A new integral representation for reconstructing the density distribution of matter in the discs of spiral galaxies using the rotation velocity curve in it

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ABSTRACT

In this paper, we propose a new integral representation for reconstructing the surface density of matter in the flat discs of spiral galaxies. The surface density is expressed through the observed rotation velocity curves of visible matter in the discs of spiral galaxies. The new integral representation is not based on the quadrature of special functions. The solution that is found is used to process and analyse observational data from several spiral galaxies. The new integral representation can be used to more accurately estimate the amount of dark matter in spiral galaxies.

Key words: methods: analytical – methods: numerical – Galaxy: disc – dark matter.

1 INTRODUCTION

Let us consider a model of a spiral galaxy with all matter distributed in a thin disc (the thick disc model is neglected). In Section 6, we take into account the components of a spherical galactic halo.

Let us consider the axisymmetric model in cylindrical coordinates (\(\rho, z, \varphi\)).

In the disc, we assume that the dust-like matter has stationary rotation (i.e. there is equality between the absolute values of the gravitational and centrifugal forces):

\[
\frac{\partial \Phi(\rho, z = 0)}{\partial \rho} = \frac{V^2(\rho)}{\rho},
\]

(1)

Here, \(V(\rho)\) is the rotation velocity curve of the matter and \(\Phi(\rho, z)\) is the Newtonian gravitational potential of the dust matter, which can be expressed as

\[
\Phi(\rho, z) = \int_0^\infty R \, dR \int_{-\pi}^{\pi} d\varphi \frac{-G \sigma(R)}{\sqrt{\rho^2 + R^2 - 2\rho R \cos \varphi + z^2}}.
\]

(2)

Here, \(\sigma(\rho)\) is the surface density distribution of matter in the disc.

The function \(\sigma(R)\) should be restricted to ensure the convergence of integral (2). It is necessary that all forces tend to zero at infinity, which corresponds to both sides of equation (1) converging to zero. We assume that this condition is satisfied. Our calculation has confirmed the validity of this assumption.

We pose the following problem. Express the distribution of the matter surface density, \(\sigma(\rho)\), in a disc from the rotation velocity curve of the matter \(V(\rho)\) in that disc.

The potential \(\Phi(\rho, z)\) is a function of \(\sigma(\rho)\)

\[
\Phi(\rho, z)_{|z=0} = 2\pi G \sigma(\rho),
\]

(3)

where \(G\) is the gravitational constant.

Thus, we can obtain an integral equation expressing the velocity V by the integral of the surface density \(\sigma\). This equation can be inverted (see Toomre 1963) to express the surface density \(\sigma\) through the rotation velocity curve of the disc:

\[
\sigma(\rho) = \int_0^\infty dk \, J_0(k\rho) \tilde{\sigma}(k);
\]

(4)

\[
\tilde{\sigma}(k) = \frac{k}{2\pi G} \int_0^\infty dR \, J_1(kR)V^2(R).
\]

(5)

Here, \(J_0(x)\) and \(J_1(x)\) are Bessel functions.

The expression (5) for the function \(V(R)\) should be restricted in order for the integral (4) to converge. We consider that \(V(R)\) acts as a constant at infinity (see below), but even in that case the integral (4) converges. This (convergence) is especially true for cases where \(V(R)\) is assumed to be an asymptotically decreasing function (see...
We denote the gradient of the square of the velocity as a function \( f(R) \):

\[
f(R) = \frac{f^2(R)}{R}.
\]

Substituting the integral definition of Bessel functions of zero order,

\[
J_0(x) = \frac{1}{\pi} \int_0^\pi e^{i x \cos \alpha} d\alpha,
\]

into equation (8) gives

\[
2\pi G \frac{\sigma}{R} = \int_0^\pi d\alpha \int_0^\pi d\beta \int_{-\infty}^{\infty} dk \int_0^\infty dR \frac{\delta(R \cos \alpha + R \cos \beta)}{2\pi} \frac{f(R)}{\pi}.
\]

Here, we have taken into account the fact that \( J_0(x) \) is an even function. Thus, the integration over \( k \) in equation (8) can be done by changing the limits of integration to go from \(-\infty \) to \( \infty \) and by multiplying the result by a factor of \( 1/2 \).

Let us now consider the integral representation of the Dirac delta function,

\[
\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i k x} dk,
\]

and substitute it into equation (11):

\[
2\pi G \frac{\sigma}{R} = \int_0^\pi d\alpha \int_0^\pi d\beta \int_{-\infty}^{\infty} dR \delta(R \cos \alpha + R \cos \beta) \frac{f(R)}{\pi}.
\]

Here, we take into account the integration rules of the complicated delta function,

\[
\delta[g(R)] = \frac{\delta(R - R_0)}{|g(R_0)|},
\]

where \( R_0 \) are the simple roots of the equation \( g(R_0) = 0 \). Also, we should find that \( R_0 \geq 0 \), which means that the integration region with respect to \( \alpha \) and \( \beta \) reduces to the following two sectors:

1. \[ \alpha \in (0; \pi/2); \quad \beta \in (\pi/2; \pi) \]
2. \[ \alpha (\pi/2; \pi); \quad \beta \in (0; \pi/2) \]

By renaming the variables \( \varphi \) and \( \beta \) in these sectors, we can rewrite the integral (12) to the following form:

\[
2\pi G \frac{\sigma}{R} = \frac{2}{\pi} \int_{0}^{\pi/2} d\varphi \int_{0}^{\pi/2} d\beta \frac{f(R_{\varphi})}{\sin \beta} \frac{\cos \alpha}{\sin \beta}.
\]

We assume that for \( \beta \to 0 \) in equation (14), the function \( f \) tends to zero fast enough.

This expression (14) is the desired result, which is without the deficiencies mentioned in Section 1.

Using

\[
V^2(R_{\varphi}, \rho) = V^2(R_{\varphi}, R_0, R_0, \rho) = \frac{f(R_{\varphi})}{\sin \beta} \frac{\cos \alpha}{\sin \beta},
\]

we can rewrite equation (14) in the following form:

\[
2\pi G \frac{\sigma}{R} = \frac{dU(\rho)}{d\rho}.
\]
Using this expression, it is easy to obtain an expression for the total mass of matter inside the disc radius $\rho$ by integrating equation (15) by parts:

$$GM(\rho) = \int_0^\rho 2\pi G\rho\sigma(\rho)\,d\rho = U(\rho) - \int_0^\rho U(\rho)\,d\rho.$$  \tag{16}$$

3 CHECKING THE METHOD

For basic testing of our new integral representation, we use the following approach. We integrate the expression (1) with respect to $\rho$:

$$H_1(\rho) = \int_0^\rho \frac{V^2(\rho)}{\rho} \,d\rho.$$ \tag{17}

However, an analogous expression can be obtained from equation (2) as

$$H_2(\rho) = U(\infty) - \frac{2}{\pi} \int_0^\infty dR Z_\mu G\sigma(\rho) \int_0^{\pi/2} \frac{\,d\gamma}{\sqrt{(R - \rho)^2 + 4R^2 \sin^2 \gamma}},$$

$$\gamma = \varphi/2$$ \tag{18}

or as

$$H_2(\rho) = \frac{2}{\pi} \int_0^\infty dR Z_\mu G\sigma(\rho) \int_0^{\pi/2} \frac{\,d\gamma}{\sqrt{(R - \rho)^2 + 4R^2 \sin^2 \gamma}} \left[ 1 - \frac{R}{\sqrt{(R - \rho)^2 + 4R^2 \sin^2 \gamma}} \right].$$ \tag{19}

The integrals (18) and (19) do not have non-integrable singularities and therefore they can easily be found numerically.

The expressions for the functions $H_1(\rho)$ and $H_2(\rho)$ ought to be identical, as they must be equal to the expression $\Phi(\rho, z = 0) - \Phi(0, z = 0)$. Thus, we can judge the practical precision of our new integral representation by evaluating and comparing the expressions for $H_1(\rho)$ and $H_2(\rho)$ [using our new method to calculate $\sigma(R)$].

To evaluate and compare these expressions, and thus to put our new method to use, we have written a simple numerical code, the details of which are described in Appendix A. Fig. 1 shows an example of such a comparison, where we have used the sample function,

$$V(x) = \frac{10x}{(1 + 100x^2)^{3/2}},$$ \tag{20}

as the basis for our velocity data.

This function satisfies the constraints required for convergence of the integral (5), as discussed in Section 1.

In the figure, this velocity function and $\sigma(x)$ are calculated by using our new integral representation. The two functions $H_1(x)$ and $H_2(x)$, which we wish to compare, are also plotted.

As can be seen from Fig. 1 (and as is confirmed by detailed inspection of the figure), the two functions $H_1(x)$ and $H_2(x)$ agree very closely. This is also the case for other sample velocity functions that we have tested. Thus, we conclude that our new integral representation is both valid and accurate, even when performing the integrations numerically.

4 SPECIAL CASES AND ASYMPTOTIC DISTRIBUTION OF THE DENSITY OF MATTER

In some special cases, expression (14) can be reduced to an ordinary quadrature and we can find the asymptotic behaviour of $\sigma(\rho)$.

(i) Consider the simplest case for the distribution function $f(R)$ and the rate $V(R)$:

$$f(R) = \begin{cases} f_0 & \text{for } R \in [0; R_1] \\ 0 & \text{for } R \in (R_1; \infty). \end{cases}$$

$$V(R) = \begin{cases} \sqrt{f_0 R} & \text{for } R \in [0; R_1] \\ \sqrt{f_0 R_1} & \text{for } R \in (R_1; \infty). \end{cases}$$ \tag{21}

This function satisfies the constraints required for convergence of the integral (5), as discussed in Section 1. The integral over $\beta$ in expression (14) can easily be evaluated and we can obtain the quadrature as

$$2\pi G\sigma(\rho) = \frac{2 f_0}{\pi} \int_{\alpha_1}^{\pi/2} \ln \left[ 1 + \frac{1}{\rho \cos \alpha / R_1 - \rho} \right] \,d\alpha,$$

$$\alpha_1 = \begin{cases} \arccos(R_1/\rho) & \text{for } \rho \leq R_1 \\ 0 & \text{for } \rho > R_1 \end{cases}$$ \tag{22}

or, in another form,

$$2\pi G\sigma(\rho) = \frac{2 R_1 f_0}{\pi \rho} \int_0^{x_1} \ln \left[ \frac{1 + \sqrt{1 - x^2}/x}{\sqrt{1 - (\rho R_1/\rho)^2}} \right] \,dx,$$

$$x_1 = \begin{cases} \rho / R_1 & \text{for } \rho \leq R_1 \\ 1 & \text{for } \rho > R_1 \end{cases}.$$ \tag{23}

From expression (22), we find that the asymptotic behaviour at $\rho R_1 \to 0$ is $\sigma(\rho) \propto \ln (R_1/\rho)$.

From expression (23), we find that the asymptotic behaviour at $\rho R_1 \to \infty$ is $\sigma(\rho) \propto (R_1/\rho)$. This is connected with the non-rigid rotation of matter around the central axis; for the case of rigid rotation, we should have $V(R) \propto R$ at $R \to 0$ (see equation 21).
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(ii) We now consider another simple case, as follows:

\[
f(R) = \begin{cases} \frac{f_0 R}{R_1} & \text{for } R \in [0; R_1] \\ 0 & \text{for } R \in (R_1; \infty) \end{cases},
\]

\[
V(R) = \begin{cases} \sqrt{f_0/(2R_1)} R & \text{for } R \in [0; R_1] \\ \sqrt{(f_0/R_1)^2} & \text{for } R \in (R_1; \infty) \end{cases}.
\]

This function satisfies the constraints required for convergence of the integral (5), as discussed in Section 1. In this case, the analogue of the integral (22) has the form:

\[
2\pi G \sigma(\rho) = \frac{2f_0}{\pi} \int_{a_2}^{\pi/2} \sqrt{1 - (\rho \cos \alpha/R_1)^2} \, d\alpha,
\]

\[
a_2 = \begin{cases} 0 & \text{for } \rho \leq R_1 \\ \arccos(R_1/\rho) & \text{for } \rho > R_1 \end{cases}.
\]

This integral is expressed through the elliptic integral of the second type, where the asymptotic behaviour at \(\rho R_1 \rightarrow 0\) is non-singular and we have \(\sigma(\rho) \rightarrow \text{const}\). At infinity, the asymptotic behaviour remains the same; that is, at \(\rho R_1 \rightarrow \infty\), we have \(\sigma(\rho) \propto (R_1/\rho)\).

5 OBSERVATIONAL DATA

By using the method described above, we have restored the matter surface density profiles for the discs of four spiral galaxies, based on their observed rotation curves. The four galaxies are NGC 2841, 7217, 7331 and 5533. The general characteristics of the galaxies are listed in Table 1 and the restored surface density distributions are presented in Fig. 2.

To compute the behaviour of the curve \(V(R)\) at large \(R > R_{\text{max}}\) (where the velocity distribution is unknown), we use the following two functions:

(i) \(V = V(R_{\text{max}}) = \text{const};\)

(ii) \(V = V(R_{\text{max}}) \sqrt{R_{\text{max}}/R}.\)

The second case corresponds to a Keplerian tail in the velocity distribution of matter, and the quantities corresponding to this case are marked in Fig. 2 by a prime.

The galaxies considered are all giant spiral galaxies of Sab–Sb type, and all of them are classified as unbarred. This is important because the axisymmetry of the galaxies signifies regular circular rotation of the gas, making it possible to measure their rotation curves from a single spectral observation per galaxy if the spectrograph long slit is aligned with the disc isophote major axis. We account for the components of the spherical halo in Section 6. The rotation curves have been obtained by measuring the line-of-sight velocities of the gas. Because the gas is a cold dynamical subsystem and it is confined to thin discs, its rotation traces the gravitational potential distribution very well. For the inner parts of the rotation curves, we have used spectral observations of the ionized (warm) gas. These observations have a good spatial resolution (typically about 1 arcsec, which corresponds to a few dozen parsec for three of the four galaxies). For the outer parts, we have used radio observations of the neutral hydrogen at 21 cm, which has a spatial resolution that is worse by an order of magnitude compared to the optical resolution. However, neutral hydrogen discs are usually more extended than stellar discs, so this allows us to trace the rotation curves (and thus the gravitational potential distribution associated with them) much farther from the galactic centres than where the stars are seen. Table 2 lists the sources from the literature that we have used to construct the combined rotation curves for our four galaxies.

It is interesting to compare the restored density distributions of the gravitating matter with the distributions of the stellar component in the galaxies considered. In all the galaxies, the stars are the dominating baryonic content. For this purpose, we have used surface photometry data from the literature for our galaxies. NGC 7331 and 2841 have extended radial profiles of the surface brightness in the near-infrared (2 mkm) photometric band \(K\), published in the survey by Muñoz-Mateos et al. (2009). We studied the other two galaxies earlier in the \(I\) band (for NGC 5533, see Sil’chenko, Burenkov & Vlasyk 1998) and in the \(I\) band (for NGC 7217, see Sil’chenko & Afanasiev 2000). To derive a density distribution from the brightness distribution or to calculate the brightness distribution corresponding to some density distribution, we need to know the mass-to-light ratio in the specified photometric band, appropriate for the stellar population of a particular age and metallicity. At the same time, the latter characteristics are only known for the discs of a few nearby spiral galaxies. For NGC 7217 and 5533, we have studied the radial distributions of the mean age and metallicity of the stellar populations in their discs by analysing long-slit spectral data from the reducer SCORPIO of the Russian 6-m telescope (Afanasiev & Moiseev 2005). So, for these galaxies, we are able to use the appropriate mass-to-light ratio values obtained in the evolutionary synthesis models by Percival et al. (2009) – recalculated by us for the reduced Salpeter initial mass function (IMF), and so slightly increased with respect to the models by Percival et al. (2009), which used the Kroupa IMF. For the other two galaxies, NGC 2841 and 7331, the disc stellar population parameters are unknown. For these, we have used the empirical calibrations of the mass-to-light ratio values versus broad-band colours from Bell et al. (2003) – one versus \(g - i\) in NGC 2841 and the other versus \(B - R\) in NGC 7331.

Table 1. Global parameters of the galaxies.

<table>
<thead>
<tr>
<th>Type (NED)⁴</th>
<th>NGC 2841</th>
<th>NGC 5533</th>
<th>NGC 7217</th>
<th>NGC 7331</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_{25}, \text{arcsec (LEDA)}⁵)</td>
<td>244</td>
<td>93</td>
<td>117</td>
<td>314</td>
</tr>
<tr>
<td>(R_25, \text{kpc})</td>
<td>16.6</td>
<td>19.3</td>
<td>9.3</td>
<td>23</td>
</tr>
<tr>
<td>Distance, Mpc (NED)</td>
<td>14</td>
<td>43</td>
<td>16.5</td>
<td>15</td>
</tr>
<tr>
<td>(M_2 (\text{LEDA}))</td>
<td>−20.84</td>
<td>−21.6</td>
<td>−20.48</td>
<td>−21.56</td>
</tr>
<tr>
<td>(V_{\text{sys}} \sin i, \text{km s}^{-1}, (\text{LEDA, H}1))</td>
<td>295</td>
<td>206</td>
<td>143</td>
<td>237</td>
</tr>
<tr>
<td>(M_{\text{HI}} (10^8 M_\odot)^6)</td>
<td>2.8</td>
<td>30</td>
<td>0.7</td>
<td>8.2</td>
</tr>
</tbody>
</table>

⁴NASA/IPAC Extragalactic Database.
⁵Lyon-Meudon Extragalactic Database.
⁶From Bosma (1981) for NGC 2841 and 7331 and from Noordermeer et al. (2005) for NGC 5533 and 7217.
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Figure 2. Observed and calculated data for the galaxies: (a) NGC 2841; (b) NGC 7217; (c) NGC 7331; (d) NGC 5533. The horizontal axis is the radial distance in arcsec. On the vertical axis plots, we show the following: (A) $V(R_{\text{max}})$; (B) $M_{\text{max}} = M(R_{\text{max}})$; (C) $\sigma_{\text{max}} = \sigma(0) \approx 3.432 M_\odot / P_c^2$; (C') $\sigma(0) \approx 3.406 M_\odot / P_c^2$; (B') $M(x)/M_{\text{max}}$ and (C') $\sigma(x)/\sigma_{\text{max}}$ for $V(R > R_{\text{max}}) = V(R_{\text{max}}) \sqrt{R_{\text{max}}/R}$; (D) a vertical line marking the end of the stellar disc to the 25th blue isophote.

(a) NGC2841: $R_{\text{max}} = 63.240 P_c$,

A) $V(R_{\text{max}}) \approx 294 km/sec$,

B) $M_{\text{max}} = M(R_{\text{max}}) \approx 1.22 \cdot 10^{12} M_\odot$,

C) $\sigma_{\text{max}} = \sigma(0) \approx 3.432 M_\odot / P_c^2$,

C') $\sigma(0) \approx 3.406 M_\odot / P_c^2$.

(b) NGC7217: $R_{\text{max}} = 9600 P_c$,

A) $V(R_{\text{max}}) \approx 305 km/sec$,

B) $M_{\text{max}} = M(R_{\text{max}}) \approx 1.94 \cdot 10^{11} M_\odot$,

C) $\sigma_{\text{max}} = \sigma(0) \approx 6.220 M_\odot / P_c^2$,

C') $\sigma(0) \approx 6.050 M_\odot / P_c^2$.

(c) NGC7331: $R_{\text{max}} = 37.230 P_c$,

A) $V(R_{\text{max}}) \approx 238 km/sec$,

B) $M_{\text{max}} = M(R_{\text{max}}) \approx 4.93 \cdot 10^{11} M_\odot$,

C) $\sigma_{\text{max}} = \sigma(0) \approx 1.883 M_\odot / P_c^2$,

C') $\sigma(0) \approx 1.854 M_\odot / P_c^2$.

(d) NGC5533: $R_{\text{max}} = 77.875.2 P_c$,

A) $V(R_{\text{max}}) \approx 226 km/sec$,

B) $M_{\text{max}} = M(R_{\text{max}}) \approx 9.34 \cdot 10^{11} M_\odot$,

C) $\sigma_{\text{max}} = \sigma(0) \approx 2.877 M_\odot / P_c^2$,

C') $\sigma(0) \approx 2.867 M_\odot / P_c^2$. © 2012 The Authors, MNRAS 420, 3071–3080

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Table 2. References for the rotation curves and photometric profiles.

<table>
<thead>
<tr>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 2841 rot(Hα+[N II]), Afanasiev &amp; Sil’chenko (1999)</td>
</tr>
<tr>
<td>rot(HI), Begeman (1987), Begeman, Broeils &amp; Sanders (1991)</td>
</tr>
<tr>
<td>density(HI), Bosma (1981)</td>
</tr>
<tr>
<td>phot, Muñoz-Mateos et al. (2009)</td>
</tr>
<tr>
<td>NGC 5533 rot(Hα+[N II]), Sil’chenko et al. (1998)</td>
</tr>
<tr>
<td>rot(HI), Noordermeer et al. (2005)</td>
</tr>
<tr>
<td>density(HI); Broeils &amp; van Woerden (1994)</td>
</tr>
<tr>
<td>phot, Sil’chenko et al. (1998), Noordermeer &amp; van der Hulst (2007)</td>
</tr>
<tr>
<td>NGC 7217 rot(Hα+[N II]), Zasov &amp; Sil’chenko (1997)</td>
</tr>
<tr>
<td>rot(HI), Noordermeer et al. (2005)</td>
</tr>
<tr>
<td>density(HI), Noordermeer et al. (2005)</td>
</tr>
<tr>
<td>phot, Noordermeer &amp; van der Hulst (2007)</td>
</tr>
<tr>
<td>NGC 7331 rot(Hα+[N II]), Afanasiev, Sil’chenko &amp; Zasov (1989)</td>
</tr>
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<td>rot(HI), Begeman (1987), Begeman et al. (1991)</td>
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<td>phot, Muñoz-Mateos et al. (2009)</td>
</tr>
</tbody>
</table>

Earlier, we found two large-scale stellar discs in NGC 7217, which had stellar populations with different characteristics (Sil’chenko et al. 2011). Inside $R \approx 45$ arcsec, a thin stellar disc with a mean stellar age of 5 Gyr and a mean metallicity of $[Z/H] = -0.2$ dominates. At larger distances from the centre, we can see a thicker and more extended star-forming stellar disc with a younger ($\langle T \rangle = 1–2$ Gyr) and very metal-poor ($[Z/H] \leq -0.4$) stellar population. The border between the two discs is narrow, so a jump in the mass-to-light ratio is noticeable at $R \approx 50$ arcsec. The matter density profile (Fig. 2) demonstrates a break at the same radius. NGC 5533 is also a multicomponent stellar system; however, the limits of the subsystems do not coincide in the brightness and density profiles. All the subsystems are more extended as presented by the gravitating matter density distribution. The surface photometry allows us to distinguish a bulge-dominated region within $R < 20$ arcsec, an inner stellar disc at $R = 20–60$ arcsec and an outer stellar disc of low surface brightness at $R > 70$ arcsec (Sil’chenko et al. 1998). By applying the software ULYSS of Koleva et al. (2009) to the long-slit spectral data obtained using SCORPIO on the 6-m telescope, we have succeeded in deriving the stellar population characteristics up to $R \approx 50$ arcsec, but only for the inner stellar disc. The results are shown in Fig. 3. We can see that at $R > 20$ arcsec, or beyond the bulge-dominated area, both the age and metallicity demonstrate flat radial dependences. So, for the inner stellar disc in NGC 5533, we can use the mean values of the stellar population characteristics at $R > 20$ arcsec (i.e. $\langle T \rangle = 2$ Gyr and $[Z/H] = -0.4$). We have taken the model mass-to-light ratio for these stellar population parameters from Percival et al. (2009). Then, we have added 0.15 in the logarithm to recalculate it with the reduced Salpeter IMF, instead of the Kroupa IMF used by Percival et al. (2009) (the recommendations are taken from Bell et al. 2003).

So, in Fig. 4, we compare the observed surface brightness profiles with those calculated from the matter density profiles reconstructed from the rotation curves. If the main contributor to the gravitational potential of a galaxy is a stellar disc, the two profiles must coincide. What have we discovered from Fig. 4? We can see that the different galaxies demonstrate different contributions of their stellar discs to the total gravity. In NGC 2841 and 7217, the inner parts of both profiles are in agreement; indeed, in these galaxies, within $R < 9$ kpc in NGC 2841 and within $R < 4$ kpc in NGC 7217, the stellar discs are the main contributors to the total gravitational potentials. In the outer parts of the galaxies, a ‘hidden mass’ reveals itself – the profiles reconstructed from the rotation curves go strongly above the observed surface brightness profiles. However, in the other two galaxies, NGC 5533 and 7331, the observed and calculated profiles diverge everywhere in the galaxies. In NGC 5533, the ‘hidden mass’ dominates in the gravitational potential over the whole galaxy extension – this is the same conclusion that we have already made in

Figure 3. Radial dependences of the mean age and mean metallicity of the stellar populations in NGC 5533, which we have obtained with the long-slit spectral data from the reducer SCORPIO of the Russian 6-m telescope.
our previous work, based on the analysis of the rotation curve shape (Sil’chenko et al. 1998). NGC 7331 represents a more intriguing situation: the observed central brightness exceeds our expectations that we have from the low visible rotation velocity. We think that non-circular gas motions are significant in the centre of this galaxy, so the rotation curve derived from the single long-slit cross-section along the isophote major axis does not reflect the circular speed that characterizes the gravitational potential. We have also previously expressed a suspicion about the tri-axiality of the gravitational potential shape in NGC 7331 (Sil’chenko 1999).

What properties of NGC 2841 and 7217 might be related to the dominance of their large-scale stellar discs in their total gravity? We can make a couple of suggestions. First, both galaxies are classified as isolated, and these are included in the list of isolated galaxies by Karachentseva (1973). Secondly, there is a noticeable difference in the neutral hydrogen content of NGC 2841 and 7217, on the one hand, and NGC 5533 and 7331, on the other hand. NGC 5533 and 7331 are more gas-rich (see Table 1). In Fig. 5, we compare the radial profiles of the gravitating matter density, which we have reconstructed from the rotation curves, to the neutral hydrogen.

Figure 4. A comparison of the observed surface brightness profiles for our four galaxies (symbols), with those based on the gravitating matter surface density profiles restored from the rotation curves for the outer rotation curve asymptotic taken as a constant value (lines), calculated by assuming that all the gravitating matter is the stellar component of the galactic discs.
surface density radial distributions (at the logarithmic scales) for all four galaxies. While in NGC 2841 and 7217 the neutral hydrogen contributes less than one per cent of the total surface density, in NGC 5533 and 7331 its contribution is as high as a few per cent over the whole extension of the galactic discs. Moreover, the general character of the gas density and matter density profiles is the same: they look like a quasi-exponential decreasing function, which is very typical for stellar and gaseous galaxy discs. Hoekstra, van Albada & Sancisi (2001), for example, have previously remarked that the ‘hidden mass’ distribution in spiral galaxies is similar to the ‘visible’ gas distribution. In our work, we have found that the gravitating matter surface density distribution agrees better with that of the stellar component in gas-poor galaxies. Thus, this is in line with the hypothesis of Pfenniger, Combes & Martinet (1994) that the ‘hidden mass’ in spiral galaxies is, in fact, very cold molecular gas, which cannot be observed in accessible electromagnetic radiation spectral domains.

6 ACCOUNTING FOR THE COMPONENTS OF A SPHERICAL HALO OF THE GALAXY

In all previous sections, we have ignored the fact that much of the matter of the galaxy, in principle, can be concentrated in a spherically symmetric halo, which has a mass function \( M_{\text{sphere}}(r) \).
Initially, we do not know the distribution of the spherical mass \( M_{\text{sphere}}(r) \); this feature is a free parameter model.

Because of the additivity of the potential \( \Phi \) (in the Newtonian approximation), it can be divided into two parts (the disc and a spherical component):

\[
\Phi(\rho, z) = \Phi_{\text{disc}}(\rho, z) + \Phi_{\text{sphere}}(r); \quad r^2 = \rho^2 + z^2. \tag{26}
\]

Equation (1) in the equatorial plane \( (z = 0) \) can be rewritten in as

\[
\frac{\partial \Phi_{\text{disc}}(\rho, z = 0)}{\partial \rho} = \frac{V^2(\rho) - GM_{\text{sphere}}(\rho)/\rho}{\rho}. \tag{27}
\]

Hence, all subsequent arguments (starting with equation 1) amount to replacing the observed features \( V(\rho) \) as

\[
V(\rho) \rightarrow \sqrt{V^2(\rho) - GM_{\text{sphere}}(\rho)/\rho}. \tag{28}
\]

Here, we are free to vary the expression \( GM_{\text{sphere}}(\rho)/\rho \) in the range from zero to a value approximately equal to \( V^2(\rho) - GM_{\text{disc}}(\rho)/\rho \), where \( GM_{\text{disc}}(\rho) \) denotes the mass that is concentrated in the disc to a certain radius \( \rho \). For example, from a certain radius \( \rho \) the radius of the observed stellar disc to the end of the 25th blue isophote, it is possible to put \( V^2(\rho) - GM_{\text{sphere}}(\rho)/\rho = GM^\text{tot}_{\text{disc}}/\rho \), where \( GM^\text{tot}_{\text{disc}} \) is the total mass of matter in the galactic disc. This will correspond to the termination of the matter density in the disc at this radius (25th blue isophote).

7 CONCLUSIONS

In this paper, we have considered a new integral representation for reconstructing the matter surface density in the flat discs of spiral galaxies. Our method (equations 14 and 15) contains only a twofold integral in the limit from 0 to \( \pi/2 \). At first, we suppose that all matter is concentrated in the disc of the galaxy, and we apply the method for reconstructing the surface density of matter in four spiral galaxies (NGC 2841, 7217, 7331 and 5533). We have compared our results with the distributions of stellar matter. This comparison allows us to make some conclusions about the possible nature of the ‘hidden mass’ in these galaxies. Because the radial distributions of the ‘hidden mass’ that we have derived correlate with the radial neutral hydrogen distributions, this is an argument in favour of the hypothesis that the hidden mass phenomenon at galaxy scales is connected with the cold gas content. In principle, this is undetectable using the electromagnetic radiation. In Section 6, we have discussed the possibility of accounting for the components of spherical haloes.

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APPENDIX A: NUMERICAL INTEGRATION OF OUR NEW INTEGRAL REPRESENTATION

As described in the main text, we wish to use our new integral representation to find distributions of surface density and mass throughout a galactic disc (i.e. equations 15 and 16). Because of the observational nature of our input data (the velocity function), our only way to solve these equations is to use numerical methods. In this appendix, we describe and test our numerical code, which has been developed for this purpose.

The fundamental integral of our method (equation 15) does not have non-integrable singularities. However, the integrand might be ill-defined at the integration endpoints. Because of this, we use the open-ended second Euler–Maclaurin integration formula as our basic integration algorithm. Then, we use increasing numerical resolutions and Romberg’s method to achieve the highest possible accuracy (see the discussion on these techniques and further references in, for example, Press et al. 1992). We use these methods with enough iterations until the result has satisfactorily converged.

In addition to these basic integration techniques, a few other tricks are needed. Our basic data for integration will usually be a discrete set of velocity data in the disc as a function of the radius, \( \rho \). Obviously, equation (15) is a function of the velocity function, but because the velocity function is expressed through the intermediate variable \( R_0 = \rho (\cos \alpha / \sin \beta) \) (see equations 14 and 15), we will often need values of the velocity function not explicitly tabulated by our observational data. To solve this issue, we need to use interpolation for velocity data in between observational data points and we also need to make some assumptions about the behaviour of the

REFERENCES


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velocity function at points beyond the tabulated range of observational data.

To obtain velocity data for points in between the observational data points, we have used simple linear interpolation (attempts at using higher-order interpolations were made, but yielded no increased accuracy). Furthermore, we have made the assumptions that the velocity at the central axis is zero and that the velocity beyond the outermost observed point is a constant function with a value equal to that of the outermost observed point.

Finally, we have tested our numerical implementation of the new integral representation by performing a number of convergence tests. We define an average error measure as

\[ E_N(x) = \frac{1}{n} \sum_{i=1}^{n} \frac{|x_{N}^i - x_{N+1}^i|}{x_{N+1}^i}, \]  

(A1)

where \( x \) denotes the function we are investigating, \( n \) denotes the number of data points (in \( \rho \)) and \( N \) denotes the number of iterations we use in Romberg’s method.\(^4\) The data in Fig. A1 use this error measure for the two basic basic functions \( U(\rho) \) and \( \sigma(\rho) \) (cf. equation 15) for varying the number of iterations. These two functions are the most basic functions of our new integral representation. If the method is correctly numerically implemented, the average error must converge to zero as we increase the numerical resolution (the number of Romberg iterations).

As the input data for this figure, we have used the test velocity function equation (20). However, note that similar results are obtained if actual observational velocity data are used. From the figure, it is clearly seen that both \( U(\rho) \) and \( \sigma(\rho) \) are converging towards zero with an increasing number of iterations. This is exactly what we expect and we thus conclude that the numerical implementation of our new integral representation is correct.

Finally, we note that for all the numerical results presented in this paper, convergence tests have been made to confirm that the presented data have converged to their limiting values.

\(^4\)That is, \( N + 1 \) indicates a higher resolution than \( N \).