulation $S_{150}$, which demonstrates the weak impact of these factors on the characteristics of the LSS.

The third simulation (hereafter referred to as $S_{90}$) consists of $512^3$ DM particles in a box of $L_{\text{box}} = 50 \, h^{-1}$ Mpc on a side. This was performed at the Astrophysical Institute Potsdam by M. Steinmetz. With such parameters, the obtained mass resolution is $7.8 \times 10^4 \, h^{-1} M_\odot$ and the force resolution is $3 \, h^{-1}$ comoving kpc, which is better than the resolutions achieved in the previous simulations. However, in this case, the important influence of large-scale perturbations is partly suppressed. The comparison of results obtained for these three simulations reveals the impact of small-scale perturbations on the properties of the LSS. In particular, in this simulation we see the more complex internal structure of the selected LSS elements and the earlier formation of high-density clouds. Also, we can trace the initial steps of the LSS formation up to redshifts $z \sim 12–14$.

2.1 Selection of high-density clouds

Using the MST code, as described in Doroshkevich et al. (2004), we have selected from the full distribution of DM particles a set of clouds with three threshold parameters, which restrict their density and richness. These are the threshold overdensity, $\delta_{\text{thr}}$, and the minimal and maximal richness, $N_{\text{min}}$ and $N_{\text{max}}$, of selected clouds:

$$
\delta_{\text{thr}} = (4\pi\Omega_\Lambda^2 (n_\rho)/3)^{-1}, \quad N_{\text{min}} \leq N_\rho \leq N_{\text{max}},
$$

$$
\langle \rho \rangle = (5, 8.6, 10^3) h^2 \, M_\odot \, \text{Mpc}^{-3}, \quad \langle n_\rho \rangle \approx 8 \times 10^{10} h^2 M_\odot \, \text{Mpc}^{-3}.
$$

Here, $\ell_{\text{thr}}$ is the maximal length of the edge of the tree within a selected cloud, and $\langle \rho \rangle$ and $\langle n_\rho \rangle$ are the mean comoving density and number density of DM particles in the simulations. Using these parameters, we can reliably identify clouds even of a complex irregular shape. Using the definition of the MST, the distances between all neighbouring points inside the cloud are less than the threshold distance, $\ell \leq \ell_{\text{thr}}$. Therefore, $\delta = \rho/\langle \rho \rangle \geq \delta_{\text{thr}}$. In other words, the selected clouds are bound by the surfaces of constant overdensity, $\delta = \delta_{\text{thr}}$.

In this paper, we are mainly interested in the mildly non-linear evolution of the LSS elements with moderate overdensity. Such elements dominate at the early period of the LSS formation, and we can expect to observe these as the Lyman-$\alpha$ forest. Hence, the threshold overdensities that we adopt in this paper are not very high: $\delta_{\text{thr}} = 1$ and $\delta_{\text{thr}} = 1.76$.

The other important threshold parameters are the minimal and maximal richness of the selected clouds. The first ensures the reliability and stability of the characteristics of the selected clouds. The second allows us to exclude from the analysis amorphous multi-connected clouds formed at later stages of the LSS evolution. Such extremely rich clouds appear as a result of the integration of less rich clouds, and they represent the elements of the well-known network of the LSS. Evidently, such clouds cannot be described by just a few local characteristics.

For these simulations, we use the following minimal and maximal richness of the clouds under investigation:

$$
N_{\text{min}} = 30, \quad N_{\text{max}} = 5000, \quad \text{for } S_{150},
$$

$$
N_{\text{min}} = 6.5 \times 10^3, \quad N_{\text{max}} = 10^6, \quad \text{for } S_{90}.
$$

This approximately corresponds to the same range of cloud masses:

$$
M_{\text{min}} \approx 5 \times 10^{11} h^{-1} M_\odot, \quad M_{\text{max}} \approx 8 \times 10^{13} h^{-1} M_\odot.
$$

Later, we refer to the samples selected with $M_{\text{min}} \leq M_\rho \leq M_{\text{max}}$ as samples of compact clouds. For some tests, we also use samples with $M_\rho > M_{\text{min}}$ (a full sample) and $M_\rho > M_{\text{max}}$ (a sample of rich clouds).

The selection of clouds with the core-sampling approach is described in Section 4.

2.2 Characteristics of clouds

In this paper, clouds are characterized by their principal axes, determined by their inertia tensor. Such a rough description allows us to take into account the global shape of the clouds and to estimate their degree of filamentarity and sheetness without the introduction of any additional parameters.

Sometimes, much more refined techniques are used for selection and discrimination of the DM filaments and walls (see, for example, discussions in Doroshkevich et al. 2004; Aragon-Calvo et al. 2007; Zhang et al. 2009). However, the advantages of such an approach are problematic because of the very complex shapes, many branches and strong non-homogeneity of the matter distribution, typical for both observed and simulated LSS elements. In contrast to our approach, such refined methods are inevitably multiparametric, and therefore the description of clouds selected in this way becomes more complicated. Such an approach can be efficient in providing a more accurate description of the local environment of considered objects.

Here, we consider several global characteristics of clouds: their mass (richness) and velocity ($M_\rho$ and $U$, respectively), the three components of their angular momenta $J_i$ and the comoving sizes along the three principal axes, $L_i \geq L_j \geq L_k$, determined through their inertia tensors.

As is well known (see, for example, the review by Schäfer 2009) the angular momenta of the clouds are mainly generated by the tidal interactions with the surrounding medium – tidal torque theory (TTT; Peebles 1969; Doroshkevich 1970; White 1984). The angular momentum of a cloud depends upon its (random) shape. The mean angular momentum of clouds is small, $\langle J_i \rangle \sim 0$, because of the random orientation, and $\langle J_i^2 \rangle$ provides more stable characteristics of clouds. For a regular ellipsoidal volume with half-axes $a_1 \geq a_2 \geq a_3$, the Zel’dovich theory gives

$$
\langle J_i \rangle = 0, \quad \langle J_i^2 \rangle \propto \langle a_i^2 - a_j^2 \rangle^2, \quad i \neq j \neq k.
$$

Thus, $\langle J_i^2 \rangle \geq \langle J_i^2 \rangle$ (Demiański & Doroshkevich 2004).

In simulations, the angular momenta of clouds and the contribution of bulk (turbulent) motions are characterized by the functions:

$$
J_i = N_p^{-1} \sum_{n=1}^{N_p} \psi_i, \quad \psi_i = \epsilon_{ikl}(\mathbf{v}_m - \mathbf{v}_c)(\mathbf{x}_m - \mathbf{x}_c).
$$

Here, $\epsilon_{ikl}$ is the totally antisymmetric unit tensor, $N_p$ is the number of particles in the cloud and $\mathbf{x}_m$ and $\mathbf{v}_m$ and $\mathbf{x}_c$ and $\mathbf{v}_c$ are the coordinates and velocities of the $m$th particle and the centre of mass, respectively.

To characterize the internal dynamics of matter accumulated by a cloud and its degree of relaxation along the principal axes, we also...
consider three velocity dispersions, $\sigma_i$, and three angular momenta of particles, $j_i$:

$$j_i = N_p^{-1} \sum_{m=1}^{N_p} |\psi_i|.$$  \hspace{1cm} (6)

For a regular ellipsoidal volume with half-axes $a_1 \geq a_2 \geq a_3$, the Zel’dovich theory gives, in contrast to equation (4),

$$\langle j_i^2 \rangle \propto (a_i^2 + a_j^2)^2, \quad i \neq k \neq l.$$ \hspace{1cm} (7)

As is well known, within anisotropic clouds of collisionless particles, the relaxation occurs independently along each principal axis of the cloud. For example, the observed galactic walls and filaments are almost relaxed along their shortest axes but have no time to relax along their longest axis. Thus, here we characterize the degree of relaxation of $N_p$ particles compressed within a cloud by the parameters:

$$w_i = 1 + \frac{1}{N_p} \sum_{m=1}^{N_p} (v_m - v_i), (x_m - x_i), (|x_m - x_i|).$$ \hspace{1cm} (8)

Here, $\sigma_i$ is the velocity dispersion and the index $i$ shows that we consider the relaxation along the $i$th principal axis.

Evidently, the functions $w_i$ discriminate between contributions of random and regular motions. Indeed, for a cloud with relaxed matter distribution and random velocities of particles, we can expect that $|w_i - 1| \ll 1$. This is because, for each particle, the probabilities of finding $(v_m - v_i)$ and $(x_m - x_i)$ are close to each other. In contrast, for the regular expansion or compression of matter $(v_m - v_i)$ and $(x_m - x_i)$ are correlated. So, $|w_i - 1| \sim 1$ seems to be much more probable. The case when $w_i \geq 1$ or $w_i \leq 1$ corresponds to the domination of the expansion or compression of matter along the $i$th axis.

The core-sampling approach was applied only for the first simulation and is described in Section 4.

2.3 Comparison of the simulations with the Zel’dovich approximation

In this paper, we consider the properties of DM clouds selected in the same manner in three simulations. The simulations are performed with different codes, in different boxes and with different resolutions. The cloud selection is performed with a complex algorithm and with the set of thresholds presented in Section 2.1. Therefore, before we start our analysis, we estimate the representiveness and reliability of the samples of selected clouds. For this purpose, we compare some integral characteristics of selected clouds with predictions of the Zel’dovich approximation (Zel’dovich 1970; Shandarin & Zel’dovich 1989; Demiański & Doroshkevich 1999, 2004), which reliably describes the mildly non-linear evolution of perturbations.

For this aim, here we use three characteristics of samples of clouds: the matter fraction accumulated by the LSS $\langle f(M) \rangle$, the PDFs of masses of selected clouds $P(M)$ and the redshift evolution of the velocity of clouds as a whole $\langle U(z) \rangle$. The last test is more sensitive to the large-scale perturbations, while the first and second tests are sensitive to the small-scale perturbations, the non-linear processes of matter condensation within clouds and the procedure of cloud selection.

The shapes of the PDFs for cloud sizes and internal velocities can also be obtained on the basis of Zel’dovich theory. However, here we note only that for these functions and the dark dark matter (CDM)-like power spectrum, the Gaussian PDF of the initial perturbations is transformed to the exponential PDF (see sections 2.4 and 3.3 in Demiański & Doroshkevich 2004 for more details). The exponential shape of these PDFs in our simulations is clearly seen in Figs 1 and 2. In Section 3.3, we compare predictions of the Zel’dovich theory with the simulated angular momentum and turbulent motions (equations 4 and 7).

![Figure 1](https://academic.oup.com/mnras/article-abstract/414/3/1813/1034245)

**Figure 1.** The PDFs of cloud sizes $\ell_i$ along the longest, middle and shortest principal axes (top, middle and bottom panels, respectively) are plotted for redshifts $z = 1, 2$ and $3$ (points, stars and squares, respectively) for clouds selected in simulation $S_{150}$ with the threshold overdensity $\delta = 1.76$ and richness given by equation (3). Here, $x_i = \ell_i/\ell_1$.

![Figure 2](https://academic.oup.com/mnras/article-abstract/414/3/1813/1034245)

**Figure 2.** For the simulation $S_{150}$, the PDFs of the reduced velocity dispersion, $c_{1,2,3}$, along the longest, middle and shortest principal axes (top, middle and bottom panels, respectively) are plotted for redshifts $z = 1, 2$ and $3$ (points, stars and squares, respectively) for clouds selected with the threshold overdensity $\delta = 1.76$ and richness given by equation (3). Here, $x_i = c_i/\langle c_k \rangle$, $k = 1, 2, 3$. 

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2.3.1 Evolution of the cloud velocity

The non-linear processes of matter relaxation and the collapse of the compressed matter strongly distort velocities of particles within clouds, but only weakly change the velocities of clouds as a whole. As noted in Demianski & Doroshkevich (1999), for the Gaussian initial perturbations, the PDF of cloud velocity remains Gaussian with the velocity dispersion, closely allied to that described by the linear theory. Thus, according to the Zel’dovich approximation, the velocity dispersion along any principal direction is

\[ \sigma_v = \frac{H(z)}{1+z} \beta(z) \sigma_i \sqrt{\frac{3}{\pi}} \quad \text{and} \quad \beta(z) = -1 + \frac{z}{B(z)} dB(z)/dz, \]

\[ B^{-3} \approx 1 + \frac{2.2 \Omega_m}{1 + 1.2 \Omega_m} [(1+z)^3 - 1], \]

\[ H^2 = H_0^2 \left[ \Omega_m (1+z)^3 + \Omega_k \right], \]

\[ \sigma^2 = \frac{1}{2 \pi} \int_0^\infty p(k) dk. \]  

(9)

Here, \( H \) is the Hubble parameter, \( H_0 = 100 \, h \, \text{km s}^{-1} \, \text{Mpc}^{-1} \), \( B(z) \) describes the growth of perturbations in the linear theory for \( \Lambda \)CDM cosmology, \( p(k) \) and \( H(z) \sigma_i \) are the power spectrum and the amplitude of initial velocity perturbations, respectively, and \( \Omega_m \) and \( \Omega_k \) are matter and dark energy density parameters, respectively. For both our simulations with the moderate resolution, we have approximately

\[ \sigma_v \approx \frac{2.07}{\Omega_m h^2} \sigma_i \text{Mpc}, \]

\[ B \approx 1.27[1 + (1+z)]^{-1/3}, \]

and for \( z \geq 1 \) we obtain

\[ H(z) \approx H_0 \sqrt{\Omega_m (1+z)^3}, \quad B(z) \approx 1.27(1+z)^{-1}, \]

\[ \sigma_\text{i} \approx 353 \, \text{km s}^{-1} (1+z)^{-1/2} (\sigma_\text{i}/0.9). \]  

(10)

Using the matter velocities in the simulation \( S_{150} \), we obtain for the one-dimensional (1D) velocity dispersion of clouds at \( z \geq 1 \):

\[ \sigma_{\text{cl}} \approx 325 \, \text{km s}^{-1} (1+z)^{-1/2} (1 \pm 0.04). \]  

(11)

For the high-resolution simulation \( S_{60} \), we obtain

\[ \sigma_{\text{cl}} \approx 315 \, \text{km s}^{-1} (1+z)^{-1/2} (1 \pm 0.04). \]  

(12)

The differences (~8 and 11 percent, respectively) between the expected (equation 10) and simulated (equations 11 and 12) dispersions characterize the impact of the large-scale perturbations, suppressed in the simulations because of the finite box size.

For clouds selected in the simulation \( S_{150} \) with the threshold overdensity \( \delta_{\text{thr}} = 1.76 \) and richness \( 3 \times 10^{12} h^2 \, M_{\odot} \leq M_{\text{cl}} \leq 5 \times 10^{14} h M_{\odot} \) at redshifts \( z = 1, 2, 3 \), the PDFs for the cloud velocities, \( P_i(\{U_i\}/\sigma_i), \) at \( i = 1, 2, 3 \), are similar to each other with high precision and are well fitted by the function

\[ P_i(x_i) \approx 0.77 \exp(-x_i^2/2), \quad x_i = \{U_i\}/\sigma_i. \]  

(13)

Small variations around the Gaussian fit (13) depend on peculiarities of the selected sample of clouds and can also indicate the weak mass dependence of the measured dispersions.

2.3.2 Matter fraction accumulated by the LSS

To test the simulations and our cloud-selection algorithm, we compare the expected and measured matter fraction accumulated by the LSS. According to the Zel’dovich theory, the fraction of matter expanding along all three axes is \( f_{\text{exp}} \approx 0.53 \approx 0.125 \). Therefore, the fraction of matter accumulated by all LSS elements is

\[ f_{\text{acc}} \approx 0.875(1 - z/z_i), \quad z \ll z_i. \]  

(14)

This approximate relation slightly overestimates the total mass accumulated by pancakes, filaments and clouds. More accurate but multiparametric and more complex estimates can be found in Demianski & Doroshkevich (1999, 2004).

For the simulation \( S_{150} \), we obtain for \( \delta_{\text{thr}} = 1 \) at \( z \leq 3 \)

\[ f(M \geq 5 \times 10^{11} h^{-1} M_{\odot}) \approx 0.73(1 - z/5.7). \]  

(15)

For the high-resolution simulation \( S_{60} \), we see earlier matter concentration and for \( \delta_{\text{thr}} = 1 \), at \( z \leq 6 \), we obtain

\[ f(M \geq 2.5 \times 10^{10} h^{-1} M_{\odot}) \approx 0.80(1 - z/10), \]

\[ f(M \geq 5 \times 10^{11} h^{-1} M_{\odot}) \approx 0.67(1 - z/10). \]  

(16)

The moderate differences between the measured (equations 15 and 16) and expected (equation 14) values demonstrate the moderate impact of the selection parameters used, mostly the minimal mass. The earlier matter condensation in the high-resolution simulation \( S_{60} \) compared with the simulation \( S_{150} \) illustrates the impact of the small-scale perturbations represented by the power spectrum of simulation \( S_{60} \).

2.3.3 Mass function of clouds

For high-density virialized DM clouds, the Press–Schechter mass distribution is usually reproduced in simulations. However, we see here that for strongly anisotropic partly relaxed clouds, selected with moderate overdensity, the simulated mass functions are well approximated by a power law. A simple qualitative explanation of such a distribution is offered by the Zel’dovich theory.

According to this theory, the process of compact object formation is determined by three conditions:

\[ \Delta q_i - B(z) \Delta S_i = 0, \quad i = 1, 2, 3. \]

The function \( B(z) \leq 1 \) was introduced in equation (9). \( \Delta S_i \) are the relative displacements of two points separated by the comoving distance \( \Delta q_i \) along the \( i \)th principal direction. This means that the expected collapsed volume, \( V_q \), and corresponding mass, \( M_q \), may be qualitatively estimated as

\[ M_{\text{cl}} \propto V_q = \Delta q_1 \Delta q_2 \Delta q_3 \propto V_i = \Delta S_1 \Delta S_2 \Delta S_3, \]

\[ \Delta S_i > q_{\text{min}} \geq 0. \]  

(17)

This condition ensures that the collapse proceeds along all three directions and restricts the volume \( V_i \).

For any \( M_0 \), the probability of finding the mass of collapsed clouds in the range \( M_0 - \Delta M \leq M \leq M_0 + \Delta M \) is proportional to the probability of finding \( \Delta S_i \geq q_{\text{min}} \geq 0 \), and the volume \( V_i \) in the corresponding range \( V_0 - \Delta V \leq V_i \leq V_0 + \Delta V \). This simple approach allows us to characterize the shape of the PDF \( P(V_i) \propto P(M_i) \) but its normalization (in equation 17) requires additional assumptions. A more detailed description of the complicated process of cloud formation inevitably leads to much more complex multiparametric expressions (see, for example, Demianski & Doroshkevich 2004).

In turn, the PDF \( P(V_i) \) can be directly obtained from the PDF \( P(\Delta S_i) \). For this purpose, we use the simple three-step numerical method: (i) we construct the three-dimensional (3D) sample of \( \Delta S_i \) in accordance with the theoretical PDF \( P(\Delta S_i) \); (ii) we select the 1D subsample of \( V_i \) in accordance with conditions (17); (iii) we construct the PDF \( P(V_i) \).
The PDF $P(\Delta S)$ is Gaussian (Demiański & Doroshkevich 1999, 2004) with dispersion $\sigma_s(\Delta q)$ and weak correlations:

$$\xi_{ik} = \langle \Delta S_i, \Delta S_k \rangle / \sigma_i^2 \leq 1/3, \quad i \neq k.$$ 

With this PDF, we have constructed $10^6$ realizations of $\Delta S_i$ and we have found that the PDF $P(V_i)$ can be fitted by a power law, $P(V_i) \propto V_i^\kappa$. The slope $\kappa$ slightly varies depending on the considered range of $V_i \geq V_{\text{min}}$. Thus, for three progressively decreasing $V_{\text{min}} = V_1$, 0.1$V_1$, and 0.01$V_1$, the selected subsamples contain 30 407, 50 025 and 164 103 elements with the slopes of PDFs $\kappa \sim -2.5$, $-1.8$ and $-1.5$, respectively. These variations indicate that this slope increases for larger $V_i$.

In all the following simulations considered, the PDFs of the masses of selected clouds are fitted by power laws with $\kappa = -2.4$ and $-2.2$ (equations 19, 20 and 22), which is in the range of expected values $-1.8 \geq \kappa \geq -2.5$. These variations of $\kappa$ indicate mainly the impact of the resolution achieved in simulations and the range of selected masses.

## 3 PROPERTIES OF THE SIMULATED DM LSS

As is well known, the formation of the LSS begins at high redshifts and its evolution continues up to now. The matter fraction accumulated within the LSS elements increases with time, as a result of both the accretion of matter on to the earlier formed LSS elements and the creation of new low-mass LSS elements and bridges, leading to the successive integration of the LSS elements into multi-connected richer elements. At small redshifts, these processes lead to the creation of the joint network of compressed matter. The high efficiency of cloud integration demonstrates the correlated characteristic of the LSS formation, and the interaction of small- and large-scale perturbations. Here, we present some quantitative characteristics of these processes obtained for the LSS formed by DM particles only.

In some important aspects, these clouds differ from the LSS formed by both the DM and baryonic components and observed as the Ly$\alpha$ forest. In turn, both these structures differ from the LSS formed by luminous matter (galaxies) and observed in large galactic surveys. None the less, all these LSS elements are dominated by the DM component, and their evolution is driven by the same initial perturbations. So, we can expect that the most fundamental properties of the LSS evolution can be established through an analysis of the simulated DM LSS.

In this section, we consider clouds selected with the threshold overdensity, $\delta_{\text{thr}} = 1.76$, which allows us to partly suppress the formation of the very massive multi-connected clouds at redshifts $z \leq 2$. Later, we present results obtained for the sample of compact LSS elements with DM masses $10^{12} \leq M \leq 5 \times 10^{14} M_\odot$.

For the high-resolution simulation $S_{150}$, we can obtain a more detailed description of the process of matter condensation within various samples of high-density clouds. Thus, we have found that within the LSS clouds there is a population of high-density cores with $\delta \geq 10$. Such an early formation of high-density low-mass clouds was also found by Diemand et al. (2008).

### 3.1 Basic characteristics of compact clouds

The set of selected compact clouds can be characterized by the redshift evolution of their comoving number density, $\langle n_{\text{cl}}(z) \rangle$, their masses, $\langle M_{\text{cl}}(z) \rangle$, the fraction of particles accumulated by clouds, $f_{\text{com}}$, and the three comoving sizes $\langle L_i(z) \rangle$ ($i = 1, 2, 3$) determined along the three principal axes of each cloud.

For the sample of $N_{\text{thr}} = 11 000–14 000$ clouds selected in the simulation $S_{150}$, with $\delta_{\text{thr}} = 1.76$ and $M_{\text{min}} \leq M_{\text{cl}} \leq M_{\text{max}}$, with variations smaller than 10 per cent, we have:

$$\langle n_{\text{cl}}(z) \rangle \approx 3.6 \times 10^{-3} \text{Mpc}^{-3},$$

$$\langle M_{\text{cl}}(z) \rangle \approx 3 \times 10^{12} h^{-1} M_\odot,$$

$$f_{\text{com}} = \langle n_{\text{cl}}(z) \rangle / (\rho) \approx 0.13,$$

$$\langle L_i(z) \rangle \approx \ell_i \sqrt{\sigma}, \quad v = M_{\text{cl}} / (M_{\text{cl}}(z)).$$

Here, $\ell_i$ are reduced cloud sizes with

$$\langle \ell_1 \rangle = 1 h^{-1} \text{Mpc},$$

$$\langle \ell_2 \rangle = 0.5 h^{-1} \text{Mpc},$$

$$\langle \ell_3 \rangle = 0.2 h^{-1} \text{Mpc}.$$

All these parameters characterize the selected sample of the LSS elements. Mass dependence of the cloud sizes reflects the complex structure of the selected clouds and implies their partial correlation. Their weak dependence on redshift characterizes the balance between the creation of new elements and matter outflow to the more massive complex LSS elements, caused by the creation of bridges between earlier formed clouds.

The PDFs of the functions $\ell_i$ plotted in Fig. 1 are also weakly dependent upon redshift. The exponential decline of the PDFs at $x_i = \ell_i / (\ell_i) \geq 1$ reflects the rare formation of asymmetric objects with large $\ell_i$, while at $x_i \leq 1$ the shape of the PDFs depends upon conditions used for the cloud selection.

The PDF of cloud masses, $P_5(M_{\text{cl}}(z))$, is surprisingly independent of redshift but depends upon the range of considered velocity dispersion of compressed matter (see Section 3.5). For all clouds, this can be approximated by the power law

$$P_5(v) = 0.25 v^{-2}, \quad v = M_{\text{cl}} / (M_{\text{cl}}(z)),$$

which is close to the predictions of the Zel'dovich theory (Section 2.3.3). It is interesting that here we do not see the exponential decline of the PDF, $P_5(x)$, predicted by all theoretical models and observed in the luminosity function of galaxies. In the simulations, such a decline appears only at higher redshifts for clouds selected with larger threshold overdensity. It can be expected that these differences reflect the complex structure of richer clouds formed partly by integration of earlier formed clouds as a result of the origin of the low-density bridges between them.

Let us note again that both the mean values $\langle L_i \rangle$ and PDFs for the cloud sizes, $P(x_i = \ell_i / (\ell_i))$, plotted in Fig. 1, do not change with redshift. These results show that for clouds selected in this way, their mean sizes in real space increase with time $\propto (1 + z)^{-1}$ along each of the three principal axes. This means that the mean cloud overdensity does not change. Therefore, the possible dissipation of the LSS elements as a result of the matter expansion is not as essential.

The characteristics of compact clouds obtained in the simulation $S_{500}$ are similar to those presented above. Let us only note that for this simulation we find some excess of low-mass clouds compared with the simulation $S_{150}$. Indeed, instead of equation (19) we obtain for this simulation the PDF

$$P_5(v) \propto v^{-2.4}, \quad v = M_{\text{cl}} / (M_{\text{cl}}).$$

This confirms some excess of less massive clouds. This PDF differs from equation (19) but it is in the range of predictions of the Zel'dovich theory (Section 2.3.3) and is similar to that obtained...
for colder less massive clouds (equation 35). This difference shows that some properties of the selected clouds are sensitive to such factors as the possible impact of the gaseous component, the initial realization of perturbations and the criteria of the cloud selection.

An analysis of the high-resolution simulation S_{00} allows us to obtain a more detailed description of the evolution of compact clouds. As noted above, in this simulation we see the formation of a rich population of high-density cores with the overdensity $\delta_{\text{int}} \geq 10$ and $M_d \geq 10^{10} M_\odot$. These cores have steep density profiles $\rho \propto r^{-2}$ and accumulate up to 40 per cent of matter at $z = 1$. They are usually incorporated into less dense clouds.

For the selected LSS elements, we can trace their evolution in a wider range of redshifts, $1 \leq z \leq 6$. Thus, instead of equation (18) we obtain

$$n_{\text{cl}}(z) \approx 6.3 \times 10^{-3}(1 - 0.1z) \text{Mpc}^{-3},$$

$$\langle M(z) \rangle \approx 2.3 \times 10^{11}(1 - 0.07z) h^{-1} M_\odot,$$

$$f_{\text{com}} \approx 0.18(1 - 0.16z).$$

$$L_i = \ell_i \sqrt{\ell}, \quad \nu = M_d/\langle M_\text{cl}(z) \rangle,$$

$$\langle \ell_1 \rangle = 0.65(1 + 0.25z) h^{-1} \text{Mpc},$$

$$\langle \ell_2 \rangle = 0.2(1 + 0.5z) h^{-1} \text{Mpc},$$

$$\langle \ell_3 \rangle = 0.11(1 + 0.5z) h^{-1} \text{Mpc}. \quad (21)$$

At $z = 3$, these sizes are similar to equation (18), but at all redshifts we see a slow continuous matter compression. This difference reflects both the impact of small-scale perturbations and the more detailed description of the LSS evolution achieved in the high-resolution simulation.

The PDFs for the cluster sizes and their masses are also similar to the previous PDFs. In particular,

$$P_\nu(v) \propto \left\{ \begin{array}{ll} v^{-2.0}, & z \leq 3, \\ v^{-2.2}, & z > 4, \end{array} \right\} \nu = M_d/\langle M_d \rangle, \quad (22)$$

which is also consistent with the predictions of the Zel’dovich theory. For larger threshold overdensity, the theoretically expected exponential decline of the mass function can be seen for high $z$, when the majority of clouds are well separated. For the mass function of high-density haloes selected within this simulation, the exponential decline is clearly seen at all redshifts.

### 3.2 Velocity dispersion of compressed matter

The velocity dispersion of particles within selected clouds depends on three factors: the retained Hubble expansion, the enhanced random initial velocities of particles and the velocity created in the course of matter relaxation. All these factors are closely linked with the cloud mass, and therefore the measured velocity dispersion also depends upon mass. The relative contributions of these components vary with time; however, for collisionless DM particles, the rate of these variations depends upon the degree of matter compression and relaxation achieved along each of the principal axes. This means that the velocity dispersions along these axes are different and must be determined separately along the longest ($\sigma_1$), middle ($\sigma_2$) and shortest ($\sigma_3$) principal axes of each cloud.

It is important that these velocity dispersions are much smaller than the dispersions $\sigma_i$ introduced in equation (11). This difference is caused by the strong correlation of the particle velocities at the scales of clouds. The simple analysis shows that the dispersion of initial velocities within clouds decreases with the cloud mass, which stimulates the formation of low-mass high-density clouds.

For clouds selected in the simulation $S_{150}$ with $\delta_{\text{int}} = 1.76$ and $M_{\text{min}} \leq M_d \leq M_{\text{max}}$, the velocity dispersions along the three principal axes, $\sigma_i$, slowly vary with time, but they depend upon the cloud richness:

$$\sigma_i = c_i \nu^{\alpha_i}, \quad \alpha_1 = 0.5, \quad \alpha_2 = 0.4, \quad \alpha_3 = 0.3. \quad (23)$$

The mean values of the reduced velocity dispersions $c_i$ are

$$\langle c_1 \rangle = 58 \text{ km s}^{-1}, \quad \langle c_2 \rangle = 66 \text{ km s}^{-1},$$

$$\langle c_3 \rangle = 80 \text{ km s}^{-1}. \quad (24)$$

and their PDFs, $P(x_i = c_i/\langle c_i \rangle)$, are plotted in Fig. 2 for three redshifts $z = 1, 2$ and 3. The very weak redshift evolution of these PDFs verifies the self-similar characteristic of the simulated LSS evolution.

These PDFs demonstrate the expected exponential decline $P(x_i) \propto \exp (- x_i/0.3)$ for $x_i \geq 1$. At the same time, there are cold clouds with $c_i \leq \langle c_i \rangle$.

For the simulation $S_{00}$, the velocity dispersions, $\langle c_i \rangle$, are $\sim 20$ per cent smaller than in equation (23). This is consistent with the excess of low-mass clouds noted in equation (20). It confirms the sensitivity of the characteristics of the low-mass clouds to the mass resolution, the impact of the gaseous component and the realization of the initial velocity field. The PDFs of the velocity dispersions are also similar to those plotted in Fig. 2.

For the simulation $S_{50}$, for the same mass range, at redshifts $1 \leq z \leq 5$ the mean velocity dispersions are described by relation (23) with

$$\alpha_1 = 0.4, \quad \alpha_2 = 0.3, \quad \alpha_3 = 0.4.$$

$$\langle c_1 \rangle \approx 105(1 - 0.15z) \text{ km s}^{-1},$$

$$\langle c_2 \rangle \approx 95 \text{ km s}^{-1}, \quad \langle c_3 \rangle \approx 80 \text{ km s}^{-1}. \quad (25)$$

Their PDFs are similar to those plotted in Fig. 2.

The differences between estimates, equations (23) and (24) and equation (25), can be related to the influence of small-scale perturbations, resulting in a more rapid relaxation and more complex internal structure of selected compact clouds. This is seen as the formation of high-density cores within these clouds and as a decrease of power indices $\alpha_1$ and $\alpha_2$ in equation (25) compared with equation (23). In turn, the action of these factors increases the reduced velocity dispersions $c_1$ and $c_2$ (equation 25) compared with equation (24). At the same time, for both simulations, $\alpha_3$ and $c_3$ remain similar.

### 3.3 Rotation and turbulent motions of the compressed matter

In the partly relaxed clouds, particles move on closed orbits, which generates both the turbulent motions, $j_i$ (equation 6), and the angular momenta of clouds as a whole, $J_i$ (equation 5), along the three principal axes. Evidently, for isotropic simulations, $J_i = 0$. Later, we consider the characteristics of the absolute values of angular momenta $|J_i|$ only. As before, these functions depend upon cloud richness. For clouds selected in the simulation $S_{150}$, we have

$$|J_i| = \mu_i g_i, \quad j_i = \tau_i g_i,$$

$$g_1^2 = (L_1^2 + L_2^2) (\sigma_1^2 + \sigma_2^2),$$

$$g_2^2 = (L_1^2 + L_3^2) (\sigma_1^2 + \sigma_3^2). \quad (26)$$

Here, $\mu_i$ and $\tau_i$ characterize the dimensionless reduced angular momenta and intensity of the turbulent motions, respectively. $L_i$ and $\sigma_i$ (equations 18 and 23) are the size and velocity dispersion
of clouds, respectively. The mean values of the reduced angular momenta are found to be independent of \( z \) and
\[
\langle \mu_1 \rangle = 0.08, \quad \langle \mu_2 \rangle = 0.15, \quad \langle \mu_3 \rangle = 0.15.
\] (27)

The small cloud rotation around the longest axis and the exponential shapes of PDFs,
\[
P(x) \approx 0.12 \exp(-x), \quad x = \mu_i / \langle \mu_i \rangle,
\] (28)
are consistent with the TTT predictions.

The mean values of the reduced intensity of turbulent motions are also found to be independent of \( z \) and
\[
\langle \tau_1 \rangle = 0.64, \quad \langle \tau_2 \rangle = 0.76, \quad \langle \tau_3 \rangle = 0.65.
\] (29)
Their PDFs are fitted by the functions
\[
P(x) \approx 0.14 \exp[-(x_i - 1)^2/0.14], \quad x_i = \tau_i / \langle \tau_i \rangle.
\] (30)

For the total momentum of the clouds, \( J \), and for the total intensity of turbulent motions, \( j \),
\[
J = \sqrt{J_1^2 + J_2^2 + J_3^2} = \mu g_i, \\
j = \sqrt{j_1^2 + j_2^2 + j_3^2} = \tau g_i, \\
g_i^2 = (L_i^2 + L_j^2 + L_k^2) (\sigma_1^2 + \sigma_2^2 + \sigma_3^2),
\] (31)
we obtain
\[
\langle \mu \rangle = 0.17, \quad \langle \tau \rangle = 0.8.
\] (32)

The PDFs for the functions \( \mu \) and \( \tau \) are plotted in Fig. 3. The obtained value of \( \langle \tau \rangle \) corresponds to the domination of elliptical trajectories of particles with the ratio of axes \( b/a \approx 0.57 \). This is consistent with the ratios of the cloud sizes (equation 18).

For the simulation \( S_{180} \), instead of equations (27), (29) and (32), we have, at \( 1 \leq z \leq 6 \) and for the same mass range:
\[
\langle \mu_1 \rangle = 0.08(1 - 0.07z), \quad \langle \tau_1 \rangle = 0.57, \\
\langle \mu_2 \rangle = 0.1(1 - 0.07z), \quad \langle \tau_2 \rangle = 0.60(1 - 0.08z), \\
\langle \mu_3 \rangle = 0.09(1 - 0.12), \quad \langle \tau_3 \rangle = 0.57(1 - 0.09z), \\
\langle \mu \rangle = 0.13(1 - 0.08z), \quad \langle \tau \rangle = 0.7(1 - 0.08z).
\] (33)

A comparison of \( \langle \mu \rangle \) with \( \langle \tau \rangle \) in equations (27), (29), (32) and (33) shows that only \( \sim 15-20 \) per cent of the angular momenta of individual particles are transformed into the angular momentum of a cloud as a whole. This characterizes the efficiency of the generation of angular momentum according to the TTT.

These results are also consistent with the predictions of the Zel’dovich theory (equations 4–7) and indicate the significant contribution of initial random velocities in the turbulent motions within compressed clouds.

### 3.4 Relaxation of the compressed matter

The three parameters \( w_1, w_2 \) and \( w_3 \) introduced in equation (8) allow us to discriminate between the regular expansion and the compression of matter along the principal axes, and to estimate the degree of matter relaxation. As noted in Section 2.2, \( w_i \geq 1 \) indicates the domination of matter expansion, while \( w_i \leq 1 \) corresponds to the domination of matter compression along the \( i \)th principal axis. For the simulation \( S_{180} \) and for redshifts \( z = 1, 2 \) and 3, the mean values \( \langle w_i \rangle \) are listed in Table 1 and their PDFs are plotted in Fig. 4. These results indicate that for our samples of selected clouds the matter compression along all three axes dominates, but it is progressively decelerated with time. The symmetry of the PDF \( P(w_i) \) points out that for our selected clouds both the compression and expansion along the longest axis are almost equally typical. These estimates only weakly depend upon the richness of clouds.

Another manifestation of the retained expansion along the longest axis is the tight correlation of velocity dispersion and size
\[
\sigma_1 \propto L^{0.8},
\]
with the deviation of \( \sim 10 \) per cent. This expression is similar to the Hubble flow. For the middle and the shortest axes, such correlations are very weak.

These results confirm that along the longest principal axis the relatively slow relaxation is accompanied by the retained expansion of clouds, which persists for some time after cloud formation. In contrast, along the middle and shortest principal axes, the moderate retained compression of clouds predominates. These results are consistent with the strongest matter compression along the shortest principal axis (which is responsible for pancake formation) and the successive transformation of prolate pancake-like clouds into elongated filamentary-like clouds.

To estimate the degree of matter relaxation along the shorter axis, we can use the fraction of clouds, \( f_{rel}(z) \), for which \( |1 - w_i(z)| \leq 0.5 \). The estimates of this fraction are listed in Table 1 for three

<table>
<thead>
<tr>
<th>( w_i(z) )</th>
<th>( z = 1 )</th>
<th>( z = 2 )</th>
<th>( z = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1 )</td>
<td>0.05</td>
<td>-0.2</td>
<td>-0.42</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>-0.4</td>
<td>-0.53</td>
<td>-0.62</td>
</tr>
<tr>
<td>( w_3 )</td>
<td>-0.2</td>
<td>-0.42</td>
<td>-0.55</td>
</tr>
<tr>
<td>( f_{rel} )</td>
<td>0.8</td>
<td>0.44</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 1. Degree of relaxation of selected clouds.
redshifts. The rapid growth of this fraction with time illustrates the progressive relaxation of the compressed matter along the shortest principal axis. The fraction of matter relaxed along the longest principal axis is only half of that along the other two axes. For the simulation $S_{500}$, the functions $|w_i(z)| - 1$ and $f_i(z)$ are similar to those discussed above.

The more complex internal structure of clouds selected from the high-resolution simulation $S_{50}$ leads to a more rapid relaxation of the compressed matter. Thus, we have for the matter fraction with $|1 - w_i(z)| \leq 0.5$

$$f_{\text{rel}}(z = 5) \approx 0.6, \quad f_{\text{rel}}(z = 3) \rightarrow 0.9.$$  \hfill (34)

These results illustrate the impact of small-scale perturbations and strong links between the resolution achieved in the simulation and the processes of cloud formation and relaxation.

### 3.5 Colder and hotter clouds

The strong link between the velocity dispersions and richness of clouds (equation 23) illustrates the large-scale correlation of the initial velocity field. None the less, the scatter around the mean dispersions (equation 23) is large, and differences between the properties of colder and hotter clouds deserve special investigation.

In the simulation $S_{500}$, the subpopulation of colder clouds was selected from the full sample of clouds by the condition that $\sigma_z \leq 40\,\text{km\,s}^{-1}$. It contains about half of the clouds with the lower richness and the steeper mass function $P_c(M_{\text{cl}}/\langle M_{\text{cl}} \rangle)$:

$$\langle M_{\text{cl}}(z) \rangle \approx 7 \times 10^{11}\,h^{-1}\,\text{M}_\odot,$$

$$P_c(M_{\text{cl}}) \propto (M_{\text{cl}}/\langle M_{\text{cl}} \rangle)^{-2.7}.$$  \hfill (35)

The subpopulation of hot rich clouds with $\sigma_z \geq 40\,\text{km\,s}^{-1}$ is characterized by larger mean richness and shallower mass function, $P_{h_c}[M_{\text{cl}}(z)/\langle M_{\text{cl}}(z) \rangle]$. $\langle M_{\text{cl}}(z) \rangle \approx 3.2 \times 10^{12}\,h^{-1}\,\text{M}_\odot$, $P_{h_c}(M_{\text{cl}}) \propto (M_{\text{cl}}/\langle M_{\text{cl}} \rangle)^{-1.8}$.  \hfill (36)

Both colder and hotter subpopulations are dominated by partly relaxed clouds expanded along the longest principal axis and compressed along the middle and shortest axes. For these, the numerical estimates of $|w_i|$ are similar to those listed in Table 1. For the subpopulations of hotter clouds, the fraction of relaxed clouds with $|1 - w_i(z)| \leq 0.5$ is close to $f_{\text{rel}}$ listed in Table 1, while for the colder clouds this fraction is smaller by a half.

### 4 CORE-SAMPLING APPROACH

For a more detailed comparison of the simulated matter distribution with the observed characteristics of the LSS and the Ly$\alpha$ forest, it is convenient to use the core-sampling approach (Doroshkevich et al. 2001, 2004; Demiański et al. 2006). With this approach, we retain only particles within the selected sample of clouds. We divide the simulated box into a system of rectangular cores of comoving size $d_{\text{core}} \times d_{\text{core}} \times l_{\text{box}}$, and we consider the properties of particles situated within each core.

All particles within a core are projected on the core axis and, using the 1D cluster analysis, are collected into clumps. For clumps selected in this way, we can determine several characteristics, such as the mass, $M_{\text{cl}}$, and the comoving surface density, $q_{\text{cl}} = M_{\text{cl}}/d_{\text{core}}^2$. Other characteristics relate to the clump position and velocity along the core. These are the comoving clump separation, $D_{\text{eq}}$, and their size (thickness), $t_{\text{eq}}$, and overdensity, $\delta_{\text{cl}} = q_{\text{cl}}/q_{\text{thr}}(\rho)$, the relative velocity of neighboring clumps, $|\delta U|$, the dispersions of clump velocity, $\sigma_v$, and particle velocity within clumps, $\sigma_c$. We can also estimate the degree of matter relaxation, $w_{\text{cl}}$ (equation 8), and the dispersion of the transverse velocity of matter within clumps, $\sigma_{\text{tr}}$.

The core-sampling analysis utilizes four parameters: the comoving size of the core ($d_{\text{core}}$), the comoving threshold linking length used in the 1D cluster analysis ($l_{\text{thr}}$) and the minimal and maximal richness of clumps ($q_{\text{thr}}$ and $q_{\text{max}}$, respectively) retained for the analysis. The results of the analysis depend mainly upon the choice of $d_{\text{core}}$. The last two parameters allow us to remove from the sample poor clumps, which arise as a result of the random intersection of the core and a cloud periphery, and a small number of extremely rich clumps with $q \geq q_{\text{max}}$, which strongly distort the mean measured characteristics.

Here, we apply the core-sampling approach to investigate the full sample of clouds selected in the simulation $S_{500}$ with the threshold overdensity $\delta_{\text{thr}} = 1.76$. In our analysis, we use

$$d_{\text{core}} = 1.0\,h^{-1}\,\text{Mpc}, \quad l_{\text{thr}} = 0.5\,h^{-1}\,\text{Mpc},$$

$$q_{\text{thr}} = M_{\text{cl}}/d_{\text{core}}^2 = 3 \times 10^{11}\,\text{M}_\odot\,h^{-1}\,\text{Mpc},$$

$$q_{\text{max}} = 10q_{\text{thr}}.$$  \hfill (37)

These samples contain $\sim(1.3–1.5) \times 10^5$ clumps. Naturally, variations of these parameters change the fraction of low- and high-mass clumps in the selected sample and also the main characteristics of the sample of clumps. However, for a wide set of limits (equation 37), we see a weak redshift dependence of the mean characteristics and their PDFs, which confirms the self-similar characteristic of the LSS evolution.

In many respects, this approach and its limitations are similar to those appearing in the analysis of the Ly$\alpha$ forest (Demiański et al. 2006) and the pencil beam observations of the LSS (Doroshkevich...
et al. 2001, 2004). Of course, the criteria of clump selection in observations and simulations are not identical. Indeed, if the Doppler parameter of the forest line coincides with the velocity dispersion $\sigma_z$, then the evolutions of the DM surface density $q(z)$ and the column density of neutral hydrogen, $N_H$, are very different, as they are driven by different factors. Thus, we cannot directly compare simulated results with observations. However, even such an analysis allows us to reveal the qualitative similarity of the observed and simulated LSS evolution and to demonstrate the impact of the main factors that influence the evolution of the observed LSS.

### 4.1 Dynamical characteristics of the DM LSS

The redshift evolution of the dynamical characteristics of the simulated DM clumps obtained with the core-sampling approach are found to be similar to those obtained in section 3. Thus, with the threshold parameters (equation 37) and for $3 \geq z \geq 1$, for the velocity dispersion of clumps, $\sigma_U$, and for the relative velocity of neighbouring clumps, $\delta U$, we obtain

$$\langle \sigma_U \rangle \approx 308 \text{ km s}^{-1} (1 + z)^{-1/2} (1.0 \pm 0.05),$$

$$\langle |\delta U| \rangle \approx 272 \text{ km s}^{-1} (1 + z)^{-1/2} (1.0 \pm 0.03).$$

So, the defined $\sigma_U$ and the value (equation 10) measured for all particles are close to each other. The PDF for the dispersion of the 1D clump velocity, $P(x_u = |U|/\sigma_U)$, plotted in Fig. 5 (top panel), is close to the Gaussian PDF. However, at all redshifts, there is some excess of clumps ($\sim 6$ per cent) with extremely low $\sigma_U$.

Both velocity dispersions of the compressed matter, measured with the core-sampling approach along the core, $\langle \sigma_z \rangle$, and in the transverse directions, $\langle \sigma_w \rangle$, are weakly dependent upon redshift, which is similar to the behaviour of $\sigma_z$ (equation 23). Thus, we obtain

$$\langle \sigma_z \rangle \approx \langle \sigma_w \rangle \approx 49 \text{ km s}^{-1} (1 \pm 0.2).$$

The random orientation of the principal axes of the clouds and the probing core leads to an intermixture of velocities along the three principal axes. The velocity dispersion measured in transverse directions, $\langle \sigma_w \rangle$, is the same as $\langle \sigma_z \rangle$ measured along the core.

The PDFs of the velocity dispersion of matter compressed within clumps, $P(x_u = \sigma_z/\sigma_U)$, are plotted in Fig. 5 (middle panel) for three redshifts, $z = 1, 2$ and $3$. As is seen from this figure, for $x_u \geq 0.5$ these PDFs are similar to each other, and they are well fitted by the exponential function

$$P(x_u) \approx 1.5 \exp(-x_u/0.75).$$

However, the fraction of clumps with $x_u \leq 0.1$ progressively decreases with time from $\sim 25$ per cent at $z = 3$ and down to 10 per cent at redshifts $z = 1$. This fraction can be related to the low-mass unrelaxed clumps formed as a result of the intersection of cores with the complex low-density periphery of the selected 3D clusters.

As before, the mean value $\langle w_{cr} \rangle \approx 1$ characterizes the symmetry of the matter compression and expansion along the core in the sample of selected clumps. However, the PDF $P(w_{cr})$ plotted in Fig. 5 (bottom panel), characterizing the degree of matter relaxation along the core, shows some variation with redshift. Using this PDF, we can estimate the fraction of clumps, $f_{rel}(z)$ with $0.5 \leq w_{cr} \leq 1.5$, which are almost relaxed along the core at redshifts $z = 3, 2$ and 1, as follows:

$$f_{rel}(3) = 0.56, \quad f_{rel}(2) = 0.64, \quad f_{rel}(1) = 0.7.$$

These fractions are similar to the fraction of clusters relaxed along the shortest axis listed in Table 1, which demonstrates the moderate impact of the macroscopic motions along the core. This result seems to be natural because of the moderate difference between the fraction of matter relaxed along the shortest and longest principal axes (Table 1 and Fig. 4).

### 4.2 Spatial characteristics of the DM LSS

The simulated overdensity of clumps, $\langle \delta_{cr} \rangle$, the surface density, $\langle q_{cr}(1 + z)^{-2} \rangle$, and the comoving thickness of clusters, $\langle t_{cr} \rangle$, are found to be weakly dependent upon the redshift:

$$\langle q_{cr}(1 + z)^{-2} \rangle \approx 4 \times 10^{11} \text{ h M}_{\odot} \text{ Mpc}^{-3} (1.0 \pm 0.2),$$

$$\langle t_{cr} \rangle \approx 1 \text{ h}^{-1} \text{ Mpc} (1.0 \pm 0.1),$$

$$\langle \delta_{cr} \rangle = \frac{q_{cr}}{t_{cr}(\bar{\rho})(1 + z)^2} \approx 5(1.0 \pm 0.15).$$

Here, $\langle \bar{\rho} \rangle$ is the mean comoving density (equation 1).

The redshift evolution of the comoving cluster separation,

$$\langle D_{sep} \rangle \approx 16.2 \text{ h}^{-1} \text{ Mpc} (1.0 \pm 0.1),$$

is also weakly dependent upon redshift, but it strongly depends upon threshold parameters (equation 37) determining the clump selection procedure.

For the simulation $S_{50}$, the mean comoving separation between clumps is

$$\langle D_{sep} \rangle \approx 7 \text{ h}^{-1} \text{ Mpc}.$$

The difference between this value and equation (44) illustrates the impact of the box size (the limited core length). The same result as equation (45) is obtained when we cut a ‘box’ of 50 $\text{h}^{-1}$ Mpc from the simulation $S_{50}$.

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**Figure 5.** The PDFs of cluster velocity, $P(x_u)$, the velocity dispersion within clusters, $P(x_u)$, and the degree of relaxation, $P(w_{cr})$, are plotted for redshifts $z = 1, 2$ and $3$ (points, stars and squares, respectively) for clusters selected within cores under conditions (37). Here, $x_u = |U|/\sigma_U$ and $x_u = \sigma_u/\sigma_z$. 

The PDFs of the DM surface density, $P(q_\alpha/(q_\alpha))$, and of the object separation, $P(D_{\text{sep}}/(D_{\text{sep}}))$, also weakly depend on redshift and are well fitted by exponential functions:

$$P(x_q) \approx 0.8 \exp(-x_q/0.8), \quad x_q = q_\alpha/(q_\alpha) \geq 0.2,$$

$$P(x_{q_{\text{sep}}}) \approx 0.9 \exp(-x_{q_{\text{sep}}}), \quad x_{q_{\text{sep}}} = D_{\text{sep}}/(D_{\text{sep}}).$$

These results show that, with the core-sampling approach, the redshift evolution of the simulated LSS is well characterized by the slow regular variations of its mean characteristics, while the corresponding PDFs practically do not change at all. This fact verifies again the self-similar characteristic of the LSS evolution, at least for its richer elements formed by the DM component of the Universe. This is consistent with the observed evolution of the Ly\(\alpha\) forest (Demiash et al. 2006).

### 5 SUMMARY AND DISCUSSION

Many branches and strongly non-homogeneous matter distribution typical for the richer LSS elements determine their complex multi-connected structure and do not allow us to characterize them in any simple way. Thus, we are compelled to restrict our analysis to the sample of compact DM clouds with a moderate richness and overdensity. The simple procedure of the sample selection and the basic characteristics of clouds are described in Section 2. A more refined technique, proposed for the selection and discrimination of the DM filaments and walls (see, for example, discussions in Doroshkevich et al. 2004; Aragon-Calvo et al. 2007; Zhang et al. 2009), can be efficient mainly for the solution of some special problems.

In many respects, the evolution of the selected population is typical for all LSS elements but some discussed characteristics depend upon the procedure of sample selection. First of all, this relates to the spatial characteristics of clouds.

As is well known, the LSS evolution is determined by the accretion of diffuse matter, the creation of poorer clouds and the integration of clouds into larger clouds. These processes lead to rapid matter concentration within the population of large multiconnected clouds and to the successive network formation. Thus, in the simulation $S_{10}$, the matter fraction accumulated by richer clouds with $M_\alpha \geq 10^5 h^{-1} M_\odot$ increases by a factor of 5 between redshifts $z = 3$ and $z = 0$, while the fraction of particles accumulated by clouds with $5 \times 10^4 \leq N_\alpha \leq 10^3 h^{-1} M_\odot$ increases by a factor of 1.5 only. This means that at $z \leq 3$ the LSS evolution is dominated by the successive integration of the earlier formed LSS elements into the richest elements. The evolution of compact clouds with moderate richness is determined by a balanced action of coalescence and formation of new clouds.

The existence of the population of DM clouds with moderate richness is in itself quite interesting. In many respects, its slow evolution leads to the stability of measured cloud characteristics, which are distorted mainly by the successive formation of high-density cores. However, at higher redshifts, the influence of the process of integration of clouds becomes less significant. The creation of new clouds and the accretion of diffuse matter dominate, and the characteristics of clouds (equation 21) evolve more rapidly.

The analysis of the simulated DM LSS allows us to clarify some important factors that cause its evolution, and to link these with the power spectrum of the initial perturbations and with the observed evolution of the Ly\(\alpha\) forest. The limited mass resolution ($M \sim 2 \times 10^{10} M_\odot$ and $M \sim 7 \times 10^9 M_\odot$ in analysed simulations) prevents the formation of low-mass clouds, comparable with the majority of the observed Ly\(\alpha\) absorbers. This means that our results cannot be directly compared with the observed characteristics of the Ly\(\alpha\) forest. However, such an analysis allows us to find some characteristics of evolution of the simulated DM LSS elements, which is very interesting in itself. It is important also that the main basic features of this evolution are similar in many respects to the observed properties of the Ly\(\alpha\) forest. This allows us to explain some peculiarities of the forest evolution discussed in Section 1.

#### 5.1 Main inferences

The most important features of the evolution of the selected DM LSS elements can be summarized as follows:

(i) weak redshift variations of the basic characteristics of the selected sample of DM clouds, such as their mean comoving size and velocity dispersion;

(ii) measured PDFs of the basic characteristics of the LSS elements are weakly dependent upon the redshift, which implies the self-similar characteristic of the LSS evolution;

(iii) a significant degree of relaxation of the compressed matter along the shortest principal axis and retained matter expansion along the longest principal axis;

(iv) weaker rotation of clouds along their longest axis;

(v) some measured cloud characteristics depend on the procedure of cloud selection.

#### 5.2 Impact of simulation parameters

The comparison of the characteristics of clouds obtained for three simulations with different box sizes and resolutions shows that some of them are sensitive to the resolution, the box size and the impact of baryonic component. In particular, the complex internal structure of clouds selected in the high-resolution simulation $S_{50}$ leads to a more complex evolution of the basic characteristics of clouds, compared with the results obtained for lower-resolution simulations. In the simulation $S_{10}$, clouds are partially fragmented into high-density cores, which distorts their dynamical properties. The fraction of high-density virialized clumps increases with the resolution achieved in the simulation. However, even a large mass fraction (up to 40 per cent) of these high-density clumps does not affect strongly the main qualitative inferences: the self-similar evolution of the characteristics of clouds and the exponential decline of their PDFs.

The most prominent impact of resolution is detected in the cloud size along the middle and shortest axes $\ell_{2,3}$ (equation 21), the velocity dispersion along the longest principal axis (equation 25) and the relaxation parameter, while the other properties of clouds are similar in all three simulations.

In the simulation $S_{50}$, the box size affects the separation between clouds (equations 44 and 45). The box size of $150 h^{-1}$ Mpc should be enough for the stable reproduction of both these properties.

#### 5.3 Self-similarity of the DM LSS evolution

The most interesting result of our analysis is the self-similar characteristic of the evolution of the DM LSS, which is manifested at $z \leq 4.5$ as the surprising stability of PDFs of the basic characteristics of clouds found with both the 3D analysis in Section 3 and with the core-sampling approach in Section 4. This result is obtained for three simulations performed in different ways and with different numbers of particles, resolution and realization of the initial particle positions and velocities. These features are also similar to the redshift independence of the PDFs of the observed characteristics.
of the Lyα forest (Demiański et al. 2006). These analogies point to the universal character of the detected self-similarity.

Another manifestation of this self-similarity is the universality of the NFW density profiles within simulated DM haloes (see, for example, Navarro et al. 1995, 1996, 1997). The self-similar characteristic of the DM halo formation is seen as the regular redshift variations of the internal structure of individual haloes. However, the complex process of formation and relaxation of the high-density LSS elements and haloes can be described analytically as a self-similar process only for the simplest cases (see, for example, Fillmore & Goldreich 1984; Gurevich & Zybina 1995; Sikivie, Tkachev & Wang 1997; Nusser 2001).

For objects formed in the course of mildly non-linear compression, the self-similar characteristic of evolution is predicted by the Zel’dovich approximation (Zel’dovich 1970; Shandarin & Zel’dovich 1989), where the evolution is described by a product of time- and space-dependent functions. The statistical description of the DM LSS based on the Zel’dovich theory (Demiański & Doroshkevich 1999, 2004) clearly demonstrates the expected self-similarity.

The simulated evolution of DM clouds is a strongly deterministic process driven mostly by the used realization of initial perturbations. The statistical properties of clouds are determined by the correlation and structure functions expressed through the simulated power spectrum. Therefore, we can expect that the self-similarity of the simulated evolution is caused by the properties of the power spectrum responsible, in particular, for the formation of clouds.

Such self-similarity naturally appears for the power spectrum approximated by a power law. Indeed, in this case all correlation and structure functions are described by the time-dependent amplitudes and standard space-dependent functions. This usually leads to the self-similar evolution of the basic statistical characteristics of the DM LSS. The power spectra used in the simulations considered here are well approximated by a power law, which explains the self-similarity found.

Of course, this inference is related to the limited range of scales of the power spectrum, and therefore to the limited range of cloud richness. However, we can expect that this range extends down to scales typical for the majority of forest elements, which in turn explains the weak redshift variations of the observed PDFs of the forest characteristics. In contrast, the more complex process of formation of the largest clouds and the network of high-density LSS elements is driven by the large-scale part of the power spectrum, which determines the formation of low-density bridges between high-density clouds. It is convenient to analyse these mildly non-linear processes on the basis of the Zel’dovich approximation (Demiański & Doroshkevich 1999, 2004).

5.4 Evolution of the velocity dispersions and the degree of relaxation of clouds

The second important result of our analysis relates to the evolution of the internal structure of clouds. Both the moderate-mass resolution achieved in simulations and the moderate richness of the dominant population of clouds prevent a more detailed description of the anisotropic matter distribution within clouds. However, the high force resolution achieved in the simulations considered here allows us to estimate, with reasonable precision, the evolution of the velocity dispersion and the degree of relaxation of the compressed matter.

The mean values of the velocity dispersions \( \sigma_1, \sigma_2, \sigma_3 \) (equation 23), and even their PDFs presented in Fig. 2 for three redshifts, are similar to each other. However, the PDFs of the degree of relaxation presented in Fig. 4 demonstrate significant differences in the characteristics of the velocity components. Indeed, at all redshifts, the excess of larger values of \( w_1 \geq 1 \) indicates a noticeable contribution of the retained matter expansion and a moderate degree of relaxation along the longest axis. At the same time, the excess of smaller values of \( w_2 \leq 1 \) demonstrates the domination of matter compression along the middle axis and the progressive transformation of a pancake-like cloud into a filamentary-like cloud. The degree of relaxation along this axis is moderate, but it is larger than that along the longest axis.

In contrast, the more symmetric shape of the PDF \( P(w_3) \) in Fig. 4 indicates that a similar fraction of matter is expanded and compressed along the shortest axis. This figure shows a progressive point concentration near the centre, which reflects the progressive randomization of velocities and the growth of the degree of relaxation of the compressed matter. An evaluation of the fraction of relaxed clouds and the corresponding matter fraction \( f_{rel} \) (Table 1) characterizes these processes quantitatively.

Similar characteristics of the velocity dispersions are also obtained with the simulations \( S_{500} \) and \( S_{200} \). The most serious quantitative divergences are found for the characteristics of low-mass clouds; these are discussed in Section 3.

It is interesting that the estimated state of relaxation of clouds with the core-sampling approach in equation (42) is very close to that obtained with the 3D analysis. This fact shows that the random orientation of the clouds with respect to the line of sight results in the intermixing of thermal and large-scale bulk velocities, but does not distort strongly the measured velocity dispersions and the degree of relaxation of compressed matter.

Special analysis confirms that the subpopulations of colder and hotter clouds with \( \sigma_1 \leq \langle \sigma_i \rangle \) and \( \sigma_1 \geq \langle \sigma_i \rangle \), respectively, contain comparable fractions of matter with different mean richness and mass functions. Both subpopulations are dominated by partly relaxed clouds slowly expanding along the longest principal axis. The colder clouds represent the population of anisotropic low-mass clouds. Both the shape of their mass function and the PDFs of other characteristics are strongly dependent on the parameters used for the selection of the considered DM clouds. In contrast, for a population of richer clouds, the PDFs of cloud characteristics only weakly depend on the parameters that select these clouds.

A strong difference between velocity dispersions \( \sigma_1 \) (equation 9) and \( \sigma_1 \) (equation 23), related to the motion of clouds as a whole and to velocities within clouds, is caused by the large-scale spatial modulation of the initial velocity field in regions of cloud formation. The same effect is seen in estimates of the relative velocities of neighbouring clusters, \( \Delta U \) (equation 39).

5.5 Angular momentum of the LSS elements

Recently, the angular momenta of the LSS elements have been widely discussed and compared with the alignments and angular momenta of galaxies and their DM haloes (see, for example, Vitvitska et al. 2002; Patiri et al. 2006; Aragon-Calvo et al. 2007, 2010; Lee & Erdogdu 2007; Park et al. 2007; Cuesta et al. 2008; Paz et al. 2008; Slosar et al. 2009; Zhang et al. 2009; Jimenez et al. 2010; Lovell et al. 2010). The main inferences of the TTT and, in particular, the preferential orientation of the angular momenta along the shortest axes of the LSS elements are basically confirmed. The influence of the anisotropic matter infall into the LSS elements on their angular momenta has been discussed by Doroshkevich (1973) for baryonic component, and by Bernando et al. (2002) for DM
particles. However, the analysis of simulations shows that, for the DM component, the contribution of this effect is moderate. Our results obtained in Section 3.4 are consistent with TTT predictions.

However, a large scatter is found between the shape of the host LSS element and the angular momenta of galaxies and their DM haloes incorporated in that element. This scatter is expected and can be caused by the action of several factors.

Thus, a comparison of the functions \( J \) and \( j \) (equation 31) shows that the process of generation of angular momentum of DM clouds is only moderately efficient (less than 20 per cent). This fact, in itself, suggests a weak correlation between the angular momenta of the host LSS element and high-density low-mass subclouds, and it basically explains the scatter mentioned above. Moreover, a more detailed analysis of the process of halo formation (Vivitskva et al. 2002) shows strong evolution of the angular momentum of DM haloes and its dependence upon the angular momenta of merged satellites. Perhaps a stronger correlation can be found between properties of the host cloud and the system of DM haloes and its satellites.

Let us note also that this scatter is partly enhanced by uncertainties in the determination of the complex shapes of the LSS elements. This is especially important for observations when the shape of such elements can be found only approximately.

### 5.6 DM LSS elements and the Ly\(\alpha\) forest

The main observed characteristics of the Ly\(\alpha\) forest have been analysed in many publications (see, for example, Demiański et al. 2006 and references therein). Detailed discussions of the observed characteristics of the Ly\(\alpha\) absorbers can be found, for example, in Kim, Cristiani & D’Odorico (2002b) and Kim et al. (2002a, 2004).

There are some essential differences between the sample of simulated DM clouds and the observed Ly\(\alpha\) absorbers. The most important of these are as follows.

(i) The Ly\(\alpha\) forest is related to LSS elements with sizes, masses and other parameters much smaller than those simulated.

(ii) The Ly\(\alpha\) forest is observed for larger neutral hydrogen column density, \(N_{\rm HI} \geq 10^{12} \text{cm}^{-2}\), which depends upon the UV background and is not directly linked with the DM component of absorbers.

(iii) The evolution of the observed Ly\(\alpha\) absorbers depends upon the poorly known UV background, and can significantly differ from the evolution of the simulated DM LSS.

These differences prevent a direct quantitative comparison of characteristics of the simulated DM LSS and the observed Ly\(\alpha\) forest. However, the absorbers are dominated by the DM component, and so a qualitative comparison of their properties and evolution with that simulated allows us to clarify some problems of the forest evolution.

First of all, the self-similar characteristic of the evolution of the Doppler parameter, \(b\), is reflected in the surprising stability of its PDFs (Demiański et al. 2006). The \(b\) parameter strongly depends upon the DM component and we can expect that the most important aspects of its evolution are closely correlated with the characteristics of the evolution of DM clouds.

The weak redshift dependence of the Doppler parameter and the high degree of matter relaxation along the line of sight for the majority of the Ly\(\alpha\) absorbers are consistent with the results obtained in Sections 3.3, 3.5 and 4.1. It is also important that the complex shape of the PDF \(P(b)\) is similar to the PDF \(P(\sigma_s)\) plotted in Fig. 2.

Our analysis in Section 4.2 cannot reproduce the criteria of the Ly\(\alpha\) forest selection, and in particular the redshift evolution of clump separation (equations 44 and 45) does not represent the evolution of the linear number density of the forest elements. The most important among the unknown factors are the disintegration of richer forest elements into a system of high-density subclouds, linked by extended low-density bridges, and variations of the ionizing UV background. The comparison of the LSS evolution in simulations \(S_{1825}\) and \(S_{305}\) confirms that the small-scale perturbations accelerate the disintegration of the LSS elements and their internal structure becomes more complex. These problems need to be studied in more detail with more representative and high-resolution simulations.

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