Gamma-ray bursts in the comoving frame

G. Ghirlanda,1⋆ L. Nava,2 G. Ghisellini,1 A. Celotti,2 D. Burlon,3 S. Covino1 and A. Melandri1

1INAF – Osservatorio Astronomico di Brera, Via E. Bianchi 46, I-23807 Merate, Italy
2SISSA – via Bonomea, 265, I-34136 Trieste, Italy
3Max Planck Institut für Extraterrestrische Physik, Giessenbachstraße 1, D-85478 Garching, Germany

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ABSTRACT
We estimate the bulk Lorentz factor $\Gamma_0$ of 31 gamma-ray bursts (GRBs) using the measured peak time of their afterglow light curves. We consider two possible scenarios for the estimate of $\Gamma_0$: the case of a homogeneous circumburst medium or a wind density profile. The values of $\Gamma_0$ are broadly distributed between few tens and several hundreds with average values $\sim 138$ and $\sim 66$ for the homogeneous and wind density profile, respectively. We find that the isotropic energy and luminosity correlate in a similar way with $\Gamma_0$, i.e. $E_{\text{iso}} \propto \Gamma_0^2$ and $L_{\text{iso}} \propto \Gamma_0^2$, while the peak energy $E_{\text{peak}} \propto \Gamma_0$. These correlations are less scattered in the wind density profile than in the homogeneous case. We then study the energetics, luminosities and spectral properties of our bursts in their comoving frame. The distribution of $L_{\text{iso}}^\prime$ is very narrow with a dispersion of less than a decade in the wind case, clustering around $E_{\text{iso}}^\prime \sim 5 \times 10^{48}$ erg s$^{-1}$. Peak photon energies cluster around $E_{\text{peak}}^\prime \sim 6$ keV. The newly found correlations involving $\Gamma_0$ offer a general interpretation scheme for the spectral energy correlation of GRBs. The $E_{\text{peak}}^\prime-E_{\text{iso}}$ and $E_{\text{peak}}^\prime-L_{\text{iso}}$ correlations are due to the different $\Gamma_0$ factors and the collimation-corrected correlation, $E_{\text{peak}}^\prime-E_\gamma$ (obtained by correcting the isotropic quantities for the jet opening angle $\theta_j$), can be explained if $\theta_j^2 \Gamma_0 = \text{constant}$. Assuming the $E_{\text{peak}}^\prime-E_\gamma$ correlation as valid, we find a typical value of $\theta_j \Gamma_0 \sim 6-20$, in agreement with the predictions of magnetically accelerated jet models.

Key words: radiation mechanisms: non-thermal – gamma-ray burst: general.

1 INTRODUCTION

The discovery of the afterglows of gamma-ray bursts (GRBs; Costa et al. 1997) allowed to pinpoint their position in the X-ray and optical bands. This opened a new era focused at measuring the spectroscopic redshifts of these sources. The present1 collection of GRBs with measured $z$ consists of 232 events. In 132 bursts of this sample (updated in this paper), the peak energy $E_{\text{peak}}^\text{iso}$ of their $\nu F_\nu$ prompt emission $\gamma$-ray spectrum could be constrained. In turn, for these bursts it was possible to calculate the isotropic equivalent energy $E_{\text{iso}}$ and luminosity $L_{\text{iso}}$. The knowledge of the redshifts showed that two strong correlations exist between the rest frame peak energy $E_{\text{peak}}$ and $E_{\text{iso}}$ or $L_{\text{iso}}$ (also known as the ‘Amati’ and ‘Yonetoku’ correlations – Amati et al. 2002; Yonetoku et al. 2004, respectively).

The reality of these correlations has been widely discussed in the literature. Some authors pointed out that they can be the result of observational selection effects (Band & Preece 2005; Nakar & Piran 2005; Butler et al. 2007; Butler, Kocevski & Bloom 2009; Shahmoradi & Nemiroff 2011) but counter-arguments have been put forward arguing that selection effects, even if surely present, play a marginal role (Ghirlanda et al. 2005, 2008; Bosnjak et al. 2008; Nava et al., 2008; Amati, Nava & Ghirlanda 2009; Krimm et al. 2009). The finding that a correlation $E_{\text{peak}}(t)-E_{\text{iso}}(t)$ exists when studying time-resolved spectra of individual bursts is a strong argument in favour of the reality of the spectral energy correlations, (Ghirlanda, Nava & Ghisellini 2010b; Ghirlanda et al. 2011) and motivates the search for the underlying process generating them. Even if several ideas have been already discussed in the literature, there is no general consensus yet, and a step forward towards a better understanding both of the spectral energy correlations and the underlying radiation process of the prompt emission of GRBs is to discover what are the typical energetics, peak frequencies and peak luminosities in the comoving frame.

The physical model of GRBs requires that the plasma emitting $\gamma$-rays should be moving relativistically with a bulk Lorentz factor $\Gamma_0$ much larger than unity. The high photon densities and the short time-scale variability of the prompt emission imply that GRBs are

*E-mail: giancarlo.ghirlanda@brera.inaf.it
1http://www.mpe.mpg.de/jcg/grbgen.html

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optically thick to pair production which, in turn, would lead to a strong suppression of the emitted flux, contrary to what observed. The solution of this compactness problem requires that GRBs are relativistic sources. From this argument lower limits \( \Gamma_0 \geq 100 \) are usually derived (Lithwick & Sari 2001). The first observational evidences supporting this scenario were found in the radio band where the ceasing of the radio flux scintillation (few weeks after the explosion as in GRB 970508; Frail et al. 1997), allowed to estimate \( \Gamma \) of a few. This value corresponds to the late afterglow phase, when the fireball is decelerated almost completely by the interstellar medium (ISM) and is characterized by a much smaller bulk Lorentz factor than the typical \( \Gamma_0 \) of the prompt phase.

Large Lorentz factors imply strong beaming of the radiation we see. We are used to consider GRB intrinsic properties \( (E_{\text{peak}}, E_{\text{iso}}, L_{\text{iso}}) \) for the bursts with measured redshifts, but still an important correction should be applied. Our aim is to study the distributions of \( E_{\text{peak}}, E_{\text{iso}}, L_{\text{iso}} \) and the spectral energy correlations \( (E_{\text{peak}} - E_{\text{iso}} \text{ and } E_{\text{peak}} - L_{\text{iso}} \text{ in the comoving frame, accounting for the } \Gamma_0 \text{ factor}. \)

The estimate of \( \Gamma_0 \) is possible by measuring the peak of the afterglow (Sari & Piran 1999) and has been successfully applied in some cases (e.g. Molinari et al. 2007, Gruber et al. 2011) and more extensively recently by Liang, Yi & Zhang (2010) in the optical and X-ray band. Other methods allow us to set lower limits (Abdo et al. 2009a,b; Ackermann et al. 2010) mainly by applying the compactness argument to the high-energy emission recently detected in few GRBs at GeV energies by the Fermi satellite (see Hascoet et al. 2011; Zhao, Li & Bai 2011; Zou, Fan & Piran 2011, for more updated calculation on these lower limits on \( \Gamma_0 \)). Conversely, upper limits (Zou & Piran 2010) can be derived by requiring that the forward shock emission of the afterglow does not appear in the MeV energy band.

The paper is organized as follows. In Section 2 we discuss the relativistic corrections that allow us to derive the comoving frame \( E_{\text{peak}}, E_{\text{iso}}, L_{\text{iso}} \) from the rest frame \( E_{\text{peak}}, E_{\text{iso}}, L_{\text{iso}} \). In Sections 3 and 4 we derive a general formula for the estimate of \( \Gamma_0 \) from the measurement of the time of the peak of the afterglow emission. In Section 5 we present our sample of GRBs and in Section 6 our results, which are finally discussed in Section 7. Throughout the paper we assume a standard cosmology with \( h = \Omega_\Lambda = 0.7 \) and \( \Omega_0 = 0.3 \).

8. **2 FROM THE REST TO THE COMOVING FRAME**

In this section we derive the Lorentz transformations to pass from rest frame quantities to the same quantities in the comoving frame. This is not trivial, since, differently from the analogue case of blazars, the emitting region is not a blob with a mono-directional velocity, but a fireball with a radial distribution of velocities. Therefore, an observer located on axis receives photons from a range of viewing angles, complicating the transformations from rest frame to comoving quantities. We are interested to three observables: the peak energy \( E_{\text{peak}} \), the isotropic equivalent energy \( E_{\text{iso}} \) and the isotropic equivalent peak luminosity \( L_{\text{iso}} \). Dealing with isotropic equivalent quantities, we can assume that the emitting region is a spherical shell with velocities directed radially. We also assume that the comoving frame bolometric intensity \( I' \) is isotropic. We then adopt the usual relation between observed \( I \) and comoving \( I' \) bolometric intensity:

\[
I = I' \delta^4; \quad \delta = \frac{1}{\Gamma(1 - \beta \cos \theta)}.
\]

where \( \delta \) is the Doppler factor and \( \theta \) is the angle between the velocity vector and the line of sight. The received flux is

\[
F = 2\pi I' \int_0^{\pi/2} \delta^4 \sin \theta \, d\theta.
\]

Since the fluence \( F \) is a time-integrated quantity, we have

\[
F \propto \int_0^{\pi/2} \delta^4 \sin \theta \, d\theta \text{, i.e. one power of } \delta \text{ less.}
\]

\( E_{\text{peak}} \): this quantity can be derived from the time-integrated spectrum, or can be the spectral peak energy of a given time interval. In this paper we will use the time-integrated \( E_{\text{peak}} = E_{\text{obs}}(1 + z) \). The received flux \( dF/d\theta \) (i.e. the flux integrated in time) from each annulus of same viewing angle \( \theta \) is \( dF/d\theta \propto \sin \delta^4 \). For \( \theta \rightarrow 0 \) the Doppler factor is maximum, but the solid angle vanishes, while for \( \theta > 1/\Gamma \) the solid angle is large, but \( \delta \) is small. Therefore, there will be a specific angle \( \theta \) for which \( dF/d\theta \) is maximum. This is given by

\[
\cos \theta = \beta + \frac{2}{5\Gamma^2}.
\]

At this angle the beaming factor is

\[
\delta = \frac{5}{3} \Gamma.
\]

We then set \( E_{\text{peak}} = E_{\text{obs}}/(5\Gamma^3) \).

\( E_{\text{iso}} \): this is proportional to the fluence \( F \), and the relation between the observed and comoving quantity is

\[
\frac{E_{\text{iso}}}{E_{\text{iso}}'} = \frac{F}{F'} = \frac{\int_0^{\pi/2} \delta^4 \sin \theta \, d\theta}{\int_0^{\pi/2} \sin \theta \, d\theta} = \frac{\Gamma^3}{3}.
\]

We then set \( E_{\text{iso}} = E_{\text{iso}}'/\Gamma \).

\( L_{\text{iso}} \): this is proportional to the flux \( F \), so the ratio \( L_{\text{iso}}/L_{\text{iso}}' \) is

\[
\frac{L_{\text{iso}}}{L_{\text{iso}}'} = \frac{F}{F'} = \frac{\int_0^{\pi/2} \delta^4 \sin \theta \, d\theta}{\int_0^{\pi/2} \sin \theta \, d\theta} \approx \frac{4}{3} \Gamma^2.
\]

We then set \( L_{\text{iso}} = L_{\text{iso}}/(4\pi^2/3) \) (in agreement with Wijers & Galama 1999).

9. **3 ESTIMATE OF THE BULK LORENTZ FACTOR \( \Gamma_0 \)**

In the thin-shell regime (i.e. for \( T_{90} < t_{\text{peak obs}} \), condition satisfied for almost all bursts in our sample), the standard afterglow theory predicts that the peak of the bolometric afterglow light curve corresponds to the start of the fireball deceleration. The deceleration radius is commonly defined as the radius at which the swept-up material \( m(r_{\text{dec}}) \) is smaller by a factor \( \Gamma_0 \) than the initial shell’s rest mass \( M_0 = E_0/\Gamma_0 c^2 \). Usually, the deceleration time \( t_{\text{dec}} \) is estimated as

\[
t_{\text{dec}} = r_{\text{dec}}/(2c\Gamma_0^2) \text{ (Sari & Piran 1999). This relation is approximate, since it does not consider that the Lorentz factor is decreasing. Some authors consider this relation to estimate } \Gamma_0 \text{ from the peak time of the afterglow light curve (Sari 1997; Sari & Piran 1999), while other authors consider that } t_{\text{dec}} = r_{\text{dec}}/(2c\Gamma_0^2), \text{ where approximately } \Gamma_0 \approx 2\Gamma(r_{\text{dec}}) \text{ (Molinari et al. 2007).}
\]

We propose here a detailed and general calculation of \( \Gamma_0 \) which extends the estimate to the generic case of a circumburst density profile described by \( n = n_0 r^{-\nu} \). We use the shape of the light curve in two different power-law regimes: the coasting phase when \( r \ll r_{\text{dec}} \) and \( \Gamma(r) = \Gamma_0 \), and the deceleration phase when \( r_{\text{dec}} \ll r \ll r_{\text{PSR}} \) (where \( r_{\text{PSR}} \) marks the start of the non-relativistic regime). During the deceleration regime, the evolution of the Lorentz factor is described by the self-similar solution found by Blandford & McKee (1976):

\[
\Gamma = \sqrt{\frac{(17 - 4\nu)E_0}{(12 - 4\nu)m(r)c^2}}.
\]
The relation between the radius and the observed time is obtained by integrating the differential equation \(dr = 2c\Gamma^2(r)dr\) and by considering the exact evolution of \(\Gamma\) with \(r\). From equation (6),

\[
L_{iso} = \frac{4}{3} \Gamma^2 L_{iso}' = \frac{4}{3} \Gamma^2 \frac{dE_{dis}}{dt},
\]

where the dissipated comoving energy \(E_{dis}'\) is given by (Panaitescu & Gamma 2000)

\[
E_{dis}' = (\Gamma - 1)m(r)c^2.
\]

Only a fraction \(\epsilon_c\) of the dissipated energy is radiated. We assume that this quantity is small and does not affect the dynamics of the fireball (adiabatic regime). Equation (8) holds until the emission process is efficient (fast-cooling regime).

During the cooling phase \(\Gamma = \Gamma_0 \gg 1\) and the luminosity (denoted by \(L_{iso,c}\)) is

\[
L_{iso,c} = \epsilon_c \frac{4}{3} \Gamma_0^3 \frac{d\rho(r)}{dr} = \epsilon_c \frac{4}{3} \Gamma_0^3 \epsilon^2 4\pi \tau e^{-(\epsilon-\eta)} n_0 m_p.
\]

Since in this phase the Lorentz factor is constant and equal to \(\Gamma_0\), the relation between the radius and the observed time is

\[
r = 2ct\Gamma_0^2.
\]

As a function of time, the luminosity is

\[
L_{iso,c} = \epsilon_c \frac{4}{3} \Gamma_0^3 \epsilon^2 \frac{d\rho(r)}{dr} = \epsilon_c \frac{4}{3} \Gamma_0^3 \epsilon^2 4\pi \tau e^{-(\epsilon-\eta)} n_0 m_p.
\]

(10)

For a homogeneous density medium \((\epsilon = 0)\) the light curve rises as \(r^2\). The luminosity is instead constant when \(\epsilon = 2\), which corresponds to the stellar wind density profile.

To derive the luminosity during the deceleration phase we start again from equations (8) and (9). However, in this case \(\Gamma\) is decreasing according to equation (7) (but still \(\Gamma \gg 1\)). We derive

\[
L_{iso,2} = \epsilon_c \frac{4}{3} \Gamma_0^2 \epsilon^2 \frac{d\rho(r)}{dr} = \epsilon_c \frac{4}{3} \Gamma_0^2 \epsilon^2 \frac{d\rho(r)}{dr} = \epsilon_c \frac{4}{3} \Gamma_0^2 \epsilon^2 4\pi \tau e^{-(\epsilon-\eta)} n_0 m_p.
\]

(11)

The first term of the sum in square brackets can be written as

\[
\frac{d\rho(r)}{dr} = (3 - s) \frac{m(r)}{r} \Gamma^2 c.
\]

The second term of the sum becomes

\[
\frac{d\Gamma}{dr} = -\frac{3 - s}{2} \frac{m(r)}{r} \Gamma^2 c.
\]

During the deceleration,

\[
t = \frac{1}{2c} \int \frac{dr}{\Gamma^2} = \frac{r}{2c(4s - 1)\Gamma^2},
\]

where we have used \(\Gamma(r)\) given in equation (7).

For \(\Gamma_0 \gg 1\) the initial energy content of the fireball \(E_0 = E_{k,iso} + M^0 c^2 \approx E_{k,iso}\), where \(E_{k,iso}\) is the isotropic kinetic energy powering the expansion of the fireball in the ISM during the afterglow phase. If the radiative efficiency \(\eta\) of the prompt phase is small, \(E_{k,iso}\) can be estimated from the energetics of the prompt as \(E_{k,iso} = E_{iso,\eta}\). We obtain

\[
L_{iso,2} = \epsilon_c \frac{4}{3} \Gamma_0^2 \epsilon^2 \frac{d\rho(r)}{dr} = \epsilon_c \frac{4}{3} \Gamma_0^2 \epsilon^2 \frac{d\rho(r)}{dr} = \epsilon_c \frac{4}{3} \Gamma_0^2 \epsilon^2 4\pi \tau e^{-(\epsilon-\eta)} n_0 m_p.
\]

(12)

The peak time of the light curve is the time when the cooling phase ends and the deceleration phase starts and can be estimated by setting \(L_{iso,2}(t_{peak}) = L_{iso,2}(t_{peak})\):

\[
t_{peak} = \left[\frac{(17 - 4\xi)(3 - s)E_{iso}}{2\pi \eta \Gamma_0^3 \epsilon^2 - \eta (12 - 4\xi)(4 - \eta) \Gamma_0^2 - \eta} \right]^{1/(3 - s)},
\]

(13)

and inverting this relation to obtain the initial Lorentz factor as a function of the peak time:

\[
\Gamma_0 = \frac{(17 - 4\xi)(3 - s)E_{iso}}{2\pi \eta \Gamma_0^3 \epsilon^2 - \eta (12 - 4\xi)(4 - \eta) \Gamma_0^2 - \eta} \frac{1}{(8 - 2s)},
\]

(15)

where \(t_{peak}\) is the peak of the afterglow light curve in the source rest frame, i.e. \(t_{peak} = t_{peak,obs}/(1 + z)\), and it will be indicated as \(t_{p,z}\) hereafter.

While a wind density profile (hereafter W: wind ISM) is expected from a massive star progenitor that undergoes strong wind mass losses during the final stages of its life (Chevalier & Li 1999), it is not possible at the present stage to prefer W to the homogeneous ISM case (H, hereafter). We already showed (Nava et al. 2006) that the collimation corrected \(E_{peak} - E_{\gamma}\) correlation (so called ‘Ghirlanda’ correlation; Ghirlanda, Ghisellini & Lazzati 2004) has a smaller scatter and a linear slope when computed under the assumption of the W compared to the H case. It is, therefore, important to compare the estimates of \(\Gamma_0\) and of the comoving frame energetics in these two possible scenarios. The most extensive study of Liang et al. (2010) estimated \(\Gamma_0\) mostly from the peak of the afterglow light curve in the optical band and in few cases from a peak in the X-ray band. They considered only the H case and found a strong correlation between \(\Gamma_0\) and the GRB isotropic equivalent energy \(E_{iso}\).

Equation (11) predicts that the afterglow light curve is flat in the deceleration phase, with no peaks in the W density case \((s = 2)\). However, this equation neglects pre-acceleration of the circumburst matter due to the prompt emission itself, which can have important consequences, as we discuss below.

### 4 HOMOGENEOUS OR WIND DENSITY PROFILE?

In the following we will find the initial bulk Lorentz factor \(\Gamma_0\) for bursts showing a peak in their early afterglow light curve. In the simple case of an homogeneous circumburst density, we expect that the afterglow luminosity \(L_{iso} \propto \Gamma^3\), and therefore \(L_{iso} \propto \Gamma^3\) when \(\Gamma = \Gamma_0\) constant (equation 11). It can be questioned if, in the case of a wind density profile, such a peak occurs, or if the initial light curve is flat (i.e. \(\Gamma^3\)), as suggested by equation (11) when \(s = 2\).

The derivation leading to equation (11) assumes that the circumburst medium is at rest when the fireball impacts through it (i.e. it is an external shock). Instead, since the electrons in the vicinity of the burst scatter part of the prompt emission of the burst itself, some radial momentum has to be transferred to the medium (as suggested by Beloborodov 2002). If the velocity acquired by the circumburst matter becomes relativistic, then the fireball will produce an internal shock when passing through the medium, with a reduced efficiency.

To illustrate this point, let us consider an electron at a distance \(r\) from the burst, scattering photons of the prompt emission of energy \(E_{peak} = \epsilon m_e c^2\). In the Thomson limit of the scattering process, this electron will scatter a number \(\tau\) of prompt photons given by

\[
\tau = \sigma_\gamma n_\gamma \Delta r = \frac{\sigma_\gamma L_{iso,\gamma} c_{\text{burst}}}{4\pi \epsilon r \epsilon m_e c^2} = \frac{\sigma_\gamma E_{iso}}{4\pi \epsilon r \epsilon m_e c^2}.
\]

(16)

To evaluate the distance \(r\) up to which this process can be relevant, consider at what distance the electrons make a number \(\tau = (m_e/m_p)\)\(s\) scatterings, namely the distance at which the electrons and their associated protons are accelerated to \(\gamma = 2\):

\[
r(\gamma = 2) \approx \left[ \frac{\sigma_\gamma E_{iso}}{4\pi \epsilon r \epsilon m_e c^2} \right]^{1/2} \sim 1.9 \times 10^{15} E_{iso,53}^{1/2} \text{ cm}
\]

(17)
where $E_{\text{iso},53} = 10^{53} E_{\text{iso}}$ erg. This distance must be compared with the deceleration radius $r_{\text{dec}}$ in the case of a wind density profile corresponding to a mass-loss $M$ and a velocity $v_w$ of the wind:

$$n(r) = \frac{M}{4\pi r^2 v_w} = 3.16 \times 10^{35} \frac{M_{-5}}{v_{w,8} \Gamma_{0.2}} r^{-2},$$

(18)

where $\eta$ is the efficiency of conversion of the kinetic energy to radiation ($L_{\text{iso}} = \eta L_{\text{iso}}$). Therefore, it is possible to have a pre-acceleration of the circumburst matter up to a distance comparable to (but less than) the deceleration radius. In this case, we expect to have a very early rising afterglow light curve (corresponding to relatively inefficient internal shocks between the fireball and the pre-accelerated circumburst medium), followed by a flat light curve and then a decay.

We conclude that the absence of a flat early light curve does not exclude (a priori) a wind density profile. This gives us a motivation to explore both cases (i.e. homogeneous and wind density profile) even if the bursts in our sample all show a peak in the afterglow light curve (and thus a rising phase).

Note that the same pre-acceleration can occur if the density is homogeneous. In this case, again, we expect the very early afterglow to be less efficient than what predicted without pre-acceleration, leading to a rising phase even harder than $t^2$.

5 THE SAMPLE

Since we want to study the energetics, luminosities and peak energies of GRBs in the comoving frame, our first requirement is to know the redshift $z$. Then we also need that the spectral peak energy $E_{\text{peak}}^{\text{iso}}$ has been determined from the fit of the prompt emission spectrum. Most of these bursts have been localized by the Burst Alert Telescope (BAT; Barthelmy et al. 2005) on-board the Swift satellite, but only for a few of them BAT could determine $E_{\text{peak}}^{\text{iso}}$ (due to its limited energy range, 15–150 keV). Most of the $E_{\text{peak}}^{\text{iso}}$ were determined by the Konus–Wind satellite (Aptekar et al. 1995) or, since mid-2008, by the Gamma Burst Monitor (GBM; Meegan et al. 2009, with energy bandpass 8 keV–35 MeV) on-board the Fermi satellite. Our sample of GRBs with $z$ and constrained $E_{\text{peak}}^{\text{iso}}$ (and consequently with computed $E_{\text{iso}}$ and $L_{\text{iso}}$) is updated up to 2011 May. It contains 132 GRBs with $z$, $E_{\text{peak}}^{\text{iso}}$ and $L_{\text{iso}}$. We have $L_{\text{iso}}$ for all but one of these bursts.

Within this sample, we searched the literature for bursts with evidence of the peak of the afterglow or an estimate of the $\Gamma_0$ factor.

(i) Liang et al. (2010, hereafter L10) measured the peaks in the optical light curves of GRBs and then estimated $\Gamma_0$ for the H case. From L10 we collected nine measurements of $t_{p,z}$, L10 also collected other estimates of $t_{p,z}$ from the literature (their table 6) from which we get other four values of this observable. Therefore, from L10 we collected 13 estimates of $t_{p,z}$ from the optical light curves.

(ii) Two GRBs, not included in the sample of L10, that show a peak in their optical afterglow light curves are taken from Ghisellini et al. (2009).

(iii) L10 searched for bursts with evidence of the afterglow peak up to 2008 December. Our sample of bursts with redshifts, $E_{\text{peak}}^{\text{iso}}$ and isotropic energies/luminosities extends to 2011 May. We searched in the literature for $t_{p,z}$ of bursts after 2008 December, and in 10 cases we could build the light curve with available published data (that will be presented in a forthcoming paper – Melandri et al., in preparation). Our systematic search of the literature resulted in other two GRBs with a peak in the optical light curve.

Our sample is thus composed of 27 GRBs with an estimate of $t_{p,z}$ obtained from their optical light curves. All these are long GRBs.

The sample is presented in Table 1 where we show the relevant properties of these bursts used in the following sections. Columns 1 and 2 show the GRB name and its redshift, column 3 the rest frame peak energy $E_{\text{peak}}^{\text{iso}}$, and columns 4 and 5 the isotropic equivalent energy $E_{\text{iso}}$, and luminosity $L_{\text{iso}}$, respectively. In column 6 it is reported the rest frame $t_{p,z}$ from which we compute the $\Gamma_0$ factor in the H case (column 7) and in the W case (column 8) assuming a typical density value $n_0 = 3 \times 10^3$ (for the H and W, respectively) and a typical radiative efficiency $\eta = 0.2$. We note from equation (15) that the resulting $\Gamma_0$ is rather insensitive to the choice of $n_0$ and $\eta$ both in the H case [i.e. $\Gamma_0 \propto (n_0 \eta)^{-1/4}$] and in the W case [i.e. $\Gamma_0 \propto n_0^{-1/4}$].

There are also four GRBs, detected by the Large Area Telescope (LAT) on-board Fermi at GeV energies, showing a peak in their GeV light curves (Ghisellini et al. 2010). The interpretation of the GeV emission as afterglow (Barniol-Durán & Kumar 2009, Ghirlanda et al. 2010b, Ghisellini et al. 2010) is however debated (Ackermann et al. 2010; Piran & Nakar 2010). Among these bursts there is also the short/hard GRB 090510 whose $\Gamma_0$ is derived from the modelling of the GeV light curve (Ghirlanda, Ghisellini & Nava 2010a). However, this burst also shows a clear peak in the optical at $\sim 300$ s after the GRB onset (De Pasquale et al. 2010) which questions the afterglow interpretation of the GeV emission.

The three LAT bursts with $t_{p,z}$ measured from the GeV light curve and the short GRB 090510 are shown separately in Table 1. These events have the smallest $t_{p,z}$ in our sample and, therefore, the largest $\Gamma_0$ values (see Table 1). This is expected since, as discussed in Ghisellini et al. (2010), the detection in the GeV energy range by LAT seems to be a characteristic of GRBs with the largest values of $E_{\text{peak}}^{\text{iso}}$. Besides, the possible measure of $t_{p,z}$ in the optical range is limited by the time delay of the follow-up of GRBs in this band, although several GRBs have been repointed in the optical band by Ultraviolet Optical Telescope on-board Swift. In the end, there could be a selection bias on the bursts with a peak in the GeV energy range, coupled with the debated interpretation of the GeV emission as afterglow. For these reasons, in the next sections we will present the results of the study of the correlations between the GRB energetics and $\Gamma_0$ both including and excluding these bursts. In all our quantitative analysis we always excluded the short GRB 090510 which is only shown for comparison with the properties of the 27 long GRBs.

In our sample we do not include upper limits on $t_{p,z}$ which are those bursts observed early in the optical whose light curve is decaying up to several days without any sign of a peak. Several of these cases can be found in the literature and they would provide lower limits on the value of $\Gamma_0$. However, it is hard to define an appropriate sample of upper limits on $t_{p,z}$ derived from the optical band because of the lack of a unique follow-up programme dedicated to the systematic observations of GRB afterglows.

6 RESULTS

In this section we first show the distributions of the $\Gamma_0$ factors computed in the H and W and show the correlation of $\Gamma_0$ with the
isotropic energy $E_{iso}$ and luminosity $L_{iso}$. Then we show how the distributions of $E_{peak}$, $E_{iso}$ and $L_{iso}$ change when they are corrected for the $\Gamma_0$ factor, i.e. how they appear in the comoving frame ($E'_{peak}$, $E'_{iso}$, $L'_{iso}$). In doing this, we always consider the two estimates of $\Gamma_0$ in the H and W to compare the different distributions of the spectral parameters. Finally, we present the rest frame $E_{peak} - E_{iso}$ and $E_{peak}$--$L_{iso}$ correlations (updated here with 132 and 131 GRBs up to 2011 May) and, for those bursts in our sample with measured $\Gamma_0$, we show where they cluster in these planes when the beaming corrections $[E'_{peak} = E_{peak}/(3\Gamma^2)$, $E'_{iso} = E_{iso}/\Gamma$, $L'_{iso} = L_{iso}/(4\Gamma^2/3)]$ are applied.

For all the reasons outlined in Section 5, in the following we consider (i) the optical sample of 27 GRBs with measured $\Gamma_0$, $E'_{peak}$, $E'_{iso}$ and $L'_{iso}$, whose $t_{p,\times}$ is measured from the optical light curve and (ii) the extended sample of 30 GRBs which includes the three long GRBs with a peak in the GeV which, if interpreted as afterglow emission, allows us to estimate the largest $\Gamma_0$ in our sample.

### 6.1 $\Gamma_0$ distributions

Fig. 1 shows the distributions of the $\Gamma_0$ factors of the 27 GRBs of our sample (with $t_{p,\times}$ measured from the optical light curve – Table 1) computed in the H (solid histogram) and W (hatched histogram) cases, respectively. The two distributions are fitted with Gaussian functions and the central value and dispersion are reported in Table 2. The average $\Gamma_0$ factor is $\sim 138$ in the H case and $\sim 66$ in the W case. In both the H and W cases, the distribution of $\Gamma_0$ is broad, spanning nearly one decade.

#### 6.2 $E_{iso}$--$\Gamma_0$, $L_{iso}$--$\Gamma_0$, $E_{peak}$--$\Gamma_0$ correlations

In this section we explore the presence of correlations between the rest frame GRB properties (i.e. the peak energy $E_{peak}$, the isotropic equivalent energy $E_{iso}$ and luminosity $L_{iso}$) and the $\Gamma_0$ factor.

In the upper panels of Fig. 2 we show the isotropic energy $E_{iso}$ and luminosity $L_{iso}$ (open red circles and filled green squares, respectively) as a function of $\Gamma_0$ in both the H and W cases (left- and right-hand panels, respectively). In the bottom panels of Fig. 2 we show the peak energy $E_{peak}$ as a function of $\Gamma_0$ in the H (left-hand panel) and W (right-hand panel) cases.

The Spearman rank correlation coefficients and associated chance probabilities are reported in Table 3. We model the correlations with a power law: $log Y = m log \Gamma_0 + q$ (with $Y = E_{iso}$, $Y = L_{iso}$ or $Y = E_{peak}$) and list the best-fitting parameters in Table 3. We fit this
model to the data points (shown in Fig. 2) with the bisector method. The choice of this fitting method, instead of the least square Y versus X method that minimizes the vertical distances of the data from the fitting line, is motivated by the large dispersion of the data and the absence of any physical motivation for assuming that \( \Gamma_0 \) or instead \( E_{\text{iso}} \) and \( L_{\text{iso}} \) or \( E_{\text{peak}} \) are the independent variable (Isobe et al. 1990).

In a recent work, Lv, Zou & Lei (2011) derive a correlation \( \Gamma_0 \propto E_{\text{iso}}^{0.92} \), similar to that found in L10. Such a flat correlation is obtained because \( \Gamma_0 \) is fitted versus \( E_{\text{iso}} \) (or \( L_{\text{iso}} \)). As described above, the large scatter of the correlations and the lack of any physical reason for assuming either \( \Gamma_0 \) or \( E_{\text{iso}} \) (or \( L_{\text{iso}} \)) as the independent variable require instead that these correlations are fitted with the bisector method. This gives different correlation slopes with respect to those reported in L10 and Lv et al. (2011). Moreover, in our sample we only consider bursts with firm estimates of \( E_{\text{peak}} \) and do not include those GRBs which are fitted by a simple power law in the BAT energy range but whose peak energy is derived through a Bayesian method, based on the properties of bright BATSE bursts (Butler et al. 2007).

We find that there are strong correlations between the spectral peak energy and isotropic energy/luminosity with \( \Gamma_0 \). The slopes of these correlations are rather insensitive to the circumburst profile adopted in deriving \( \Gamma_0 \) (H or W) and are similar for \( E_{\text{iso}} \) and \( L_{\text{iso}} \) (\( E_{\text{iso}} \propto \Gamma_0^2 \) and \( L_{\text{iso}} \propto \Gamma_0^3 \)). A roughly linear correlation exists between \( E_{\text{peak}} \) and \( \Gamma_0 \): \( E_{\text{peak}} \propto \Gamma_0 \) (bottom panels in Fig. 2).

The dispersion of the data points around the best-fitting correlations (shown by the solid and dashed lines in Fig. 2) is modelled with a Gaussian and its \( \sigma_{\text{cr}} \) is given in Table 3. The less dispersed correlation is between the luminosity \( L_{\text{iso}} \) and \( \Gamma_0 \) (with \( \sigma_{\text{cr}} = 0.07 \)).

We finally verified that there is no correlation between the GRB duration \( T_{90} \) and \( \Gamma_0 \) (chance probability \( P = 0.3 \) and 0.7 for the H and W cases) and between the redshift \( z \) and \( \Gamma_0 \).

### 6.3 Comoving frame \( E_{\text{peak}}' \), \( E_{\text{iso}}' \), \( L_{\text{iso}}' \) distributions

In Figs 3, 4 and 5 we show the distributions of the comoving frame peak energy, isotropic equivalent energy and luminosity. In Fig. 3 we show the distribution of the peak energy: the sample of 132 GRBs with measured redshifts and known \( E_{\text{peak}} \) is shown with the dashed line and the subsample of 30 GRBs of this work for which we could estimate \( \Gamma_0 \) is shown with the red hatched histograms. These distributions represent \( E_{\text{peak}} \), i.e. the peak energy in the rest frame of the sources.

The distributions of the comoving peak energy [derived as \( E'_{\text{peak}} = E_{\text{peak}}/\Gamma_0 \) (5\( \Gamma_0/3 \))] are shown by the (cyan) fitted and hatched (purple) histograms in Fig. 3 for the H and W cases, respectively, considering the 27 GRBs which show a peak in the optical light curve. Fig. 3 shows also the fits with Gaussian functions; their parameters are reported in Table 2.

There is a reduction of the dispersion of the distribution of the peak energy from the rest frame to the comoving one. The comoving frame \( E_{\text{peak}}' \) clusters around \( \sim 6 \) and \( \sim 3 \) keV in the H and W cases, respectively, with dispersions of nearly one decade, i.e. narrower than the dispersion of \( E_{\text{peak}} \).

Fig. 4 shows the distribution of the isotropic energy \( E_{\text{iso}} \) for all the 132 GRBs with known \( z \) and measured \( E_{\text{peak}} \) (dashed line) and for the 30 GRBs with an estimate of \( \Gamma_0 \) (hatched red histogram). The \( E_{\text{iso}} = E_{\text{iso}}/\Gamma_0 \) distributions are shown with the solid filled (cyan) histogram and the hatched (purple) histogram for the H and W cases. These distributions are obtained with the 27 GRBs with a
peak in the optical light curve. The three GRBs with a peak in the GeV light curve are only shown for comparison (open red circles and filled green squares). The three GRBs with peak in the GeV light curve are shown with the grey symbols, but are not included in the fits shown here. The short GRB 090510 with both a peak in the GeV and a delayed peak in the optical (see Table 1) is shown by star symbols connected by the dashed (grey) line. The larger value of $\Gamma_0$ is that derived from the peak in the GeV light curve. Bottom panels: peak energy $E_{\text{peak}}$ for the H case (left-hand panel) and W case (right-hand panel) as a function of $\Gamma_0$. The solid line is the best-fitting correlation. The correlation coefficient and the slope and normalization of the best-fitting correlations are reported in Table 3.

Finally, in Fig. 5 we show the distribution of $E_{\text{iso}}$ for the 131 GRBs in the sample (dashed line), the distribution of $L_{\text{iso}}$ for the 30 GRBs with estimated $\Gamma_0$ (red hatched histogram) and the comoving frame $L'_{\text{iso}} = L_{\text{iso}}/(4\pi$ $\Gamma_0^2/3$) distribution (solid filled cyan and hatched purple histograms for the H and W cases, respectively, obtained with the 27 GRBs with a peak in the optical light curve). Interestingly, we find a strong clustering of the comoving frame distribution of $L'_{\text{iso}}$. For the H case we find (see Table 2 for the values of the Gaussian fits) an average $L'_{\text{iso}} \sim 10^{48}$ erg s$^{-1}$ with a small dispersion (0.47 dex), while when using the $\Gamma_0$ computed in the wind density profile (W) case we find an almost universal value of $L'_{\text{iso}} \sim 5 \times 10^{48}$ erg s$^{-1}$ with a dispersion of less than 1 order of magnitude around this value (hatched purple histogram and dashed purple line in Fig. 5).
with measured $z$ and $E_{\text{peak}}$ and $L_{\text{iso}}$. We show the corresponding $E_{\text{peak}}$–$E_{\text{iso}}$ and $E_{\text{peak}}$–$L_{\text{iso}}$ correlations in Fig. 6 (left- and right-hand panels, respectively). The best-fitting correlation parameters (obtained with the bisector method) are reported in Table 4. We find that $E_{\text{peak}} \propto E_{\text{iso}}^{0.56}$ (dashed line in Fig. 6) with a scatter $\sigma = 0.24$ (computed perpendicular to the best-fitting line and modelled with a Gaussian function). The other correlation is $E_{\text{peak}} \propto L_{\text{iso}}^{0.50}$ with a slightly larger scatter $\sigma = 0.3$. The $1\sigma$, $2\sigma$ and $3\sigma$ dispersion of the correlations are shown with the shaded stripes.

Fig. 6 also shows the comoving frame $E_{\text{peak}}'$ and $E_{\text{iso}}'$ (left-hand panel) and $E_{\text{peak}}'$ and $L_{\text{iso}}'$ (right-hand panel) for the 30 GRBs of our sample with an estimate of $\Gamma_0$ in the H case. The 27 GRBs with a peak in the optical are shown with the cyan filled squares. Fig. 7 shows the same correlations ($E_{\text{peak}}$–$E_{\text{iso}}$ and $E_{\text{peak}}$–$L_{\text{iso}}$ in the left- and right-hand panels, respectively) for the W case. We note that in both the H and W cases there is a clustering of the points around typical values of $E_{\text{peak}}$, $E_{\text{iso}}$ and $L_{\text{iso}}$. Table 4 reports the correlation analysis among the comoving frame quantities.

7 DISCUSSION AND CONCLUSIONS

We have considered all bursts with measured $E_{\text{peak}}$ and known redshift up to May 2011 (132 GRBs). Among these we have searched in the literature for any indication of the peak of the afterglow light curve $t_{p,z}$ suitable to estimate the initial bulk Lorentz factor $\Gamma_0$. Our sample of bursts is composed by 27 GRBs with a clear evidence of $t_{p,z}$ in the optical light curve. We have derived the peak energy $E_{\text{peak}}'$, the isotropic energy $E_{\text{iso}}'$, and the isotropic peak luminosity $L_{\text{iso}}'$ in the comoving frame. To this aim we have derived the general formula for the computation of $\Gamma_0$ (Section 3) considering two possible scenarios: a uniform ISM density profile ($n = \text{const.}$, H) or a wind density profile ($n \propto r^{-2}$, W).

For the wind case, the $\Gamma_0$ distribution (Fig. 1 and Table 2) is shifted at somewhat smaller values ($\langle \Gamma_0 \rangle \sim 66$) than the same distribution for the homogeneous density case ($\langle \Gamma_0 \rangle \sim 138$). The distribution of $E_{\text{peak}}'$ is relatively narrow and centred around $\sim 6$ or $\sim 3$ keV for the W and H cases (Fig. 3 and Table 2). The distribution of $L_{\text{iso}}'$ (Fig. 5) clusters, especially for the wind case, in a very narrow range (much less than a decade), around $5 \times 10^{50}$ erg s$^{-1}$. While the distribution of $E_{\text{iso}}'$ (Fig. 4) is broader and centred at $3 \times 10^{51}$ erg. $E_{\text{iso}}'$ and $L_{\text{iso}}'$ correlate with $\Gamma_0$ ($\propto \Gamma_0^{1.2}$) both for the wind and the homogeneous case) and the correlation is stronger (with a scatter $\sigma = 0.07$) for the wind case. Finally, the duration of the burst, as expected, does not correlate with $\Gamma_0$.

The correlations that we have found are strong despite they are defined with a still small number of GRBs. We expect that with the increase of the number of GRBs with measured $t_{p,z}$ and well-determined spectral properties (i.e. $E_{\text{peak}}$, $E_{\text{iso}}$ and $L_{\text{iso}}$) the slope and normalization of these correlations might change.

For comparison we also considered four GRBs with a peak in the GeV light curve. If the GeV emission is interpreted as afterglow (Barniol-Duran & Kumar 2009; Ghirlanda et al. 2010b; Ghisellini et al. 2010), the measure of $t_{p,z}$ at early times in the GeV range allows us to estimate their $\Gamma_0$ that are consistent with the correlations found using only the bursts with $t_{p,z}$ observed in the optical. Although not a proof, this is a hint in favour of the afterglow origin of the GeV emission.

These results are schematically summarized in the first column of Table 5. The second column of the same table reports some...
Figure 5. Isotropic luminosity distributions in the rest frame (dashed histogram) for the sample of 131 GRBs with known redshift and constrained $E_{\text{peak}}^{\text{obs}}$. The hatched histogram shows the 30 GRBs of our sample for which we have an estimate of the peak of the afterglow. The beaming-corrected distribution of $L_{\text{iso}}'$ is shown by the solid filled histogram and hatched purple histogram for the H and W cases for the 27 GRBs with a peak in the optical light curve. The four bursts with a peak in the GeV light curve are shown for comparison with the hatched and filled grey histograms.

Figure 6. Homogeneous ISM (H). Left: $E_{\text{peak}}' - E_{\text{iso}}$ correlation in the rest frame (crosses and red circles) for 132 GRBs with $z$ and fitted $E_{\text{peak}}$ updated up to 2011 May. Right: $E_{\text{peak}}' - L_{\text{iso}}$ correlation with 131 GRBs. In both panels, the best-fitting correlation is shown by the dashed line and its 1σ, 2σ, 3σ scatters are shown by the shaded region. The comoving frame $E_{\text{peak}}'$ and $E_{\text{iso}}$ (left) and $E_{\text{peak}}'$ and $L_{\text{iso}}$ (right) of 30 GRBs [red open circles (left-hand panel) and green open circles (right-hand panel)] in our sample (Table 1) with an estimate of the $\Gamma_0$ factor are shown with the filled cyan square symbols (27 events with $t_p,z$ in the optical light curve) or grey filled square (the three long GRBs with a peak in the GeV light curve). The short GRB 090510 is also shown with a star symbol and the low-luminosity GRB 060218 (with $\Gamma_0 \sim 5$; Ghisellini, Ghirlanda & Tavecchio 2007) is shown with an open circle.
Table 4. Results of the fit of the $E_{\text{peak}}$–$E_{\text{iso}}$ and $E_{\text{peak}}$–$L_{\text{iso}}$ correlations updated in this paper up to May 2011. The Spearman correlation coefficient $\rho$ and the chance probability $P_{\text{chance}}$ are given with the slope $m$ and normalization $q$ of the least-squares fit.

<table>
<thead>
<tr>
<th>Correlation</th>
<th># GRBs</th>
<th>$\rho$</th>
<th>$P_{\text{chance}}$</th>
<th>$m$</th>
<th>$q$</th>
<th>$\sigma_{sc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{peak}}$–$E_{\text{iso}}$</td>
<td>132</td>
<td>0.8</td>
<td>$10^{-30}$</td>
<td>0.56</td>
<td>0.02</td>
<td>26.06 ± 1.14</td>
</tr>
<tr>
<td>$E_{\text{peak}}$–$E_{\text{iso}}$</td>
<td>30</td>
<td>0.76</td>
<td>$10^{-6}$</td>
<td>0.58</td>
<td>0.07</td>
<td>28.26 ± 3.74</td>
</tr>
<tr>
<td>$E_{\text{peak}}$–$L_{\text{iso}}$</td>
<td>131</td>
<td>0.77</td>
<td>$3 \times 10^{-26}$</td>
<td>0.49</td>
<td>0.04</td>
<td>23.03 ± 1.84</td>
</tr>
<tr>
<td>$E_{\text{peak}}$–$L_{\text{iso}}$</td>
<td>27</td>
<td>0.76</td>
<td>$3 \times 10^{-6}$</td>
<td>0.65</td>
<td>0.08</td>
<td>31.53 ± 4.36</td>
</tr>
<tr>
<td>$E_{\text{peak}}$–$L_{\text{iso}}$</td>
<td>30</td>
<td>0.8</td>
<td>$10^{-7}$</td>
<td>0.57</td>
<td>0.06</td>
<td>27.14 ± 3.37</td>
</tr>
</tbody>
</table>

Table 5. Schematic summary of our results and their implications for the case of a wind density profile. We have assumed that both $E_{\text{iso}}$ and $L_{\text{iso}}$ scale as $\Gamma^2$, instead of $\Gamma^2$.

<table>
<thead>
<tr>
<th>Our results</th>
<th>Implications</th>
<th>If $\theta^2 \Gamma \sim \text{const}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{peak}} \sim \text{const}$</td>
<td>$E_{\text{peak}} \propto \Gamma$</td>
<td></td>
</tr>
<tr>
<td>$E_{\text{iso}} \propto \Gamma^2$</td>
<td>$E_{\text{iso}} \propto E_{\text{peak}}^2$</td>
<td>$E_{\gamma} = \theta^2 E_{\text{iso}} \propto \Gamma \propto E_{\text{peak}}$</td>
</tr>
<tr>
<td>$L_{\text{iso}} \propto \Gamma^2$</td>
<td>$L_{\text{iso}} \propto E_{\text{peak}}^2$</td>
<td>$L_{\gamma} = \theta^2 L_{\text{iso}} \propto \Gamma \propto E_{\text{peak}}$</td>
</tr>
<tr>
<td>$T_{90}$ not $f(\Gamma)$</td>
<td>$T_{90} \propto \Gamma$</td>
<td>$E_{\gamma} \sim \text{const}$</td>
</tr>
<tr>
<td>$L_{\text{iso}} \sim \text{const}$</td>
<td>$E_{\text{iso}} / L_{\text{iso}} \propto T_{90} \propto \Gamma$</td>
<td>$L_{\gamma} \sim E_{\gamma} / T_{90} \sim 1 / \Gamma$</td>
</tr>
</tbody>
</table>

Figure 7. Wind ISM (W). Same as Fig. 6.

immediate implications of these results. Since $E_{\text{peak}}' \propto E_{\text{peak}} \Gamma_0$ is contained in a narrow range, all bursts emit their radiation at a characteristic frequency in their comoving frame, irrespective of their bulk Lorentz factor. Furthermore, we can assume that $E_{\text{peak}} \propto \Gamma_0$, and this, together with the quadratic dependence on $\Gamma_0$ of $E_{\text{iso}}$ and $L_{\text{iso}}$, yields the ‘Amati’ and the ‘Yonetoku’ relations. They are the result of a different $\Gamma_0$ factors. Indeed, at the extremes of the $E_{\text{peak}}$–$E_{\text{iso}}$ and $E_{\text{peak}}$–$L_{\text{iso}}$ correlations we find GRB 060218 which has the lowest $\Gamma_0 \sim 5$ (inferred from its X-ray and optical properties – Ghisellini et al. 2007), while at the upper end (corresponding to the largest peak energies and isotropic energetics and luminosities) there is GRB 080916C which has the largest...
$\Gamma_0 = 880$. The fact that the $E_{\text{peak}}-E_{\text{iso}}$ and $E_{\text{peak}}-L_{\text{iso}}$ correlations could be a sequence of $\Gamma_0$ factors has been also proposed by Dado, Dar & De Rujula (2007) based on different assumptions.

If all bursts had the same jet opening angle, then $L'_p = \theta^2 L_{\text{iso}}$, and the (logarithmic) width of the $L'_{\text{iso}}$ distribution would be the same of the (more fundamental) $L'_p$ distribution. On the other hand, we have some hints that very energetic and luminous GRBs tend to have narrower opening angles (e.g. Firmani et al. 2005). It is this property that makes the collimation corrected $E'_p$ and $L'_p$ quantities to correlate with $E_{\text{peak}}$ in a different way (i.e. different slope) than in the Amati and Yonetoku relations (Ghirlanda et al. 2004; Nava et al. 2006).

We are then led to propose the following ansatz: the opening angle of the jet inversely correlates with the bulk Lorentz factor $\theta_1 \propto \Gamma_0^{-\alpha}$. There are too few GRBs in our sample with measured $\theta_1$ to find a reasonable value for the exponent $\alpha$, but it is nevertheless instructive to explore the case $\alpha = 1/2$, leading to $\theta_1^2 \Gamma_0 = \text{constant}$. If we assume this relation, we find, for the collimation corrected $E'_p$:

$$E'_p = \theta_1^2 E_{\text{iso}} \propto \Gamma_0 \propto E_{\text{peak}}. \quad (20)$$

This is the ‘Ghirlanda’ relation in the wind case (Nava et al. 2006). Similarly, for the collimation corrected luminosity (Ghirlanda, Ghisellini & Firmani 2006):

$$L'_p = \theta_1^2 L_{\text{iso}} \propto \Gamma_0 \propto E_{\text{peak}}. \quad (21)$$

Another important consequence of our ansatz is that, in the comoving frame, the collimation corrected energetic $E'_p$ becomes constant:

$$E'_p = \frac{\theta_1 E_{\text{iso}}}{\Gamma_0} = \text{constant}. \quad (22)$$

This allows us to ‘re-interpret’ the constancy of $L'_{\text{iso}}$ as a consequence of the constant $E'_p$:

$$L'_{\text{iso}} \sim \frac{E'_p}{\theta_1^2 \Gamma_0} = \frac{E_{\text{iso}}}{\theta_1^2 \Gamma_0} = \text{constant}. \quad (23)$$

In other words, in the comoving frame, the burst emits the same amount of energy at the same peak frequency, irrespective of the bulk Lorentz factor. For larger $\Gamma_0$, the emitting time in the comoving frame is longer (by a factor $\Gamma_0$ if the observed $T_{90}$ is the same), so the comoving luminosity is smaller. But since the jet opening angle is also smaller (for larger $\Gamma_0$), the isotropic equivalent luminosity turns out to be the same. These consequences are listed in the third column of Table 5.

Interestingly, we note that the general formula for the estimate of the jet opening angle

$$\theta_1 \propto \left( \frac{t_{\text{obs}}}{t_{\gamma, \text{obs}}} \right)^{3(\gamma-2)/(8-2\gamma)} \left( \frac{n_0}{n_{\gamma, \text{obs}}} \right)^{1/(8-2\gamma)}, \quad (24)$$

with $s = 0$ for the homogeneous case and $s = 2$ for the wind case, can be combined with equation (15) to give

$$\theta_1^2 \Gamma_0 \propto \left( \frac{t_{\text{obs}}}{t_{\gamma, \text{obs}}} \right)^{3(\gamma-2)/(8-2\gamma)}. \quad (25)$$

The product $\theta_1^2 \Gamma_0$, then depends only on two observables, i.e. the time of the peak of the afterglow $t_{\gamma, \text{obs}}$ and the time of the jet break $t_{\text{obs}}$, and it is independent of the redshift $z$ and the energetic $E_{\text{iso}}$, as well as of the density profile normalization $n_0$ and radiative efficiency $\eta$. If also the product $\theta_1^2 \Gamma_0 = \text{const}$, then we can derive both $\theta_1 \propto (t_{\text{obs}}/t_{\gamma, \text{obs}})^{3(\gamma-2)/(8-2\gamma)}$ and $\Gamma_0 \propto (t_{\text{obs}}/t_{\gamma, \text{obs}})^{(3-\gamma)/(4-\gamma)}$. If the ansatz $\theta_1^2 \Gamma_0 = \text{const}$ will prove to be true, then by simply multiplying the peak time and the jet break time of the afterglow light curve we could estimate both $\theta_1$ and $\Gamma_0$ for any GRB.

In our sample, only for four bursts we can estimate the jet opening angle from the measure of the jet break time of the optical light curve. Their small number does not make possible to directly test the existence of a relation between $\Gamma_0$ and $\theta_1$. However, an estimate of the jet opening angle can be possible by assuming that all bursts in our sample are consistent with the ‘Ghirlanda’ relation. Fig. 8 shows the estimated $\theta_1$ as a function of $\Gamma_0$. Stars (squares) refer to angles derived under the assumption of a H (W). To estimate the jet opening angles we considered the most updated ‘Ghirlanda’ correlation, which comprises 29 GRBs with measured jet break time (Ghirlanda et al. 2006). For the homogeneous density profile, the relation has the form $\log E_{\text{peak}} = -32.81 + 0.70 \log E_{\text{iso}}$, while in the case of a W the relation becomes $\log E_{\text{peak}} = -50.08 + 1.04 \log E_{\text{iso}}$. Given the large scatter of the data points in Fig. 8, we fitted both $\theta_1$ versus $\Gamma_0$ and $\theta_1$ versus $\Gamma_0$; we obtain $\theta_1 \propto \Gamma_0^{-0.22}$ and $\Gamma_0 \propto \theta_1^{-2.32}$ for the H case (dashed lines in Fig. 8) and $\theta_1 \propto \Gamma_0^{-0.52}$ and $\Gamma_0 \propto \theta_1^{-1.14}$ for the W case (dot–dashed line in Fig. 8). We conclude that our ansatz $\theta_1 \propto \Gamma_0^{-1/2}$ is consistent with, but not proven by, this analysis.

An interesting exercise is to estimate the product $\theta_1 \Gamma_0$. From the observational point of view of $\theta_1 \Gamma_0 \gg 1$ at the end of the prompt phase, so that the decrease of $\Gamma$ in the afterglow phase, due to the interaction of the GRB fireball with the ISM, gives rise to a jet break when $\theta_1 \Gamma_0 \sim 1$.

Some numerical simulations (Komissarov et al. 2009) of jet acceleration have shown that a magnetic-dominated jet confined by an external medium should have $\theta_1 \Gamma_0 \ll 1$. This value is inconsistent with typical values of $\theta_1$ and $\Gamma_0$: in the case of an homogeneous wind density profile the typical $\theta_1 \sim 0.1$ radians (Ghirlanda et al. 2007), while in the case of a wind density profile $\theta_1 \sim 0.07$ radians. Combining these values with the average values of $\Gamma_0$ estimated in this paper (Table 1) we find $\theta_1 \Gamma_0 \sim 14$ (5) for the H (W) case.
These are approximate values: the sample of GRBs with measured $\theta_j$ (Ghirlanda et al. 2007) contains only four bursts of the sample of events of the present paper with estimated $\Gamma_0$. However, though somehow speculative, we can derive $\theta_j$ for the 32 GRBs of our sample assuming the $E_{\text{peak}} - E_{\gamma}$ correlation in the H case (Ghirlanda et al. 2004) or in the W (Nava et al. 2006) case. In Fig. 9 we show the distributions of the product $\theta_j\Gamma_0$ in the H case (blue histogram) and in the W case (purple histogram). We note that both are centred around typical values of 20 and 6 (for the H and W cases, respectively). These values are in good agreement with the results of recent simulations of (i) a magnetized jet confined by the stellar material that freely expands when it breaks out the star (Komissarov, Vlahakis & Koenigl 2009; Tchekhovskoy, Narayan & McKinney 2010) or (ii) a magnetized unconfined split-monopole jet (Tchekhovskoy, McKinney & Narayan 2009; Tchekhovskoy, Narayan & McKinney 2010). A possible test of these two scenarios could be short GRBs where the absence of the progenitor star would prefer model (ii) for the jet acceleration. In our sample only the short/hard GRB 090510 is present. No jet break was observed for this event, and in general we do not yet know if short GRBs follow the same $E_{\text{peak}} - E_{\gamma}$ correlation of long ones.

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REFERENCES

Abdo A. A. et al., 2009b, Nat, 462, 331
Barthelmy S. D. et al., 2005, Space Sci. Rev., 120, 143
Blandford R. D., McKee C. F., 1976, Phys. Fluids, 19, 1130
Costa E. et al., 1997, Nat, 387, 783
Friel D. et al., 1997, Nat, 389, 261

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