A cosmic speed-trap: a gravity-independent test of cosmic acceleration using baryon acoustic oscillations

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ABSTRACT

We propose a new and highly model-independent test of cosmic acceleration by comparing observations of the baryon acoustic oscillation (BAO) scale at low and intermediate redshifts: we derive a new inequality relating BAO observables at two distinct redshifts, which must be satisfied for any reasonable homogeneous non-accelerating model, but is violated by models similar to $\Lambda$ cold dark matter, due to acceleration in the recent past. This test is fully independent of the theory of gravity (general relativity or otherwise), the Friedmann equations, cosmic microwave background and supernova observations: the test assumes only the cosmological principle, and that the length-scale of the BAO feature is fixed in comoving coordinates. Given realistic medium-term observations from the Baryon Oscillation Spectroscopic Survey, this test is expected to exclude all homogeneous non-accelerating models at $\sim 4\sigma$ significance, and can reach $\geq 7\sigma$ with next-generation surveys.

Key words: dark energy – large-scale structure of Universe.

1 INTRODUCTION

In the last 10–15 yr, the $\Lambda$ cold dark matter ($\Lambda$CDM) model has been established as the standard model of large-scale cosmology; the model is an excellent match to many observations including the anisotropies in the cosmic microwave background (CMB) measured by WMAP (Komatsu et al. 2011) and other experiments, the large-scale clustering of galaxies (Percival et al. 2010), the Hubble diagram for high-$z$ supernovae (hereafter SNe; Guy et al. 2010; Conley et al. 2011), and the abundance and baryon fraction of rich clusters of galaxies (Allen, Evrard & Mantz 2011).

Despite these great observational successes, the model appears unnatural since 96 per cent of the universe’s mass-energy is not observed, but is only inferred from fitting the observations. Also, the dark sector contains at least two apparently unrelated components, dark matter and dark energy; recent reviews of dark energy are given in Frieman, Turner & Huterer (2008) and Linder (2008).

The most direct evidence for cosmic acceleration comes from the Hubble diagram of Type Ia SNe (Guy et al. 2010; Conley et al. 2011), which shows that SNe at $0.3 \lesssim z \lesssim 0.9$ are fainter, relative to local SNe, than can be accommodated in any Friedmann–Robertson–Walker model without dark energy. A model-independent approach has also been given by Shapiro & Turner (2006), who show that the SN results require accelerated expansion at $z < 0.4$ at around the $5\sigma$ significance level without assuming the Friedmann equations.

However, there are some possible loopholes in the SN results: since they are fundamentally based on brightness measurements, the interpretation could be affected by either unexpected evolution of the mean SN properties over cosmic time, or some process which removes photons en route to our telescopes, such as peculiar dust or more exotic effects such as photon–dark matter interactions. The simplest such effects with monotonic time dependence are strongly disfavoured by SN observations at $z > 1$ (Riess et al. 2007), but more complex time-dependent effects could still leave these loopholes open.

Independent of SNe, there is powerful support for dark energy from observations of the anisotropies in the CMB (Larson et al. 2010; Komatsu et al. 2011) and the large-scale clustering of galaxies (Percival et al. 2010), but this is dependent on assuming general relativity (GR) and the Friedmann equations; if both these hold, the model parameters are tightly constrained by CMB and large-scale structure (LSS) data, and the expansion history $a(t)$ must match $\Lambda$CDM models within a few per cent. However, in alternative gravity theories, we cannot make model-independent statements from the CMB or LSS: clearly any successful modified-gravity model should eventually be consistent with these observations, but the model space of modified gravity is large and the calculations non-trivial; thus, in non-GR models, we cannot necessarily use the CMB and LSS observations to make any definite statement about recent acceleration.

The accelerated expansion is so startling that it is desirable to test it via multiple routes with a minimum number of model assumptions. A very direct test of acceleration has been proposed using the ‘cosmic drift’, which is the small change in redshift for fixed
object(s) over time (e.g. Liske et al. 2008); the predicted change is \( \frac{dz}{dt} = (1 + z) H_0 - H(z) \). However, this effect is tiny over human time-scales, of the order of cm s\(^{-1}\) yr\(^{-1}\), and will probably require over 20-yr baseline to get a convincing detection.

Here we propose a new and robust test for cosmic acceleration based only on the cosmic ‘standard ruler’ in the galaxy correlation function: in the standard model, this is a feature created by baryon acoustic oscillations (BAOs) in the baryon-photon fluid before recombination (e.g. Peebles & Yu 1970); this was analysed in more detail by Eisenstein & Hu (1998) and Meiksin, White & Peacock (1999), then first detected in 2005 by Eisenstein et al. (2005) in SDSS data, and Cole et al. (2005) using 2dFGRS. The length of this ruler, hereafter \( r_s \), depends only on matter and radiation densities and is accurately predicted from CMB observations at \( \approx 153 \pm 2 \) Mpc (Komatsu et al. 2011). Many recent studies (e.g. Eisenstein, Seo & White 2007b; Shoji, Jeong & Komatsu 2009; Abdalla, Blake & Rawlings 2010; Tian et al. 2011) have shown how precision measurements of this BAO scale from huge galaxy redshift surveys can provide powerful constraints on the properties of dark energy, and test for the evolution of dark energy density; more details are given in Section 2.

However, in this paper, we do not assume any gravity theory or the actual length-scale of this feature, only that we can observe some feature at a specific length-scale imprinted on the galaxy distribution at high redshift, which expands with the Hubble expansion and remains a constant ruler in comoving coordinates. We then derive an inequality relating observations comparing this ruler at low and intermediate redshifts, which is satisfied in any reasonable non-accelerating model, but is violated by accelerating models approximating \( \Lambda \)CDM. In more detail, we use the radial component of the BAO feature at \( z = 0.75 \) to constrain the product \( H(z) r_s \), and then we compare to the spherical-averaged BAO feature at low redshift \( z_s \approx 0.2 \), which is related to the average of \( H(z) \) at \( 0 \leq z \leq z_s \). Then, assuming any non-accelerating model, we derive a strict upper limit on the ratio of these. Models approximating standard \( \Lambda \)CDM predict a result which violates this inequality by a substantial amount \( \sim 10-20 \) per cent, depending on cosmological parameters and redshift. Future large redshift surveys should be able to measure this ratio to \( \pm 2 \) per cent precision: assuming our inequality is significantly violated as predicted, we can then exclude all homogeneous non-accelerating models regardless of Friedmann equations, gravity theory or details of the expansion history.

The plan of this paper is as follows. In Section 2, we review the basic features and observables of BAOs. In Section 3, we derive the new inequality relating BAO observables for non-accelerating models. In Section 4, we discuss future observations and related issues, and we summarize our conclusions in Section 5.

2 OBSERVATIONS OF THE BAO FEATURE

The BAO feature (Eisenstein & Hu 1998; Meiksin et al. 1999) is a bump in the galaxy correlation function \( \xi(r) \), or equivalently a decaying series of wiggles in the power spectrum \( P(k) \), corresponding to a comoving length denoted by \( r_s \), created by acoustic waves in the early universe prior to decoupling (see Basset & Hlozek 2010 for a recent review). In the standard model, its length-scale is essentially set by the distance that a sound wave can propagate prior to the ‘drag epoch’ at \( z_d \approx 1020 \), denoted by \( r_s(z_d) \), and this length depends only on physical densities of matter \( \Omega_m h^2 \) and baryons \( \Omega_b h^2 \) (together with radiation density \( \Omega_r h^2 \) which is pinned very precisely by the CMB temperature). In the standard model, the relative heights of the acoustic peaks in CMB anisotropies constrain \( \Omega_m h^2 \) and \( \Omega_b h^2 \) well (Komatsu et al. 2011), which leads to a prediction \( r_s \approx 153 \) Mpc comoving with approximately 1.5 per cent precision. This predicted length does not rely on the assumption of a flat universe, since the relative CMB peak heights constrain the various densities reasonably well without assuming flatness. However, the CMB-predicted length \( r_s \) does depend on assuming standard GR, and several assumptions about the mass-energy budget including standard neutrino content, negligible early dark energy, no late-decaying dark matter, negligible admixture of isocurvature perturbations, etc. However, in the rest of this paper, we leave \( r_s \) as an arbitrary comoving scale, which cancels later.

The BAO feature provides a standard ruler which can be observed at low to moderate redshifts using very large galaxy redshift surveys; in the small angle approximation and assuming we observe a redshift shell which is thin compared to its mean redshift \( z \), there are two primary observables derived from a BAO survey: first, the angle on the sky subtended by the BAO feature transverse to the line of sight, \( \Delta \theta(z) = r_s / (1 + z) D_A(z) \), where \( D_A(z) \) is the conventional (proper) angular diameter distance to redshift \( z \); and, secondly, the difference in redshift along one BAO length along the line of sight, \( \Delta z_A(z) = r_s / H(z) c \) (e.g. Blake & Glazebrook 2003; Seo & Eisenstein 2003). We note that calculating comoving galaxy separations from observed positions and redshifts requires a reference cosmology; hence, a difference between the true and reference cosmologies will produce an error in the inferred \( r_s \); however, any error in the reference model cancels to first order in the dimensionless ratios \( r_s / D_A(z) \) and \( r_s / H(z) c \), so both of these ratios can be well constrained with minimal theory dependence by measuring BAOs in a galaxy redshift survey.

The ability to independently probe \( D_A(z) \) and \( H(z) \) is a powerful advantage of BAOs over other low-redshift cosmological tests. Furthermore, a redshift survey useful for BAOs can also measure the growth of structure via redshift-space distortions and thus test for consistency with GR, though we do not consider this here.

However, in practice, current galaxy redshift surveys are not quite large enough to robustly measure the BAO feature separately in angular and radial directions (though there are tentative detections, e.g. Gaztanaga, Cabre & Hui 2009). The current measurements primarily constrain a spherically averaged scale, called \( D_v \), which is defined by Eisenstein et al. (2005) as

\[
D_v(z) \equiv \left[ \frac{(1 + z)^2 D_A(z)^2}{H(z)} \right]^{1/3} ; \tag{1}
\]

this is essentially a geometric mean of two transverse directions and one radial direction. Observations using the redshift surveys 2dFGRS and SDSS-II have measured the dimensionless ratio \( d(z) \equiv r_s / D_A(z) \) at low redshifts (Kazin et al. 2010; Percival et al. 2010), which we discuss later. We note that as \( z \rightarrow 0 \), \( D_v(z) \rightarrow c z / H_0 \); however, this approximation is not very useful in practice, since we cannot measure the BAO feature at very low redshift \( z < 0.02 \) where corrections of the order of \( z^2 \) are unimportant. We give a better approximation below in Section 4.2.

In practice, the BAO feature is not a sharp spike but a hump in \( \xi(r) \) of width approximately 15 per cent of \( r_s \), so there are several subtle effects in actually extracting the scale \( r_s \) from a redshift survey: we discuss these in more detail in Section 4.1. However, for the purposes of this paper, we only need to assume that \( r_s \) is a constant comoving length to \( \sim 1 \) per cent at redshift \( z \lesssim 0.8 \), so these precision details are relatively unimportant for the rest of this paper.
3 THE COSMIC SPEED-TRAP

Here we derive a new inequality which we denote by the ‘cosmic speed-trap’, which must be satisfied by any reasonable non-accelerating model, but is violated by ΛCDM and other accelerating models. We start off by assuming an arbitrary non-accelerating model, and deriving a lower limit for \(D_V(z_1)\) in terms of the value of \(H(z_1)\) at a higher redshift \(z_2\). Then, we form a ratio of BAO observables which eliminates \(H(z_2)\) and \(r_s\), and we obtain the speed-trap inequality (15) which forms our main new result.

3.1 An inequality for \(D_V\) in non-accelerating models

Here we derive an inequality for \(D_V(z)\) which is satisfied in any non-accelerating model, but may be violated by acceleration.

First, we define as usual \(a\) to be the cosmic expansion factor relative to the present day with \(a_0 = 1\), redshift \(z\) by \(1 + z ≡ a^{-1}\), and the Hubble parameter \(H(a) = \dot{a}/a\), where the dot represents time-derivative. Then we have the expansion rate

\[
\dot{a} = \frac{aH(a)}{1 + z};
\]

if the expansion of the universe was non-accelerating, then \(\dot{a}\) is non-positive and the function above must be non-increasing with time or \(a\), therefore non-decreasing with increasing \(z\). Therefore, if we consider any two redshifts \(z_1 < z_2\), in any non-accelerating universe, then

\[
\frac{H(z_1)}{1 + z_1} < \frac{H(z_2)}{1 + z_2}.
\]

Assuming only the cosmological principle, any observed violation of this inequality is a direct proof that the expansion has accelerated, on average, between the earlier epoch \(z_2\) and the later epoch \(z_1\), without reference to any specific theory of gravity or geometry.

A concordance ΛCDM model does violate this inequality due to the recent positive acceleration: a minimum value of \(H(z)/(1 + z)\) occurred at \(z_{ac} = \sqrt{\Omega_m/\Omega_m - 1}\); for the concordance value \(\Omega_m ≈ 0.27\), this gives \(z_{ac} ≈ 0.75\), and \(H(z_{ac})/(1 + z_{ac}) ≈ 0.85H_0\).

The expansion rate \(H(z)/(1 + z)\) is shown in Fig. 1 for a few representative models: it is notable that the value of \(H(z)/(1 + z)\) remains within a few per cent of its minimum in the range \(0.5 < z < 1.2\), and it rises rather sharply at low redshift; for the concordance model, it only crosses the half-way value between the minimum and the present-day \(H_0\) at the modest redshift of \(z ≈ 0.17\), and three-quarters of the speedup has occurred since \(z ≈ 0.31\). Thus, the actual speedup of the expansion rate is quite concentrated at rather low redshift; this becomes relevant later.

Next, we suppose we have a measurement of \(H(z_1)\) at an earlier epoch \(z_1\); for a non-accelerating model, we now derive a lower limit on \(D_V(z_1)\) at a later epoch \(z_2\) where \(z_1 < z_2\).

The comoving radial distance to redshift \(z_1\) is

\[
D_R(z_1) = \frac{c}{H(z_1)} \int_0^{z_1} \frac{1}{H(z)} \, dz.
\]

If the universe is non-accelerating and \(z_1 < z_2\), we can rearrange inequality (3) into \(1/H(z) ≥ (1 + z_1)/(H(z_1)(1 + z_1))\); substituting this, we have

\[
D_R(z_1) ≥ \frac{c(1 + z_1)}{H(z_1)} \ln(1 + z_1).
\]

The proper angular diameter distance \(D_\Lambda(z)\) is defined by

\[
(1 + z)D_\Lambda(z) ≡ |R_c| S_l \left( \frac{D_R(z)}{|R_c|} \right) = D_R(z) \frac{S_l(x) x}{x_1},
\]

where \(|R_c|\) is the curvature radius of the universe in comoving Mpc, \(x = D_R(z)/|R_c|\), and the function \(S_l(x) = \sinh x, x, \sin x\) for the cases \(k = −1, 0, +1\), respectively, where \(k\) is the sign of the curvature.

For the other term in \(D_V\), we use a similar inequality for \(1/H(z_1)\) to the above, which is

\[
\frac{c z_1}{H(z_1)} > \frac{c z_1 (1 + z_2)}{H(z_2)(1 + z_2)};
\]

substituting both the above into equation (1), we obtain the inequality

\[
D_V(z_1) ≥ \frac{c(1 + z_2)}{H(z_1)} \left[ \frac{z_1 \ln(1 + z_1)}{1 + z_1} \right]^{1/3} \left( \frac{S_l(x_1) x_1}{x_1} \right)^{2/3},
\]

where \(x_1 = D_R(z_1)/|R_c|\) as above.
This inequality is strict for any non-accelerating and homogeneous universe with a Robertson–Walker metric, independent of details of the expansion history or the gravity model. This is not so useful on its own, but we will see in the next section how to combine observables to cancel the $z_1$ dependence.

We note that the factor $(S(x)/x)^{2/3} = 1$ exactly for flat models, and is $\geq 1$ for open models (so open models always strengthen the inequality); the factor is $\leq 1$ for closed models which weakens our inequality, but only by a small amount if we consider sufficiently low redshift $z$, since the effect of curvature on distances only enters to third order in $z$; at small $x$ and $k = \pm 1$, we have

$$\left(\frac{S(x)}{x}\right)^{2/3} \approx 1 - \frac{x^2}{9};$$

therefore, we need an upper limit on $x$ for closed models. We get a firm limit as follows, using an upper bound on $D_R$ and a lower bound on $R_C$ for closed models.

To limit $D_R$, we can use the non-acceleration inequality (3) between $z = 0$ and an upper redshift $z_1$ to get $V(H(z)) \leq 1/(H_0(1 + z))$, which now leads to an upper bound on $D_R(z_1)$ in terms of $H_0$, $D_R(z_1) \leq (c/H_0)\ln(1 + z_1) \leq c\bar{H}_0$, for any non-accelerating model. This gives $x_1 \leq c\bar{H}_0/R_C$.

We may also obtain a lower bound on $R_C$ as follows: in a closed model, it is clear from equation (6) and $\sin x \leq 1$ that $D_R(z)$ cannot exceed $R_C/(1 + z)$ regardless of the expansion history $H(z)$. If we take, for example, $R_C = 0.6c\bar{H}_0$ and $H_0 = 70\text{ km s}^{-1}\text{ Mpc}^{-1}$, this leads to $D_R(z) = 3 \leq 642\text{ Mpc}$, only 0.4 times the concordance value of 1638 Mpc. However, observed angular sizes of $z \sim 3$ galaxies already convert to rather small physical sizes based on the concordance model, and making them smaller by another factor $< 0.4$ appears to be seriously discrepant. We therefore exclude closed models with $R_C < 0.6c\bar{H}_0$.

A stronger lower bound may be obtained with other methods: for example, the luminosity distance $D_L(z) = 1.5$ measured from SNe (Riess et al. 2007) agrees well with the concordance model, and if we adopt a lower bound 0.8 times the concordance value, we obtain $R_C > 0.84c\bar{H}_0$. However, to remain fully independent of SN data, we do not use this below. A stronger limit should also be possible in future using angular BAO measurements at $z \sim 3$, for example, from the HETDEX or Baryon Oscillation Spectroscopic Survey (BOSS) projects.

However, for the following, we take $R_C \geq 0.6c\bar{H}_0$ as a conservative gravity-independent lower limit for closed models. This leads to a firm upper limit $x \leq \ln(1 + z_1)/0.6$ for closed non-accelerating models, which we use below.

### 3.2 The observable speed-trap

The above inequality (8) relates the volume distance $D_V(z)$ at low redshift to the Hubble constant $H(z_2)$ at a higher redshift. Neither of these quantities is directly observable at present, but it is possible to measure both of them relative to the BAO length-scale $r_5$; then, dividing these two cancels the length-scale $r_5$ and gives a ratio measurement. Applying the $D_V$ inequality above gives us a limit which must be satisfied by any reasonable non-accelerating model, but is found to be violated by an expansion history close to $\Lambda$CDM, for a range of suitable choices of $z_1 \sim 0.2, z_2 \sim 0.75$.

The Hubble parameter $H(z_2)$ may be measured using the radial BAO scale (along the line of sight) in a redshift shell near $z_2$; for a thin shell and ignoring redshift-space distortion effects, this gives the observable

$$\Delta z_2 = \frac{r_5H(z_2)}{c}.$$  \hspace{1cm} (10)

In practice, it is useful to divide by $1 + z_2$ and define

$$y(z_2) = \frac{\Delta z_2}{1 + z_2} = \frac{r_5H(z_2)}{c(1 + z_2)}.$$  \hspace{1cm} (11)

since this $y$ is rather close to a constant over a substantial range of redshift in a $\Lambda$CDM model (as shown in Fig. 1), and we will see that it has a convenient cancellation below.

Using the redshift survey SDSS-II, Percival et al. (2010) have already measured the dimensionless ratio

$$d(z) = r_5/D_V(z)$$  \hspace{1cm} (12)

at redshifts $z = 0.2$ and 0.35, and also a combined ratio at $z = 0.275$. (We discuss the numerical results later.)

We now form the ratio of observables $z_1d(z_1)/y(z_2)$ which gives, from the definitions above,

$$\frac{z_1d(z_1)}{y(z_2)} = \frac{c(1 + z_1)}{H(z_1)}\frac{z_1}{D_V(z_1)},$$

assuming only that $r_5$ is a fixed comoving ruler independent of $z$.

If we now assume that the universe has never accelerated below redshift $z_2$, we may apply inequality (8) for $D_V(z_1)$; this cancels the $z_1$ factors, giving the inequality

$$\frac{z_1d(z_1)}{y(z_2)} \leq \left[\frac{z_1(1 + z_1)}{(\ln(1 + z_1))^2}\right]^{1/3}\left(\frac{x_1}{S(x_1)}\right)^{2/3}.$$  \hspace{1cm} (13)

It is more convenient to rearrange this to put the square bracket term on the left-hand side, and define the quantity $X_S$ (‘excess speed’)

$$X_S(z_1, z_2) = \frac{z_1d(z_1)}{y(z_2)}\left[\frac{(\ln(1 + z_1))^2}{z_1^2(1 + z_1)}\right]^{1/3} \left(\frac{x_1}{S(x_1)}\right)^{2/3},$$

where $X_S$ is a ratio of observables, and $x_1 = D_R(z_1)/R_C$ as before. (Note one may cancel some powers of $z_1$ on the left-hand side, but leaving them as above makes both terms in $X_S$ well behaved as $z_1 \rightarrow 0$.)

This inequality forms the main result of our paper, our cosmic speed-trap, which must be obeyed for any chosen values $z_1$ and $z_2$ with $z_1 \leq z_2$, given the following conditions:

(i) The universe is nearly homogeneous and isotropic with a Robertson–Walker metric.

(ii) The redshift is due to cosmological expansion and $c$ is constant.

(iii) $r_5$ is the same comoving length at $z_1$ and $z_2$.

(iv) The expansion has never accelerated in the interval $0 < z < z_2$.

If the speed-trap is observationally violated, $X_S > (x_1/S(x_1))^2/3$ at high significance, one or more of assumptions (i)–(iv) above must be false, independent of gravity theory or Friedmann equations. To apply this test, we also require an upper bound on the right-hand side, that is, an upper bound on $x_1$ for closed models, which we derive below [this is not strictly a fifth ‘assumption’, since it follows from observational data assuming (i), (ii) and (iv) given above].

In inequality (15), $X_S$ is formed from a ratio of two dimensionless BAO observables $d(z_1)$ and $y(z_2)$, while the right-hand side is close to 1 with a weak dependence on curvature: the effect of curvature on the low-redshift $D_V(z_1)$ is folded into the factor containing $S(x_1)$ on the right-hand side. As noted above, this is exactly 1 for flat models and is always $< 1$ for open models, so open models always
Figure 2. This figure shows both sides of inequality (15) as a function of redshift \( z \). The solid lines show the right-hand side of inequality (15), that is, the upper limit on \( X_S \) for non-accelerated models, assuming respective curvature radii \( R_C \) = −0.6, −1.0, \( \infty \), +1.0, +0.6 in units of \( c/H_0 \) (bottom to top). The dashed lines show the predicted values of \( X_S(z_1, z_2) \) for varying \( z_1 \) at fixed \( z_2 = z_{\text{acc}} \), for the same four models as in Fig. 1. The three dashed lines show flat \( \Lambda \)CDM models with \( \Omega_m = 0.24 \) (upper), 0.27 (thick), 0.31 (lower). The triple-dot–dashed line shows \( \omega_0 \)CDM with \( \Omega_m = 0.32 \), \( w = -0.85 \).

tighten the speed-trap. For closed (positively curved) models, the \( S_t \) factor is >1, which weakens the trap slightly; however, at low redshift \( z_1 \), this is a small effect as follows: from the discussion in Section 3.1, for closed models, we found a conservative lower limit \( R_C \geq 0.6c/H_0 \); this leads to \( x_1 \leq \ln(1 + z_1)/0.6 \); thus, for example, the right-hand side is \( \leq 1.013 \) for \( z_1 = 0.2 \). The top solid curve in Fig. 2 shows the resulting upper limit on the right-hand side of equation (15) assuming the very conservative limit \( R_C \geq 0.6c/H_0 \), while the next-to-top solid curve shows the limit assuming \( R_C \geq 1.0c/H_0 \).

Thus, if actual observations reveal that \( X_S \gtrsim 1.02 \) with good significance, the cosmic speed-trap ‘flashes’: if so, we can then rule out all homogeneous non-accelerated models regardless of the detailed expansion history or gravity model.

In the above equation (15), the square bracket term in \( X_S \) is given to first order by \( (1 + z_1)^{-1} \). Higher order terms are small, and a quadratic approximation is not an improvement; a slightly better approximation is \( (1 + 0.65z_1)^{-1} \) which is accurate to 0.2 per cent for \( z_1 \leq 0.3 \). Note that the right-hand side of equation (15) has no dependence on \( z_2 \); the curvature radius \( R_C \) has no effect on the observable \( y(z_2) \) since \( y \) is purely a line-of-sight measurement. Therefore, we may choose to measure \( y(z_2) \) anywhere, but if the real universe is accelerating, then the observed \( X_S \) will be maximal when \( z_2 \) is close to the past minimum of \( y(z_2) \), at \( z_2 \approx z_{\text{acc}} \).

3.3 Predictions for \( \Lambda \)CDM

In Fig. 2, we show predictions for \( X_S(z_1, z_2) \) as a function of \( z_1 \) for three \( \Lambda \)CDM models (dashed lines) and one \( \omega_0 \)CDM model (triple-dot–dashed line), from substituting equation (13) into equation (15) and evaluating \( H(z) \) and \( D_V(z) \) for the models. For each of these plotted curves, \( z_2 \) is set to \( z_{\text{acc}} \) for that model. The non-accelerating upper limit for \( X_S \) (the right-hand side of equation 15) is shown as the solid lines for several assumed values of curvature radius \( R_C \).

We see from Fig. 2 that if the real universe has followed an expansion history \( H(z) \) similar to \( \Lambda \)CDM prediction, inequality (15) will be violated if \( z_1 \) is reasonably small and \( z_2 \) is near \( z_{\text{acc}} \sim 0.75 \). Essentially, the accelerated expansion between \( z_2 \) and \( z_1 \) causes the value of \( H(z)/(1 + z) \) to be larger at \( z < z_1 \) than in the past at \( z_2 \), as shown in Fig. 1; this makes \( D_V(z_1) \) smaller and \( d(z) \) larger, compared to any non-accelerating model with the same \( H(z) \), so \( X_S \) violates the limit in equation (15).

As noted above, to maximize the violation, we should choose \( z_2 \) to minimize the observed value of \( y(z_2) \), that is, the redshift \( z_{\text{acc}} \) where \( H(z)/(1 + z) \) had its past minimum; for a \( \Lambda \)CDM model with \( \Omega_m = 0.27 \), the actual minimum is at \( z_{\text{acc}} \approx 0.75 \), but the theoretical \( y(z) \) is within 2 per cent of its minimum value over a rather broad window \( 0.5 \leq z \leq 1.1 \); so for an observational application of the test, \( z_2 \) may be whatever is most convenient observationally within this range, with only marginal weakening of the trap.

Turning to the variation in \( X_S \) with \( z_1 \), the predicted value of \( X_S \) is maximal at \( z_1 = 0 \) (with a value of 1.185 for our reference model C), and slowly declines with \( z_1 \); thus, lower \( z_1 \) is better both to maximize lever arm in our speed-trap and to minimize curvature uncertainty. However, for practical observations, \( z_1 \) cannot be too small since we need sufficient cosmic volume to get a robust detection of the acoustic feature in the galaxy correlation function \( \xi(r) \) or power spectrum \( P(k) \); therefore, there is a trade-off between \( X_S \) which declines with \( z_1 \); and the curvature uncertainty also favours smaller \( z_1 \), but the available cosmic volume for measuring \( d(z_1) \) grows with \( z_1 \). Thus, for an observational application of the speed-trap, there is an optimal window around 0.15 \( \leq z_1 \leq 0.35 \).

Taking the example values \( z_1 = 0.1, 0.2, 0.3 \), the concordance model predicts \( X_S(z_1, 0.75) = 1.145, 1.113, 1.088 \), respectively.
Table 1. Cosmological parameters for the four example models discussed in the text; model C is the baseline concordance model, while the others are selected to roughly span the current 2σ allowed range in \( \Omega_m \) and \( w \). All are flat and have \( H_0 \) adjusted to give very similar values of \( \xi_z \) consistent with WMAP, and therefore have similar values of \( t_0 \).

<table>
<thead>
<tr>
<th>Model</th>
<th>( \Omega_m )</th>
<th>( H_0 ) (km s(^{-1}) Mpc(^{-1}))</th>
<th>( w )</th>
<th>( t_0 ) (Gyr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.27</td>
<td>70.0</td>
<td>−1</td>
<td>13.86</td>
</tr>
<tr>
<td>L</td>
<td>0.24</td>
<td>72.5</td>
<td>−1</td>
<td>13.82</td>
</tr>
<tr>
<td>H</td>
<td>0.31</td>
<td>67.1</td>
<td>−1</td>
<td>13.91</td>
</tr>
<tr>
<td>W</td>
<td>0.32</td>
<td>64.6</td>
<td>−0.8</td>
<td>13.98</td>
</tr>
</tbody>
</table>

also note that the value of \( \xi_z \) is fairly sensitive to the value of \( \Omega_m \); taking example cases from Table 1 with \( \Omega_m = 0.24, 0.27, 0.31 \) to bracket the plausible range, we find that \( \xi_z(0, z_{acc}) = 1.225, 1.185, 1.143 \), while \( \xi_z(0.2, z_{acc}) = 1.142, 1.113, 1.081 \), respectively. For each model, \( \Delta z \approx 1 \) approximately halves from \( z_1 = 0 \) to \( z_1 \approx 0.27 \). This is because the rate of acceleration grows with time after \( z_{acc} \), so \( \Delta z \) has stronger than linear dependence on \( \Delta z \).

We note here that the prediction for \( \xi_z \) is independent of \( H_0 \) if all \( \Omega_m, \Omega_\Omega, \Omega_k \) and \( w \) are held fixed. However, since our example models are approximately CMB-matched, a correlation appears, because raising \( \Omega_m \) and/or \( w \) compared to the concordance model requires lowering \( H_0 \) to remain consistent with the CMB; raising \( \Omega_m \) or \( w \) also leads to weaker acceleration and thus lowers \( \Delta z \). Thus, \( \Delta z \) at a fixed redshift is positively correlated with \( H_0 \) in CMB-matched Friedmann models.

We also note that for accelerating models \( \Delta z \) remains a few per cent greater than 1 for the case \( z_1 = z_1 \); this occurs because \( \chi(z_1) \) measures the instantaneous expansion rate at \( z_1 \), while \( d(z_1) \) depends on the average expansion rate at redshifts below \( z_1 \), which is larger. In principle, we could use this to test acceleration by measuring \( d \) and \( y \) from a single survey at \( z_1 = z_1 \), but in practice the curvature uncertainty probably disfavours this (see Section 4.5 for more discussion).

### 4 DISCUSSION

In this section, we discuss various aspects of the test above, including possible shifts in length \( r_s \), useful approximations for \( D_V(z) \), observational issues, the relation to the Alcock–Paczynski ratio and the effect of giant-void models.

#### 4.1 Possible shifts in \( r_s \)

In applying the speed-trap, clearly assumptions (i) and (ii) above are very basic; if future observations show the speed-trap is observationally violated, we need to be confident that assumption (iii) on constancy of \( r_s \) is valid to around \( \sim 2 \) per cent, in order to reject general homogeneous non-accelerating models with high confidence.

We now consider some details which may actually give rise to a significant shift in comoving \( r_s \) between redshifts \( z_1 \) and \( z_2 \); the main such effects are galaxy bias, non-linear growth of structure, redshift-space distortions (Kaiser 1987; Hamilton 1992), and possible effects due to the hump(s) sitting on a sloping background power spectrum, etc.

We first note that there is non-negligible evolution in \( r_s \) at high redshift between \( z_3 \approx 1020 \) and \( z \approx 10 \), as shown in fig. 1 of Eisenstein et al. (2007b); the initial BAO bump is only in the baryons and photons, and the peak shifts slightly as the dark matter and baryon perturbations align together at later times; this implies the late-time BAO peak is not exactly at the sound horizon length \( r_s(z_0) \).

However, after \( z < 10 \) the density perturbations in baryons and dark matter are very similar. In most real BAO analyses, a matter power spectrum from CMBFAST or similar is used, together with a model for non-linear evolution and an arbitrary linear ‘stretch factor’ \( \alpha \), to fit observations; finally, the measurement is quoted as \( d(z) = \alpha r_s(z)dV(z) \), where \( r_s \) and \( D_V \) are both computed from the reference theoretical model. This implies that small errors in the reference model should (on average) be absorbed into an opposite shift in \( \alpha \), so the final estimate of \( d(z) \) should be unbiased. Any shift in the BAO length from \( z \sim z_1 \) to \( z \sim 10 \) is included in the reference model; therefore, \( r_s(z_0) \) forms essentially a convenient fiducial length for intercomparison between models, which is close to but not exactly the position of the low-redshift BAO peak in the correlation function. For this work, we are only interested in shifts of the BAO scale at \( z < z_1 \approx 0.75 \), so the above effect cancels.

Galaxy bias, at least in standard versions, has little effect since the BAO scale is very much larger than any scale of relevance for galaxy formation; thus, bias may affect the overall amplitude of galaxy clustering but cannot significantly shift the scale \( r_s \). Likewise, non-linear growth of structure primarily moves galaxies around on \( \sim 5 h^{-1} \) Mpc scales; this significantly blurs the bump in \( \xi(r) \), and/or erases the higher harmonics in the power spectrum, but this is almost symmetrical between inward and outward shifts: the systematic shift in the BAO scale-length is much smaller.

For the standard model, these effects have been investigated from both theory, for example, by Eisenstein et al. (2007b) and Shoji et al. (2009), and large N-body simulations, for example, by Seo et al. (2008) and Seo et al. (2010); these papers agree that systematic shifts are small, typically below the 0.6 per cent level at \( z = 0.3 \) and less at higher redshift. They also find that reconstruction methods based on velocity-field reconstruction (Eisenstein et al. 2007a) can reduce the shift to \(-0.1 \) per cent. This will become important for the next generation of ambitious planned surveys such as ESA’s Euclid (e.g. Samushia et al. 2011) or NASA’s WFIRST, which aim to achieve sub-percent precision on BAO observables in many redshift bins, but are almost negligible with respect to the speed-trap test in this paper.

We caution that there is a slight level of circular argument in the above, in that we are assuming standard cosmology to limit the shift in \( r_s \), and then using this to reject non-standard non-accelerating models; it remains possible that a model with non-standard gravity could produce a much larger shift in \( r_s \) than the standard cosmology. However, non-standard models producing a gross \( \sim 10 \) per cent shift in \( r_s \), since \( z_2 \approx 0.75 \) would almost certainly produce large levels of redshift-space distortion, and give strong inconsistencies between the angular and radial measurements of \( r_s \) at low redshift. If both the redshift-space distortion pattern and the angular measurements of \( r_s \) are measured to be consistent with standard ΛCDM, this would strongly suggest that the true shifts in \( r_s \) should not be much larger than the per cent level effects predicted by the standard model.

#### 4.2 Approximations for \( D_V \)

As an aside, we also note that in nearly-flat CDM-like models, an accurate approximation to \( D_V(z) \) at moderate redshift is given by Taylor-expanding \( 1/H(z) \) around \( z/2 \) (rather than zero), and substituting into integral (4); this makes \( z \) terms vanish, and leads to the...
approximation

\[
D_\nu(z) \approx \frac{cz}{[H(z)/H(\bar{z})]^{3/2}} + O(z^3); \tag{16}
\]

in practice, the first term is surprisingly accurate for ΛCDM models, with errors < 0.1 per cent compared to the numerical result for \( z < 0.5 \). (See Appendix A for evaluation of the third-order term, and an explanation why it is small.)

A simpler approximation is

\[
D_\nu(z) \approx \frac{cz}{[H(2z/3)]^{3/2}}; \tag{17}
\]

this is slightly less accurate than the previous approximation, but still accurate to <0.4 per cent for \( z < 0.5 \), better than the mid-term precision on observables. (For open-zero-Λ models, these approximations are less good, with errors up to 2 per cent.)

While it is straightforward to evaluate \( D_\nu \) and \( X_s \) numerically for any given model, the main value of this approximation is that it tells us that a measurement of \( \zeta d(z) \) at low redshift is quite close to a measurement of \( r_c H(2z/3)c; \) substituting this into equation (15), along with the approximation \((1 + \frac{2}{3}z_1)^{-1}\) for the square bracket term, gives simply

\[
X_s(z_1, z) \approx \frac{(1 + z_2) H(\frac{2}{3}z)}{H(z)} \frac{a(\frac{2}{3}z)}{a(z)} = \frac{a(\frac{2}{3}z)}{a(z)},
\]

and inequality (3) tells us this should be less than 1 for non-accelerated models. Unlike our upper limit (15), this expression is not rigorous, but this gives a simple and fairly accurate approximation for what \( X_s \) is measuring, that is, it is closely related to the ratio of expansion rates \( a \) at \( \frac{2}{3}z_1 \) compared to \( z_2 \).

### 4.3 Observational advantages

One possible objection to this test is that it is comparing two related but slightly different observables, that is, a spherical average scale at \( z_1 \) with a radial scale at \( z_2 \). Why have we done this, rather than comparing two measures of \( d(z) \) or two measures of \( y(z) \) at two different redshifts?

It is well known that comparing \( y(z) \) at two different redshifts provides another direct test of acceleration. The main difficulty is observational, since for our baseline model, \( y(z) \) only grows to \( 1.1y(z_{acc}) \) at rather low redshift \( z \approx 0.16 \). Furthermore, for a given survey, a radial-only measurement of \( r_i \) has a statistical error roughly \( \sqrt{3} \) worse than a spherical average measure. Even if we had a 3σ weak lensing survey at \( z \approx 0.16 \), we may not do much better than 3 per cent statistical error on \( y \), a 3σ violation, and we would like to get above 5σ for a decisive result. Using \( d(z) \) instead of \( y \) gives two substantial advantages: first, \( d(z) \) effectively measures \( H \) at \( \sim 2z_1/3 \), giving more lever arm on the low-redshift acceleration; second, a measurement of \( d(z = 0.24) \) is similar in content to a measurement of \( y(z_1 = 0.16) \). Secondly, there is the obvious gain that \( d \) uses three spatial directions instead of one. Thus, for a fixed thickness of survey shell, the former measure has around 9/4 times more available volume and three independent axes, so the cosmic variance limit should improve by a factor of \( \sqrt{27/4} \approx 2.6 \), which is a very important practical advantage.

In contrast, comparing \( d(z) \) at two different redshifts suffers from potential major uncertainty in cosmic curvature at the high redshift \( z_2 \). At \( z_2 \approx 0.75 \), there is ample available volume for a precision measurement of \( y \), and ambitious future probes such as \( Euclid \) (Samushia et al. 2011) plan to push to statistical errors \( \leq 0.75 \) per cent on both \( y \) and \( d \), in each of many bins of width 0.1 in redshift. Thus, at \( z_2 \) the cosmic variance is minimal for a wide-area survey, so the radial measure is preferable because it is independent of the curvature nuisance parameter. Also, \( d(z_2) \) depends on the full history of \( H(z) \) back to \( z_2 \), which complicates the issue of deriving an inequality.

In our proposed comparison, we have constructed a ratio \( X_s \) using \( d(z_1) \) at low redshift and \( y(z_2) \) at the higher redshift, to circumvent both of these problems: the potential cosmic-variance limits are probably around 1 per cent on \( d(0.24) \) and significantly less on \( y(0.75) \), so this test can (given ample data) deliver a stand-alone rejection of homogeneous non-accelerating models at \( \sim 7σ \) significance level. This can be further improved by using several independent redshift bins, for example, \( z_1 = 0.15, 0.25 \) and \( z_2 = 0.65, 0.75 \).

### 4.4 Future observations

As noted above, there already exist measurements of the numerator on the left-hand side of equation (14) from Percival et al. (2010); they quote values of \( d(0.2) = 0.1905 \) and \( d(0.35) = 0.1097 \), with approximately 3.3 per cent error on each. For the numerator \( z d(z) \) in inequality (15), these give 0.2 \( d(0.2) = 0.0381 \) and 0.35 \( d(0.35) = 0.0384 \).

As yet there is no available measurement of radial BAOs at \( z > 0.5 \) with which to actually calibrate our speed-trap, but these are expected soon from the recently completed AAT WiggleZ survey (Blake et al. 2010), and in a few years from the ongoing BOSS (White et al. 2011). It is currently unclear whether the final WiggleZ survey covers enough volume to separately measure the radial component as required here, but BOSS should very likely achieve this; the upper redshift limit of BOSS is \( z \approx 0.65 \), so this is close enough to \( z_{acc} \) to be useful.

For ΛCDM, the predicted value of \( y(z) \) near its minimum is approximately 0.0302 for \( \Omega_m = 0.27 \) and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \). For reasonable variations of parameters, we now show that if we assume a flat universe then \( y(z) \) is well constrained by CMB observations: it is well known that for flat models with varying \( \Omega_m, h \), there is a tight correlation between the age of the Universe, \( t_0 \), and the CMB acoustic scale \( \ell_A \) (Knox, Christensen & Skordis 2001), and it turns out that there is also a tight correlation between these and the value of \( H \) at intermediate redshift, with a pivot point occurring at \( z \approx 0.8 \) (see Fig. 1). This is partly a coincidence, because for moderate parameter variations around the concordance model, \( t_0 \) scales \( \propto \Omega_m^{-0.3} h^{-1} \), while \( \ell_A \) scales as \( \Omega_m^{0.15} h^{-0.5} \). For the value of \( H(z) \), we note that as \( z \to 0 \) these scales as \( \Omega_m^{0.5} h^{-1} \). Therefore, there exists a pivot point at intermediate redshift where \( H(z) \) scales as \( \Omega_m^{0.5} h \) (i.e. inversely to \( t_0 \)), and this pivot redshift turns out to be \( z \approx 0.85 \) for Λ models. For \( w > -1 \), the pivot redshift is somewhat lower, but for near-flat Friedmann models the value \( H(z = 0.75) \) is better constrained by WMAP data than the local \( H_0 \), and fixing \( t_0 \approx 13.75 \text{ Gyr} \) constrains \( H(z = 0.75)/(1.75) = 59.2 \text{ km s}^{-1} \text{ Mpc}^{-1} \approx 0.8 \text{ per cent} \), which in turn leads to a tight prediction for \( y(0.75) \).

\(^1\) Soon after the submission of the first version of this paper, three new measurements of the BAO feature appeared: one from 6dFGS at \( z < 0.1 \) in Beutler et al. (2011), one from WiggleZ at \( z = 0.6 \) in Blake et al. (2011), and one from SDSS at \( z = 0.55 \) in Carnero et al. (2011). All of these show good consistency with the concordance model, but do not yet measure the radial component as required here.
[As an aside, there is a corollary that if some future method could give a direct measurement of $H(z = 0.75)$ independently of $r_z$, then this would produce another strong consistency test of standard ΛCDM. This may be possible in principle using methods such as differential-age measurements of early-type galaxies, or lensing measurements with source and lens close in redshift, but this will require a major advance in precision over current data.]

Assuming some future $y$ measurement turns out at the concordance value $y(z_2) \approx 0.0302$, we would then obtain measurements $X_S \approx 1.12, 1.05$ at $z_1 = 0.2, 0.35$, respectively. The error on $y(z_2)$ must be added in quadrature to the current 3.3 per cent error on $d(z_1)$, but if the former is around 2 per cent, then we can anticipate a fairly clear violation from the $z = 0.2$ value, and a somewhat less significant violation at $z = 0.35$.

The prospects are good for improving on the current results: the projections for BOSS (White et al. 2011) are for 1 per cent precision on $d(z = 0.35)$, and precision of 1.7 per cent on $y(0.6)$. Adding the above errors in quadrature leads to around 2 per cent precision on $X_S$, with a predicted value $\approx 1.077$, thus nearly a $4\sigma$ proof of acceleration. BOSS may also do better using the larger value of $X_S$ at $z_1 \sim 0.2$, but projected precision on $d(0.2)$ is not quoted separately.

Next-generation surveys in the planning stage such as BigBOSS, Euclid or WFIRST should substantially improve on the higher redshift measurement, reaching sub-percent precision on $y(z_2)$. The low-redshift $d(z_1)$ measurement is ultimately limited by cosmic variance, but extending BOSS to the Southern hemisphere can give a straightforward improvement by a factor of $\sqrt{2}$, or probably more if denser sampling of galaxies is used. Further improvements are possible in principle using $H$- or near-infrared-selected surveys which can cover $>80$ per cent of the whole sky, compared to $\sim 50$ per cent for visible-selected surveys.

### 4.5 Comparison with the Alcock–Paczynski test

We note here that our ratio $X_S$ may be considered as a generalized version of the classic test of Alcock & Paczynski (1979), hereafter AP: the AP ratio was defined to be $R_{AP} = \Delta z / \Delta \theta$, which in our notation becomes

$$ R_{AP}(z) = \frac{(1 + z)D_A(z)H(z)}{cz} \ . $$ (19)

If we choose $z_1 = z_2$ in equation (13) above and substitute equation (1) for $D_V$, we then obtain

$$ \frac{z_1 d(z_1)}{y(z_1)} = (1 + z_1)R_{AP}(z_1)^{-2/3} \ , $$ (20)

thus, $X_S(z_1, z_2)$ contains the same information as $R_{AP}(z_2)$ combined with a function of $z_1$; substituting the above into equation (15) gives a lower limit on $R_{AP}$ for non-accelerating models, which is

$$ R_{AP}(z_1) \geq \frac{(1 + z_1)\ln(1 + z_1) S_1(z_1)}{z_1} \ . $$ (21)

It is well known that if we assume the Friedmann equations, the AP test at high redshift provides a strong test for $\Lambda$ or dark energy: however, if we drop the Friedmann connection between curvature and matter content, then at $z \geq 0.5$ the AP test becomes mostly degenerate between acceleration and curvature. At lower $z < 0.4$, we may use the approximation $(1 + z)D_A(z) \approx czH(z/2)$ from above, which leads to $R_{AP}(z_1) \approx H(z_1)/H(\frac{z_1}{2})$. This does have more sensitivity to acceleration than curvature, but is not ideal for the following reason: at small $z_1$, the AP ratio suffers from a short redshift lever arm, while at $z_1 \geq 0.4$ the ratio mainly probes the regime of sluggish acceleration at $z > 0.2$. The AP ratio at $z_1 \approx 0.4$ may provide a useful test, but will probably require sub-percent-level precision on both observables to get a decisive result.

Compared to the AP test, the use of two widely spaced redshifts in $X_S$ requires the added assumption that $r_z$ has minimal evolution between $z_2$ and $z_1$, but enables a much longer effective time lever arm, giving a larger acceleration signal while keeping the curvature sensitivity very small.

### 4.6 Inhomogeneous void models

Recently there has been some interest in models which produce apparent acceleration without dark energy, by placing us near the centre of a giant underdense spherical void, with a Lemaitre– Tolman–Bondi metric; examples are in Tomita (2009), and references therein. These models have several problems such as severe fine-tuning of our location very close to the void centre, and probable inconsistency with limits on the kinetic Sunyaev–Zel’dovich effect (Zhang & Stebbins 2011); however, it is interesting to note how $X_S$ behaves in such models. A recent confrontation of giant-void models with BAO observables has been done by Moss, Zhibin & Scott (2011): they find that void models with profiles adjusted to match SN and CMB observations have a $\Delta z_1$, which is $\geq 30$ per cent smaller at $z \sim 0.5–0.7$ compared to ΛCDM. Those specific cases would have $X_S(0.2, 0.75) \geq 1.4$, which is substantially larger than any reasonable dark energy model; thus, Moss et al. (2011) show that giant-void models matched to angular distances and the CMB appear to suffer from severe ‘overkill’ in radial BAO measurements.

The parameter space of possible void models is very large, so other void models may look more similar to ΛCDM, but we note that the test of Clarkson, Bassett & Lu (2008) can be used to test for homogeneity without assuming GR. They show that if we have both angular and radial BAO measurements spanning a range of redshift, then there is a consistency relation which must be satisfied by homogeneous models but is usually violated by giant-void models. Thus, assumption (i) above becomes observationally testable using future BAO observations, though this probably requires observations spanning more redshifts than the $X_S$ test here.

### 5 Conclusions

We have proposed a new and simple smoking-gun test for cosmic acceleration using only a comparison of the BAO feature at two distinct redshifts $z \sim 0.2$ and $z \sim 0.75$. The main result of our paper is inequality (15) relating the two dimensionless BAO observables, which must be satisfied for any homogeneous non-accelerating model, but will be observationally violated by $\approx 10$ per cent in models with an expansion history close to standard ΛCDM.

Clearly, our proposed measurement has advantages and disadvantages: the main advantages are extreme simplicity and model independence, that is, if inequality (15) is violated, we can rule out essentially all homogeneous non-accelerating models in one shot, without assuming any particular gravity theory or parametric form of $H(z)$, and independent of SN and CMB observations.

The main drawback of our test is that it is essentially one-sided: if inequality (15) is observationally violated, we have proved (given some basic assumptions) that acceleration has occurred during $0 \leq z \leq z_2$ and have a rough quantification of the amount, but no more details about the underlying cause or the details of the expansion history.
If we assume GR and the Friedmann equations hold, and that $r_c$ has the value which is accurately predicted from CMB analysis, then we have much more statistical power: future measurements of BAOs in many redshift bins may be used to reconstruct the detailed form of the expansion history $H(z)$; this can be integrated to give predictions of $D_L(z)$, and comparison with the measured transverse BAO scale gives $D_L(z)$ can constrain spatial curvature independent of the CMB, while comparison of $D_L$ with $D_V(z)$ from SNe can check the distance duality or Tolman relation ($D_V/D_L = (1+z)^3$). All of this can give much more powerful cross-checks and parameter estimates than our simplified one-sided test.

However, our proposed cosmic speed-trap seems to provide a valuable addition to the set of cosmological measurements, due to its bare minimum of assumptions. This provides a strong motivation for future improved BAO measurements specifically near redshifts $z \sim 0.25$ and $z \sim 0.75$; this should preferably include a low-redshift survey comparable or superior to BOSS in the Southern hemisphere to minimize the cosmic variance in the local measurement.

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APPENDIX A: THE APPROXIMATION FOR $D_V$

We here add a note which explains the surprisingly good accuracy of approximation (16) for $D_V(z)$ at fairly low redshift $z < 0.4$. As noted, in integral (4) for $D_V$, it is helpful to Taylor-expand the function $1/H(z)$ around the mid-point of the integral at $z_i/2$, then integrate; this naturally makes terms with odd-integer derivatives of $1/H$ integrate to zero, and leads to

$$D_V(z_i) \approx c \left[ \frac{z_i}{H(z_i)} + \frac{z_i^2}{24} \left( \frac{1}{H} \right)'' \left( \frac{z_i}{2} \right) + O \left( \frac{z_i^4}{24} \right) \right],$$

where the prime denotes $d/dz$. We now need the second derivative $(1/H')''$ evaluated at $z_i/2$. Defining the usual deceleration parameter $q$ and the jerk parameter $j$ (e.g. Alam et al. 2003) as

$$q \equiv -\frac{a^2 \dot{a}}{aH^2}, \quad j \equiv \frac{\ddot{a}}{aH^3}.$$

we can rearrange these in terms of $d/dz$ to get

$$\frac{dH}{dz} = \frac{H}{1+z} \left( 1 + q \right), \quad \frac{d^2 H}{dz^2} = \frac{H}{(1+z)^2} (j - q^2).$$

Using these we obtain

$$\frac{d^2}{dz^2} \left( \frac{1}{H} \right) = -j + 2 + 4q + 3q^2 \left( \frac{1}{1+z^2} \right).$$

For the case of flat $\Lambda$CDM models, $j = 1$ independent of parameters (assuming radiation density is negligible; Rapetti et al. 2007); thus, the numerator in equation (A4) has zeros at $q = -1$ and $q = -1$. For $\Omega_m$ near the concordance model, $q$ passed through $-1$ in the fairly recent past at $z \sim 0.3$, so the numerator is significantly smaller than unity at low redshift. This explains qualitatively the very good accuracy of approximation (16) near the concordance model, even up to significant redshifts $z \approx 0.5$.

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