Seeing measurements using the solar limb – I. Comparison of evaluation methods for the Differential Image Motion Monitor

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ABSTRACT
Differential Image Motion Monitors (DIMMs) are used not only for stellar observations but also for solar observations with the limb as the target to evaluate the seeing. In stellar observations, the differential image motions along and perpendicular to the DIMM baseline are consistently employed to evaluate Fried’s parameter. However this is not the case for solar-limb observations, in which we can measure the image motion only perpendicular to the limb. Therefore the validity of the methods so far proposed to calculate Fried’s parameter from a DIMM using the solar limb is still open to examination. We have focused on this problem and carried out DIMM observations of the solar limb using a Shack–Hartmann wavefront sensor of the Domeless Solar Telescope at Hida Observatory. Pairs of apertures in the Shack–Hartmann sensor act as multiple DIMMs that have different baselines in distance and orientation. We calculated Fried’s parameters based on three evaluating methods (those of Sarazin & Roddier, Sasiela and Conan et al.) and found that the method of Sarazin & Roddier gives a consistent value of Fried’s parameter over the set of multiple DIMMs.

Key words: atmospheric effects – Sun: general.

1 INTRODUCTION
Atmospheric seeing is one of the most crucial factors determining the spatial resolution in ground-based optical observations. The seeing depends on landforms, time, season, etc. and the pattern of fluctuation changes within a few milliseconds. Quantitative evaluation of the seeing is important for site surveys for new ground-based telescopes, and in designing an adaptive optics system that restores the flat wavefront using a deformable mirror.

One of the quantitative expressions for the seeing is Fried’s parameter $r_0$ (Fried 1965). Fried’s parameter corresponds to the effective aperture of a telescope that determines the spatial resolution; i.e. even with a large telescope the spatial resolution deteriorates comparable to the diffraction limit of a telescope with an aperture $r_0$ when the seeing condition is expressed by $r_0$. Even in good seeing conditions, $r_0$ is typically 8 cm or smaller in daytime solar observations in visible wavelengths. Therefore, for all ground-based optical telescopes with large diameters the spatial resolution is limited by the seeing.

There are several ways to measure Fried’s parameter. In this study we deal with the Differential Image Motion Monitor (DIMM): Martin (1987; Tokovinin 2002), with which we observe image motions of an object through two separated apertures. The two apertures can be made by segmenting the aperture of one telescope. The image motion is the angle-of-arrival fluctuation of an electromagnetic wavefront incident on the plane of the aperture from a target star; thus, we can estimate the spatial distribution of the power spectrum of the atmospheric turbulence from the variance of the relative motion of the images taken with the two apertures. DIMMs are used for measurements targeting not only stars but also the Sun (e.g. Beckers 2001; Özisik & Ak 2004). In observations of stars the target is a point source, while in observations of the Sun the target is mainly the solar limb, i.e. a spatially extended object. Sunspots may serve as a good target for a DIMM, but their size, shape and even presence on the disc change from day to day. Therefore for a DIMM aiming to obtain a continuous data set of the seeing for a site survey or for designing an adaptive optics system, the solar limb is the only suitable target.

Three methods were proposed for converting the variance of image motion to Fried’s parameter, i.e. those by Sarazin & Roddier (1990), Sasiela (1994) and Conan et al. (2000) for point-source DIMM observations. The differences among these methods are the mathematical representation of the tilt of a wavefront in a pupil and the definition of the pupil function. These methods have been used to analyse solar-limb observations with DIMMs; Beckers (2001)...

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used the Sarazin & Roddier method and Özışık & Ak (2004) used Sasiela’s method. However, it is not yet clear which method is suitable for a DIMM using extended objects such as the solar limb. Our aim is to check whether these methods are valid to estimate $r_0$ based on the actual solar-limb observations.

To verify whether these methods work properly for the solar limb, we realized multiple DIMMs with different baselines using a Shack–Hartmann wavefront sensor. A Shack–Hartmann sensor consists of a micro lens array and a high-speed camera. In this sensor, a telescope pupil is imaged on the micro lens array and the image of the solar limb is focused on the camera; thus images from subapertures dividing the pupil of the telescope are obtained. Each pair of subapertures acts as a DIMM, so that multiple DIMMs are realized. These multiple DIMMs work under the same atmospheric conditions, and therefore the values of $r_0$ calculated from any pair of subapertures are expected to be consistent with each other. We examine whether the aforementioned methods satisfy this condition in this study.

In Section 2, we describe the mathematical expressions for converting the variance of the differential image motion to $r_0$ and discuss the differences among the methods mentioned above. In Section 3, we describe observation of the solar limb with the Shack–Hartmann sensor and the data analysis method. In Section 4, we show the results of the analysis and discuss the characteristics of the DIMM based on our observations.

2 METHODS TO CONVERT DATA FROM A DIMM TO FRIED’S PARAMETER

In this section, we describe the methods to convert the observed variance of the image motion to Fried’s parameter $r_0$, and explain differences in the mathematical expressions of Sarazin & Roddier (1990), Sasiela (1994) and Conan et al. (2000). We assume that atmospheric non-uniformity is described by the Kolmogorov turbulence model with outer and inner scales of turbulence of infinity and zero, respectively. The power-spectrum density of the wavefront fluctuation $W_0(k)$ is described using $r_0$ (Roddier 1981) as

$$W_0(k) = \hat{\phi}(k) \hat{\phi}^*(k) = 0.0229 r_0^{-3/2} k^{-11/3} \text{ (rad}^2 \text{m}^{-1}), \tag{1}$$

where $k$ is the spatial frequency (m$^{-1}$) of the wavefront error $\phi(x)$ and $\hat{\phi}$ and $\hat{\phi}^*$ are the Fourier transform and its complex conjugate, respectively. The electromagnetic wavefront fluctuation due to atmospheric turbulence changes as a function of time. We interpret the temporal fluctuation observed by a DIMM as a spatial fluctuation by adopting Taylor’s frozen hypothesis (Taylor 1938), i.e. the temporal variation of the wavefront error at a particular spatial point is produced by the translation of a fixed pattern in the air by wind. Thus if we denote the temporal fluctuation of the image displacement from a pupil at a location $r$ as $\alpha(t)$ and the spatial fluctuation of the wavefront tilt (angle of arrival) as $\alpha(r)$, the relation $\alpha(t) \propto \alpha(t-v\tau)$ holds, where $\tau$ is time and $v$ is the wind velocity. Note that this could be a valid assumption when the exposure time is sufficiently short (Soules et al. 1996).

In a DIMM, we observe the variance of a temporal series of differential image motion of two apertures placed at $r_1$ and $r_2$, namely $\langle [\alpha(t) - \alpha(t)]^2 \rangle$, where $\langle \rangle$ denotes the temporal or spatial mean of a quantity inside. Under Taylor’s hypothesis, the variance of a temporal series can be expressed mathematically as an integral of a spatial power-spectrum density. First we express the power spectrum of the difference between the angle-of-arrival fluctuations at the displaced apertures, $\alpha(r_1) - \alpha(r_2)$; secondly we integrate it in $k$-space to obtain the variance $\langle [\alpha(r_1) - \alpha(r_2)]^2 \rangle$, i.e. $\langle [\alpha(t) - \alpha(t)]^2 \rangle$; finally, using equation (1), we obtain the relation between the variance and $r_0$.

We define an operator $\delta$, that returns a slope $\phi(x)$ in a pupil and a pupil function $P(x)$. In the case of a circular pupil with diameter $D$, $P(x)$ is given as follows:

$$P(x) = \begin{cases} 1 & \text{if } |x| \leq D/2, \\ 0 & \text{if } |x| > D/2. \end{cases} \tag{2}$$

We define a function $P_r(x)$ that describes a pupil placed at $r$, so that with Dirac’s $\delta$-function, $P_r(x)$ is written as

$$P_r(x) = P(r - x) = \int d\rho \delta((r - x) - \rho) P(\rho) = \delta(x) * P(r - x), \tag{3}$$

where ‘*’ denotes the convolution operator in $x$-space.

The angle of arrival averaged over the pupil, $\alpha(r)$, is written using equation (3) as

$$\alpha(r) = \int dx P_r(x) \delta_x[\phi(x)] = \int dx \left[ \delta(x) * P(r - x) \right] \delta_x[\phi(x)] = \delta(r) * P(r) * \delta_x[\phi(x)]. \tag{4}$$

Next, we place two apertures at $r_1$ and $r_2$, so that we can write the differential angle of arrival between the two apertures, $f(r_1, r_2)$, using equation (4) as

$$f(r_1, r_2) = \alpha(r_1) - \alpha(r_2) = \delta(r_1) * P(r_1) * \delta_x[\phi(r_1)] - \delta(r_2) * P(r_2) * \delta_x[\phi(r_2)]. \tag{5}$$

Here we define $r_2 - r_1 = B$ and $r_1 = r$ and regard $r$ as variable and $B$ as a fixed vector, i.e. in Taylor’s frozen picture a DIMM with a separation of two apertures $B$ moves under the turbulent air.

The power-spectrum density of $f_a(r) = f(r_1, r_2)$, $W_f(k)$ is expressed in the following way in use of the Fourier transform operator $\hat{\delta}$:

$$W_f(k) = |\hat{\delta} \left[ f_a(r) \right]|^2 = |\hat{\delta} \left[ \delta(r) - \delta(r + B) \right]|^2 |\hat{\delta} \left[ P(r) * \delta_x(\phi(r)) \right]|^2. \tag{6}$$

Here $\hat{\delta} \left[ \delta(r) - \delta(r + B) \right] = 1 - \exp(2\pi i k \cdot B)$. The variance of the differential image motion is then obtained from the power spectral density using the following formula:

$$\langle |f_a|^2 \rangle = \int dk W_f(k). \tag{7}$$

The definitions of the basic parameters of a DIMM are summarized in Fig. 1. The $x$-axis is set perpendicular to the solar limb, i.e. the direction of the measurement of image motion, and $\theta$ is the angle between the $x$-axis and the line connecting the two apertures.

The three methods described below are different in definition of the pupil function $P(x)$ and the operator $\delta_x$, so that $\langle |f_a|^2 \rangle$ for each method are expressed by formulae for $D, B, \theta, r_0$. The $x$-axis is set perpendicular to the solar limb, i.e. the direction of the measurement of image motion, and $\theta$ is the angle between the $x$-axis and the line connecting the two apertures. The three methods described below are different in definition of the pupil function $P(x)$ and the operator $\delta_x$, so that $\langle |f_a|^2 \rangle$ for each method are expressed by formulae for $D, B, \theta, r_0$, which are different from each other.

Z-tilt (Sasiela 1994). In this method, $P(x)$ is a circular aperture with radius $D/2$ and $\partial_x(\phi)$ is defined as the slope of the tilt term in
the Zernike polynomial fitting for \( \phi(x) \) in terms of aperture. The Fourier transform of \( P \ast \partial_x \phi(x) \) is the following:

\[
\hat{\mathcal{P}} \{ P \ast \partial_x \phi(x) \} = 4k_r^i J_{\nu}(2\pi k r) \hat{\phi}(k),
\]

where \( \hat{\phi}(k) \) is the Fourier transform of \( \phi(x) \) and \( J_n(z) \) is the first-kind \( n \)th Bessel function. Thus, the power-spectrum density of \( f_b(r) \) is written, using equations (1), (5), (6) and (8), as

\[
W_f(k) = 1.46\lambda^2 k_r^3 r_0^{-5/3} k^{-11/3} \left[ \frac{J_2(\pi k D)}{\pi k D} \right]^2 \times \{ 1 - \exp(2\pi i k \cdot B) \}^2.
\]

The integral of \( W_f(k) \) over the azimuth in \( k \)-space is given by

\[
W_f(k) = 4.58\lambda^2 k_r^3 r_0^{-5/3} k^{-2/3} \left[ \frac{J_2(\pi k D)}{\pi k D} \right]^2 \times \{ 1 - J_0(2\pi k B) + \cos(2\theta) J_2(2\pi k B) \}.
\]

G-tilt (Conan et al. 2000). In this method, \( P(x) \) is a circular aperture with radius \( D/2 \) and \( \partial_x \phi(x) \) is defined by the spatial average of \( \partial_x \phi(x) / \partial x \) in the pupil. Thus the Fourier transform of \( P(x) \) and \( \partial_x \phi(x) \) are the following:

\[
\hat{\mathcal{P}} \{ P(x) \} = \frac{2J_2(\pi k D)}{\pi k D},
\]

\[
\hat{\mathcal{P}} \{ \partial_x \phi(x) \} = -2\pi i k \hat{\phi}(k),
\]

so that the power-spectrum density of \( f_b(r) \) is written as

\[
W_f(k) = 0.0916\lambda^2 k_r^3 r_0^{-5/3} k^{-11/3} \left[ \frac{J_2(\pi k D)}{\pi k D} \right]^2 \times \{ 1 - \exp(2\pi i k \cdot B) \}^2.
\]

When we integrate \( W_f(k) \) over the azimuth in \( k \)-space, we obtain

\[
W_f(k) = 0.288\lambda^2 k_r^3 r_0^{-5/3} k^{-2/3} \left[ \frac{J_2(\pi k D)}{\pi k D} \right]^2 \times \{ 1 - J_0(2\pi k B) + \cos(2\theta) J_2(2\pi k B) \}.
\]

Figure 1. Parameters defining the DIMM geometry. The \( x \)-axis is set perpendicular to the solar limb, i.e. the direction of the measurement of the image motion. \( D \) is the diameter of the apertures, \( r \) is the position of one of the apertures, \( B \) is the distance between the two apertures and \( \theta \) is the angle between the \( x \)-axis and the line connecting two apertures. The edge of the bright area in the apertures is regarded as the solar limb.

Sarazin & Roddier (1990). In this method, the definitions of \( P(x) \) and \( \partial_x \phi(x) \) are different for the autocorrelation and cross-correlation terms, as follows. The variance of the differential image motion \( \langle |f_b(r)|^2 \rangle \) can be written as the following expression:

\[
\langle |f_b(r)|^2 \rangle = \langle |\alpha(r) - \alpha(r + B)|^2 \rangle = 2\langle |\alpha(r)|^2 \rangle - \langle |\alpha(r) \alpha(r + B)\rangle \rangle.
\]

In equation (13), \( 2\langle |\alpha(r)|^2 \rangle \) is the autocorrelation term and \(-2\langle \alpha(r) \alpha(r + B)\rangle \) is the cross-correlation term of the two apertures. These terms are reduced from the product of the Fourier transform of the \( \delta \) functions in the power-spectrum density of equation (6), i.e.

\[
|\hat{\mathcal{P}} \{ \delta(r) - \delta(r - B) \} |^2 = 1 - \exp(2\pi i k \cdot B)\}\)^2.
\]

where \( 2\langle |\alpha(r)|^2 \rangle \) is the autocorrelation term and \(-2\cos(2\pi k \cdot B) \) is the cross-correlation term.

Sarazin and Roddier assumed for the autocorrelation term that \( P(x) \) is a circular aperture with radius \( D/2 \) and \( \partial_x \phi(\phi) \) is defined as the slope of the tilt term in the Zernike polynomial fitting for \( \phi(x) \) in an aperture. Thus,

\[
\hat{\mathcal{P}} \{ P \ast \partial_x \phi(x) \} |_{\text{auto}} = 4k_r^i J_{\nu}(2\pi k D) \frac{\pi k D}{\pi k} \hat{\phi}(k).
\]

On the other hand, for the cross-correlation term, \( P(x) \) is a point source with zero area and \( \partial_x \phi(\phi) \) is defined by \( \partial_x \phi(x) / \partial x \) at the centre of the pupil. Thus

\[
\hat{\mathcal{P}} \{ P(x) \} |_{\text{cross}} = 1,
\]

\[
\hat{\mathcal{P}} \{ \partial_x \phi(x) \} |_{\text{cross}} = -2\pi i k \hat{\phi}(k).
\]

Using equations (15) and (16), the power-spectrum density of \( f_b(r) \) is written as

\[
W_f(k) = 0.0229\lambda^2 k_r^3 r_0^{-5/3} k^{-11/3} \times \left\{ 64 \left[ \frac{J_2(\pi k D)}{\pi k D} \right]^2 - 2\cos(2\pi i k \cdot B) \right\}.
\]

When we integrate \( W_f(k) \) over the azimuth in \( k \)-space, we obtain

\[
W_f(k) = 0.0719\lambda^2 k_r^3 r_0^{-5/3} k^{-2/3} \times \left\{ 64 \left[ \frac{J_2(\pi k D)}{\pi k D} \right]^2 - J_0(2\pi k B) + \cos(2\theta) J_2(2\pi k B) \right\}.
\]

To calculate the variance of the differential image motion \( \langle |f_b(r)|^2 \rangle \), we take the integral of this power spectrum over \( k \),

\[
\langle |f_b(r)|^2 \rangle = \int_0^\infty dk W_f(k) = 2 \left( \frac{\lambda}{\lambda_{\text{D}}} \right)^{5/3} \left( \frac{\lambda_{\text{D}}}{D} \right)^{1/3} \left[ 1 - \frac{1}{3} \cos^2 \theta \right].
\]

When setting \( \theta \) as 0° and 90°, we obtain the same expressions as equations (9) and (10) in Sarazin & Roddier (1990).
Comparison of the three methods. When the exposure time is sufficiently short, we can convert $\langle |f|^2 \rangle$ i.e. $\langle |\alpha(t) - \alpha(t')|^2 \rangle$ to Fried’s parameter by using the formulation obtained from these three methods. We show examples of the power spectra $W_f(k)$ for the three methods in Fig. 2. The left panel shows the case for $\theta = 0^\circ$, the right panel for $\theta = 90^\circ$. In each figure, the power spectrum in the low-frequency region is suppressed due to the finite separation of the DIMM, since wavefront errors with wavelength longer than the separation of a DIMM aperture produce only small differential motions, while the finite size of each aperture in the G-tilt and Z-tilt models suppresses the Kolmogorov spectrum in the high-frequency range due to spatial averaging in the apertures.

It is noticeable that the powers in the high-frequency range for the Sarazin & Roddier method show significant difference between $\theta = 0^\circ$ and $\theta = 90^\circ$, while those for the Z-tilt and G-tilt methods are nearly the same. Such a difference between the Sarazin & Roddier method and the others arises from the issue of whether the cross-correlation term in equation (13) is significant in the high-frequency range or not. In the Z-tilt and G-tilt models, the cross-correlation terms are multiplied by the aperture term so that the high-frequency part of this term becomes smaller than that in the Sarazin & Roddier method. When we integrate $W_f(k)$, the contributions from the cross-correlation terms in the Z-tilt and G-tilt methods are small compared with that in the Sarazin & Roddier method. Therefore, the dependence of spatial distribution of $W_f(k)$ on $\theta$ for the Sarazin & Roddier method is more significant than for the Z-tilt and G-tilt methods.

The difference between the Z-tilt and G-tilt methods is in whether we take the higher orders of the Zernike polynomials into account or not. Following Noll (1976), we define the radial order to be $m$ and the angle order to be $n$ in the Zernike series. In the case in which $n$ is an odd number greater than 1 and $m$ is 1, namely for a ‘coma aberration term’, the spatially averaged $\partial f(x)/\partial x$ over the whole aperture is not 0 and the calculated $\langle |f|^2 \rangle$ from an integration of $W_f(k)$ over $k$ is systematically greater by about 20 per cent in the Z-tilt method than in the G-tilt method, regardless of the form of the wavefront error. This difference becomes more significant as $B$ becomes smaller. Thus, when we observe a point source using a DIMM we have to choose the proper method depending on the method for determining the image position. Namely, when we define the position of an image by its centre of gravity, which is influenced by the coma aberration, we should adopt the G-tilt method. When we define the position of an image by its position of peak intensity, the Z-tilt method is more appropriate. The question of which method is more suitable for application to the sharp edge of an object like the solar limb is the subject of the following sections.

3 OBSERVATIONS AND DATA ANALYSIS

3.1 Observation system

A Shack–Hartmann wavefront sensor installed in an experimental adaptive optics system deployed in the room of the horizontal spectrometer of the Domeless Solar Telescope (DST: Nakai & Hattori 1985) at Hida observatory in Japan is used in our investigation. The tip-tilt mirror and the deformable mirror in the adaptive optics system were turned off in the observations. We monitor the images of the Shack–Hartmann sensor together with the images produced by the full aperture of the DST, which is of 60 cm diameter, so that we can check the real images influenced by the seeing. The full-aperture images are used only for subsidiary information and are not analysed in this study. Each pair of subapertures of the Shack–Hartmann sensor is regarded as a DIMM, and we analyse the image motion in each subaperture along the direction perpendicular to the limb of the Sun. Therefore, we can obtain the DIMM results as a function of $B$ and $\theta$.

In the Shack–Hartmann sensor, the pupil of the DST is divided into $8 \times 8$ subapertures with a microlens array located at a pupil image. (Each microlens has a $600 \mu m \times 600 \mu m$ aperture and a focal length of 28.2 mm.) Each subaperture corresponds to a 75 mm $\times$ 75 mm square in the real aperture and has a field of view (FOV) of $33 \times 33$ arcsec$^2$, which is imaged by a 30 pixel $\times$ 30 pixel CCD camera. Examples of solar images taken with the Shack–Hartmann sensor and the DST full aperture are shown in Fig. 3. The CCD camera of the Shack–Hartmann sensor is a DALSA CA-D6-0256, which takes 955 images per second with $260 \times 260$ pixels and 8-bit analogue-to-digital (A/D) conversion, and the images are stored on the hard drive of a PC/AT (CPU Intel Core2 DUO 6600 2.4 GHz, 2 GB memory) computer. In each run of the data acquisition we take 1000 frames during about 1 s and 10 runs are repeated in each sequence, which takes about 20 s for completion. The sequences are repeated every 10 min. The properties of the data acquisition system are listed in Table 1.
Table 1. Data acquisition properties.

<table>
<thead>
<tr>
<th>Data</th>
<th>Shack–Hartmann image</th>
<th>Full aperture image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposure time</td>
<td>1.05 msec</td>
<td>150 msec</td>
</tr>
<tr>
<td>Frame rate</td>
<td>955 Hz</td>
<td>about 5 Hz</td>
</tr>
<tr>
<td>Sequence</td>
<td>1000 frames × 10 files</td>
<td>1 frame × 10 files</td>
</tr>
<tr>
<td>Interval of sequence</td>
<td>10 min</td>
<td>10 min</td>
</tr>
<tr>
<td>FOV</td>
<td>33 arcsec × 33 arcsec</td>
<td>74 arcsec × 55 arcsec</td>
</tr>
<tr>
<td>FOV pixel⁻¹</td>
<td>1.17 arcsec</td>
<td>0.05 arcsec</td>
</tr>
</tbody>
</table>

Other

| Aperture              | 75 mm × 75 mm (square) | ∅600 mm             |
| Central obscuration   | ∅210 mm               |                     |
| Observation wavelength| 600 ± 50 nm           | 600 ± 50 nm         |

We carried out observations from 2008 May 21–2008 November 29. The number of data sets in each time/date window is shown in Table 2. As seen in this table, there is no bias in the number of observations during 1000–1600 (JST). Except for the summer season, during which we obtained fewer data sequences due to other observation programmes, there is no significant seasonal bias. After discarding (1) data for which it is difficult to define the limb because of the low brightness and (2) data in which there is a subaperture missing the limb from its FOV, we analyse 4223 out of the 5920 1-s data sets listed in Table 2.

3.2 Data reduction

3.2.1 Flat field and definition of subapertures

A flat-field image of the Shack–Hartmann sensor is obtained by averaging 1000 × 10 images taken at the quiet disc centre. During the flat-field data acquisition, which takes about 20 s, distortion and motion of the images due to the seeing and the guiding error of the telescope smear out granular structures; thus the average of the entire images provides a fairly good flat field. Fig. 4 shows the flat-field image of the Shack–Hartmann sensor (left) and the telescope aperture divided by the microlens array (right). In the right panel, the grey areas correspond to the outside of the DST aperture. Among subapertures extending over the edge of the DST pupil and the central obscuration, those where the area of the real aperture occupies less than 65 per cent of the subaperture are not used in the data reduction. The dark squares in the right panel are thus discarded subapertures and the remaining 48 white squares...
indicate the valid subapertures for our DIMM analysis. The 1000 × 10 images in a sequence targeting the limb are processed with the dark and flat-field images, and valid subaperture images are extracted from each image by referring to the grid in the flat-field image.

3.2.2 Definition of the image motion

Since the FOV of the Shack–Hartmann sensor is small, the solar limb in the FOV is approximately a straight line. Therefore the measurable motion of the solar limb is limited to the direction perpendicular to the limb. The method to determine the image motion in each subaperture is as follows. First, we define the tangent of the limb in an image produced by averaging 1000 images taken in one second. Secondly, if the absolute value of the angle between the normal line to the limb and the bottom edge of the FOV is less than \( \pi/4 \), we shift the second and subsequent frames horizontally and search for a maximum value of correlation coefficients with the first frame in a horizontal slit of 2-arcsec width crossing the centre of the FOV. If the absolute value of the angle is larger than \( \pi/4 \), we perform the same procedure in the vertical direction. We obtain the image motion with subpixel resolution by a parabolic fitting of the three pixels around the peak of the correlation coefficients.

We obtain a time series of the image motion in this way for each of the valid subapertures for each sequence (1000 frames × 10 files) and then we reduce the image motion in the horizontal or vertical directions to the image motion in the line normal to the limb by applying the cosine of the angle between the normal line to the limb and the bottom edge of the FOV. The centre of the image motion is defined for each subaperture by the mean position of the three pixels around the peak of the correlation coefficients.

For each data set taken in 1 s, we determine the parameters of each pair of subapertures, i.e. the distance between the pair of subapertures \( B \) and the angle between the line connecting subapertures and the normal to the limb, \( \theta \). Parameter \( B \) is determined as the distance between the centre of gravity of the areas of two subapertures. The distribution of \( \theta \) in our data is shown in Fig. 5. We sampled \( \theta \) almost uniformly over the range from \(-\pi/2\) to \(\pi/2\).

The error in the measurement of the differential image motion is estimated by using images of a ‘diaphragm’, which has 192 arcsec diameter, placed at the primary focus of the DST (see Fig. 6). Usually, the FOV is set at the centre of the diaphragm (‘observation’ in Fig. 6). When we set the edge of the diaphragm (‘calibration’ in Fig. 6) in the FOV of the Shack–Hartmann sensor and set the pointing of the telescope to the solar disc centre, the edge of the diaphragm can be regarded as a proxy of the solar limb. Any shift of the diaphragm images must be identical for all subapertures, since most of the light path is evacuated in the telescope and therefore the seeing effect is negligible. Differences in the image shifts between the subapertures provide an estimate of the error in our measurement of the differential image motion. We show the histogram of the differential image motion of the diaphragm from eight sequences in Fig. 7, in which the horizontal axis is variance of the differential image motion in units of (pixel)\(^2\). The variance of the differential image motion with typical seeing is about 1 arcsec\(^2\) (∼1 pixel\(^2\)). Therefore, the error is less than 0.1 per cent of the typical seeing in the variance and gives a negligibly small influence.

3.2.3 Fried’s parameter \( r_0 \)

We convert the variance of the differential image motion thus obtained to \( r_0 \) using the three methods described in Section 2. Circular apertures are assumed in these methods, but the real subapertures used here are square or even have irregular shapes strained by the edge of the aperture of the DST. We define the effective diameter,
4 RESULTS AND DISCUSSIONS

In Section 3, we computed Fried’s parameter $r_0$ using the three methods discussed in Section 2 for subapertures that have various geometrical configurations $B$ and $\theta$ where $B$ is the distance between two apertures and $\theta$ is the angle between the solar limb and the line connecting the two apertures (see Fig. 1). In Section 4, we examine the dependence of $r_0$ derived from the three methods on $(B, \theta)$. The distribution of the variance of the differential image motions $<|f_B|^2>$ on the $(B, \theta)$ plane obtained from the entire data set is shown in Fig. 8, in which the values of $<|f_B|^2>$ are normalized by the mean value for each data set. It is remarkable that the variance tends to be larger for smaller $|\theta|$, i.e. the variance is larger for differential motion in the direction of the baseline of the DIMM than for motion perpendicular to the baseline of the DIMM when $B/D$ is larger than approximately 2. This anisotropy can be understood as a consequence of the statistical properties of the influence of turbulent eddies on a DIMM as discussed in Sarazin & Roddier (1990).

To examine the distribution of $r_0$ on $(B, \theta)$ using the whole data set, it is necessary to normalize $r_0$ using a representative $r_0$ in each data sequence. We defined the representative $r_0$ for each data sequence, $<r_0>$, as the median of $\sqrt{D^2}$ (see Fig. 11).

The histogram of $r_0$ on $(B, \theta)$ plane calculated with the Sarazin & Roddier, G-tilt and Z-tilt methods is shown in Fig. 9(a), (b) and (c), respectively. Since the real $r_0$ does not depend on $B$ and $\theta$, a uniform distribution of $r_0/\langle r_0 \rangle$ is expected in Fig. 9. In this figure, the result obtained by the Sarazin & Roddier method gives a fairly uniform distribution of $r_0/\langle r_0 \rangle$ over the $(B, \theta)$ plane. On the other hand, the results obtained by the G-tilt and Z-tilt methods show notable deviations from uniformity. This means that Fried’s parameter calculated with these two methods has an error depending on the orientation of the two subapertures and the solar limb. The top and bottom panels in Fig. 10 show $\langle r_0 \rangle/\langle r_0 \rangle(\theta)$ and $\langle r_0 \rangle/\langle r_0 \rangle(B)$ against $\theta$ and $B$, respectively, where $\langle r_0 \rangle/\langle r_0 \rangle(\theta)$ is $\langle r_0(B, \theta) \rangle/\langle r_0 \rangle$ averaged over $B$ and $\langle r_0 \rangle/\langle r_0 \rangle(B)$ is $\langle r_0(B, \theta) \rangle/\langle r_0 \rangle$ averaged over $\theta$. Black +, blue × and red * symbols stand for the Sarazin & Roddier, G-tilt and Z-tilt methods, respectively. The dependence of $r_0$ on $B$ and $\theta$ is remarkable in the results of the Z-tilt and G-tilt methods, but it is much smaller for the Sarazin & Roddier method. It is notable that the variation of $\langle r_0 \rangle/\langle r_0 \rangle$ from the Sarazin & Roddier method is less than 10 per cent over the different geometrical configurations of the two apertures, even though Sarazin & Roddier expressions are an approximation working only for $B > 2.5D$ (Sarazin & Roddier 1990).

The histogram of $\langle r_0 \rangle$ for the 4223 data sets is shown in Fig. 11 for the three methods, where black, blue, and red lines show the results from the Sarazin & Roddier, G-tilt and Z-tilt methods, respectively. Insofar as we take the value from the median of the spatial distributed $r_0$, the differences in the histogram between the three methods are small. Only a small difference between the Z-tilt and the other two methods is recognizable; its origin was discussed in Section 2.

The reason why only the Sarazin & Roddier method gives a uniform distribution of $r_0$ is that the cross-correlation term in the Sarazin & Roddier method is larger than in other methods, as mentioned in Section 2. We can show using Fig. 2 and equations (10), (12) and (18) that the contribution of the cross-correlations terms in the G-tilt and Z-tilt methods is negligible at high frequencies, and then only lower frequency components change with $\theta$. On the other hand, in the Sarazin & Roddier method higher frequency components depending on $\theta$ have a non-negligible contribution. As a result, we found that the only Sarazin & Roddier method compensates for the non-uniform distribution of the observed variance $<|f_B|^2>$ and results in a uniform distribution of $r_0$ over the $(B, \theta)$ plane.

When the spatial variation of the angle of arrival in the subapertures is small, the tilt component of the fluctuation is dominant so that the observed images are only shifted and not degraded. When this is the case for all subapertures, the relative motion of the two images give the precise differential shifts of the angle of arrival. Therefore, the spectrum of the difference of the angle of arrival is significant even at higher frequencies. This is the case supposed in the Sarazin & Roddier method. On the other hand, when angle-of-arrival fluctuations in the subapertures are not negligible, the observed images are degraded. Since such image degradation is caused by averaging through turbulence, differential image
motion between degraded images is reduced in comparison with the degradation-free case. As a consequence, high-frequency components of the differential angle of arrival tend to be small. This is the case in the G-tilt and Z-tilt methods.

Figure 9. Spatial distribution of $r_0/r_0^*$ in the $(B, \theta)$ plane, calculated by (a) Sarazin & Roddier, (b) G-tilt and (c) Z-tilt methods.

Figure 10. Spatial distribution of $r_0/r_0^*$ averaged over (a) $B$ and (b) $\theta$ for the results shown in Fig. 9. Black ‘+’, blue ‘x’ and red ‘*’ symbols show the results from the Sarazin & Roddier, G-tilt and Z-tilt methods, respectively.

The reason why the Sarazin & Roddier method is suitable for this observation can be explained from a comparison between sub-aperture size and Fried parameter. In the result for our observation, the Fried parameter is about 40 mm while the subaperture size is 75 mm. The observed image with $D/r_0=1.9$ is considered to be nearly diffraction-limited. Therefore, we think that our observation conditions are best represented by the Sarazin & Roddier method.

In conclusion, the method of Sarazin & Roddier, which provides a consistent Fried’s parameter for different conditions of $(B, \theta)$, is suitable for a DIMM using the solar limb as a target.

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