Dark matter merging induced turbulence as an efficient engine for gas cooling

Joaquin Prieto,¹* Raul Jimenez¹,² and Jose Martí¹

¹Institute of Cosmos Sciences, Universitat de Barcelona (IEEC-UB), c. Martí Franquès 1, E-08028 Barcelona, Spain
²Institució Catalana de Recerca i Estudis Avançats, Passeig Lluís Companys, 23, 08010 Barcelona, Spain

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ABSTRACT

We have performed a cosmological numerical simulation of primordial baryonic gas collapsing on to a $3 \times 10^7$ M⊙ dark matter (DM) halo. We show that the large scale baryonic accretion process and the merger of few $\sim 10^6$ M⊙ DM haloes, triggered by the gravitational potential of the biggest halo, are enough to create supersonic ($\mathcal{M} > 10$) shocks and develop a turbulent environment. In this scenario, the post-shocked regions are able to produce both H$_2$ and deuterated H$_2$ molecules very efficiently, reaching maximum abundances of $n_{H_2} \sim 10^{-2} n_H$ and $n_{HD} \sim \text{few} \times 10^{-4} n_H$, enough to cool the gas below 100 K in some regions. The kinetic energy spectrum of the turbulent primordial gas is close to a Burgers spectrum, $\tilde{E}_k \propto k^{-2}$, which could favour the formation of low-mass primordial stars. The solenoidal-to-total kinetic energy ratio is $0.65 \lesssim R_k \lesssim 0.7$ for a wide range of wavenumbers; this value is close to the $R_k \approx 2/3$ natural equipartition energy value of a random turbulent flow. In this way, turbulence and molecular cooling seem to work together in order to produce potential star formation regions of cold dense gas in primordial environments. We conclude that both the mergers and the collapse process on to the main DM halo provide enough energy to develop supersonic turbulence which favours the molecular coolant formation: this mechanism, which could be universal and the main route towards the formation of the first galaxies, is able to create potential star-forming regions at high redshift.

Key words: turbulence – stars: formation – galaxies: formation – large-scale structure of Universe

1 INTRODUCTION

In the currently accepted paradigm to describe the Universe, a cold dark matter (ΛCDM), the formation of bound dark matter (DM) structures takes place via hierarchical growth. While this build-up is driven by DM accretion on to an already virialized DM halo, the process of merging is equally important. The effect of DM mergers in building up the DM halo has been studied in detail both numerically and analytically. Due to new hydrodynamical simulations that have achieved enough resolution, the effect of merging of baryons is being taken into account, including also the important process of transfer of baryonic matter between DM merging haloes.

One aspect of merging that has not been studied in detail is the development of (supersonic) turbulence in the merging haloes due entirely to the DM merging process and its consequent effect on the environment. The current paradigm to power turbulence in the interstellar medium is mainly through the process of supernova (SN) explosions (Norman & Ferrara 1996). However, it is possible that the DM merging process itself does power the initial phase of turbulence and thus star formation.

Using numerical simulations, from the cosmological initial condition, Wise & Abel (2007) and Greif et al. (2008) have discussed the generation of turbulent motions in primordial gas through the virialization process of haloes of mass $\approx 10^8$ and $\approx 5 \times 10^8$ M⊙, respectively. They have shown that supersonic turbulent motions, which partially ionize the primordial gas, allow for an efficient formation of H$_2$ and deuterated H$_2$ (HD) molecules and efficient cooling to a gas temperature below $\sim 100$ K in some regions. They have argued that these turbulent low-temperature regions could be sites of efficient star formation.

The result of Greif et al. (2008) is somehow different from the conclusions of Johnson & Bromm (2006) who have analytically shown that in order to ionize primordial gas through the DM virialization process a halo of mass above $10^8$ M⊙ is needed. This apparent contradiction is easily understood because Johnson & Bromm (2006) took into account the fixed halo average circular velocity for the velocity shock waves instead of a velocity distribution for primordial gas (which can take values higher and lower than the average circular velocity). Summarizing, Greif et al. (2008) have shown

*E-mail: joaquin.prieto.brito@gmail.com

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that the DM virialization process of haloes with mass \( \geq 10^7 \, M_\odot \) can generate supersonic turbulent media (triggered by both hot and cold gas accretion) where coolants (\( H_2 \) and HD molecules) are generated efficiently. However, despite the interesting insights into the development of turbulence in cosmological simulations of primordial gas made by the simulation mentioned above, the authors have not paid attention to the impact of the halo-merger process as a turbulence inductor and they have related the onset of turbulence just with the gas accretion on to the DM haloes.

On the other hand, Wise & Abel (2007) mentioned that the minor and major mergers are able to produce turbulence by Kelvin–Helmholtz instabilities, as well as by gas accretion. Despite this mention, both the studies mentioned above neither characterize the turbulence with power spectrum (PS) in protogalaxy environments nor characterize the gas conditions with probability distribution functions (PDFs) in order to have a clear idea about the primordial gas conditions inside the first protogalaxies. None of the previous studies has ever done this in such kind of environments; therefore, we believe our study brings many new results to the field.

In recent years, there have been a number of works studying the development of turbulence under isothermal and non-isothermal conditions. A very interesting aspect of supersonic gas turbulence is the effect of the gas dynamics on the gas chemistry (mainly on the formation of gas coolants). Milosavljevic et al. (2011) have shown that compressive forcing turbulence leads to \( H_2 \) formation faster than in the solenoidal forcing case. This behaviour is explained by these authors as due to the broader compressive forcing density PDF which creates more \( H_2 \) molecules at initial times. The results of Milosavljevic et al. (2011) could be placed in a cosmological context and argue that if a DM halo is able to produce enough compressive supersonic turbulent motion, then it is possible to produce a large number of molecules and cool down the primordial gas very efficiently.

In this paper, we investigate in detail how turbulence is generated by both mini DM halo merging and gas collapse using a cosmological size simulation. We characterize the gas physical conditions computing PDFs of gas (density, temperature and velocity) and the gas velocity PS of this cosmological simulation, which is a study not conducted before and can be useful for idealized primordial star formation simulations. We follow the merging process in detail and investigate the onset of a turbulent medium inside the merging haloes, including very small mass haloes (\( M \sim 10^6 \, M_\odot \)). The most interesting finding of our work is not only that merging does indeed power turbulence, but also that it enhances significantly the coolants in the primordial gas, thus significantly lowering the temperature of the primordial gas, even below 100 K in some regions. Furthermore, we found that the velocity PS is nearly a Burgers spectrum, \( E_k \propto k^{-2} \), and the solenoidal-to-total kinetic energy ratio is nearly two-third of that found in idealized random fluid simulations.

The merger-inducing turbulence process can be universal and can also take place at lower redshift, thus powering the initial phases of star formation in merging haloes. This paper is structured as follows. In Section 2, we describe the methodology and follow in Section 3 with our main results and analysis. We continue with a discussion of our results in Section 4, and conclude and summarize in Section 5.

### 2 METHODOLOGY

We used the adaptive mesh refinement code RAMSES (Teyssier 2002) with a modified non-equilibrium cooling module with 21 species (including \( H_2 \), HD and LiH molecules with their cooling functions) in order to follow the chemothermal evolution of primordial gas and to study the gas gravitational collapse process in pristine environments (Prieto, Infante & Jimenez 2008). All chemical reaction rates were taken from Stancil, Lepp & Dalgarno (1996), Galli & Palla (1998) and Glover & Abel (2008).

We worked in a concordance LCDM cosmological model: \( h = 0.72 \), \( \Omega_m = 0.73 \), \( \Omega_b = 0.27 \), \( \Omega_\Lambda = 0.04 \), \( \sigma_8 = 0.9 \) and \( n_s = 0.95 \). The dynamical initial conditions were taken from MPgrafic (Prunet et al. 2008) and the initial chemical abundances were taken from Galli & Palla (1998) at \( z = 120 \) (the initial redshift of the simulation).

We first simulated a volume of \( (1 \, \text{Mpc} \, h^{-1})^3 \) of only DM, with \( 256^3 \) particles. Using the HOP algorithm (Eisenstein & Hut 1998), we identified a \( 3 \times 10^7 \, M_\odot \) halo at \( z = 10 \). In order to better resolve the formation of that halo and to study its baryon component, we re-simulated the same \( (1 \, \text{Mpc} \, h^{-1})^3 \) volume with \( 512^3 \) particles (with a particle mass of \( \sim 800 \, M_\odot \)), including gas with non-equilibrium primordial chemistry from the beginning of the simulation at \( z = 120 \). The gas dynamics were computed on a root grid of 64\(^3\) computational zones, which was geometrically refined towards the centre, increasing the spatial resolution by a factor of 2 within the central one-eighth of the volume. This refinement criterion was applied three times, generating four nested meshes (including the root grid), each with 64\(^3\) computational cells.

Inside the innermost 64\(^3\) mesh, the gas dynamics were computed using seven extra levels of adaptive mesh refinement by a factor of 2. This adaptive refinement was based on four different criteria: (i) the number density of DM particles (a finer refinement level is created in cells containing more than four DM particles); (ii) the baryonic mass density; (iii) the gas pressure gradient (for \( \Delta p / p \geq 2 \)); and (iv) the Jeans length, to satisfy Truelove’s condition (Truelove et al. 1997). The pressure gradient criterion was included in order to better resolve the turbulent flow, as discussed in Kritsuk, Norman & Padoan (2006). The three geometrical refinement levels, plus the additional seven adaptive refinement levels, give an effective spatial resolution corresponding to that achieved by a uniform mesh of 65 536\(^2\) computational elements, and corresponding to a proper size of \( \approx 1.95 \, \text{pc} \) at \( z \approx 10.0 \).

This setup was used from the beginning of the simulation, at \( z = 120 \), until \( z = 50 \). At \( z = 50 \), another geometrical refinement criterion was added (while maintaining the others): uniform spatial resolution till level 15 was imposed around the densest central region of the halo, creating a uniform mesh of 512\(^3\) elements which was refined with a \( J = 0.125 \) Jeans criterion till level 16 (due to technical reasons, the uniform refinement criterion was imposed gradually and it was completed at \( z = 30 \)). This high-refinement region, covering a proper size of \( \approx 2.0 \, \text{kpc} \) at \( z \approx 10.0 \), served the purpose of better resolving the generation of turbulent motions in the central region of the halo. The results presented in this work are based on the analysis of this central 2.0-kpc region at the end of the simulation at \( z \approx 10.0 \).

### 3 RESULTS AND ANALYSIS

The simulation presented in this work follows the baryonic matter accretion process on to a \( \approx 3 \times 10^7 \, M_\odot \) halo (hereinafter the main halo) with \( T_{\text{vir}} \approx 7770 \, \text{K} \) and \( V_{\text{vir}} \approx 14.6 \, \text{km} \, \text{s}^{-1} \).

At \( z \approx 14 \), the main halo is crossed by two interacting DM minihaloes of mass \( \sim 10^5 \, M_\odot \). These two interacting minihaloes merge and create a spinning baryonic overdensity going out the main halo central region. Due to this violent merging process, the gas...
develops a shock wave which increases the free-electron fraction. The post-shocked gas develops turbulence and creates H\(_2\) and HD molecules very efficiently behind the merged overdensity.

After the merging process, the main halo continues to accrete gas mainly by filamentary flows. The accretion process heats up the gas till \(T \gtrsim 7 \times 10^{10}\) K and accelerates it developing supersonic shocks. Near \(z \approx 12\), at the densest main halo central regions, the post-shocked primordial gas creates a large number of both H\(_2\) and HD molecules in a very efficient way. Furthermore, due to the compressional turbulent motions triggered by the roughly radial collapse, the molecular formation is enhanced as well. Because of the molecular formation process, the turbulent gas can reach temperatures even below 100 K in some regions and it is able to form potential star formation regions of dense gas \((n \gtrsim 10^4\) cm\(^{-3}\)) at low temperatures \((T \sim \text{few} \times 100\) K\).

Figs 1–3 show the \(z\)-projection of the gas number density, gas temperature and gas vorticity inside a cubic box of \(\sim 3\) kpc a side (larger than the highest refinement level box of \(\sim 2\) kpc a side) as a function of redshift. The horizontal and vertical lines in the vorticity map are features due to the lower resolution outside the highest refinement inner box (\(\sim 2\) kpc a side).

The analysis presented below is centred on both the chemothermal and the dynamical properties of the primordial gas. In this paper, we do not analyse either the gas-fragmentation process or the primordial gas clump physical properties. This kind of study is postponed to a future work.

### 3.1 Chemistry and turbulence

It is well known that below \(\sim 10^4\) K the primordial gas can be cooled down only by molecular line emissions. In this sense, the molecules are crucial for the formation of star formation regions of cold dense gas.

**Figure 1.** The \(z\)-projection of the mass-weighted gas number density at \(z = 15\) (top left-hand panel), 13 (top right-hand panel), 12 (bottom left-hand panel) and 10 (bottom right-hand panel). Top left-hand panel: at \(z = 15\), near the main halo central region, appear three interacting minihaloes; the nearest two overdensities will merge. Top right-hand panel: at \(z = 13\), after the merger, a spinning overdensity crosses the main halo central region as a supersonic bullet; the overdensity leaves a post-shock region where molecular coolants are formed very efficiently due to both the turbulent motions and the enhancement in the free-electron fraction. Bottom left-hand panel: at \(z = 12\), the spinning overdensity is leaving the main halo central region which starts to develop more overdensities due to the gas-accretion process. Bottom right-hand panel: at \(z = 10\), the gas has developed a number of overdensities near the main halo central region as a consequence of the combined effect of molecular cooling and gas dynamics in turbulent compressed regions.
The $\text{H}_2$ molecule is the most abundant molecule in primordial gas (Galli & Palla 1998). Due to its symmetry, it does not have a permanent dipolar moment and only quadrupolar rotational transitions are allowed (Abgrall, Roueff & Viala 1982).

The main path for $\text{H}_2$ formation in primordial environments is a two-step channel (Peebles & Dicke 1968):

\begin{equation}
e^− + \text{H} → \text{H}^- + h\nu, \tag{1}
\end{equation}

\begin{equation}
\text{H}^- + \text{H} → \text{H}_2 + e^−. \tag{2}
\end{equation}

The $\text{H}_2$ molecule is able to drop the unperturbed primordial gas temperature till $T \sim (2–3) \times 10^2 \text{ K}$ but $\gtrsim 100 \text{ K}$ in perturbed regions with high electron fraction, for example, in post-shocked regions of turbulent gas with high Mach number or in relic H II regions.

On the other hand, the HD molecule, also present in primordial gas, has a permanent dipolar moment which allows faster rotational transitions. Furthermore, due to its higher reduced mass, it has rotational energy levels lower than the $\text{H}_2$ levels, thus reaching lower minimum gas temperatures due to rotational transitions (Flower & Roueff 1999).

The main path for HD production in primordial gas is based on the $\text{H}_2$ molecule (e.g. Flower 2000):

\begin{equation}
\text{D}^+ + \text{H}_2 → \text{HD} + \text{H}^+. \tag{3}
\end{equation}

Due to the reasons mentioned above, the HD molecule is able to cool the unperturbed primordial gas below 100 K, but under perturbed conditions, the gas could reach the cosmic microwave background (CMB) temperature floor due to HD cooling.

### 3.1.1 $\text{H}_2$ and HD molecules

It is interesting to understand how the gas dynamics affect the molecular formation process in primordial environments and what are the consequences of this highly non-linear process.

Because $\text{H}_2$ production depends on the abundance of free electrons (and $\text{H}^−$) and HD production depends on that of $\text{H}_2$ (and $\text{D}^+$), there are two paths to increase the molecular production rate: (i) to increase the local gas density and enhance the number of reactions, which can be a natural consequence in highly turbulent environments; or (ii) to increase the number of free electrons by a given physical process, for example, an ionization front.
Figure 3. The z-projection of the z-component of gas vorticity ($\nabla \times \mathbf{v}$). The different maps correspond to the same redshifts of Fig. 1. The maps show how a turbulent environment is developed by both the merger process and the accretion process near the main halo central region.

The first path can be the result of either a gravitational collapse process or a highly turbulent environment, as mentioned above. An example for the first case is the collapse of minihaloes at high redshift (McGreer & Bryan 2008); in this case, the density enhancement due to the gravitational collapse produces enough molecules to reduce the primordial gas temperature to few $\times 100$ K. An example for the second case can be the interaction of gas flows with a (massive enough) DM halo; the flows can produce a turbulent enough environment where overdensities fill the interaction region (Heitsch et al. 2008).

In primordial environments, the second path can be the result of post-shock waves formed in the virialization process of massive enough DM haloes. Despite the fact that previous works based on simplified unidimensional models have shown that post-shock waves created through the baryonic collapse on to DM haloes with mass $M < 10^8 M_\odot$ cannot create enough free electrons in order to enhance the molecular production (Johnson & Bromm 2006), this is not completely true due to the real non-uniform baryonic velocity field, that is, for a given DM average circular velocity, the gas content of the halo is able to reach velocities higher (and lower) than its average and then it could create shocks able to increase the free-electron fraction even if the average circular velocity is not high enough. Furthermore, in the hierarchical paradigm of structure formation, it is natural that the smallest haloes interact with the biggest ones, producing a merger process which can create shock waves as well and consequently increase the free-electron fraction due to a violent merger process.

The number density versus gas temperature plane of our simulation at $z = 15$ (top left-hand panel), 13 (top right-hand panel), 12 (bottom left-hand panel) and 10 (bottom right-hand panel) is shown in Fig. 4. The figure shows number densities $n \geq 10^3$ cm$^{-3}$ corresponding to the densest gas of the simulation central regions. At $z = 15$, before the merger, the gas temperature is above $\sim 400$ K in the densest regions associated with the minihaloes. After the merger, the gas develops very high temperature regions with $T \gtrsim 10^4$ K due to the violent merger process. At $z = 13$, the high-temperature gas has $T \lesssim 10^4$ K because of the hydrogen recombination line cooling. This temperature is still enough to both dissociate the molecular coolants and enhance the ionization fraction. At this redshift, the high-density regions, $n \geq \text{few} \times 10^3$ cm$^{-3}$, with high temperature, $T \lesssim 10^4$, are associated with the resultant hot dense spinning overdensity going away from the main halo central region, while the lower density hot regions correspond to the gas expelled by the explosive merger. At the same time, due to the enhanced ionization
fraction, the post-shocked regions have had enough time to create HD molecules in order to drop the gas temperature till the CMB limit. [In our previous study of first galaxy formation in Prieto et al. (2011), the mass resolution was a factor of 8 lower than in this simulation. It is well known that in order to see the HD effect on the gas temperature, a density threshold is needed (McGreer & Bryan 2008). Due to the higher resolution of this simulation, here it is possible for the HD molecule to dominate gas cooling in some regions, including the formed gas clumps not analysed in this paper.] At later times, when the hot dense spinning overdensity is outside the main halo central region (and certainly not captured in the bottom panels of Fig. 4), a large amount of gas reaches temperatures \( T \lesssim 100 \text{ K} \) in the density range \( 10 \lesssim n/\text{cm}^{-3} \lesssim 10^3 \); potential places for star formation. These final conditions are the result of a post-shocked turbulent evolution followed by gas accretion on to the central region.

At the first glance, our simulation seems to contradict the result of Shchekinov & Vasiliev (2006). These authors found that in order to ionize the primordial gas in a halo-merger process, the halo mass should be \( \gtrsim 10^7 \, M_\odot \), whereas in our simulation, the merged haloes have masses \( \sim 10^8 \, M_\odot \). The explanation for this apparent contradiction is that the result of Shchekinov & Vasiliev (2006) is based on an isolated halo–halo interaction, whereas in our case the minihaloes collide after being accelerated by the few \( \times 10^7 \, M_\odot \) main halo; thus, the merger is much more violent than an isolated \( \sim 10^6 \, M_\odot \) halo–halo interaction.

Fig. 5 shows the number density–molecular mass fraction plane for both \( H_2 \) (top panels) and HD (bottom panels) molecules at the same four different redshifts as in Fig. 1. As mentioned above, before the merging there are a number of regions with temperature \( T \gtrsim 400 \, \text{K} \) and number density \( n \gtrsim 10^3 \, \text{cm}^{-3} \) associated with the minihaloes (consistent with Fig. 4). After the merger, both figures show that a number of molecules are destroyed and the molecular mass fraction is reduced in a number of regions (top right-hand panels). At the same time, there appear regions where molecules have continued working to reduce the gas temperature, allowing for the formation of denser zones. Approximately 40 Myr later (at \( z = 12 \)), the gas has developed very dense regions with \( n \gtrsim 10^7 \, \text{cm}^{-3} \). Furthermore, there are new regions with \( n \gtrsim 10^8 \, \text{cm}^{-3} \) at lower molecular abundance. A hundred of millions of years later there seem to be two populations of dense regions with \( n \gtrsim \text{few} \times 10^3 \, \text{cm}^{-3} \): one with temperature below 300 K and \( H_2 \) mass fraction \( f_{H_2} \gtrsim 10^{-2} \) and HD mass fraction \( f_{\text{HD}} \lesssim 10^{-5} \) and another one with temperature above 300 K and \( f_{H_2} \gtrsim 2 \times 10^{-3} \) and \( f_{\text{HD}} \gtrsim 10^{-7} \). These two different regions can be the result of two different evolutions near the main halo central region. The coolest one seems to be the result of evolution on to a post-shocked environment where the overdensities grow up in regions with an enhanced initial molecular mass fraction. The other overdensities seem to develop in regions not too much affected by the merger process presenting a molecular mass fraction one order of magnitude below (Johnson & Bromm 2006; McGreer & Bryan 2008).

### 3.1.2 Molecular cooling and gas velocity

The figures shown in the previous section suggest that the primordial gas dynamics trigger the formation of abundant \( H_2 \) and HD molecules through both the halo-merging and the baryonic collapse process. Consequently, these molecules are able to cool the gas...
Figure 5. Top panels: number density–$\text{H}_2$ mass fraction plane at the same redshifts of Fig. 4. In brown are all grid cells with temperature in the range 100–300 K, in red, $300 < T < 500$ K, in yellow, $500 < T < 1000$ K, in green, $1000 < T < 5000$ K, and in blue, $5000 < T < 10000$ K. Due to the post-shock conditions, the $\text{H}_2$ molecule is able to reach very high mass fractions, $f_{\text{H}_2} \gtrsim 0.01$, which enables the gas to reach temperatures $T \lesssim 200$ K. Bottom panels: same as the top panels, but for the HD molecule. Again, due to the post-shock conditions, the HD molecule can be produced very efficiently and it reaches $f_{\text{HD}} \lesssim 10^{-5}$. This high mass fraction allows the gas to reach temperatures $T \lesssim 100$ K.
allowing it to reach $T \lesssim 300$ K at density peaks, reaching temperatures even below 100 K in some regions.

Based on this dynamical dependence of molecular formation, it is interesting to know how the velocity modulus/Mach number is related to the number density, gas temperature and molecular mass fractions. Fig. 6 shows the molecular mass fraction–velocity modulus planes for both the H$_2$ (top panels) and the HD (bottom panels) molecules at the same redshifts of previous figures. The velocity modulus is computed as the value of $[(v_{\text{grid},x} - v_{\text{av},x})^2 + (v_{\text{grid},y} - v_{\text{av},y})^2 + (v_{\text{grid},z} - v_{\text{av},z})^2]^{1/2}$ for each grid; here $v_{\text{av},i}$ is the average gas velocity inside the analysed box of $\approx 22$ comoving kpc, that is, $(\sum v_{\text{grid},i})/N_{\text{grid}}$, where $i = x, y, z$; $v_{\text{grid},i}$ is the local grid velocity and $N_{\text{grid}}$ is the total number of grids inside the highest resolution volume. We emphasize that the analysed volume is not the entire 1 Mpc$^3$ simulation comoving box but the innermost highest resolution volume of $\approx 2$ proper kpc at $z = 10$ ($\approx 1.4$ proper kpc at $z = 15$). Keeping this in mind, the gas velocity definition of $v_{\text{gas}} = v_{\text{grid}} - v_{\text{av}}$ has all sense because it represents the local grid velocity without taking into account the gas average movement of the analysed volume. (The computed rms physical velocity through the evolution is 8.18 km s$^{-1}$ at $z = 15$, 7.06 km s$^{-1}$ at $z = 13$, 6.21 km s$^{-1}$ at $z = 12$ and 4.18 km s$^{-1}$ at $z = 10$.) Despite the main halo circular velocity $V_{\text{circ}} \approx 14.6$ km s$^{-1}$, the main halo central regions develop gas velocities above 22 km s$^{-1}$, enough to enhance the ionization fraction and produce shock waves in or at least enough to compress significantly the primordial gas and accelerate the molecular formation process. At $z = 13$, about 40 Myr after the merger process, the gas reaches velocities near 60 km s$^{-1}$ evidencing the violation of the minihalo interaction. The interaction is able to enhance the temperature till $\sim 10^4$ K, slightly increase the free-electron fraction and trigger an efficient molecular formation process.

The molecular mass fraction–Mach number planes for both H$_2$ (top panels) and HD (bottom panels) molecules at the same redshifts as in the previous figures are shown in Fig. 7. The Mach numbers were computed based on the velocity modulus defined above. From these figures, we can see that the evolution almost all primordial gas with $n > 10^3$ cm$^{-3}$ develops supersonic waves. After the merger, there are regions with Mach number near 50, evidencing the violation of the process again. At later times, the Mach numbers (and velocities) decay at the main halo central region to increase again at $z = 10$, showing how after the merger the gas relaxes and reaches Mach numbers below 10 at the densest regions, which increase again, promoted by the baryonic accretion process. These two figures show that the coolest gas has the highest molecular mass fraction but interestingly this coolest gas occupies the high-velocity/Mach number regions as a proof of the fact that the highly supersonic turbulent gas is composed of a high molecular mass fraction.

At the moment, we have a picture in which the mergers and the gas-collapse process on to the main halo trigger efficient molecular formation in high-velocity (supersonic) gas regions, allowing the gas to reach temperatures even below 100 K. Fig. 8 shows the gas temperature–velocity modulus plane (top panel) and the gas temperature–Mach number plane (bottom panel), for number density above 10 cm$^{-3}$, both at the same redshifts as before. From this figure, it is clear how the merger process creates high-density regions with temperature $\sim 10^4$ K due to its violence. At lower redshifts, this figure shows that the high-density regions with $n \gtrsim 10^7$ cm$^{-3}$ are associated with both high Mach numbers and high velocities, suggesting that the collapsed gas converging in the main halo central region is compressed at velocities $10 \lesssim v_{\text{max}} \lesssim 30$ km s$^{-1}$, which can help to increase both the molecular abundances and consequently the gas cooling. In this way, the mergers and the baryonic matter collapse process would favour the formation of cold dense primordial gas regions and create potential primordial star formation clouds.

### 3.2 Probability distribution function

In the hydrodynamical context, the density PDF $p$ gives the probability to find a fluid element with a density between $\rho$ and $\rho + d\rho$ inside the analysed volume. In other words, it quantifies the fraction of gas in a given range of density as $\int_{\rho}^{\rho + d\rho} p(\rho') d\rho'$.

Vázquez-Semadeni (1994) shows that in the high Mach number limit, without viscosity and neglecting the gravitational term, the normalized Euler equations for hydrodynamics describe a scale-invariant pressureless fluid, that is, the probability to create a relative density fluctuation in a given region is independent of the local density. As explained in Slyz et al. (2005), if we assume that in a fully developed turbulent fluid the density field is a random variable and the events of successive density increments are independent, then the central limit theorem states that the density PDF is lognormal.$^2$

The argument exposed above has been extensively studied using hydrodynamical simulations of isothermal supersonic turbulence without gravity (e.g., Kritsuk et al. 2007; Federrath, Klessen & Schmidt 2008). These simulations have shown that the density PDF is well represented by a lognormal distribution:

$$p(\ln \rho) d\ln \rho = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln\rho - \ln\mu)^2}{2\sigma^2}} d\ln \rho,$$

where $\ln \rho = -\frac{\sigma^2}{2}$.

with $\sigma$ the standard deviation of the logarithm of the density. For a formal proof of the lognormal density PDF distribution based on Pope & Ching (1993), see Nordlund & Padoan (1999).

The lognormality of the PDF is valid for isothermal turbulent fluids without gravity, but under realistic conditions, the assumption of isothermality is not well justified due to the cooling/heating processes triggered by both the chemistry and the gas dynamics. Furthermore, in less than a dynamical time, the self-gravity becomes unavoidable at high-density regions in a realistic scenario. Therefore, we do not expect a priori to find a perfect lognormal density PDF in our non-isothermal simulation with self-gravity.

Fig. 9 shows the PDF of both the gas density (top panel) and the gas temperature (bottom panel) for redshifts from 15 to 10. The PDFs (for density, temperature and velocity) were computed as the histogram of the given quantity inside equal-size bins in logarithmic scale. The density PDF shows a clear deviation from a lognormal distribution which as mentioned above is a consequence of the non-isothermal evolution with self-gravity.

It is interesting to note how the high-density tail of the PDF evolves with redshift; it takes higher values at lower redshift to show how the gas develops high-density ($n \gtrsim 10^7$ cm$^{-3}$) regions

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1 For comparison, the average circular velocity of a $10^8$-M$_\odot$ DM halo collapsed at $z \approx 10$ is $V_{\text{circ}} \approx 21.8$ km s$^{-1}$.

2 The PDF is lognormal because the random variable density is the product of independent random variables instead of the sum of random variables; in the latter case, the PDF should be a normal distribution.
Figure 6. Top panels: H$_2$ mass fraction–velocity modulus plane at $z = 15$ (top left-hand panels), $z = 13$ (top right-hand panels), $z = 12$ (bottom left-hand panels) and $z = 10$ (bottom right-hand panels). Bottom panels: same as the top panels, but for the HD molecule. The colours show the same range of temperature as in Fig. 5. Both panels show how the merger process increases the gas temperature and creates very high velocity gas which is able to create shocks and enhance the free-electron fraction favouring the molecular formation in post-shock regions. After the merger, the gas of the main halo central region decreases its velocity, but it is increased again by the baryonic accretion gas process at lower redshift. This accretion process is able to create high-velocity gas waves which compress it and increase the molecular mass fraction.
Figure 7. Top panels: H$_2$ mass fraction–Mach number plane at the same redshifts as the previous figures. Bottom panels: same as the top panels, but for the HD molecule. The colours show the same range of temperature as in Fig. 5. Both planes show that Mach numbers above 10 are reached in regions with both high molecular abundances and low temperatures. In general, the coolest regions tend to show the highest Mach numbers which relate turbulent regions with high molecular abundances, evidencing the effect of merger and supersonic turbulence on the primordial gas, that is, to increase the molecular mass fraction inducing efficient cooling.
Figure 8. Top panels: gas temperature–velocity modulus plane for $n > 10 \text{ cm}^{-3}$. In yellow, $10^1 \leq n / \text{cm}^{-3} < 10^2$; in green, $10^2 \leq n / \text{cm}^{-3} < 10^3$; in blue, $10^3 \leq n / \text{cm}^{-3} < 10^4$; and in red, $10^4 \leq n / \text{cm}^{-3} < 10^5$. Top panels: show that high-density gas with $n \gtrsim 100 \text{ cm}^{-3}$ is able to reach velocities above $v_{\text{rms}} \approx 20 \text{ km s}^{-1}$, high enough to enhance the free-electron number density and compress the gas increasing the molecular mass fraction. Bottom panels: same as the top panels, but for the gas temperature–Mach number relation. High-density gas with temperature below $10^3 \text{ K}$ shows supersonic velocities with the highest Mach numbers associated with the lowest temperature regions. The high-density regions with $n > 100 \text{ cm}^{-3}$ clearly present supersonic turbulence. This feature relates low-temperature, high Mach number regions with high-density regions, showing a clear relation among supersonic turbulence, molecular cooling and high-density regions.
Figure 9. Top panel: number density PDF for different redshifts. In black, $z = 15$; in red, $z = 13$; in green, $z = 12$; and in blue, $z = 10$. This panel shows that the density PDF does not have a symmetric lognormal distribution and a high percentage of the analysed volume corresponds to low-density, $n \lesssim 0.1 \text{ cm}^{-3}$, regions. As explained in the text, the high-density tail covers two regimes: the first one for $0.5 \lesssim n \text{ cm}^{-3} \lesssim 50$ shows an $n^{-3/2}$ behaviour (purple line) associated with a nearly isothermal collapse process; the second one, at densities $n \gtrsim 50 \text{ cm}^{-3}$, follows an $n^{-1}$ behaviour (light blue line) associated with the disc structures of collapsed gas clumps. Bottom panel: same as the top panel, but for the gas temperature. At $z = 10$, the temperature PDF shows two clear peaks consistent with a two-phase fluid of low-temperature regions coexisting with high-temperature regions. These two peaks are not so clear at $z > 13$ and develop through the gas-collapse process.

The average density inside the analysed box for all redshifts of Fig. 9 is $\bar{n} \approx (1-2) \times 10^{3} \text{ cm}^{-3}$. The density PDF shows a peak at very low densities (few $\times 10^{-3} \text{ cm}^{-3}$) instead of the average density like in the lognormal isothermal distribution. This peak is easily explained in the collapsing scenario: through the collapse process, the baryonic matter piles up near the main halo central region. While in this region the gas develops overdensities in a tiny percentage of the volume, the rest of the volume is dominated by low-density regions due to the converging gas flow on to the main halo. This dynamical behaviour creates more low-density regions at lower redshift as shown by the density PDF and favours the formation of new high-density regions.

The temperature PDF in Fig. 9 shows two clear peaks at $z = 10$, one at $\sim \text{few K}$ and another one at $\sim 10^{3} \text{ K}$. These two peaks are not easily recognized at $z > 13$ and they appear at lower redshift as a consequence of the baryonic matter accretion process. The two peaks feature the existence of two preferred gas states of high and low temperature coexisting inside the analysed volume. Again, in the collapsing scenario, the low-temperature gas can be associated with decreasing (expanded and very low) density regions – which are not in equilibrium with the CMB photons – due to the infalling gas flow on to the main halo central region. These regions contain the gas inside the voids surrounded by high-density regions due to the converging gas flow on to the DM halo through the virialization process. Between these two temperature peaks is located the $\sim \text{few} \times 10^{3} \text{ K}$ gas, where the molecular coolants are working, allowing for the formation of high-density regions at lower redshift. Of course, the molecular cooling region is a tiny percentage of the total $\sim \text{few} \times 10^{3} \text{ K}$ range which should be dominated by thermally unstable gas.

Because we have shown that the gas dynamics are related to the cooling process in primordial environments, it is interesting to study the mean gas velocity PDF of the system. Fig. 10 shows the mean gas velocity PDF for the same redshifts as analysed in previous sections. The two vertical lines are the the main halo average circular velocity (14.6 km s$^{-1}$ in purple) and the average circular velocity of a 10$^{7}$-M$_{\odot}$ halo (21.8 km s$^{-1}$ in light blue). At $z = 15$, before the minihalo merger, the gas velocity is below $\sim 20 \text{ km s}^{-1}$, a velocity too low to create shock waves. At $z = 13$, after the merger, the gas develops regions with velocities near 60 km s$^{-1}$ and it shows a longer high-velocity tail. At this redshift, above 1 per cent of the gas has mean velocity above 21.8 km s$^{-1}$. At lower redshifts, this percentage slightly increases but does not reach 2 per cent of the volume. Furthermore, below $z = 13$, the maximum velocity slightly surpasses 30 km s$^{-1}$, showing that the very high velocities triggered by the merger are replaced by moderately high velocities promoted by the gas-accretion process on to the main halo.
Figure 10. Gas velocity PDF at different redshifts. In black, $z = 15$; in red, $z = 13$; in green, $z = 12$; and in blue, $z = 10$. The vertical lines show the main halo average circular velocity of 14.6 km s$^{-1}$ (in purple) and the average circular velocity of a 10$^5$-M$_\odot$ halo (in light blue): 21.8 km s$^{-1}$. The time-evolution of the PDF shows how the minihalo-merger process creates regions with velocities near 60 km s$^{-1}$. After that the maximum gas velocity decreases to values near 30 km s$^{-1}$, mainly supported by the gas-accretion process on to the main halo.

3.3 Structure functions

In order to characterize the turbulence, we computed the second-order velocity structure function. The velocity structure function of order $p$ is defined as

$$ S_p(\ell) = \langle |v(x + \ell) - v(x)|^p \rangle \propto \ell^{\zeta(p)}, \tag{6} $$

where the velocity component $v$ is parallel (longitudinal structure function) or perpendicular (transversal structure function) to the vector $\ell$. The spatial average is over all values of the position $x$ and $\zeta(p)$ is the exponent of a power-law fit to the structure function. The second-order longitudinal structure function averaged over the central region of the simulation from $z = 15$ to 10 is plotted in Fig. 11. It is well approximated by a power law, $\ell^{1.02}$, in the range of scales $\ell = 50$–200 pc (light blue line) for the given range of redshifts.

Taking into account that after the merger the dynamics of the system are supported by the matter-accretion process at scales of $L_{\text{max}} \gtrsim 500$ pc, and the molecular cooling process is efficient (in high-density regions) at scales $L_{\text{min}} \lesssim \text{few} \times 10$ pc, the range in which the power law is valid corresponds roughly to the inertial range of the system.

In order to relate this result with the observed properties of local Galactic star-forming regions, where the turbulence is driven mainly by SN shock waves, Fig. 11 actually shows the square root of the second-order structure function next to the velocity-scaling law found in galactic molecular clouds (Larson 1979, 1981), which follow the relation $S_2(\ell) \propto \ell^{0.76}$. Interestingly, on the scale of approximately 100 pc, the velocity dispersion in the primordial gas of our simulation is comparable to that of the molecular gas in our Galaxy. However, due to the larger temperature in the primordial gas, the Mach number of the turbulence is a few times smaller than in nearby molecular clouds at the same scale.

3.4 Fourier analysis

3.4.1 Power spectrum

After we have shown that supersonic turbulence, triggered by both merger processes and gas accretion, promotes molecular formation and gas cooling in primordial environments, it is interesting to study how the kinetic gas energy is distributed in their different Fourier modes.

The kinetic energy PS is defined as

$$ \hat{E}_k = \frac{1}{2} \langle 4\pi k^2 |\theta_k|^2 \rangle, \tag{7} $$

where $\theta_k$ is the gas velocity Fourier transform, $|\theta_k|^2$ is its complex conjugate and $k \equiv |k|$ is the modulus of the wavenumber. $\hat{E}_k$ represents the kinetic energy associated with Fourier modes of wavenumber between $k$ and $k + dk$. The kinetic energy spectrum is useful to distinguish between subsonic Kolmogorov turbulence, with $\hat{E}_k \propto k^{-5/3}$, and shock-dominated Burgers turbulence, with $\hat{E}_k \propto k^{-2}$.

The energy PS for our analysed volume is shown in Fig. 12. This figure shows the kinetic energy PS from $z = 15$ to 10. The purple line shows an $\hat{E}_k \propto k^{-2}$ Burgers PS. It is clear that at $z = 15$ (black line) the gas energy spectrum follows a nearly Burgers behaviour from $k \approx 10$ kpc$^{-1}$ (physical scale $\approx 600$ pc) to $k \approx 100$ kpc$^{-1}$ (physical scale $\approx 60$ pc). Actually, the power-law exponent is a bit steeper at this redshift. At lower redshifts, the power-law exponent decreases and it reaches values near $-2.2$ at $z = 10$. This result is in good agreement with the second-order longitudinal velocity structure function shown in the previous section: for $\hat{E}_k \propto k^{-n}$, $S_2(\ell) \propto \ell^{n-1}$, which for $n \approx 2$ gives approximately $S_2(\ell) \propto \ell^1$ as shown in Fig. 11. The close to Burgers power law indicates that the turbulent motions triggered by both the merger process and the matter-accretion process on to the main halo produce supersonic velocities. This
result is in agreement with our results of Section 3.1.2, where we have shown that the primordial gas develops supersonic $M \gtrsim 10$ motions through the collapse process in high-density regions, near the main halo central region.

It is interesting to note that at high wavenumbers the PS does not decay as fast as in supersonic isothermal simulations without self-gravity (Kritsuk et al. 2007). This feature can be understood because at small scales – of the order of a few $\times 10^{10}$ in high-density regions – the gravitational potential energy is converted into kinetic energy which is distributed among the longitudinal and transverse velocity modes as we will show in the next subsection.

In this sense, in the same way as the large scale accretion process drives the turbulent and shocked environments at the main halo central region, at small scales, the gravity is able to increase the kinetic energy through a turbulent energy cascade which favours the formation of gravitationally bounded cold dense clumps (Slyz et al. 2005; Federrath et al. 2011).

As mentioned in Section 3.1.2, the supersonic turbulence and the molecular cooling seem to work together developing high-density regions: the large-scale collapse triggers the formation of supersonic turbulent regions where both H$_2$ and HD are formed by efficiently cooling down the gas and allowing gravity to dominate at few $\times 10^{10}$ pc scales. These phenomena could rise the velocity powers at small scales compared to the non-self-gravity case as shown in Fig. 12. One interesting thing to take into account for future idealized primordial star formation simulations, for example, Clark et al. (2011b), is the PS slope found in this simulation. Our $\sim -2$ slope is different from the assumed $-4$ slope in Clark et al. (2011b). This difference could be non-negligible because it could imply more fragmentation at small scales, certainly an interesting point to study.

### 3.4.2 Solenoidal versus longitudinal modes

The Helmholtz decomposition of the velocity field allows us to study the kinetic energy content in compressional modes ($\hat{v}_{i,k}$ with $\nabla \cdot \hat{v}_{i,k} = 0$) and in solenoidal modes ($\hat{v}_{i,k}$ with $\nabla \cdot \hat{v}_{i,k} = 0$). In Fourier space, these two components are defined as

$$\hat{v}_{i,k} = (\hat{v}_k \cdot \hat{k})/k^2,$$

$$\hat{v}_{i,k} = \hat{v}_k - (\hat{v}_k \cdot \hat{k})/k^2.$$

Fig. 13 shows the solenoidal-to-total kinetic energy ratio $R_k = \frac{\frac{1}{2}(4\pi k^2 \hat{v}_{i,k} \cdot \hat{v}_{i,k})}{\frac{1}{2}(4\pi k^2 \hat{v}_k \cdot \hat{v}_k)}$ from $z = 15$ to $10$. For redshifts below $15$ and for all wavenumbers shown in the plot, the solenoidal modes contain more than $50$ per cent of the total kinetic energy, that is, $R_k > 0.5$. Furthermore, it is very interesting to note that above $k \approx 30$ kpc$^{-1}$ (physical scales below $\sim 200$ pc) $R_k$ naturally takes values between $0.65$ and $0.70$, nearly a two-thirds distribution found in previous works (Federrath et al. 2011) which arises naturally in a three-dimensional space where for a given compressional direction there are two rotational directions. It seems that at $z = 15$ the gas has not distributed the energy following an equipartition rule. May be it needs the merger and the gas accretion on to the main halo in order to redistribute its energy to get the two-thirds value; in other words, the gas needs to pass through chaotic processes to achieve equipartition of energy.

In view of these results, we can say that both the minihalo merger process and the large-scale gravitational collapse process triggered by the main DM halo create a supersonic turbulent cascade which naturally distributes the kinetic energy in a ratio $E_i/E_c \approx 0.5$, where $E_i$ and $E_c$ are the kinetic energy in solenoidal and compressional modes, respectively.

### 4 DISCUSSION

In this paper, we have shown that the minihalo-merger process inside an $\approx 3 \times 10^8$ M$_\odot$ halo is able to form shock waves and supersonic turbulence in primordial gas. Furthermore, we have shown that the gas-accretion process on to the main DM halo creates more
supersonic turbulence and maintains the turbulence formed in the merger process. It is interesting that despite the halo mass $M < 10^8 \, M_\odot$ (the limit suggested by Johnson & Bromm 2006), it is possible to find gas elements with enough velocity to create shocks (due to both the mergers and the gas accretion). As mentioned above, it is not surprising if we think that the estimation of Johnson & Bromm (2006) is based on the average halo circular velocity and does not take into account the velocity distribution of a more realistic system. The inclusion of a realistic velocity distribution would allow the gas to reach velocities higher (and lower) than its average circular velocity and create shocked regions. This could be possible for haloes with masses well below $M < 10^7 \, M_\odot$ as has been previously shown by Greif et al. (2008) and through this work. However, here we have also demonstrated, for the first time, that even very low mass haloes ($M \sim 10^6 \, M_\odot$), because of the presence of a $10^5$-$M_\odot$ halo, can be perturbed enough to produce turbulence and shocks inside themselves, thus triggering more efficient cooling than without the merging process. The previous fact does not contradict the results of Shchekinov & Vasiliev (2006) and Vasiliev & Shchekinov (2008) who by one-dimensional simulations have shown that in order to ionize the primordial gas in a halo-merger process, the halo mass should be $\geq 10^7 \, M_\odot$. Despite that in our case the merged haloes have masses $\sim 10^7 \, M_\odot$, they are accelerated by an $\sim 10^6 \, M_\odot$ halo and they can reach enough velocity to produce the gas ionization.

The supersonic turbulence formed through this process favours molecular formation by two main channels: (i) the strong shocks of velocities above 22 km s$^{-1}$ (due to both gas accretion and minihalo merger) are able to enhance the ionization fraction which catalyzes both H$_2$ and HD formation, increasing their abundances in post-shocked regions with the consequent gas cooling; and (ii) the supersonic turbulence compresses the gas and increases the local gas density, favouring molecular formation, which in turn enhances the energy-loss rate in the overdensities. The first channel is possible because the electrons recombine on time-scales of $t_{\text{rec}} \geq 10$ Myr, while the molecular coolants are formed on time-scales of $t_{\text{cool}} \ll$ few Myr, under post-shock conditions in these environments. The second channel works on time-scales of $t_{\text{cool}} \leq 10$ Myr for overdensities of $n \sim 10^5$ cm$^{-3}$. A combination of these two processes, triggered by both halo merger and gas accretion, facilitates the gas to drop its temperature and to form high-density regions of $n \geq 10^6$ cm$^{-3}$ with temperatures below 300 K; in other words, it allows to create cold dense gas regions which are potential places for star formation. The gas temperature reaches values below 100 K at regions with $50 \lesssim n/c^3 \lesssim 10^5$ where the HD molecule seems to be a more efficient coolant than the H$_2$ molecule; at higher densities, gas cooling again is dominated by H$_2$ allowing temperatures $\sim (2-3) \times 10^2$ K.

The combined effect of gas dynamics (merger and accretion) and gas chemistry (molecular formation and cooling) is able to produce overdensities with high velocity and high Mach number. The highly supersonic densest regions of the main halo central region develop a density PDF with interesting features. In the low-density region, it shows a peak below the average density inside the analysed volume. This peak is formed because through the gas-accretion process a large percentage of the matter is piled up in the central region. While the accretion flows converge near the halo centre, the regions surrounding it lose gas continuously increasing the fraction of low-density volume at lower redshift.

At the other extreme, the high-density PDF shows two clear power-law regions. The first at $0.5 \lesssim n/c^3 \lesssim 50$ follows PDF $n^{-1.5}$ which is explained as the region dominated by nearly isothermal overdensities (the overdensities have to scale as $n \propto r^{-2}$ in order to reproduce the $-3/2$ exponent) before collapse. At higher densities, $n \gtrsim 50$ cm$^{-3}$, the PDF tail shows how the collapse process at large scales feeds the central region and produces the high-density structures. The breakdown of the $-3/2$ slope shows the point where the locally collapsed overdensities start to pile up the accreted material and develop rotationally supported structures as a natural consequence of angular momentum conservation. Very interestingly, the density PDF computed from our cosmological simulation has similar features to the PDF found in idealized star formation simulations, for example, Kritsuk et al. (2011).

We emphasize that the density, temperature and velocity PDF can be used to characterize the gas conditions in future idealized primordial star formation simulations inside similar high-redshift environments.

The previous evidence of supersonic environments is further supported by our Fourier analysis. The kinetic energy spectrum of the turbulent gas follows nearly Burgers behaviour, $\tilde{E}_k \propto k^{-2}$, a bit steeper at lower redshift, reaching a minimum exponent of $\approx -2.2$. This result is also supported by the second-order longitudinal structure function exponent $\xi_2 (l) = l^{1.02}$. The PS slope confirms that a merger followed by the inhomogeneous accretion process on to the main halo develops a supersonic turbulent gas with shocks through the halo virialization process. An interesting feature of this spectrum is that towards small scales it does not decay as fast as in isothermal simulations without self-gravity. This feature could be explained by taking into account the self-gravity of the densest regions: at the beginning, the kinetic energy of the system comes from large scale accretion processes. As mentioned above, after a shock is produced, the turbulent environment of the gas dynamics facilitates the gas to reach high-density, low-temperature states. Under such conditions, the gas is able to locally collapse gravitationally, transforming their gravitational energy into kinetic energy at small scales, producing the feature at high wavenumbers in the PS.

The distribution of energy in solenoidal and compressive modes shows that the solenoidal modes contain $R_e \approx 2/3$ of the total kinetic energy. This energy distribution is the consequence of a random turbulent motion which for each compressional direction, for example, $x$, presents two rotational directions, $y$ and $z$: the kinetic energy follows an equipartition rule with the same amount of energy for each direction. This energy distribution is not constant either in time or in scale, but it has well-stated values $65 \lesssim R_e \lesssim 70$ below $\sim 100$ pc.

All the Fourier information presented above can be very useful in order to set initial conditions for idealized primordial star formation simulations in the spirit of Clark et al. (2011b). For instance, these authors take an initial velocity PS with a slope $-4$. This value is much lower than our slope $\approx -2$. The inclusion of a $-2$ slope in a previous study could bring a more efficient fragmentation at small scales with possibly more efficient low-mass primordial star formation as found in Clark et al. (2011a) and Greif et al. (2011) who, using three-dimensional numerical simulations from cosmological initial conditions, have shown that a multiple system of low-mass ($\sim 0.1$–$10$ M$_\odot$) primordial stars can be formed inside DM minihaloes.

Using a one-zone model, Schneider et al. (2011) have shown that without dust there is a critical metallicity ($Z \sim 10^{-4}$ Z$_\odot$) above which gas can fragment into $\gtrsim 10$ M$_\odot$ objects. Furthermore, they found that there is a critical dust-to-gas ratio above which the gas can fragment into low-mass ($\sim 0.01$–$1$ M$_\odot$) objects. The existence of a critical metallicity above which it is possible to form low-mass stars is explained by the low-mass star formation critical metallicity theory of Bromm & Loeb (2003). Frebel, Johnson & Bromm (2007)
tested this theory with data of stars formed at the galactic halo, globular clusters and dwarf spheroidal galaxies, reaching a good agreement between data and theory. Nevertheless, the solidness of this theory has been questioned by a recently found extremely-metal-poor star which seems to violate the theory (Caffau et al. 2011) in the sense that it does not have the amount of both oxygen and carbon needed to form a low-mass star as stated in the Bromm & Loeb (2003) theory. This finding makes possible the potential observation of low-mass primordial stars as mentioned in Johnson & Khochar (2011).

This study does not take into account the possible radiation background from previously formed Population III stars. Of course, a high enough number of photons in the Lyman–Werner band could avoid the gas fragmentation due to the molecular coolant destruction (Haiman, Rees & Loeb 1997). The result of this process could be non-fragmented hot gas piled on the main-halo centre-of-mass. This configuration could be responsible for the formation of supermassive black holes at high redshifts (e.g. Shang, Bryan & Haiman 2010).

Our work takes into account the possible SN explosion from Population III stars previously formed inside the merged minihaloes. For masses of the SN progenitors in the range 15–40 M⊙, this kind of violent phenomena certainly would be able to expel the gas inside the DM minihaloes, and for massive enough SN progenitors, in the range of 140–260 M⊙, the explosion could be able to disrupt the gas inside haloes even more massive than ~107 M⊙ (Whalen et al. 2008). Because our main target is to try to understand the main processes involved in the formation of supersonic turbulence in unperturbed primordial environments, we have neglected these two important feedback.

Our cosmological initial conditions were set with σ8 = 0.9. This value is larger than the most recent measure of σ8 = 0.8 which implies that the DM haloes in our simulation collapse faster than they do in a more realistic scenario. Actually, in a more realistic scenario, the halo-collapse process would occur at z ≈ 0.89(1 + Z0.9) − 1, where Z0.9 is the collapse redshift with σ8 = 0.9. For instance, the results that we show at z = 10 should occur at z ≈ 8.8.

5 SUMMARY AND CONCLUSIONS

We have performed hydrodynamical simulations of primordial gas from cosmological initial conditions in order to study how the DM-merging process triggers the formation of turbulence in primordial environments and how the developed supersonic environment favours the formation of molecular coolants. The simulation, inside a 1 Mpc h−1 box size, follows the evolution of a 3 × 107 M⊙ DM halo from z ≈ 120 to z = 10 with a proper distance resolution Δr ≈ 1.95 pc at this redshift, covering almost five orders of magnitude in distance scales.

Our main conclusions can be summarized as follows:

(i) Both the baryonic accretion process through filaments and the DM minihalo merging on to the main halo are able to produce a supersonic (M ≳ 10) turbulent and shocked environment where the H2 and HD molecules are formed efficiently.

(ii) The non-equilibrium molecular formation triggered by the supersonic turbulent environment creates a number of regions with temperature T ≲ 300 K. Some regions with number density n ≲ 103 cm−3 reach temperatures below 100 K, evidencing the effect of the HD molecule as the main coolant. At higher densities, H2 is the more efficient coolant. All these low-temperature, high-density regions are potential star formation sites.

(iii) The Fourier analysis of the velocity field shows that the kinetic energy has a nearly Burgers spectrum, Ek ∝ k−2, albeit a bit steeper at lower redshift with a minimum exponent of ≈−2.2. This power law confirms the previous statement of supersonic and shocked environment. This spectrum approximately implies a S2(ℓ) ∝ ℓ−0.1 second-order longitudinal structure function, steeper than the S2(ℓ) ∝ ℓ−0.76 behaviour observed in local molecular clouds.

(iv) The characterization of the turbulence PS shows that it could favour the formation of low-mass primordial stars.

(v) The energy spectrum does not decay at high wavenumbers as fast as shown in isothermal simulations without self-gravity. This behaviour is explained by taking into account that after the accretion process enhances the cooling at supersonic turbulent regions, the gas is able to form gravitationally-unstable overdensities. In this way, the local gravitational collapse enhances the velocity power at scales of a few ×10 pc converting the gravitational energy into kinetic energy.

(vi) The energy in solenoidal (rotational) modes is Rk ≈ 2/3 of the total energy as in previous simulations of turbulent gas.

This paper is part of our ongoing effort (Prieto et al. 2011) to produce numerical simulations that capture enough physics to understand the process of galaxy formation at the highest redshifts in order to shed light on what mechanisms could be dominating and shaping the future evolution of galaxies. We have shown here that the presence of relatively low mass haloes (≈107 M⊙) dramatically influences its environment by creating a turbulent interstellar medium (ISM). This in turn makes the smaller mass haloes (M ∼ 106 M⊙), the most abundant ones at that redshift, to also develop turbulence in their ISM because of the merging process. The end result is that the production of coolants is enhanced, so much that even the HD molecule becomes an important coolant in some regions, thus producing regions of high density and low temperature that could be sites of star formation. We are now investigating how universal the process presented in this paper is by analysing the whole 1-Mpc-box simulation and including the low-mass and high-mass DM haloes. We will present our findings in a forthcoming paper. We can at this point speculate that, if this mechanism turns out to be universal at such high redshift, then it could also potentially work at lower redshifts, so that the initial turbulence in the ISM of a galaxy could be due to the merging process, which is universal in the standard ΛCDM scenario.

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REFERENCES

Caffau E. et al., 2011, Nat, 477, 67