Collapse of a molecular cloud core to stellar densities: the formation and evolution of pre-stellar discs

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ABSTRACT

We report results from radiation hydrodynamical simulations of the collapse of molecular cloud cores to form protostars. The calculations follow the formation and evolution of the first hydrostatic core/disc, the collapse to form a stellar core, and effect of stellar core formation on the surrounding disc and envelope. Past barotropic calculations have shown that rapidly rotating first cores evolve into ‘pre-stellar discs’ with radii up to ~100 au that may last thousands of years before a stellar core forms. We investigate how the inclusion of a realistic equation of state and radiative transfer alters this behaviour, finding that the qualitative behaviour is similar, but that the pre-stellar discs may last 1.5–3 times longer in the more realistic calculations. The masses, radii and lifetimes of the discs increase for initial molecular cloud cores with faster rotation rates. In the most extreme case we model, a pre-stellar disc with a mass of 0.22 M⊙ and a radius of ~100 au can form in a 1-M⊙ cloud and last several thousand years before a stellar core is formed. Such large, massive objects may be imaged using the Atacama Large Millimeter/Submillimeter Array. Fragmentation of these massive discs may also provide an effective route to binary and multiple star formation, before radiative feedback from accretion on to the stellar core can inhibit fragmentation. Once collapse to form a stellar core occurs within the pre-stellar disc, the radiation hydrodynamical simulations produce qualitatively different behaviour from the barotropic calculations due to the accretion energy released. This drives a shockwave through the circumstellar disc and launches a bipolar outflow even in the absence of magnetic fields.

Key words: accretion, accretion discs – hydrodynamics – radiative transfer – brown dwarfs – stars: formation – stars: low-mass – stars: winds, outflows.

1 INTRODUCTION

The first numerical calculations of the collapse of a molecular cloud core to stellar core formation, and beyond, were performed by Larson (1969) more than four decades ago. These one-dimensional radiation hydrodynamical calculations revealed the main stages of protostar formation: the almost isothermal collapse until the inner regions become optically thick, the almost adiabatic formation of the first hydrostatic core (typical radius ~5 au and initial mass ~5 Jupiter Masses M_J), the growth of this core as it accreted from the infalling envelope, the second collapse within this core triggered by the dissociation of molecular hydrogen, the formation of the stellar core (initial radius ~2 R⊙ and mass ~1.5 M_J) and, lastly, the long accretion phase of the stellar core to its final mass. More recent one-dimensional studies (e.g. Masunaga & Inutsuka 2000; Commerçon et al. 2011) have not altered this picture.

However, in one-dimension, the effects of rotation and magnetic fields cannot be examined. Larson (1972) began performing two-dimensional calculations of rotating clouds soon after his ground-breaking one-dimensional calculations. However, multidimensional calculations are considerably more time-consuming and it was not until almost two decades after the first one-dimensional models that the first two-dimensional calculations were able to follow the collapse to the formation of the stellar core (Tscharnuter 1987). These calculations showed that with rotation both the first hydrostatic core and the later-formed stellar core became rotationally flattened and allowed the shock structures in the accretion flows to be studied in detail (e.g. Tscharnuter et al. 2009).

It was not until nearly three decades after Larson’s original calculations that the first three-dimensional calculations following the collapse to stellar core formation were possible. Bate (1998) performed three-dimensional calculations of rotating molecular cloud...
cores using a barotropic equation of state to mimic the effects of a realistic equation of state and radiative transfer. He found that if the original molecular cloud core is rotating rapidly enough, then the rotationally flattened first hydrostatic core could be dynamically unstable to the growth of non-axisymmetric perturbations. It deformed into a bar, followed by the wrapping up of the ends of the bar into a disc with strong trailing spiral arms. Gravitational torques removed angular momentum and rotational support from the inner regions of the first core, quickening the onset of the second collapse and inhibiting fragmentation during the second collapse phase. The development of a bar mode and spiral structure is expected for rapidly rotating polytropic-like structures (e.g. Durisen et al. 1986). Such instabilities occur when the ratio of the rotational energy to the magnitude of the gravitational potential energy of the first core exceeds $\beta = 0.274$. Bate was also the first to point out that because a rapidly rotating first core develops into a disc before the stellar core forms, the disc forms before the star. Rather than hydrostatic cores, such structures are better described as ‘pre-stellar discs’. Once the second collapse occurred and a stellar core formed, Bate found an inner disc formed around the stellar core and grew in radius as material with larger angular momentum fell in. Bate speculated that if the calculation were able to be followed further, then the outer radius of the inner disc would eventually grow to join on to the outer disc, leaving the stellar core surrounded by a single large disc.

The next major advance in this field was to begin including the effects of magnetic fields. Tomisaka (2002) was the first to follow the collapse of a molecular cloud core to stellar densities including magnetic fields. Using two-dimensional calculations, he found that both the first core and stellar cores launched magnetically driven outflows. This work has been followed by a number of papers using three-dimensional calculations to investigate magnetized collapse and outflows (Machida et al. 2005; Machida, Inutsuka & Matsumoto 2006, 2008), and the rotation rates and magnetic field strengths in the first and stellar cores (Machida, Inutsuka & Matsumoto 2007). At the same time, the development of non-axisymmetric structure in the first core stage and the formation of a pre-stellar disc discovered by Bate (1998) and fragmentation has been studied further both without (Saigo & Tomisaka 2006; Saigo, Tomisaka & Matsumoto 2008; Saigo & Tomisaka 2011) and with magnetic fields (Machida et al. 2005; Machida, Inutsuka & Matsumoto 2010; Machida & Matsumoto 2011). With the increasing capability of observational instruments and, in particular, looking forward to the capabilities of the Atacama Large Millimeter/Submillimeter Array (ALMA), some of these works also began to investigate the observational signatures of the first core phase such as their luminosities (Saigo & Tomisaka 2006), spectral energy distributions and images (Saigo & Tomisaka 2011).

However, all three-dimensional studies listed above have used barotropic equations of state rather than realistic equations of state with radiative transfer. The first three-dimensional calculations, including radiative transfer, that followed collapse to the point of stellar core formation (but not beyond) are described by Whitehouse & Bate (2006), using the flux-limited diffusion approximation, and by Stamatellos et al. (2007), using a radiative cooling approximation. Bate (2010) investigated the evolution of both first and stellar cores using radiation hydrodynamical calculations. He found that the evolution up until stellar core formation was qualitatively similar to that obtained with a barotropic equation of state, including bar-mode instabilities and stellar core formation. However, following the formation of the stellar core, the energy released has a dramatic effect on the surrounding disc and envelope and launches a temporary outflow even in the absence of a magnetic field. Recent two-dimensional radiation hydrodynamical simulations that follow the collapse to stellar densities and well beyond as the stellar core accretes from the infalling envelope (Schönke & Tschammler 2011) show a similar effect, where the formation of the stellar core drives a temporary outflow.

The most complete three-dimensional calculations of the collapse of molecular cloud cores to date include radiation hydrodynamics and magnetic fields (Tomida et al. 2010a,b; Commerçon et al. 2010, 2011). As with the past magnetized barotropic calculations, the first core is found to launch outflows, but now the thermodynamics of the gas is more properly treated. However, these models only study the evolution of the first core and do not follow the second collapse and formation of the stellar core.

In this paper, we extend the study of Bate (2010) who presented the first three-dimensional radiation hydrodynamical calculations to follow the collapse beyond the formation of the stellar core. Bate only presented results for one particular value of the rotation rate of the molecular cloud core. Here we investigate how the evolution of the first core or pre-stellar disc varies with different rotation rates, and we compare the results obtained using radiation hydrodynamics with those obtained using a barotropic equation of state. We also study how the thermally launched outflows following stellar core formation depend on the rotation rate and how the discs evolve. Finally, we perform many of the calculations at several different numerical resolutions to test the effects of numerical resolution and examine convergence.

2 COMPUTATIONAL METHOD

The calculations presented here were performed using a three-dimensional smoothed particle hydrodynamics (SPH) code based on the original version of Benz (1990; Benz et al. 1990), but substantially modified as described in Bate, Bonnell & Price (1995), Whitehouse, Bate & Monaghan (2005), Whitehouse & Bate (2006), Price & Bate (2007), and parallelized using both OpenMP and MPI. Gravitational forces between particles and a particle’s nearest neighbours are calculated using a binary tree. The smoothing lengths of particles are variable in time and space, set iteratively such that the smoothing length of each particle $h = 1.2(m\rho)^{1/3}$, where $m$ and $\rho$ are the SPH particle’s mass and density, respectively (see Price & Monaghan 2007, for further details). The SPH equations are integrated using a second-order Runge–Kutta–Fehlberg integrator with individual time-steps for each particle (Bate et al. 1995). To reduce numerical shear viscosity, we use the Morris & Monaghan (1997) artificial viscosity with $\alpha_\nu$ varying between 0.1 and 1, while $\beta_\nu = 2\alpha_\nu$ (see also Price & Monaghan 2005).

2.1 Equation of state and radiative transfer

The calculations presented in this paper use two different types of equations of state. The first is a barotropic equation of state, almost identical to that used by Bate (1998), where the temperature of the gas depends only on its density. In this case, the thermal pressure of the gas $p = K\rho^n$. The value of the effective polytropic exponent $\eta$ varies with density as

$$
\eta = \begin{cases} 
1, & \rho \leq 1.0 \times 10^{-13} \\
7/5, & 1.0 \times 10^{-13} < \rho \leq 5.7 \times 10^{-8} \\
1.15, & 5.7 \times 10^{-8} < \rho < 1.0 \times 10^{-3} \\
5/3, & \rho > 1.0 \times 10^{-3}.
\end{cases}
$$ (1)
We take the mean molecular weight of the gas to be $\mu = 2.38$. The value of $K$ is defined such that when the gas is isothermal, $K = c_s^2$, with the sound speed $c_s = 2.04 \times 10^4$ cm s$^{-1}$, and the pressure is continuous when the value of $\eta$ changes.

The second type of calculation is performed using radiation hydrodynamics. In this case, we use an ideal gas equation of state $p = \rho T \mathcal{R} / \mu$, where $T$ is the gas temperature and $\mathcal{R}$ is the gas constant. The thermal evolution takes into account the translational, rotational and vibrational degrees of freedom of molecular hydrogen (assuming a 3:1 mix of ortho- and para-hydrogen; see Boley et al. 2007). We also include molecular hydrogen dissociation, and the ionization of hydrogen and helium. The hydrogen and helium mass fractions are $X = 0.70$ and $Y = 0.28$, respectively. The contribution of metals to the equation of state and the thermal evolution is neglected. Two temperature (gas and radiation) radiative transfer in the flux-limited diffusion approximation is implemented as described by Whitehouse & Bate (2005) and Whitehouse & Bate (2006), except that the standard explicit SPH contributions to the gas energy equation due to the work and artificial viscosity are used when solving the (semi-)implicit energy equations to provide better energy conservation. We assume solar metallicity gas, using the interstellar grain opacity tables of Pollack, McKay & Christofferson (1985) and the gas opacity tables of Alexander (1975) (the IVA King model) (see Whitehouse & Bate 2006).

### 2.2 Initial conditions

The initial conditions for the calculations are identical to those of Bate (1998) and Bate (2010). We follow the collapse of initially uniform-density, uniformly rotating, molecular cloud cores of mass $M = 1 M_\odot$ and radius $R = 7 \times 10^6$ cm. The free-fall time of the initial clouds is $t_{ff} = 1.785 \times 10^{12}$ s (56 500 yr). The ratios of the thermal and rotational energies to the magnitude of the gravitational potential energy are $\alpha = 0.54$ and $\beta$, respectively. We have performed calculations with $\beta = 0$ (non-rotating), and $\beta = 5 \times 10^{-4}$, 0.001, 0.005, 0.01 and 0.04. Typically, each calculation was performed twice, once using the barotropic equation of state and again using radiation hydrodynamics.

To satisfy the resolution criterion of Bate & Burkert (1997) that the minimum Jeans mass during the calculation contains at least $\approx 2 N_{\text{sp}}_{\text{sp}} = 100$ particles, we require at least $1 \times 10^5$ equal-mass particles. To test for convergence, we performed calculations using $1 \times 10^5$, $3 \times 10^5$, $1 \times 10^6$ and $3 \times 10^6$ equal-mass SPH particles. Only the $\beta = 0.005$ radiation hydrodynamical calculation was performed with all these resolutions, but most calculations were performed with at least two different resolutions (see Table 1). Calculations using the highest resolution of $3 \times 10^6$ SPH particles were only performed for the $\beta = 0.001$ cases and radiation hydrodynamical $\beta = 0.005$ case. In this paper, unless otherwise stated, when a particular set of initial conditions and equations of state are discussed or a figure is presented, the calculation performed with the highest resolution is used. We discuss the degree to which the simulations are numerically converged in Appendix A and at various points in Section 3.

The calculations were performed on the University of Exeter Supercomputer SGI Altix ICE 8200.

### 3 RESULTS

The collapse of each molecular cloud core up until the formation of the stellar core proceeds in a manner that is qualitatively similar to those reported from previous three-dimensional calculations without magnetic fields and using barotropic equations of state (Bate 1998; Saigo & Tomisaka 2006; Saigo et al. 2008; Machida et al. 2010; Machida & Matsumoto 2011) for both the barotropic and the radiation hydrodynamical calculations presented here.

Fig. 1 gives the evolution with time of the maximum density and temperature for clouds with different initial rotation rates, with each calculation performed using $10^6$ particles (except for the barotropic $\beta = 0.04$ case). The initial collapse is isothermal until the maximum density exceeds $10^{-13}$ g cm$^{-3}$ using the barotropic equation of state. Using radiation hydrodynamics, the evolutions to this density are also almost isothermal. However, slight heating when using radiation hydrodynamics does slow the collapse marginally, leading to the time taken to form the first hydrostatic cores being slightly longer ($\approx 0.01–0.015 t_{ff}$) than in the corresponding barotopic calculations.

In the radiation hydrodynamical calculations, as the heating rate in the central regions exceeds the rate at which the gas can cool, the gas begins to heat up and the collapse enters an almost adiabatic phase where the temperature rises as the gas is compressed. This is mimicked in the barotropic calculations by the change in the value of $\eta$ from 1 to 7/5 (equation 1). This increasing temperature leads to the formation of a pressure-supported ‘first hydrostatic core’ (Larson 1969), which can be seen in Fig. 1 when the initial collapse stalls with central (maximum) densities of $\approx 10^{-11}$ g cm$^{-3}$ and temperatures of $\approx 30–120$ K, depending on the degree of rotational support (i.e. cores that rotate more quickly have lower maximum temperatures). Again, apart from the small offset in the time of first core formation, the barotropic and radiation hydrodynamical calculations give very similar results to this point.

Without rotation ($\beta = 0$), the first core has an initial mass of $\approx 5 M_\odot$ and a radius of $\approx 5$ au (in agreement with Larson 1969). However, with higher initial rotation rates of the molecular cloud core, the first cores become progressively more oblate (Figs 2–4). For example, with $\beta = 0.005$ using radiation hydrodynamics, before the onset of dynamical instability, the first core has a radius of $\approx 20$ au and a major-to-minor-axis ratio of $\approx 4:1$ (first panel in the third row of Figs 3 and 4). With $\beta = 0.01$, the first core has a
Formation and evolution of pre-stellar discs

Figure 1. The time-evolution of the maximum density (left-hand panels) and gas temperature (right-hand panels) during the barotropic calculations (top panels) and the radiation hydrodynamical calculations (bottom panels) of the collapse of molecular cloud cores with different initial rotation rates. For each panel, the different lines are for cloud cores with $\beta = 0, 5 \times 10^{-4}, 0.001, 0.005, 0.01$ and $0.04$ from the left-hand to right-hand side. The free-fall time of the initial cloud core $t_{ff} = 1.8 \times 10^{12}$ s (56,500 yr). Each calculation was performed with $10^6$ SPH particles, except the barotropic calculation with $\beta = 0.04$ which used $3 \times 10^5$ particles.

3.1 Slowly rotating first hydrostatic cores

The evolution of the first core up until the point of stellar core formation depends on its rotation rate. Non-rotating and slowly rotating cores evolve as they accrete mass from the surrounding infalling envelope with their central densities and temperatures increasing (Figs 1 and 5, calculations with $\beta \leq 0.001$). In the radiation hydrodynamical calculations, when the central temperature exceeds $\approx 2000$ K, molecular hydrogen begins to dissociate, leading to a second hydrodynamic collapse deep within the first core (Larson 1969). The formation of the stellar core occurs just a few years after the onset of the second collapse, during which the maximum density increases from $\sim 10^{-8}$ to $\gtrsim 0.1$ g cm$^{-3}$ and the maximum temperature increases from $\approx 2000$ to $>60,000$ K. The stellar core is formed with a mass of $M_{sc} \approx 1.5 M_\odot$ and a radius of $R_{sc} \approx 2 R_\odot$.

Without rotation, the stellar core accretes the remnant of the first core in which it is embedded in $\approx 10$ yr and then accretes the envelope (Larson 1969), though with three-dimensional calculations, we only follow the calculations for $\approx 50$–$100$ yr after stellar core formation.

Although these stages are qualitatively the same in the barotropic calculations, there is a much greater difference in the evolution of the first core between the barotropic and radiation hydrodynamical calculations than for the phase of the collapse prior to first
Figure 2. Snapshots of the column density viewed parallel to the rotation axis during the evolution of the barotropic calculations of the collapse of molecular cloud cores with different initial rotation rates. From the top to bottom, the different rows are for cloud cores with $\beta = 5 \times 10^{-4}, 0.001, 0.005, 0.01$ and 0.04. Note that the spatial scale is different for each row, with each panel measuring $1600\sqrt{\beta}$ au across (i.e. from 36 to 320 au). The free-fall time of the initial cloud core $t_{ff} = 1.8 \times 10^{12}$ s (56 500 yr). Each calculation was performed with $10^6$ SPH particles, except for the $\beta = 0.001$ case which used $3 \times 10^5$ SPH particles and the $\beta = 0.04$ case which used $3 \times 10^5$ particles. The evolution is almost identical when using $3 \times 10^5$ particles or more, but the latter are slightly more detailed.

core formation. To make this clear, in Fig. 5, the time-evolution of maximum density and maximum temperature during the calculations is replotted with $t = 0$ set to the time of stellar core formation (defined as the time when the maximum density reaches $10^{-3}$ g cm$^{-3}$). This allows us to clearly see the amount of time spent between first core formation and stellar core formation in each of the calculations. The barotropic results are plotted using the dashed lines, while the solid lines give the radiation hydrodynamical
Figure 3. Snapshots of the column density viewed parallel to the rotation axis during the evolution of the radiation hydrodynamical calculations of the collapse of molecular cloud cores with different initial rotation rates. From the top to bottom, the different rows are for cloud cores with $\beta = 5 \times 10^{-4}$, 0.001, 0.005, 0.01 and 0.04. Note that the spatial scale is different for each row, with each panel measuring $1600\sqrt{\beta}$ au across (i.e. from 36 to 320 au). The free-fall time of the initial cloud core $t_{ff} = 1.8 \times 10^{12}$ s (56 500 yr). Each calculation was performed with $10^6$ SPH particles, except for the $\beta = 0.001$ and 0.005 cases which used $3 \times 10^6$ SPH particles. The evolution is almost identical when using $10^6$ or $3 \times 10^6$ particles, but the latter are slightly more detailed (see Appendix A).

results. The evolution time of the first core or pre-stellar disc is longer using radiation hydrodynamics than using the barotropic equation of state in all cases, by factors of 1.5–3 except for the most rapidly rotating case. This is because the barotropic equation of state consistently underestimates the temperature of the gas, providing less pressure support to the gas and thus allowing it to enter the second collapse phase earlier. The consistent temperature underestimate can be seen clearly in Fig. 6 which plots the maximum temperature versus maximum density for each of the radiation hydrodynamical calculations and compares this to the barotropic equation of state.
Figure 4. Snapshots of the column density viewed perpendicular to the rotation axis during the evolution of the radiation hydrodynamical calculations of the collapse of molecular cloud cores with different initial rotation rates. From the top to bottom, the different rows are for cloud cores with $\beta = 5 \times 10^{-4}$, 0.001, 0.005, 0.01 and 0.04. Note that the spatial scale is different for each row, with each panel measuring $1600/\sqrt{\beta}$ au across (i.e. 36, 50, 114, 160 or 320 au). The free-fall time of the initial cloud core $t_{ff} = 1.8 \times 10^{12}$ s (56 500 yr). Each calculation was performed with $10^6$ SPH particles, except for the $\beta = 0.001$ and 0.005 cases which used $3 \times 10^6$ SPH particles. The evolution is almost identical when using $10^6$ or $3 \times 10^6$ particles, but the latter are slightly more detailed (see Appendix A).

(dashed black line). The lines from the radiation hydrodynamical calculations almost lie on top of one another, but from densities of $10^{-11}$ to $10^{-8}$ g cm$^{-3}$ the barotropic equation of state underestimates the maximum temperature by about a factor of 2. The only exception to this is the most rapidly rotating calculation with $\beta = 0.04$. Here, the maximum temperature at a given density is lower than in the other radiation hydrodynamical calculations and more similar to the barotropic equation of state. This is because in
Formation and evolution of pre-stellar discs

2043

Figure 5. The time-evolution of the maximum density (upper panels) and gas temperature (lower panels) during the radiation hydrodynamical calculations of the collapse of molecular cloud cores with different initial rotation rates. From the left-hand to right-hand side, the different panels are for cloud cores with $\beta = 0, 5 \times 10^{-4}, 0.001, 0.005, 0.01$ and 0.04. The dashed lines give the evolution using the barotropic equation of state, while the solid lines give the evolution using radiation hydrodynamics. Time is set to zero at the end of the second dynamic collapse phase when the density reaches $10^{-3} \text{g cm}^{-3}$ which allows the lengths of the first hydrostatic core phases to be compared. It can be seen that the length of the first core phase increases with increasing rotation rate, but also that the use of radiation hydrodynamics and a realistic equation of state lengthens the first core phase relative to calculations that use the barotropic equation of state. Each calculation was performed with $10^6$ SPH particles, except the barotropic calculation with $\beta = 0.04$ which used $3 \times 10^5$ particles.

this case the first core is actually a torus and therefore the gas can cool more effectively. Returning to Fig. 5, we also see that it is the $\beta = 0.04$ case (rightmost panels) where the time-scales between first and stellar core formation are most similar for the barotropic and radiation hydrodynamical calculations (differing only by about 15 per cent rather than a factor of 1.5–3).

One possible reason that the radiation hydrodynamical calculations give greater temperatures during the first core phase than the barotropic calculations is that there may be considerable shock heating which is not taken into account with a barotropic equation of state. However, computing the entropy of the gas before the first core forms and in the bulk of the first core as it forms and evolves, we find it monotonically decreases in the radiation hydrodynamical calculations. The rate of decrease is rapid before the first core forms (when it is cooling rapidly and almost isothermal), but even after the first core forms, the gas is losing energy due to radiation. Only at the surface of the first core (around the accretion shock) does the entropy briefly increase before it radiatively cools. This is consistent with the recent calculations of Commerson et al. (2011) who find that essentially all of the energy liberated in the accretion shock is radiated away.

Instead, the reason the barotropic equation of state consistently underestimates the temperature in this density range is due to the approximation that $\eta = 7/5$ in this part of the evolution. In fact, molecular hydrogen (the dominant constituent) has a ratio of specific heat capacities of $\gamma = 5/3$ until it reaches $\sim 100 \text{K}$. Only then are the rotational degrees of freedom, which lower $\gamma$ to 7/5, excited. Again, this is apparent in Fig. 6 where it can be seen that the lines from the radiation hydrodynamical calculations are steeper than the barotropic line in the temperature range $\approx 20–150$. Above this temperature, the lines are almost parallel (i.e. $\gamma \approx 7/5$ for both), but the temperature offset (that originates in the 20–150 K range) persists. The barotropic equation of state could be improved for this part of the evolution by including a smooth transition from $\gamma = 1$ to $5/3$ and then a transition from $5/3$ to $7/5$. However, this would still not capture all of the details present in the radiation hydrodynamical equations (see below).

3.2 Rotationally-unstable first hydrostatic cores

If the first core is rotating rapidly enough that its own value of $\beta > 0.274$, then the core is dynamically unstable to the growth of non-axisymmetric structure (Bate 1998; Machida et al. 2005; Saigo & Tomisaka 2006; Saigo et al. 2008; Bate 2010; Machida et al. 2010; Tomida et al. 2010b; Machida & Matsumoto 2011; Saigo & Tomisaka 2011). For the particular initial conditions used here, this occurs for the $\beta = 0.001–0.01$ cases (see Figs 2 and 3). The torus that forms in the $\beta = 0.04$ calculations is also dynamically
The evolution of the maximum gas temperature versus maximum density for the radiation hydrodynamical collapse of molecular cloud cores with $\beta = 0$ (solid black line), $5 \times 10^{-4}$ (dotted red curve), 0.001 (short-dashed green curve), 0.005 (long-dashed blue curve), 0.01 (dot–short dashed cyan curve) and 0.04 (dot–long-dashed magenta curve). The barotropic equation of state is given by the black short-dashed lines. The axisymmetric ($\beta = 0$) case is always the hottest at a given maximum density because the collapse is quickest and the radiation is most effectively trapped. Conversely, the calculation that is the coolest has the highest rotation rate ($\beta = 0.04$). Each calculation was performed with $10^6$ SPH particles.

In each of the $\beta = 0.001$–0.005 cases, the first core begins as an axisymmetric flattened pre-stellar disc, but after several rotations, it develops a bar mode (see e.g., the second panel of the third row in Figs 2 and 3). The ends of the bar subsequently lag behind and the bar winds up to produce a spiral structure. Spiral structure removes angular momentum from the inner parts of the first core via gravitational torques (Bate 1998), the effect of which can be seen in the evolution of density and temperature in Fig. 5. For example, in the $\beta = 0.005$ case, the slow increases in central density and temperature after first core formation suddenly accelerate with the onset of the spiral structure. This occurs at about 1300 yr before stellar core formation for the radiation hydrodynamical calculation with $\beta = 0.005$ and about 600 yr before stellar core formation for the barotropic calculation. Similar accelerations are seen for both the $\beta = 0.01$ cases and also for the barotropic $\beta = 0.001$ case. Thus, the removal of angular momentum from the central regions of the core substantially accelerates the evolution of the first core towards the second collapse, which would take much longer to reach without the angular momentum redistribution (i.e. due to accretion and radiative cooling alone).

The barotropic and radiation hydrodynamical evolutions are qualitatively similar in that both display the progression from an axisymmetric core, to bar instability, to a torus as the initial rotation rate of the molecular cloud core is increased. However, radiation hydrodynamical calculations are more resistant to the bar instability than the barotropic calculations. This is directly attributable to the difference in the thermal evolution of the gas that was discussed above. Since the gas is somewhat hotter in the radiation hydrodynamical calculations, the first core is larger (more ‘puffy’) and has a lower value of $\beta$ for a given value of $\beta$ of the initial molecular cloud core. Tomida et al. (2010a) also noted that the first cores in their radiation magnetohydrodynamical calculations had higher entropies and larger sizes than when they used a barotropic equation of state. It should be noted, however, that with radiation hydrodynamics there is no one-to-one relation between temperature and density (see also Boss et al. 2000; Whitehouse & Bate 2006; Tomida et al. 2010a). This is illustrated in Fig. 7, where we plot temperature versus density from various snapshots from the $\beta = 0.005$ calculation and compare this to both the run of maximum temperature versus maximum density during the calculation and the barotropic equation of state. The red dot–dashed lines are for cuts perpendicular to the rotation axis along the $x$-axis (i.e. in the plane of the disc), while the blue long-dashed lines display the temperature along the rotation axis. Not only is the run of maximum temperature versus maximum density always greater than or equal to the temperature from the barotropic equation of state, but also the gas both in the mid-plane of the disc and along the rotation axis at the same density is even hotter. This is particularly true of the gas along the rotation axis, which is heated by the accretion shock at the surface of the first
core. Tomida et al. (2010a) give a very similar plot for a snapshot from one of their simulations and find very similar behaviour.

These hotter temperatures in the radiation hydrodynamical calculations mean that while the transition from the rotationally stable to dynamically unstable first core occurs in the range $\beta = (5–10) \times 10^{-4} \times 10^{-3}$ using the barotropic equation of state, the transition occurs at $\beta \approx 0.001$ with radiation hydrodynamics (this case undergoes an extremely weak instability, just visible in the fourth panel of the second row of Fig. 3). Similarly, while the barotropic calculation with $\beta = 0.01$ manages to produce a torus-shaped first core, the radiation hydrodynamical calculation with $\beta = 0.01$ is still definitely disc-shaped rather than torus-shaped.

As mentioned above, for the highest initial rotation rates ($\beta = 0.04$), the first core is actually a torus or ring-like structure (e.g. Norman & Wilson 1978; Cha & Whitworth 2003; Machida et al. 2005). Such a configuration is highly unstable to non-axisymmetric perturbations and, indeed, as is clearly visible in Figs 2 and 3 the rings rapidly fragment into four objects. Such a configuration is highly chaotic, and symmetry is broken quickly (due to truncation error and the use of a tree-structure to calculate gravity in the calculations). In the radiation hydrodynamical calculation, two of the fragments merge to produce a triple system, while in the barotropic calculation, all four fragments survive (at least until the calculation was stopped). Each of the fragments follows its own evolution towards the second collapse phase and stellar core formation. The calculations were stopped soon after the first fragment in each calculation produced a stellar core.

Finally, as mentioned above, first cores may evolve into pre-stellar discs with radii ranging from $\approx 5$ to $\geq 100$ au before a stellar core forms (Fig. 3). Those with radii greater than $\approx 10$ au are produced due to the angular momentum transport that occurs during the dynamical rotational instability. A further consequence is that all star formation should go through at least a brief phase when the disc mass is greater than the stellar mass. Such discs are gravitationally unstable and may evolve through spiral density waves (see e.g. the third row of Figs 3 and 8) and/or fragmentation (see e.g. the fourth row of Figs 3 and 8). This is discussed further in Section 4.

In conclusion, in switching from the simple barotropic equation of state to a realistic equation of state with radiation hydrodynamics, the qualitative evolution of the first core and its dependence on the initial rotation rate of the molecular cloud core is identical. However, the gas temperature in the more realistic calculations tends to be hotter and is not only a function of density: for example, the gas is significantly hotter in the accretion shock at the surface of the first cores. Quantitatively, the higher temperatures mean that the critical values of the initial molecular cloud core rotation rates that are required for bar instability of the first core, or the transition to a torus geometry, are somewhat higher. The value of $\beta$ must be $\approx 50$ per cent greater with radiation hydrodynamics, which translates into a rotation rate which is $\approx 25$ per cent greater (since $\beta \propto \Omega^2$, where $\Omega$ is the angular frequency of the molecular cloud core).

3.3 The effect of stellar core formation

Although the evolution of the first core or pre-stellar disc is qualitatively the same when computed with a barotropic equation of state or radiation hydrodynamics, Bate (2010) showed the evolution subsequent to the formation of the stellar core is qualitatively different. When using a barotropic equation of state, the formation of the stellar core deep within the optically-thick disc has no effect on the temperature of the gas farther out in the disc because its temperature is set purely according to its density. However, with radiation hydrodynamics, the situation is completely different.

When the second collapse occurs and produces the stellar core, the gravitational potential energy that is released is $\sim GM^2_c/R_{\text{sc}} = 4 \times 10^{43}$ erg. Since the stellar core is in virial equilibrium, approximately half of this energy is radiated away. Moreover, the stellar core rapidly begins to accrete from the first core, reaching a mass of $\approx 6 M_{\odot}$ in only a few years (i.e. more quickly than the dynamical time-scale of the large-scale disc; see below). This increases the total energy released to $\approx 3 \times 10^{44}$ erg. Bate (2010) compared this energy with the binding energy of the disc in the $\beta = 0.005$ calculation. Just before the onset of the second collapse, the binding energy of the pre-stellar disc is only $4 \times 10^{43}$ erg (estimated as $\sim GM^2_c/R_{\text{sc}}$ with $M_{\text{sc}} \approx 0.18 M_{\odot}$ and taking a mean ‘spherical’ radius of $R_{\text{sc}} \approx 15$ au). Thus, when the second collapse occurs, the disc suddenly finds itself irradiated from the inside by an energy source emitting a substantial fraction of the binding energy of the disc itself. Because the disc is extremely optically thick, this energy is temporarly trapped in the centre of the disc and heats the gas dramatically, sending a weak shockwave out along the mid-plane of the disc at a speed of a few km s$^{-1}$. However, perpendicular to the disc, the effect is even more dramatic. Because there is less material along the rotation axis, the hot gas finds it easiest to break out in this direction and a bipolar outflow is launched. Whereas the wave within the disc decays as it travels, leaving the bulk of the disc gravitationally bound, the gas forming the bipolar outflow has velocities up to 10 km s$^{-1}$ and travels out into the infalling envelope to distances in excess of 50 au in less than 50 yr. Using two-dimensional radiation hydrodynamical calculations, Schönke & Tschamnter (2011) have recently reported similar behaviour, although the outflow in their simulations is less bipolar. By discarding the central regions of one of their calculations, they were able to follow the outflow for a few tens of thousands of years and found that after reaching $\approx 500$ au, the material fell back in to reform a disc.

Bate (2010) mainly discussed the $\beta = 0.005$ calculation as a typical example. Here we examine how the effect of stellar core formation on the surrounding disc depends on the initial rotation rate of the molecular cloud core and, thus, on the degree of rotation that the first core/pre-stellar disc has. Figs 9–14 illustrate the evolution of calculations with $\beta = 5 \times 10^{-4}$–0.01 following stellar core formation. The $\beta = 0$ case is not discussed further since the situation is as Larson (1969) described it. This case remains spherically symmetric, and although the stellar core irradiates the first core from within, the radiation liberated is not sufficient to stop the spherically-symmetric accretion flow on to the stellar core. We do not discuss the $\beta = 0.04$ case following stellar core formation since each of the three fragments is qualitatively similar to the first core obtained in the $\beta = 0.001$ case and therefore we assume that each of these cores would evolve in a similar manner.

Figs 9 and 10 provide snapshots of the column density in the directions parallel to the rotation axis and perpendicular to the rotation axis, respectively. The shockwave propagating outwards through each of the first cores/discs is clearly visible in Fig. 9. For the $\beta = 5 \times 10^{-4}$ and 0.001 cases, the shockwave reaches the outer edge of the disc before the calculations are stopped. For the $\beta = 0.005$ and 0.01 cases, we were not able to follow the calculations this long and the shockwave was only followed for 10–15 au. In Fig. 10, both the propagation of the shockwaves through the discs and the bipolar outflows launched perpendicular to the discs...
Figure 8. Snapshots of the density-weighted temperature viewed parallel to the rotation axis during the evolution of the radiation hydrodynamical calculations of the collapse of molecular cloud cores with different initial rotation rates. From the top to bottom, the different rows are for cloud cores with $\beta = 5 \times 10^{-4}$, 0.001, 0.005, 0.01 and 0.04. Note that the spatial scale is different for each row, with each panel measuring $1600\sqrt{\beta}$ au across (i.e. from 36 to 320 au). The temperature scale differs between the top three rows and the lower two rows in order to enhance the low-temperature structure in the cases with greater rotation. The free-fall time of the initial cloud core $t_{ff} = 1.8 \times 10^{12}$ s (56 500 yr). Each calculation was performed with $10^6$ SPH particles, except for the $\beta = 0.001$ and 0.005 cases which used $3 \times 10^6$ SPH particles. The evolution is almost identical when using $10^6$ or $3 \times 10^6$ particles, but the latter are slightly more detailed (see Appendix A).
Figure 9. Snapshots of the column density viewed parallel to the rotation axis during the evolution of the radiation hydrodynamical calculations of the collapse of molecular cloud cores with different initial rotation rates. The shockwaves propagating outwards through the discs following stellar core formation are clearly visible. From the top to bottom, the different rows are for cloud cores with $\beta = 5 \times 10^{-4}$, 0.001, 0.005 and 0.01. Note that the spatial scale is different for each row, with each panel measuring $1600\sqrt{\beta}$ au across (i.e. from 36 to 160 au). The free-fall time of the initial cloud core $t_{\text{ff}} = 1.8 \times 10^{12}$ s (56 500 yr). Each calculation was performed with $10^6$ SPH particles, except for the $\beta = 0.001$ and 0.005 cases which used $3 \times 10^6$ SPH particles. The evolution is almost identical when using $10^6$ or $3 \times 10^6$ particles, but the latter are slightly more detailed (see Appendix A).

are clearly visible, the latter reaching distances of 15–35 au. The maximum distance that an outflow was followed was 60 au in the $\beta = 0.005$ case, approximately 50 yr after the outflow began (Bate 2010).

Each of Figs 11–13 gives density, velocity and temperature profiles, both in the disc plane and perpendicular to the disc plane (i.e. along the rotation axis), at four characteristic times during the evolution following stellar core formation. They also provide the radial mass profiles. Fig. 11 shows the evolution of the barotropic $\beta = 0.005$ calculation which can be compared with the results from the radiation hydrodynamical calculations in Figs 12 and 13 with $\beta = 5 \times 10^{-4}$ and 0.005, respectively. We do not provide figures for $\beta = 0.001$ and 0.01 since they are qualitatively similar to the cases with $\beta = 5 \times 10^{-4}$ and 0.005, respectively.

In all calculations, the stellar core forms with a radius of $R_\ast \approx 0.01$ au (i.e. $\approx 2R_\odot$) and a mass of a few $M_\odot$ (see e.g. Fig. 11, top left-hand and top middle panels). Using the barotropic equation of state (Fig. 11), the radii of the first core (actually a disc) and stellar core are clearly visible in the top middle panel which gives the radial velocity profiles just after stellar core formation. Gas falling on to the first core from the envelope at speeds of $\approx 2–4$ km s$^{-1}$ is visible at radii of $\approx 10–30$ au, while gas falling on to the stellar core much...
Figure 10. Snapshots of the column density viewed perpendicular to the rotation axis during the evolution of the radiation hydrodynamical calculations of the collapse of molecular cloud cores with different initial rotation rates. The bipolar outflows, launched after stellar core formation, are clearly visible. From the top to bottom, the different rows are for cloud cores with $\beta = 5 \times 10^{-4}, 0.001, 0.005$ and 0.01. Note that the spatial scale is different for each row, with each panel measuring $1600\sqrt{\beta}$ au across (i.e. 36, 50, 114 or 160 au). The free-fall time of the initial cloud core $t_{ff} = 1.8 \times 10^{12}$ s (56,500 yr). Each calculation was performed with $10^6$ SPH particles, except for the $\beta = 0.001$ and 0.005 cases which used $3 \times 10^6$ SPH particles. The evolution is almost identical when using $10^6$ or $3 \times 10^6$ particles, but the latter are slightly more detailed (see Appendix A).

more rapidly (up to 7–10 km s$^{-1}$) is apparent at radii of $\approx 0.01$–0.3 au. In less than a year (second and third rows of Fig. 11), an inner disc builds up around the stellar core (radius $\approx 0.07$ au in the middle panel of the second row and $\approx 0.2$ au in the middle panel of the third row) and the radial extent of the region undergoing dissociation and collapse on to the stellar core and inner disc is reduced. Several years after the stellar core has formed (bottom row of Fig. 11), the outer edge of the inner disc has completely merged into the outer disc (the remnant of the first core) and the azimuthally averaged radial velocity in the disc plane is approximately zero out to $\approx 30$ au. The formation of an inner disc around the stellar core, which grows in radius as gas with higher angular momentum falls in through the region of molecular hydrogen dissociation, and the eventual merger of the inner and outer discs are just as was first discussed by Bate (1998), and more recently studied by Machida et al. (2010) and Machida & Matsumoto (2011) who also included magnetic fields.

Using radiation hydrodynamics, the evolution is initially similar to the barotropic cases. For example, take the $\beta = 5 \times 10^{-4}$ case (Fig. 12). In the top middle panel, the first core and stellar cores are clearly visible in the radial velocity profiles due to the accretion shocks at their surfaces. An inner disc builds up around the stellar
core as material with higher angular momentum falls inwards and the radial extent of the dissociating, collapsing region decreases (middle panel of the second row).

However, as the inner disc grows in radius and begins to merge with the remnant of the first core, a bipolar outflow is launched vertically at about 5 km s\(^{-1}\) (dashed line in the middle panel of the third row of Fig. 12), while a lower velocity shock propagates outwards along the disc mid-plane at \(\approx 1\) km s\(^{-1}\). All of this evolution occurs within just 4–5 yr of the stellar core forming. At 20 yr after the stellar core formation, the outflow has travelled more than 20 au and the shockwave in the disc has bursted out of the edge of the disc (middle panel of the bottom row of Fig. 12; see also Fig. 10).

The outflow and the shockwave do not vary greatly with the degree of rotation of the first core/pre-stellar disc (cf. Figs 12 and 13). In all cases, the outflow is launched (vertically) at a radius of \(\approx 0.5\) au from the stellar core, just before the outer edge of the inner disc merges (in the disc plane) with the remnant of the first core/pre-stellar disc at \(\approx 1\) au and the infalling region of dissociating molecular hydrogen disappears (see e.g. the middle panel of the second row in Fig. 13). The peak speed of the outflow is slightly larger when the first core is rotating more rapidly, ranging from \(\approx 5\) km s\(^{-1}\) for the \(\beta = 5 \times 10^{-4}\) case to \(\approx 10\) km s\(^{-1}\) for the \(\beta = 0.01\) case. The outflows have been followed for long enough in the \(\beta = 0.005\) case to clearly see the decay (cf. the dashed lines in the middle panels of the third and fourth rows of Fig. 13). This outflowing material will not leave the system; it will eventually stall and fall back on to the protostar and disc (Schönke & Tschamntter 2011). It is worth noting that because the flux-limited diffusion method used to calculate the radiative transfer in the calculations presented here assumes that the material radiates with maximal efficiency, it may overestimate the cooling and hence underestimate the temperatures in the optically-thin outflowing material. Thus, the outflow may be even stronger with a more accurate method of radiative transfer.
Figure 12. The evolution of the radiation hydrodynamical $\beta = 5 \times 10^{-4}$ calculation of the collapse of a molecular cloud core following stellar core formation. Each of the four rows shows the state of the protostar at different times. The left-hand panels provide the radial density profile perpendicular to the rotation axis (solid line, averaged in azimuth), the density profile along the rotation axis (dashed line) and the cumulative mass profile (dotted line). The middle panels give the radial velocity profiles perpendicular to (solid line) or along (dashed line) the rotation axis. The right-hand panels show the radial temperature profiles perpendicular to (solid line) or along (dashed line) the rotation axis.

The shockwave propagating outwards through the disc travels with a speed of 1–2 km s$^{-1}$ in all cases. In the $\beta = 5 \times 10^{-4}$ and 0.001 calculations, the shockwave is followed until it has reached the edge of the first core/pre-stellar disc. The combination of the outflow and the outward propagating wave in the disc plane evacuates most of the gas from within a few au of the protostar, leaving the stellar core surrounded by a broad torus, rather than a disc. This is apparent in the column density plots in Fig. 9, as well as the mass and density profiles in the bottom rows of Figs 12 and 13. These holes would eventually fill in again as the disc evolves. In the low-resolution calculations, the mass resolution is poor and the ‘holes’ around the stellar cores are in fact holes (devoid of SPH particles). However, in the high-resolution $\beta = 0.001$ and 0.005 cases (each of which uses $3 \times 10^6$ particles), it is found that a small amount of gas does remain within the holes, and this gas continues to slowly accrete on to the stellar core.

In Fig. 14, we plot the mass with density $>10^{-4}$ g cm$^{-3}$ versus time for the $\beta = 0.001$ and 0.005 cases with different resolutions. It can be seen that the mass of the stellar core at which the feedback dramatically curtails the accretion decreases from $\approx 10^{11}$ down to $6 M_J$ with increased resolution. For a few years, the stellar cores grow at a rate of $\approx 10^{-3} M_J$ yr$^{-1}$ (even with the highest resolutions). With low resolution ($\leq 1 \times 10^6$ SPH particles), the accretion on to the stellar cores ceases entirely after the launching of the outflows due to the formation of ‘holes’ in the inner few au of the discs. However, using $3 \times 10^6$ SPH particles, it is found that the stellar cores do continue to accrete during this time, but at much reduced accretion rates of $2 \times 10^{-5} M_J$ yr$^{-1}$ for the $\beta = 0.001$ calculation and $1 \times 10^{-5} M_J$ yr$^{-1}$ for the $\beta = 0.005$ calculation (measured between 20 and 50 yr after stellar core formation; Fig. 14).

Although the rate of convergence with increasing resolution is relatively slow, strong outflows are launched in all cases (see Appendix A). The slow convergence is due to the strong interplay between the energy released by the formation of the stellar core and the launching of the outflow which decreases accretion and, thus, reduces the energy that is released. If the resolution is poor,
the stellar core accretes more material before the energy released feeds back and manages to stop the accretion, whereas with higher resolution, the accretion of a small amount of mass can feed thermal energy into the infalling material more quickly and, thus, inhibit further accretion.

4 DISCUSSION

4.1 The lifetime of the first core/pre-stellar disc phase

As mentioned in Section 3.2, the introduction of radiative transfer and a realistic equation of state increases the lifetimes of the first core phase compared with that obtained using our barotropic equation of state by factors of 1.5–3 (Fig. 5). This is important, because the longer the first core phase lasts, the more easy it will be to observe. The lifetimes obtained using radiative transfer range from $\approx 400$ yr (with no rotation) to $\approx 3000$ yr (for $\beta = 0.01$), whereas using the barotropic equation of state they range from $\approx 100$ to $\approx 1500$ yr. As discussed above, the difference is due to the higher temperatures (and thus higher pressures) that are obtained using the realistic physics rather than the barotropic equation of state, which slows the evolution towards the second collapse phase. This lengthening of the lifetimes with radiation hydrodynamics compared to barotropic calculations was also observed by Tomida et al. (2010a). The lengthening occurs for both the non-rotating first cores and in the cores that undergo non-axisymmetric instabilities. In the latter, once the first cores or pre-stellar discs develop the bar instability, they very quickly evolve to high central densities and temperatures when computed using the barotropic equation of state (Fig. 5, fourth and fifth panels across where the maximum density increases rapidly from $10^{-11}$ to $10^{-7} \text{ g cm}^{-3}$ and the maximum temperature from 100 to 2000 K). Using the more realistic physics, this evolution takes three to eight times longer. This is because the hotter temperatures weaken the strength of the spiral arms generated by the instability (cf. the third rows of Figs 2 and 3) and, thus, decrease the rate of angular
The mass of the stellar core (gas with density \( > 10^{-4} \text{ g cm}^{-3} \)) measured from the time of its formation. The two panels show the results for the radiation hydrodynamical \( \beta = 0.001 \) (left-hand panel) and \( \beta = 0.005 \) (right-hand panel) calculations of the collapse of molecular cloud cores performed using \( 10^5 \) (long-dashed line), \( 3 \times 10^5 \) (short-dashed line), \( 10^6 \) (dotted line) and \( 3 \times 10^6 \) (solid line) particles. With low resolutions, the radiative feedback stops accretion on to the stellar cores, but for the highest resolutions, the accretion continues at a low level \((1-2) \times 10^{-5} \text{ M}_\odot \text{ yr}^{-1}\).

**Figure 14.** The mass of the stellar core (gas with density \( > 10^{-4} \text{ g cm}^{-3} \)) measured from the time of its formation. The two panels show the results for the radiation hydrodynamical \( \beta = 0.001 \) (left-hand panel) and \( \beta = 0.005 \) (right-hand panel) calculations of the collapse of molecular cloud cores performed using \( 10^5 \) (long-dashed line), \( 3 \times 10^5 \) (short-dashed line), \( 10^6 \) (dotted line) and \( 3 \times 10^6 \) (solid line) particles. With low resolutions, the radiative feedback stops accretion on to the stellar cores, but for the highest resolutions, the accretion continues at a low level \((1-2) \times 10^{-5} \text{ M}_\odot \text{ yr}^{-1}\).

The pre-stellar disc phase will be more easily observed in more rapidly rotating molecular cloud cores because the lifetime of the phase is longer. Resolved observations of the pre-stellar disc phase (e.g. using the ALMA) would also be easier and more interesting for the more rapidly rotating cases because the disc will be much larger (up to \( \approx 50-100 \) au in radius rather than \( \approx 5-10 \) au for rotationally-stable first cores) and substructure may be visible (i.e. the presence of spiral density waves). Recently, Suigo & Tomisaka (2011) investigated the observability of pre-stellar discs using the ALMA and concluded that the ALMA should be able to image objects in the nearest star-forming regions. For the particular initial conditions used in this paper, the lifetime of a pre-stellar disc as a function of the initial cloud rotation rate is plotted in Fig. 15. As pointed out recently by Tomida et al. (2010b), the lifetime of the first core phase also depends on the mass of the molecular cloud core such that the lifetimes are longer in lower mass molecular cloud cores.

For initial core masses of \( \approx 0.1 \text{ M}_\odot \), the accretion on to the first core/pre-stellar disc is not sufficient to drive the object into the second collapse phase. Instead, the central regions of the first core/disc evolve to higher temperatures primarily through radiative cooling. As the first core radiates energy, it contracts and heats up, eventually exceeding 2000 K when the dissociation of molecular hydrogen begins, but this process can take in excess of \( 10^4 \) yr. Thus, the first core/pre-stellar disc phase is more likely to be observed for molecular cloud cores that have higher rotation rates and lower masses.

Despite this, observing the pre-stellar disc phase with the ALMA will be challenging. Lifetimes of even \((4-10) \times 10^3 \times 10^4 \) yr are still only 2–5 per cent of the estimates of Class 0 lifetimes of \( 2 \times 10^5 \) yr (Hatchell et al. 2007; Evans et al. 2009), so that for every 100 Class 0 object there should only be a few pre-stellar discs. The first place to look is likely to be the so-called very low luminosity objects with luminosities of less than 0.1 L\(_\odot\) (e.g. Young et al. 2004; Crapsi et al. 2005; Bourke et al. 2006; Huard et al. 2006), some of which have recently been proposed as candidate first cores (Chen et al. 2010; Enoch et al. 2010).

**Figure 15.** The dependence of the pre-stellar disc (first core) properties on the initial rotation rate of the progenitor molecular cloud core, \( \beta \). We plot the masses (black solid line) and radii (red dotted line) of the pre-stellar cores, measured just before the second collapse to form the stellar core occurs, and the lifetime (blue dashed line) of the pre-stellar disc (i.e. the time between the formation of the first hydrostatic core and the formation of the stellar core). The mass, radius and lifetime all increase with larger initial molecular cloud core rotation rates. In the more rapidly rotating cases, the pre-stellar disc accumulates a significant fraction of the total cloud mass before the stellar core forms. Note that the most rapidly rotating \( \beta = 0.04 \) case has been excluded because it forms four fragments rather than a single pre-stellar disc.

### 4.2 Disc-to-stellar mass ratios

As first discovered by Bate (1998), a rotating first core actually leads to a disc that forms before the star. These discs can range from \( \approx 5 \) au in radius for very slowly rotating first cores (cf. Larson 1969) to \( \geq 100 \) au for cores that undergo rotational instabilities (Bate 2010; Machida et al. 2010; Machida & Matsumoto 2011).

Because the disc forms first, there is always a phase of star formation when the disc-to-stellar mass ratio is greater than unity. For example, in the original calculation of Bate (1998), the disc had a mass of 0.08 M\(_\odot\) when the stellar core formed with an initial mass of only 1.5 M\(_\odot\), giving a disc-to-stellar mass ratio \( \gtrsim 50 \). Although some quite massive discs have been discovered around some young stars (e.g. the 0.8-M\(_\odot\) disc around a 3.5-M\(_\odot\) star; Hamidouche...
2010), such cases are rare (e.g. Boissier et al. 2011) and the usual assumption is that the circumstellar disc mass is always likely to be less than the stellar mass. While this is almost certainly true later in the evolution of a protostar (e.g. Class I or Class II low-mass objects and Herbig Ae/Be stars), in the very earliest phases, the disc-to-star mass ratio can greatly exceed unity.

With the initial conditions used in this paper, the disc mass when the stellar core forms increases with the rotation rate of the progenitor molecular cloud core. This is because a greater fraction of the 1-M⊙ molecular cloud core has sufficient angular momentum to settle outside the typical 5 au radius of a non-rotating first core. Because more mass settles away from the centre of the object, the accretion from the infalling envelope is less effective at increasing the central temperature (i.e. mass receives support from rotation as well as thermal pressure). Thus, the first core phase lasts longer and the masses of the first cores/pre-stellar discs at the time of stellar core formation range from \( \approx 0.03 \) M⊙ for \( \beta = 0 \) to \( \approx 0.22 \) M⊙ for \( \beta = 0.01 \) (see Fig. 15 and the dotted lines in the left-hand panels of Figs 12 and 13). In the most extreme case we have modelled here, when the stellar core forms with an initial mass of \( \approx 2 \) M⊙, the disc is more than 100 times more massive than the star! Note that the initial rotation rate of the molecular cloud core is not extreme for this case. Goodman et al. (1993) find that the values of \( \beta \) for observed molecular cloud cores range from 0.002 to 1, with typical values of \( \beta \sim 0.02 \). Therefore, it is quite possible that when pre-stellar discs are observed (e.g. with the ALMA), they could have substantial masses (e.g. up to one-fourth of the mass of an entire 1-M⊙ molecular cloud core) without a stellar core having formed.

4.3 Disc fragmentation

The potential for high disc-to-stellar mass ratios also implies that some of these discs may fragment very early in the star formation process. Indeed, for our \( \beta = 0.01 \) case (and, of course, the \( \beta = 0.04 \) case), this is exactly what happens, with the disc fragmenting to produce two additional ‘first cores’ even before the original first core/disc has undergone second collapse to produce a stellar core. The fate of such fragments is of course not clear. Multiple fragments may merge with each other before each of them undergoes second collapse. They may also spiral in to the central object and be disrupted before they undergo second collapse. However, those that survive will produce the seeds of binary or multiple stellar systems.

As pointed out by Whitehouse & Bate (2006), Krumholz (2006), Krumholz, Klein & McKee (2007), Bate (2009) and Offner et al. (2009), radiative feedback from newly formed protostars heats their surrounding gas and inhibits fragmentation. In star cluster formation simulations (e.g. Bate 2009; Offner et al. 2009), this radiative heating dramatically reduces the number of objects formed compared to using a barotropic equation of state. Some of this reduction comes from a reduction in the frequency of disc fragmentation because the discs are heated and stabilized by the radiation from their central objects (Bate 2009).

However, as shown here in the \( \beta = 0.01 \) case, fragmentation of the pre-stellar disc resulting from a rapidly rotating first core phase can occur before the central stellar core is formed. Thus, even if radiative feedback can completely stop disc fragmentation after the stellar core forms, disc fragmentation before stellar core formation may be a common route to forming binary and multiple systems.

4.4 Implications of radiative feedback following stellar core formation

The radiative impact of stellar core formation on the surrounding disc, driving a shockwave through the disc and launching an outflow, may have several wider implications. These were discussed by Bate (2010), so we only briefly list them here. First, in future studies, it will be important to examine the relative roles of radiative heating due to stellar core formation and magnetic fields in the launching of a protostellar jet. Secondly, although the outburst and the dramatic decrease in the stellar core accretion rate discussed in this paper is a transient associated with stellar core formation, it might reoccur if the accretion rate to the stellar core increases again and, thus, may potentially be a source of episodic accretion and outbursts. Thirdly, such outbursts may contribute to the production of the chondrules found in meteorites (Grossman et al. 1988).

5 CONCLUSIONS

We have presented results from three-dimensional radiation hydrodynamical calculations that follow the collapse of a molecular cloud core through the formation and evolution of the first hydrostatic core and beyond the formation of the stellar core. We find the evolution before the formation of the stellar core is qualitatively similar to that found in the past using barotropic equations of state. As found in past studies, the evolution of the first hydrostatic core depends on the initial rotation rate of the cloud. Non-rotating or slowly rotating clouds produce small spherical or weakly flattened first cores with radii of \( \approx 5–10 \) au. More rapidly rotating clouds (\( \beta \approx 0.001–0.01 \)) produce highly-oblate rapidly rotating first cores that undergo dynamic rotational instabilities to produce large discs (radii \( \approx 10–100 \) au) before the formation of a stellar core. These objects are better described as pre-stellar discs than first cores. Yet more rotation typically results in fragmentation, via a torus-shaped first core if the initial cloud is axisymmetric. Quantitatively, using radiation hydrodynamics with a realistic equation of state produces first cores that are somewhat hotter, slightly more stable to rotational instabilities, and longer lived (by factors of 1.5–3) than using our barotropic equation of state.

The masses and radii of the pre-stellar discs produced before the stellar core increase with the initial rotation rate of the molecular cloud core. Their lifetimes also increase with rotation rate, and for lower mass molecular cloud cores (see also Tomida et al. 2010b). With high rotation rates, these pre-stellar discs can have radii up to \( \approx 100 \) au and contain in excess of 0.2 M⊙ of gas (up to one-fourth of the mass of the entire molecular cloud core) and exist for several thousands of years before stellar core formation. Such objects may be resolvable by the ALMA. Since the initial mass of the stellar core which forms within these discs is just a few Jupiter masses, the disc-to-star mass ratios can exceed 100 for a short period of time.

Fragmentation of a pre-stellar disc prior to stellar core formation may be an effective way to produce binary and multiple star systems because the radiative feedback from the stellar core which may inhibit later fragmentation of a massive disc is absent.

After the formation of a stellar core within the disc, the dissociating gas from the inner regions of the disc falls towards the stellar core and builds a smaller disc around the stellar core. This inner disc increases with radius as material with higher angular momentum falls in, and eventually merges with the inner edge of the outer disc.

Although this inner/outer disc structure and the earlier evolution of the first core phase are similar for barotropic and radiation hydrodynamical calculations, we find that the subsequent evolution is
qualitatively different in radiation hydrodynamical calculations. In barotropic calculations, the formation of the stellar core deep inside the first core (or pre-stellar disc) has no effect on the surrounding disc because the temperature of the gas is simply set by the density of the gas. However, with radiation hydrodynamics, the energy released by the formation of the stellar core within the optically-thick disc is similar to the binding energy of the disc. This heats the region surrounding the stellar core and, at approximately the same time as the inner disc and outer discs merge, a shockwave forms in the merger region and propagates outwards through the disc, while a bipolar outflow is launched perpendicular to the rotation axis, dramatically decreasing the accretion rate on to the stellar core. The properties of the outflow do not seem to vary greatly with the initial rotation rate of the molecular cloud core or the pre-stellar disc, but the speed of the outflow is somewhat greater for higher rotation rates and the shockwave which propagates outwards through the disc reaches the outer edge of the disc more quickly for smaller discs. It remains to be seen how the inclusion of magnetic fields alters this thermally driven outburst.

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APPENDIX A: NUMERICAL CONVERGENCE OF THE RADIATION HYDRODYNAMICAL CALCULATIONS

As mentioned in Section 2.2, for the different initial conditions and equations of state discussed in this paper, most of the calculations were performed with several different numerical resolutions to check for numerical convergence of the results. The numbers of SPH particles used were $1 \times 10^5$, $3 \times 10^5$, $1 \times 10^6$ and $3 \times 10^6$, although as summarized in Table 1, not all cases were performed with all resolutions. The only type of calculation performed with all four resolutions was the $\beta = 0.005$ case with radiation hydrodynamics, but the evolution of three other initial conditions performed with radiation hydrodynamics was studied with two ($\beta = 0.01$ and
Examining the numerical convergence of the lifetime and evolution of the first core/disc. We plot the time-evolution of the maximum density (upper panels) and gas temperature (lower panels) during the radiation hydrodynamical calculations of the collapse of molecular cloud cores with different initial rotation rates. From the left-hand to right-hand side, the different panels are for cloud cores with $\beta = 0.001$, $0.005$, $0.01$ and $0.04$. The different line types are for calculations performed using $1 \times 10^5$ (long-dashed line), $3 \times 10^5$ (dotted lines), $1 \times 10^6$ (short-dashed lines) and $3 \times 10^6$ (solid lines) SPH particles. Time is set to zero at the end of the second dynamic collapse phase when the density reaches $10^{-3}$ g cm$^{-3}$ which allows the length of the first hydrostatic core phases to be compared. It can be seen that the lifetime of the first core phase is well converged for all except the lowest resolution calculations for the $\beta = 0.005$ case and 0.001 cases. This is because the lifetime of the first core/disc is determined by the dynamic rotational instability for $\beta \geq 0.005$, whereas for $\beta = 0.001$, viscous evolution probably plays a significant role.


In Fig. A1, we plot images of the first cores/pre-stellar discs at different resolutions. For the $\beta = 0.001$ case, the first core is right at the boundary of rotational stability, with a very weak spiral instability visible in the highest resolution case. For the $\beta = 0.005$ case, all resolutions except the lowest ($1 \times 10^5$ particles) resolve the rotational instability. As the resolution is increased, the sharpness of the spiral density waves improves and the pre-stellar disc becomes slightly smaller, but the structure definitely seems to be converging. For the $\beta = 0.01$ case, there are only two resolutions. Both produce a central object surrounded by a disc of radius $\approx 90$ au which fragments. The higher resolution case forms two fragments, while the lower resolution case has only one fragment. Thus, while the number of fragments has not converged, the size of the pre-stellar disc and the fact that it does fragment seems to be robust. Finally, both resolutions of the $\beta = 0.04$ case produce ring-type first cores which fragment into four separate cores. However, with four objects the subsequent evolution becomes chaotic. In the $3 \times 10^5$ particle calculation, the four fragments survive (at least until the first undergoes second collapse), but in the $1 \times 10^5$ particle calculation two of the objects merge, producing a triple system (see the bottom rows of Figs A2 and 3).

Following stellar core formation, as discussed in detail in Section 3.3, a bipolar outflow is launched perpendicular to the disc and a shockwave propagates outwards along the plane of the disc. These outflows and shockwaves occur in all of the radiation hydrodynamical simulations, with the exception of the spherically-symmetric $\beta = 0$ case. This result does not appear to depend on resolution (see Fig. A3). However, as discussed in Section 3.3 and illustrated in Fig. 14, the initial mass of the stellar core and its accretion rate are affected by resolution.

In summary, we conclude that the lifetimes of the first cores/pre-stellar discs are well converged for initial conditions that produce rotationally-unstable pre-stellar discs, but that for more slowly rotating cores (i.e. $\beta = 0.0005$ and 0.001), the lifetimes are lower limits. We also find that the qualitative outcomes (rotationally-stable first core, pre-stellar disc with spiral arms, fragmenting pre-stellar disc and torus) are numerically converged but the details (e.g. how many fragments) may not be.
Figure A2. Snapshots of the column density viewed along the rotation axis at the point of stellar core formation (defined as when the maximum density reaches $1 \times 10^{-3} \text{ g cm}^{-3}$). From the top to bottom, the different rows are for molecular cloud cores with $\beta = 0.001, 0.005, 0.01$ and 0.04. Note that the spatial scale is different for each row. From the left-hand to right-hand side, the columns are for calculations performed with $1 \times 10^5, 3 \times 10^5, 1 \times 10^6$ and $3 \times 10^6$ SPH particles. For each rotation rate, the evolution is qualitatively the same when using $3 \times 10^5$ particles or more.

Figure A3. Snapshots of the column density viewed perpendicular to the rotation axis 25 yr after stellar core formation (defined as when the maximum density reaches $1 \times 10^{-3} \text{ g cm}^{-3}$) in a $\beta = 0.005$ molecular cloud core. From the left-hand to right-hand side, the different panels are for radiation hydrodynamical calculations performed with $1 \times 10^5, 3 \times 10^5, 1 \times 10^6$ and $3 \times 10^6$ SPH particles. The bipolar outflow and the shockwave propagating through the disc are similar in all cases, regardless of the resolution.

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