Reversal magnetic chirality of solar active regions and a possible dynamo model

Hongqi Zhang

National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China

ABSTRACT

The transfer of magnetic chirality in solar active regions is related to the emerging magnetic flux ropes generated in the subatmosphere. This analysis has been presented based on the calculation of the injection of magnetic helicity at the solar surface. As the long-term evolution of the accumulated magnetic helicity is followed, it is found that the transfer of reversal magnetic helicity of active regions is a complex process, and is not monotonic with the same sign. It is found that the dominant contribution of helicity occurs mainly in the fast-developing stage of active regions.

By considering the hemispheric trends of magnetic helicity, it is proposed that the reversal helicity in solar active regions has two possible causes: local generation in the convection zone, and trans-equatorial processes in the subatmosphere. In addition to the mirror-symmetrical reverse of twisted magnetic field in the convection zone, the possibility of a trans-equatorial process is discussed in this paper.

Key words: Sun: activity – Sun: dynamo – Sun: magnetic topology.

1 INTRODUCTION

The evolution of the magnetic field in solar active regions provides some basic information on the generation of the field in the subatmosphere. One can follow the evolution of the magnetic field of solar active regions on the solar surface to diagnose its possible properties in the subatmosphere. The helicity is an important quantity to measure the topology of the magnetic field in the convection zone before it emerges on the solar surface. It can be inferred in several ways, based on measurements of the magnetic field in the solar atmosphere (Zhang 2006). A pioneering study was made by some authors based on the transfer of magnetic helicity \( H_m = \int A \cdot B \, d^3x \) in the solar atmosphere (e.g. Berger & Field 1994; Chae 2001) and on the mean factor of the force-free field \( \vec{\alpha} = (\nabla \times \vec{B}) \cdot \vec{B}/B_\perp^2 \) (or the mean current helicity density \( \vec{J}_c = (\nabla \times \vec{B}) \cdot \vec{B}/B_\perp^2 \)) in solar active regions (e.g. Seehafer 1990). The consistency between the two helicity parameters in solar active regions has been analysed by Liu & Zhang (2006), Zhang (2006) and Zhang et al. (2010a).

It is found that the mean current helicity in most solar active regions in the northern (southern) hemisphere tends to show a negative (positive) sign. This sign rule in the magnetic helicity was first discovered by Hale et al. (1919) from the Hα pattern of active regions. It was analysed statistically by Ding, Hong & Wang (1987) and has been confirmed in the last 20 years (Seehafer 1990; Pevtsov, Canfield & Metcalf 1995; Abramenko, Wang & Yurchishin 1996; Bao & Zhang 1998; Hagino & Sakurai 2005; Xu et al. 2007).

The magnetic helicity can be considered as a measure of the mirror asymmetry of solar magnetic fields in the Sun. It is generated, according to the mean-field solar dynamo model, by solar differential rotation, the action of the Coriolis force on the turbulent motion of plasma in the solar convection zone, and the meridional circulation (e.g. Berger & Ruzmaikin 2000; Kuzanyan, Bao & Zhang 2000; Kleeorin et al. 2003; Choudhuri, Chatterjee & Nandy 2004; Zhang et al. 2006). In comparison with observational results, the mirror-symmetrical reverse of the magnetic helicity of solar active regions relative to the preferred hemisphere trends at different phases of the solar cycle has been theoretically demonstrated (Choudhuri et al. 2004; Xu et al. 2009). A similar simulation for the distribution of current helicity in the full Sun has been provided by Yeates, Mackay & van Ballegooijen (2009).

The statistical imbalance of the magnetic helicity of solar active regions in both hemispheres with solar cycles was discovered by Zhang et al. (2010b), who analysed a series of vector magnetograms of solar active regions observed at Huairou Solar Observing Station in China over more than 20 years. It is consistent with the results of Tiwari, Venkatakrishnan & Sankarasubramanian (2009) based on the analysis of 43 sunspots in a period of a solar cycle.

Zhang et al. (2010b) found that the distribution of the magnetic helicity in solar active regions results in the following observational evidence: magnetic (electric current) helicity and twist patterns are, in general, anti-symmetric with respect to the solar equator. The helicity pattern is more complicated than Hale’s polarity law for...
sunspots. Areas of the ‘wrong’ sign have been found at the ends of the butterfly wings as well as at their very beginnings. The average amplitude of the helicity does not show any significant dependence on the solar cycle phase. The maximum value of helicity, at the surface at least, seems to occur near the edges of the butterfly diagram of sunspots.

A study of the long-term evolution of magnetic helicity transfer in solar active regions could be an important way to diagnose the source of magnetic helicity from the subatmosphere, because large active regions are probably located at the solar surface for several months and where a sequence of magnetic flux emerges with the helicity. In this paper, we present the transfer of magnetic helicity from the individual solar active regions (NOAA 10484, 10486 and 10488 and their evolution in the following solar rotation cycles) in both hemispheres in the decaying phase of solar cycles to analyse the possible relationship between these active regions. We also propose a possible model of the relevant changing magnetic helicity at the large scale of the solar hemisphere and the relationship with the solar dynamo.

2 OBSERVATIONAL EVIDENCE OF REVERSAL HELICITY FROM THE SUBATMOSPHERE

To analyse the possible reversal magnetic helicity in both hemispheres presented by Zhang et al. (2010b), Fig. 1 shows the 195-Å solar active regions NOAA10484, 10486 and 10488, and the corresponding magnetogram observed by the SOHO satellite. These super-active regions occurred on the solar disc from 2003 October 18 to November 4 and produced a number of unexpected eruptive events. An inverse sigmoid configuration was found in the difference of 195-Å images in AR 10486 at 10:14–10:36 UT on 2003 October 28, which is an index of magnetic helicity in the active region. This is consistent with the analysis by Zhang, Liu & Zhang (2008).

NOAA 10484, 10486 and 10488 were new developing active regions in the solar rotation cycle in the chart of 2003 November 4, and AR 10484 and 10486 decayed significantly in the subsequent solar rotation cycles in the chart of 2003 December 1 (see Fig. 2). The emerging sequence of these active regions in late October was presented by Zhou et al. (2007). Active regions 10484 (L355, N4) and 10488 (L288, N8) formed in the northern hemisphere, and their transverse component of the magnetic field rotated counterclockwise. Liu & Zhang (2006) found that the negative magnetic helicity was transported from the subatmosphere into the corona in AR 10488. AR 10486 (L293, S15) formed in the southern hemisphere. The transfer of magnetic helicity from AR 10486 was analysed by Zhang et al. (2008), who detected a negative magnetic helicity from the active re-
region with a strong anticlockwise rotation of sunspots in 2003 October 25–30. It was found that the magnetic flux of AR 10486 was about 20 per cent of the total flux of the Sun as inferred from the MDI synoptic chart on 2003 November 4. It is comparable with the total flux of AR 10484 and 10488 in the northern hemisphere. The relationship between the accumulation of magnetic helicity and the sign of current helicity in AR10486 and 10488 has been analysed by Liu & Zhang (2006) and Zhang et al. (2008), who found that the mean current helicity density of these active regions also shows a negative sign. Active regions NOAA 10484, 10486 and 10488 show a negative sign of magnetic helicity and are located in both solar hemispheres.

To analyse the evolution of magnetic helicity in such active regions in detail, we calculated the magnetic helicity injection of active regions NOAA 10484, 10486 and 10488 and subsequent regions in the following solar rotation cycles until these regions become enhanced networks. The injection of magnetic helicity in these active regions is calculated by the local correlative tracker (LCT) method (Chae 2001) with MDI 96-min magnetograms. Our result for active region NOAA 10488 is slightly different from that of Liu & Zhang (2006), because they performed the helicity accumulation with the high time cadence data sequence. In spite of these differences, the evolution of injective magnetic helicity in the active regions is roughly the same.

Fig. 3 shows the transfer of magnetic helicity in active regions NOAA 10484, 10501 and 10520 in the southern hemisphere. These active regions occurred at almost the same location on the solar surface during continuous solar rotation cycles (see Fig. 2). The amount of negative magnetic helicity injected from the solar subatmosphere is found to be \(-8.7 \times 10^{43} \text{ Mx}^2\) for 2003 October 20–27, \(-4.3 \times 10^{43} \text{ Mx}^2\) for 2003 November 16–23 and \(1.2 \times 10^{43} \text{ Mx}^2\) for 2003 December 12–20.

Fig. 4 shows the transfer of magnetic helicity in active regions NOAA 10486, 10508 and 10523-10524 in the southern hemisphere. These active regions occurred at almost the same location on the solar surface during continuous solar rotation cycles (see Fig. 2). The amount of negative magnetic helicity injected from the solar subatmosphere is found to be \(-8.7 \times 10^{43} \text{ Mx}^2\) for 2003 October 20–27, \(-4.3 \times 10^{43} \text{ Mx}^2\) for 2003 November 16–23 and \(1.2 \times 10^{43} \text{ Mx}^2\) for 2003 December 12–20.

Fig. 5 shows the transfer of magnetic helicity in active regions NOAA 10488, 10507 and 10525 in the northern hemisphere. These active regions occurred at almost the same location on the solar surface during continuous solar rotation cycles (see Fig. 2). The amount of magnetic helicity injected from the solar subatmosphere is found to be \(-1.8 \times 10^{43} \text{ Mx}^2\) for 2003 October 26–November 1, \(5.5 \times 10^{43} \text{ Mx}^2\) during 2003 November 21–28 and \(0.4 \times 10^{43} \text{ Mx}^2\) during 2003 December 19–25.

Figs 3–5 show the long-term injection of magnetic helicity from active regions. The magnetic helicity is injected with the same sign (negative) in active regions NOAA 10484-10501-10520 and 10486-10508-10523-10524, as seen in Figs 3 and 4, while NOAA 10488-10507-10525 has the opposite sign of helicity in the different periods, as seen in Fig. 5. The injection processes of magnetic helicity from the subatmosphere show two tendencies: either monotonic or mixed in a given region at the solar surface.

### 3 Reverse Helicity in the Subatmosphere

Comparing the helicity sign in active regions NOAA 10484–10501–10520, 10486–10508–10523–10524 and 10488–10507–10525 with the hemispheric helicity rule, it is found that active regions NOAA 10484-10501-10520 and 10486-10508-10523-10524 show a negative sign and deviate from the hemispheric helicity rule. This reversal accumulation of magnetic helicity is consistent with the distribution of the mean reversal helicity of solar active regions in the period of solar cycle 23 in fig. 2 of Zhang et al. (2010b), and
Figure 5. The injection rate (left) and accumulation (right) of magnetic helicity in the regions of Fig. 2 in the northern hemisphere.

this distribution is not completely random. It is proposed that the transferred reversal magnetic helicity in solar active regions has two possible sources. (i) The local generation of magnetic helicity from the subatmosphere. This is the normal case for the mirror-symmetric dynamo (cf. Kleeorin et al. 2003; Kuzanyan et al. 2003). A possible statistical mechanism of emerging magnetic flux loops with magnetic helicity was proposed by Longcope, Fisher & Pevtsov (1998). (ii) A trans-equatorial dynamo wave from the other subhemisphere. The possible channels in the subatmospheres are shown in Fig. 2, marked by dashed lines.

To analyse the magnetic helicity in the two hemispheres, two different topological twist patterns of magnetic lines of force are shown in Fig. 6. Fig. 6(a) shows a typical schematic pattern of the twisted magnetic field generated in the subatmosphere. The twisted magnetic lines of force in the subatmosphere exhibit mirror symmetry relative to the solar equator. It is morphologically consistent with the normal models of a solar dynamo, even if some authors propose reversal mirror-symmetrical models for studying the helicity formation process at a different phase of the solar cycle (Choudhuri et al. 2004; Xu et al. 2009; Popova & Nefedov 2010). Fig. 6(b) shows another possibility in the generation of magnetic helicity in the solar subatmosphere. The magnetic lines of force twist with the same handedness in the subatmosphere of both hemispheres.

As the twist component of the magnetic field transfers on the order of the Alfvén speed \( V_A = B/\sqrt{\mu_0 \rho} \) (Alfvén 1942), the trans-equatorial time of magnetic helicity in the subatmosphere can be estimated. Fig. 7 shows the possible relationship between the depth and time of magnetic helicity transfer along the trans-equatorial magnetic loops connecting different regions in both subhemispheres immediately. It is assumed that both regions are located at latitude \( \pm 25.5^\circ \), the magnetic field is \( 10^3 \) G, and the distribution of mass density with depth of the Sun comes from the models of Allen (1973), Bahcall & Ulrich (1988) and Cox (1999). As the magnetic field is \( 10^5 \) G in the deep subatmosphere (Choudhuri 1989), the transfer time of helicity is of the order of a few hours only.

4 REVERSAL HELICITY AND POSSIBLE DYNAMO MODEL

The main contribution to the global magnetic helicity of the Sun comes from active regions. It is likely that the magnetic helicity from active regions is generated in the solar dynamo process. Models of the solar dynamo are normally based on observations of the magnetic field (e.g. Babcock & Babcock 1955; Parker 1955; Leighton 1969) and meridional circulation (e.g. Wang, Nash & Sheeley 1989a,b) on the solar surface. Owing to the opacity of the solar atmosphere, we still know very little about the generation of the magnetic field inside the Sun. Even though various models (such as the Babcock–Leighton flux transport model, Wang, Sheeley & Nash 1989; the \( \alpha \Omega \) dynamo, Parker 1955) are possible candidates (Jouve et al. 2008), a simple model of the solar dynamo can also be basically described in terms of the toroidal magnetic field \( B \) and the toroidal component of the magnetic potential \( A \), which are assumed to be axisymmetric. Equations for these quantities can be obtained from the general mean-field electrodynamics by averaging over the radial cross-section of the convection zone, so that both quantities are functions of the latitude \( \theta \) and time \( t \). The simplest case, by Parker (1955), refers to the behaviour of a dynamo wave
near a given latitude \( \theta \), and the equations assume the form
\[
\frac{\partial A}{\partial t} + V \frac{\partial A}{\partial \theta} = \alpha B + \frac{\partial^2 A}{\partial \theta^2},
\]
(1)
\[
\frac{\partial B}{\partial t} + V \frac{\partial B}{\partial \theta} = D \frac{\partial A}{\partial \theta} + \frac{\partial^2 B}{\partial \theta^2},
\]
(2)
where \( V \) is the large-scale flow, \( D \) is the dynamo number \( (D \sim \Omega (\partial, r), \text{where } \Omega \text{ is the angular velocity and } r \text{ is in the direction of the solar radius}) \) and \( \lambda \) is the turbulent diffusivity. \( D(\Omega) \) and \( \alpha \) are topologically important parameters relative to the stretch and twist of the magnetic field in the solar dynamo model. The solar differential rotation is related to the quantity \( D(\Omega) \). The magnetic helicity as a quantity relates to the \( \alpha \) effect (the toroidal magnetic field to the poloidal one), and characterizes the hooking coefficient of the magnetic lines of force (Kleeorin et al. 2003; Zhang et al. 2006).

The magnetic helicity is a global quantity, while the mean helicity density reflects the basic handedness of the helicity, as its spatial configuration can be neglected. The magnetic helicity density \( AB \) in the solar dynamo process can be analysed according to equations (1)–(2), which relate to the product of toroidal and poloidal magnetic flux, and propagates in a similar way to the dynamo wave. In comparison with the sunspot butterfly diagram, the mean magnetic helicity density actually reflects a comprehensive effect of magnetic helicity in solar active regions and its evolution with solar cycles.

Magnetic diffusion with solar cycles was proposed by Leighton (1964), while the turbulent diffusion of magnetic helicity depending on the evolution of the magnetic field can be estimated based on the analysis of the solar dynamo (cf. Kleeorin & Rogachevskii 1999; Subramanian & Brandenburg 2004; Zhang et al. 2006).

As indicated by Xu et al. (2009), the total magnetic field and magnetic potential break down into sums of large-scale and small-scale components. In this model, the large-scale components of the magnetic field and magnetic potential have only toroidal components, whose product is not conserved and must be balanced through the density of the small-scale magnetic helicity. Because the magnetic and current helicity are proportional to each other in the approximation, and we are basically interested in their qualitative behaviour over the course of a cycle, we can plot them in the same form without necessarily specifying which of these two helicities is considered.

The magnetic helicity density can be written in the form (see Appendix and Xu et al. 2009)
\[
AB = ab \exp(2\gamma t + 2i\kappa \theta)
= \left( \frac{\alpha}{\kappa |D|} \right)^{1/2} \exp(2\gamma t + 2i\kappa \theta + \frac{i\pi}{4}).
\]
(3)

To analyse the possibility of the same sign of magnetic helicity in both hemispheres, the magnetic helicity density in the solar convection zone has been separated into two components: the normal helicity component \( h^{\text{nor}}_m \) and an oscillating component \( h^{\text{osc}}_m \) in both hemispheres. The form of the large-scale mean magnetic helicity density in the solar hemispheres is written in the form
\[
h_m = h^{\text{nor}}_m + h^{\text{osc}}_m = A B + h^{\text{nor}}_m \sin(\omega_\kappa t + \kappa_\kappa \theta + \phi_\phi),
\]
(4)
where \( \omega_\kappa, \kappa_\kappa \) and \( \phi_\phi \) are parameters of the oscillating component of helicity. One can choose \( \omega_\kappa, \kappa_\kappa \) and \( \phi_\phi \) as they are free parameters.

For simplicity, the phase difference of \( \pi/4 \) between the magnetic field and helicity density in equation (3) is ignored. It is assumed that \( \phi_\phi = \pi/4, \kappa_\kappa = \kappa \) and the oscillatory component occurs at low (Figs 8a and b) and high (Fig. 8c) latitudes in both hemispheres. It is assumed that \( \omega_\kappa = i\gamma \) in Figs 8(a) and (c), and \( \omega_\kappa = i3\gamma \) in Fig. 8(b). Fig. 8(a) is found to be basically consistent with the observational fig. 2 of Zhang et al. (2010b). Fig. 8(b) shows a high oscillatory frequency at lower latitudes, while Fig. 8(c) shows oscillation at high latitudes.

As a phase difference of \( \pi/4 \) between the magnetic field and helicity density in equation (3) has been considered, the pattern of

Figure 8. Latitude–time distribution of the magnetic helicity (grey-scale) superposed onto the analogous distribution of the toroidal magnetic field (the solid and dashed contours correspond to positive and negative values of \( B \)). The dimensionless time is plotted on the horizontal axis. The phase difference of \( \pi/4 \) between the magnetic field and helicity in equation (3) has been ignored.
the magnetic helicity with solar cycles is more complex in Fig. 9 than in Fig. 8(a), with \( \phi_b = \pi/4, \kappa, \omega, = \kappa, \omega_b = 1\gamma, \) and the oscillatory component occurs at low latitudes only, but the basic tendency is the same.

5 RESULTS AND DISCUSSION

To analyse the same sign of magnetic helicity of active regions in the northern and southern hemispheres in the decaying phase of the Sun, an oscillating component of magnetic helicity in both hemispheres with the evolution of the solar cycle has been introduced. Possible reasons for the generation of helicity with the same sign in both hemispheres are: (i) the transfer of the magnetic helicity (twist of lines of force) from one hemisphere to the other along the trans-equatorial magnetic field in the subatmosphere owing to its imbalance in the two hemispheres, and (ii) the local generation of reversal twisted magnetic lines of force in the northern or southern hemispheres alone (Kuzanyan et al. 2003). From the above discussions in this paper, a believable form is that the large-scale trans-equatorial form in the subatmosphere.

Following the analysis of the long-term transfer of magnetic helicity in solar active regions, some of the main results are as follows.

(i) The accumulation of magnetic helicity in active regions is a complex process and is not always monotonic because some active regions are located at the solar surface for several months. The major contribution of the accumulation of magnetic helicity generally occurs in the fast-developing stage of active regions. 

(ii) Even if the trend of the hemispheric rule of magnetic helicity holds in the solar atmosphere, the formation of large-scale reversal helicity in solar active regions is an interesting topic. In addition to the possibility of the local generation of reversal magnetic helicity of solar active regions within (northern or southern) hemispheres in the convection zone, another possibility is that the magnetic helicity of solar active regions transfers from different hemispheres in the convection zone, such as from the southern (northern) hemisphere to northern (southern) one, to form the reversal magnetic helicity relative to the hemispheric helicity rule.

(iii) The generation of the mirror-symmetrical component of magnetic helicity was proposed in models of a solar dynamo (e.g. Berger & Ruzmaikin 2000; Kleeorin et al. 2003; Choudhuri et al. 2004; Zhang et al. 2006), although these models cannot explain the formation of the same sign of large-scale magnetic helicity in the northern and southern hemispheres in the butterfly diagram of fig. 2 in Zhang et al. (2010b) in almost the same period of solar cycles. To analyse the statistical evolution and imbalance of magnetic helicity from the subatmosphere with solar cycles, simple oscillatory components of magnetic helicity have been proposed based on observational evidence.

The eruption process of the magnetic field from the solar atmosphere brings the magnetic helicity of solar active regions into interplanetary space. The balance of helicity in both hemispheres can be seriously distorted, even if the dynamo and turbulent convection mechanism produce helicity of opposite signs across the equator and absolute amounts of helicity are comparable.

In order to study the basic properties of the solar dynamo, the symmetrical and unsymmetrical components of the magnetic helicity of solar active regions with regard to the solar cycle in both hemispheres need to be observationally confirmed in detail.

ACKNOWLEDGMENTS

The author would like to thank Drs D. Sokoloff, K. Kuzanyan, V. Pipin, H. Xu, Y. Gao and S. Yang for their kind discussions concerning this study. This study was supported by grants from the National Basic Research Program of China (2011CB811400), the Chinese Academy of Sciences (KJCX2-EW-T07) and the National Natural Science Foundation of China (10733020, 10921303 and 41174153).

REFERENCES

Alfvén H., 1942, Nat, 150, 405
Berger M., Field G., 1984, J. Fluid Mech., 147, 133
Hagino M., Sakurai T., 2005, PASJ, 57, 481

APPENDIX A: DETAILED DESCRIPTION OF THE SOLUTION OF EQUATION (3)

We can assume the form of a travelling wave in equations (1) and (2) as

\[ A = a \exp(\gamma t + i\kappa \theta), \]  
\[ B = b \exp(\gamma t + i\kappa \theta). \]  

(A1)  

(A2)

Under the assumption of the mirror dissymmetry of magnetic helicity \( h_\alpha = AB \) with solar cycle, the simple solution and the corresponding picture of magnetic helicity are presented by Xu et al. (2009). The solution of equations (1) and (2) gives the mirror-asymmetrical component of magnetic helicity relative to the equator with opposite sign. Equations (1) and (2) can be written in the form

\[ \begin{align*}
\gamma A + i\kappa V A &= ab - \lambda \kappa^2 a, \\
\gamma B + i\kappa V B &= i\kappa DA - \lambda \kappa^2 b.
\end{align*} \]  

(A3)

The dispersion relation can be written

\[ (\gamma + \lambda \kappa^2 + i\kappa V)^2 = i\kappa D\alpha, \]  

(A4)

and thus

\[ \gamma = -\lambda \kappa^2 + i\kappa V \pm \sqrt{i\kappa D\alpha}. \]  

(A5)

For \( D > 0 \), we have

\[ \gamma = -\lambda \kappa^2 - i\kappa V \pm (\kappa |D|\alpha)^{1/2} \frac{1 + i}{\sqrt{2}}, \]  

(A6)

and thus as \( \text{Re} \gamma > 0 \)

\[ \gamma = -\lambda \kappa^2 - i\kappa V + (\kappa |D|\alpha)^{1/2} \frac{1 + i}{\sqrt{2}}, \]  

(A7)

For \( D < 0 \), we have

\[ \gamma = -\lambda \kappa^2 - i\kappa V \pm (\kappa |D|\alpha)^{1/2} \frac{-1 + i}{\sqrt{2}}, \]  

(A8)

and thus as \( \text{Re} \gamma > 0 \)

\[ \gamma = -\lambda \kappa^2 + i\kappa V + (\kappa |D|\alpha)^{1/2} \frac{1 - i}{\sqrt{2}}. \]  

(A9)

For maximum \( \text{Re} \gamma \), then

\[ -\lambda \kappa^2 + (\kappa |D|\alpha)^{1/2} = \text{Max}, \]  

(A10)

and

\[ \frac{d\gamma}{dr} = -2\lambda \kappa + i \frac{1}{2} (\kappa |D|\alpha)^{-1/2} = 0. \]  

(A11)

We find

\[ \kappa = \left( \frac{|D|\alpha}{2^{1/3}} \right)^{1/3}. \]  

(A12)

\[ \gamma = -\lambda \left( \frac{|D|\alpha|}{2^{2/3}} \right)^{1/3} - i \left( \frac{|D|\alpha}{2^{1/3}} \right) V + \left( \frac{|D|\alpha^{2/3}}{2^{2/3}} \right) (1 - i) \]  

\[ = \left( \frac{|D|\alpha^{2/3}}{2^{2/3}} \right) (-\lambda + 1 - i) - i \left( \frac{|D|\alpha^{1/3}}{2^{1/3}} \right) V. \]  

(A13)

If \( \lambda = 1 \), then

\[ \gamma = -\frac{1}{2} \left( \frac{|D|\alpha^{2/3}}{2^{2/3}} \right) \left( \frac{|D|\alpha^{1/3}}{2^{1/3}} \right) V. \]  

(A14)

If \( b = 1 \), then

\[ a = \kappa + \lambda \kappa^2 + i\kappa V. \]  

(A15)

The magnetic helicity density can be written in the form

\[ A B = ab \exp(2\gamma t + 2i\kappa \theta) \]  

\[ = \frac{\alpha}{\gamma + \lambda \kappa^2 + i\kappa V} \exp(2\gamma t + 2i\kappa \theta) \]  

\[ = \frac{2^{1/2}/\alpha}{(1 - i)(\kappa |D|\alpha)^{1/2}} \exp(2\gamma t + 2i\kappa \theta) \]  

\[ = \frac{\alpha^{1/2}}{2^{1/2}(\kappa |D|\alpha^{1/2})} (1 + i) \exp(2\gamma t + 2i\kappa \theta) \]  

\[ = \left( \frac{\alpha}{\kappa |D|} \right)^{1/2} \exp \left( 2\gamma t + 2i\kappa \theta + \frac{i\pi}{4} \right). \]  

(A16)

This paper has been typeset from a \TeX/\BibTeX file prepared by the author.