Temporal variations in the acoustic signal from faculae

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ABSTRACT

The integrated brightness of the Sun shows variability on time-scales from minutes to decades. This variability is mainly caused by pressure mode oscillations, by granulation and by dark spots and bright faculae on the surface of the Sun. By analysing the frequency spectrum of the integrated brightness, we can obtain greater knowledge about these phenomena. It is shown how the frequency spectrum of the integrated brightness of the Sun in the frequency range from 100 to 3200 μHz shows clear signs of granulation, faculae and p-mode oscillations, and that the measured characteristic time-scales and amplitudes of the acoustic signals from granulation and faculae are consistent with high-resolution observations of the solar surface. Using 13 years of observations of the Sun’s integrated brightness from the Variability of solar IRadiance and Gravity Oscillations (VIRGO) instrument on-board the SOHO satellite, it is shown that the significance of the facular component varies with time and that it has a significance above 0.99 around half the time. Furthermore, an analysis of the temporal variability in the measured amplitudes of the granulation, faculae and p-mode oscillation components in the frequency spectrum reveals that the amplitude of the p-mode oscillation component shows variability that follows the solar cycles, while the amplitudes of the granulation and facular components show signs of quasi-annual and quasi-biennial variabilities, respectively.

Key words: Sun: faculae, plages – Sun: granulation – Sun: helioseismology – Sun: oscillations.

1 INTRODUCTION

The frequency spectrum of observations of the integrated brightness of the Sun is often referred to as the solar acoustic spectrum (Iben & Mahaffy 1976), as it is mainly used to study pressure or sound waves inside the Sun. These sound waves can be observed on the surface of the Sun as oscillation patterns which can be represented as spherical harmonics of the eigenfunctions of the sound waves inside the Sun. These sound waves lead to a series of distinct peaks in the solar acoustic spectrum (see Fig. 1), and by studying the frequencies, amplitudes and linewidths of these peaks, we can through helioseismology obtain knowledge of the physics that takes place inside the Sun (see e.g. Gough et al. 1996), but more information are hidden in the solar acoustic spectrum. At lower frequency, the solar acoustic spectrum contains signals from spots, granulation and faculae. In fact, the acoustic signal from granulation and faculae continues up to the frequencies of the p-mode oscillations and even to higher frequencies and therefore this signal is often referred to as the acoustic background. Studies of Sun-like stars have shown that other stars show a similar acoustic background (see e.g. Michel et al. 2008; Chaplin et al. 2010).

Harvey (1985) could be the first to suggest that the solar acoustic background was modelled with a number of exponentially decaying functions. The idea is that the signal from e.g. granulation is understood as a random signal with some memory and thus its autocorrelation function should be given by an exponentially decaying function defined by a characteristic time-scale and an amplitude (Baudin et al. 2007). Long uninterrupted observations from the South Pole and from the SOHO satellite have later revealed that the solar acoustic background contains signals from many more phenomena than just granulation. Harvey et al. (1993) identified a total of six different components in the acoustic spectrum from observations of the Ca II K line in the solar atmosphere from the South Pole. The identified components were, from lower to higher frequency, non-periodic fluctuations attributed to active region evolution, non-periodic fluctuations attributed to granulation overshoot, non-periodic fluctuations attributed to chromospheric bright points, periodic fluctuations attributed to the photosphere (i.e. p-mode oscillations) and periodic fluctuations attributed to the chromosphere (i.e. chromospheric oscillations). Whereas the signals from granulation and p-mode oscillations are reported in all studies, the signals associated with the chromosphere are only reported in the study by Harvey et al. (1993), as this is the only study that analyses the acoustic background in observations of the Ca II lines. The feature between the granulation signal and the p-mode oscillations has on
the other hand also been reported in most studies, especially studies
based on intensity measurements, but an agreement on the origin of
these features has not been reached.

Aigrain, Favata & Gilmore (2004) analysed observations of solar
irradiance from the VIRGO instrument on SOHO and identified not
only a granulation component in the frequency spectrum, but also
components from active regions, super- and meso-granulations and
bright points. The characteristic time-scale of the component that
Aigrain et al. (2004) identified as bright points is similar to the
characteristic time-scale of the component that Harvey et al. (1993)
identified as chromospheric bright points. Vázquez Ramíó, Régulo
& Roca Cortés (2005) identified the same component in integrated
light observations from VIRGO, but speculated that this component
might be due to a second granulation population – as shall be shown
later, this is not likely.

Bright points and faculae are related phenomena and they are both
related to the magnetic network on the solar surface. The difference
is that whereas bright points are clearly seen in the inter-granular
lanes, faculae are seen inside the granules, and whereas bright
points can maintain their brightness over several tens of minutes
(Berger, Rouppe van der Voort & Lofdahl 2007), faculae change
significantly on granular evolution timescales (De Pontieu et al.
2006). For this reason, it is more likely that the component between
the granulation and p-mode oscillation components is caused by
faculae rather than bright points.

In the picture of Berger et al. (2007), faculae are defined as ‘the
edges of granules’ seen through the ‘forest’ of the magnetic field
plages and the network’. The faculae do in other words occur in a
localized region of the granules and it is thus clear that the
characteristic time-scale of faculae will be a bit shorter than that
of granulation. There are no quantitative studies that directly relate
the time-scale of granulation to the time-scale of faculae based
on high-resolution observations. A qualitative argument that the
characteristic time-scale of faculae is shorter than that of granulation
is given in the small movie in fig. 9 of De Pontieu et al. (2006).
This small movie of a 3D radiative magneto-convective simulation
shows three distinct granules that live throughout the full length of
the movie (330 s) and a number of faculae on these granules. If one
looks e.g. at the facular at (0.3, 1.4 arcsec), it is clearly visible at \( t =
0 \text{s} \), but completely gone at \( t = 90 \text{s} \). At \( t = 120 \text{s} \) a new facular starts
to evolve just to the right of where the other was located, and at \( t =
210 \text{s} \) this facular has its peak brightness. In other words, though
the evolution of faculae follows the evolution of the granules, it is
clear from the movie that the same granule can, during its lifetime,
contain a number of faculae – i.e. the characteristic time-scale of
faculae is shorter than that of granulation.

A few studies (Trampedach et al. 1998; Ludwig 2006; Ludwing
et al. 2009) have used hydrodynamical models of stellar atmos-
pheres to simulate acoustic spectra of Sun-like stars, but so far no
such studies have been made using magneto-hydrodynamical mod-
els, which is needed in order to reproduce the facular component.

In this paper we will use observations from the green channel
on the VIRGO instrument on-board SOHO (Fröhlich et al. 1997)
to analyse the acoustic signature of faculae. We will show how
the characteristic time-scale and amplitude that we measure of this
component in the acoustic background are consistent with measure-
ments based on high-resolution observations of the solar surface.
We also analyse any temporal variations in the different components
in the solar acoustic background – including the facular component.

The paper is arranged as follows. In Section 2 we give a short
definition of the acoustic background and present the model that we use to model the granulation, the faculae and the p-
mode oscillations components in the acoustic spectrum. This model
is used in the analysis in Section 3, where we also describe a sta-
tistical test of the significant of the facular component in the acoustic
background. Section 4 presents the results of the analysis, and con-
cluding remarks are found in Section 5. Appendix A describes a test based on Monte Carlo simulations of the statistical method
developed in Section 3.

2 THEORY

The original formulation of the solar acoustic background by Harvey
(1985) is the following:

\[
f(\nu) = \frac{4\nu^2 \tau}{1 + (2\pi\nu\tau)^2},
\]

where \( f(\nu) \) is the power density at frequency \( \nu \), \( \sigma \) is the amplitude
of the signal and \( \tau \) is the characteristic time-scale. Equation (1) is
normalized to Parseval’s theorem so that the variance of the signal
in the time domain equals the sum of \( f(\nu) \) over all frequencies:

\[
\sigma^2 = \int f(\nu) \, d\nu,
\]

which means that \( \sigma \) in equation (1) does in fact reflects the variance
of the signal in the time domain (Baudin et al. 2007). Note that
every component in the acoustic background will contribute with the
signal given by equation (1) to the spectrum. The spectrum will
thus be a sum of individual signals from i.e. granulation and faculae
each defined by unique values of \( \sigma \) and \( \tau \).

Unfortunately, the original model by Harvey (1985) fails to repro-
duce the observed solar acoustic background for frequencies higher
than the atmospheric acoustic cut-off frequency. The reason for
this is that granulation cannot be modelled with turbulent cascades
(Nordlund et al. 1997) as it is done in the drift model by Harvey
(1985). Turbulence shows a distribution with a slope of around −2
in power, convection on the other hand has a lower limit in the time
domain on which changes can take place. This means that on small
time-scales (or at high frequency) convection is not noisy, whereas
turbulence is, and therefore the acoustic background decays with
a slope smaller than $-2$. In the Sun the decay rate turns out to be around $-4$ at frequencies higher than the atmospheric acoustic cut-off frequency. Taking this into account, the Harvey model extends to

$$f(v) = \frac{4\sigma^2\tau}{1 + (2\pi\nu\tau)^2 + (2\pi\nu)^2}. \quad (3)$$

Karoff (2008) used this model to successfully describe the granulation and facular components in the solar acoustic background between $400 \mu\text{Hz}$ and up to $8000 \mu\text{Hz}$, which is close to the Nyquist frequency.

Harvey et al. (1993) used another approach to solve the problem with the fast decline at high frequency and considered the slope as a free parameter. When this is done, the decline will often be faster than $-2$ and this will solve the problems at high frequency. In this way, Harvey et al. (1993) found slopes of $-5.6$ and $-5.0$ for the granulation and the bright point components, respectively. The slope gives the decay rate, which ‘calibrates’ the amount of memory in the process responsible for the noise (Harvey et al. 1993) and there is no reason that this value should be exactly $-2$, or that it should be the same for the different components. On the other hand, there is a physical argument as to why the slope should not be the same at low and high frequencies (which were given above), so allowing the slope to be a free parameter only partly solves the problem.

We have therefore chosen another approach and modelled only the lower frequency part of the spectrum (up to $3200 \mu\text{Hz}$). This means that no extra component is needed in order to model the high-frequency part. The following model is thus used to model the granulation and facular signals in the solar acoustic background between 100 and $3200 \mu\text{Hz}$:

$$f(v) = \frac{4\sigma^2\tau}{1 + (2\pi\nu\tau)^2}. \quad (4)$$

The envelope of the p-mode oscillations shows variability on a weekly time-scale with an amplitude of 6.2 per cent, which is properly mainly due to changes in the damping rates of the oscillation modes (Kjeldsen et al. 2008). The envelope of the p-mode oscillations therefore needs to be modelled along with the acoustic background in order to disentangle changes in the damping rates of the oscillation modes from changes in the acoustic background. A Gaussian envelope is therefore also included in the model in order to model the envelope of the p-mode oscillations, but as the high frequency part of the spectrum is not analysed it is not necessary to include a white noise background. Of course, the background will have some effect on the amplitudes that are measured of the granulation and facular components, whereas the damping rate in the VIRGO observations is generally around 3 orders of magnitudes smaller than the granulation and facular amplitudes, this effect is negligible.

A Gaussian has here been chosen for the envelope of the p-mode oscillations, but one or two Lorentzians could also have been chosen (Harvey et al. 1993; Lefebvre et al. 2008). This would however not affect the final results significantly and it is not clear which functional form that agrees best with the behaviour of the Sun and other Sun-like stars in general (see e.g. the simulations by Houdek et al. 1999).

Figs 2 and 3 show examples of fits to the solar acoustic background using the models described above either containing or not containing a facular component. The observations and the calculation of the spectrum are described in Section 4.

3 ANALYSIS

The data analysis consists of three steps: calculating the power density spectra; modelling the power density spectra and evaluating the significance of the different measured parameters and the goodness of the different models.

3.1 Observations

For the analysis we use a 13-yr time series from the green channel on the VIRGO instrument SOHO (Fröhlich 2009). There are some gaps in this time series especially around the SOHO ‘vacation’ and recent measurements do suffer from increased noise level. In the analysis performed here, we have divided the full 13-yr time series into substrings of one month each. In order to get the lowest probability for artefacts such as gaps and increased noise levels to affect the data analysis, we have performed a discrimination of both bad data points and bad substrings. Bad data points were identified by calculating a logarithmic running box variance log($\sigma_i$):

$$\log(\sigma_i) = \log \left( \frac{1}{N} \sum_{j=i-w}^{i+w} \left[ x_j - \mu \right]^2 \right). \quad (5)$$

where $x_j$ is the $j$th observation, $\mu$ is the mean value of the observations between $i - w$ and $i + w$ and $w$ is the width of the running box.
box which was set equal to 10 points \((N = 21)\). Using this formulation, all data points with a logarithmic running box variance that deviated more than 5σ from the mean were then removed. This removed around 5 percent of the data points. The reason for using the logarithmic variance is that many data points show artificially low scatter.

After removing the bad data points, all monthly substrings with a duty cycle lower than 95 percent were also removed. This left us with 119 monthly substrings out of 147 possible. We did test that the relative high demands for selecting good data points did not affect the results. The test showed slightly diverging results when using either the 119 good monthly substrings or all the 147 substrings, but the differences were insignificant.

The power density spectra were calculated using least squares (Lomb 1976; Karoff 2008). Each spectrum was normalized by the effective observation length given as the reciprocal of the area under the window function in order to convert the spectra into power density spectra.

### 3.2 Modelling the acoustic background

Acoustic backgrounds have earlier been modelled by smoothing the spectra with a Gaussian running mean, with a width of a few hundred \(\mu\text{Hz}\), and then assuming that the differences between the smoothed observed spectrum and the model followed a normal distribution, which allowed a comparison utilizing least squares (see e.g. Karoff 2008; Kjeldsen et al. 2008; Chaplin et al. 2010). Though this works in practice, there are no mathematically formal arguments for that the differences between the smoothed observed spectrum and the model should follow a normal distribution. A method for modelling the observed spectra that is statistically valid and that does not require the spectra to be smoothed with a Gaussian running mean is therefore presented below.

In order to model the solar acoustic background, we make use of maximum likelihood estimators (see e.g. Anderson, Duvall & Jeffries 1990, and references therein). The purpose is to calculate a logarithmic likelihood function \(\ell\) between \(N\) independent measurements \(x_i\), i.e. power density at a given frequency, and a model \(f_j\) given by a set of parameters \(\lambda\). Adopting the notation of Toutain & Appourchaux (1994) and Appourchaux, Gizon & Rabello-Soares (1998), the logarithmic likelihood function can be calculated as

\[
\ell = - \sum_{i=1}^{N} \ln p(x_i, \lambda), \tag{6}
\]

where \(p(x_i, \lambda)\) is the probability density function. The position of the minimum of \(\ell\) in \(\lambda\) space will give us the most likely value of \(\lambda\) (Appourchaux et al. 1998). The so-called formal error bars can be calculated as the diagonal elements of the inverse of the Hessian matrix \(h\):

\[
h_{ij} = \frac{\partial^2 \ell}{\partial \lambda_i \partial \lambda_j}. \tag{7}
\]

In order to calculate the probability density function, we use the formulation by Appourchaux (2004) where a binned version of the original spectrum is compared to the model. The formulation by Appourchaux (2004) can be used on any model \(f_j\) and not just for calculating the significance of peaks in the spectrum. We thus calculate the binned spectrum \(\mathcal{S}_i(x, n)\) by summing \(x_i\) over \(n\) bins:

\[
\mathcal{S}_i(x, n) = \frac{1}{n+1} \sum_{k=-n/2}^{k+n/2} x_k. \tag{8}
\]

Following Appourchaux (2004), the probability density function is then given as

\[
p \left[ \mathcal{S}_i(x, n), f_j(\lambda) \right] = \frac{\mu_i}{\Gamma(v_i)} \mathcal{S}_i(x, n)^{v_i-1} e^{-\mathcal{S}_i(x, n)}, \tag{9}
\]

where \(\Gamma\) is the Gamma function and \(\mu_i\) and \(v_i\) are given as

\[
\mu_i = \frac{1}{\sum_{k=-n/2}^{k+n/2} f_k(\lambda)} (n + 1) \sum_{k=-n/2}^{k+n/2} f_k(\lambda), \tag{10}
\]

and

\[
v_i = \frac{2 \left[ \sum_{k=-n/2}^{k+n/2} f_k(\lambda) \right]^2}{\sum_{k=-n/2}^{k+n/2} f_k(\lambda)}. \tag{11}
\]

\(n\) appears in \(\mu\) and \(v\) in order to get the right amplitudes.

### 3.3 Testing the facular hypothesis

Using the formalism above, the model in equation (4), either containing or not containing a facular component, can be compared to an observed spectrum. The model containing a facular component is based on a total of nine free parameters — i.e. three from granulation, three from faculae and three from the \(p\)-mode oscillations — whereas the model not containing a facular component is based on only six free parameters. Having the maximum likelihood of each model from equation (6), we are then able to calculate the statistical significance of the facular component in the acoustic spectrum by calculating the logarithmic likelihood ratio \(\Lambda\):

\[
\ln \Lambda = \ell(\lambda_{p+q}) - \ell(\lambda_p). \tag{12}
\]

In our case, \(p\) equals 6 and \(q\) equals 3. Following Wilks (1938) (see also Appourchaux et al. 1998), we can then compare the value of \(-2 \ln \Lambda\) to a \(\chi^2\) distribution with \(q\) degrees of freedom and in this way calculate the significance of the facular component in the acoustic spectrum.

### 4 RESULTS

Two kinds of results are presented: one set of results on the average spectrum of the 119 one-month substrings and one set of results on the temporal variability in the spectra of the individual substrings. The analysis is made by binning 100 frequency bins \((n = 100)\), which gives a frequency resolution in the binned spectra of around \(39 \mu\text{Hz}\). Error bars for all parameters were calculated as the diagonal elements of the inverse of the Hessian matrix (see equation 7).

#### 4.1 Average results

The average spectrum from the 119 one-month substrings is modelled with a model containing a facular component and a model not containing a facular component. The results from the modelling are given in Table 1 and the average observed spectra with the models overlaid are shown in Figs 2 and 3. As the decay rates were treated as free parameters in both models, the decay rates for the granulation component are different in the two models. This leads to differences in the models at high frequency as the obtained decay
rate of 1.8 ± 0.1 for the granulation component in the model without a facular component is too small to describe the acoustic background at high frequency.

4.2 Comparison to high-resolution observations

A number of studies have used high-resolution observations of the solar surface to measure the amplitude and time-scale of granulation and faculae, but in general there is little agreement between the different results. The reason for this is probably differences in the way the amplitude and the time-scale are measured, but also differences in the resolution of the observations, differences in the solar activity level at the time of the observations and whether the observations were taken close to the disc centre or close to the limb. The high-resolution observations can therefore only be used as a rough comparison to the seismic results.

If a typical size of granules (or cell area) of 1.2 Mm² (Del Moro 2004) is assumed, then around 2.5 million granules are found on the visible solar surface. The contrast of a typical granule is around 20 per cent (Wedemeyer-Böhm & Rouppe van der Voort 2009). The typical contrast of faculae is around 50 per cent at the peak distance from the disc centre (μ ≃ 0.6) (Berger et al. 2007) and the relative fraction of granules that show faculae is around 1.8 per cent (Berger et al. 1995).

If each granule on the solar surface is assumed to cause a signal of relative amplitude δIL/L at a random time and the entire solar surface is covered with granules, then this will cause a time series of integrated sunlight with a variance of (Ludwig 2006; Ludwig et al. 2009)

$$\sigma \approx \left( \frac{N}{\sqrt{N_{\text{granule}}}} \right)_{\text{granule}},$$

(13)

where $$N_{\text{granule}}$$ is the number of granules present on the visible solar surface at any given time.

The amplitudes of the granulation component were measured to be 73.0 ± 1.6 and 85.8 ± 1.5 ppm for the model containing a facular component and the model not containing a facular component, respectively. This is consistent with the theoretical estimate of around 126 ppm that is found by dividing 20 per cent with the square root of 2.5 million granules.

The amplitude of the facular component was measured to be 61.5 ± 1.9 ppm. This agrees roughly with the theoretical estimate of around 42 ppm obtained by multiplying ~50 per cent with the square root of ~1.8 per cent and dividing it with the square root of 2.5 million granules.

The characteristic time-scales of convection were measured to be 214.3 ± 2.9 and 219.0 ± 4.9 s for the model containing a facular component and the model not containing a facular component, respectively. This is consistent with the coherence time-scale between 156 and 246 s found by Del Moro (2004). For faculae a characteristic time-scale of 65.8 ± 2.5 s was measured which is consistent with the fact that it should be shorter than the granulation time-scale.

The amplitude and characteristic time-scale of the granulation and facular components in the acoustic spectra are consistent with the high-resolution measurements. This indicates that the interpretation that the second bump in the acoustic spectrum originates from faculae is consistent – though the comparison alone does not make the argument. The interpretation is also based on the relative relations between the amplitude and characteristic time-scale of granulation, faculae and p-mode oscillations.

Equation (13) shows why the component in the acoustic spectrum between the granulation and p-mode oscillation components does not originate from a second granulation population as suggested by Vázquez Ramíó et al. (2005). This suggestion is based on the work by Del Moro (2004) who finds, using high-resolution images of quiet granulation, a second population of granules with a shorter coherence time, but also smaller brightness contrasts. The fact that this second population of granules is both more numerous and has smaller brightness contrasts than ordinary granules means that it should not be visible in the acoustic spectrum as the numerator of equation (13) would be smaller and the denominator would be larger than for ordinary granulation.

4.3 Statistical significance of the facular signal

The significance of the facular component in the average observed spectrum can be calculated from the likelihood ratio between the two different models. The significance was measured to be 83 per cent. This means that though the model with the facular component and a total of nine free parameters does describe the average observed spectrum better than the model without the facular component and six free parameters, it does not describe it significantly better. Visually, it is clear that the fit in Fig. 2 is better than the fit in Fig. 3, but given the statistic of the observed spectra and the number of free parameters it is not significantly better.

In order to analyse why the facular component is not significant in the average observed spectrum, we calculate the significance of the facular component in the spectrum of each individual one-month substring. The decay rates and characteristic time-scales for the granulation and facular components and the frequency of maximum power for the p-mode oscillation component had to be fixed to the values given in the first column of Table 1. If this was not done, the location and slope of the exponential decaying function would change in order to make up for changes in amplitude. This means that the models used for modelling the 119 individual one-month substrings contained three and four free parameters, respectively.

The logarithmic likelihood ratios between the models with and without a facular component for the 119 individual one-month substrings are shown in Fig. 4. It is seen that most of the logarithmic likelihood ratios are between 0 and 10 with a few scatter points below and above. The figure does reveal a few points with logarithmic likelihood ratios below 0. A negative logarithmic likelihood ratio
means that in these months the model without the facular component describes the observed spectrum better than the model with the facular component or, in other words, that adding more free parameters to the model makes a worse agreement between the observed spectrum and the model for these months. The reason for this is that the decay rates of the granulation components are not the same in the two different models, and for some months the slow decay rate in the model without the facular component describes the observed spectrum better than the fast decay rate in the model with the facular component. These scatter points with logarithmic likelihood ratios below 0 could be the reason for the facular component not being significant in the average observed spectrum.

Clearly, a lot of variability is seen in Fig. 4, but no trend known to follow the visibility or amplitude of faculae is seen. In particular, no clear variability that follows the solar cycle is seen (see Fig. 5 for comparison). The reason for this could either be that the measured ratios are not accurate enough to show intrinsic facular variability or that the acoustic signal from faculae depends stronger on the number of available granules than on the fraction or contrast of faculae (see equation 14).

Fig. 6 shows the distribution of the significances of the facular signal measured in the 119 one-month substrings assuming that the logarithmic likelihood ratios follow a $\chi^2$ distribution with one degree of freedom. This distribution shows a clear increase towards a significance of one that falls off rapidly down to a significance of around 0.97 and then has a long tail down to smaller significances (also smaller significances than what is shown in the figure). The conclusion from the figure is that the acoustic signal from faculae is significantly present in the observed solar acoustic background 47 per cent of the time.

The same effect is seen in the spectrum calculated from the full 13 years of observations – i.e. the acoustic signal from faculae is not significantly present in the spectrum. Again, the reason for this is most likely that the signal is not present all the time, which reduces the significance of the signal when analysing the full 13 years of observations.

4.4 Temporal variation of p-mode oscillations

The frequencies, amplitudes and linewidths of the individual p-mode oscillation modes in the acoustic spectrum are known to change with the solar cycle (see e.g. Libbrecht & Woodard 1990; Chaplin et al. 2000) and we therefore also expect the average shape and position of the p-mode oscillation envelope to change with the solar cycle. The temporal variability of the height and width of the p-mode oscillation envelope is shown in Fig. 7 for the models with and without a facular component (the position of the envelope is kept fixed in both models). There is a good agreement between the results from the two different models – with a linear Pearson correlation coefficient of 0.95 (0.97 for the smoothed curves) for the height and 0.83 (0.70 for the smoothed curve) for the width.

A clear anticorrelation is seen between the height of the envelope and the solar cycle (see Fig. 5 for comparison) so that the height has a minimum around 2000, which coincides with the peak of the solar cycle. The anticorrelation between the height of the envelope and the solar cycles is consistent with other studies (i.e. Kjeldsen 2012).
et al. 2008; García et al. 2010) and can be explained by the fact that the lifetime of the p-mode oscillation is known to decrease with increasing solar activity (Chaplin et al. 2003). The envelope width does not show any correlation with the solar cycle. The position of the envelope, which is not analysed here, as it was fixed to the average value, does only show small changes compared to the height and width (García et al. 2010).

The uncertainties on the parameters measured in the average spectrum are comparable in size to the uncertainties measured in the individual one-month substrings. This reflects that the scatter seen in e.g. Figs 7 and 8 is much larger than the uncertainties on the individual measurements – in other words, it reflects that the parameters does show intrinsic variability on a monthly time-scale.

4.5 Temporal variation of granulation

The temporal variability of the amplitude of the granulation component does not show any clear correlation with the solar cycle, neither for the model with the facular component nor for the model without the facular component. The amplitudes of the granulation component show an offset between the different models and only some agreement is seen in the temporal variability – with a linear Pearson correlation coefficient of 0.74 (0.84 for the smoothed curves). Instead, some signs of a quasi-annual modulation are seen, especially in the results from the model with the facular component.

A similar quasi-biennial variability was seen in the residuals of the p-mode oscillation frequency shifts in the study by Fletcher et al. (2010). In Fig. 10 we therefore compare the residuals of the measured amplitudes of the facular component with the residuals of the p-mode oscillation frequency shifts from Fletcher et al. (2010). In order to do that we have subtracted a third-order polynomial from the measurements in Fig. 9 and binned the one-month measurements to the temporal resolution of the residuals of the p-mode oscillation frequency shifts (182.5 d). The two sets of measurements show a correlation with a linear Pearson correlation coefficient of 0.34. The reason that no strong correlation is seen could however be the relative large uncertainties on both sets of measurements and thus it cannot be ruled out that the amplitude of the facular component and the p-mode oscillation frequency shifts have the same origin.

Figure 7. Temporal variability of the height (left) and width (right) of the p-mode oscillation envelope for the model with a facular component (top) and without a facular component (bottom). The solid black lines show a Gaussian running mean with a width of 0.4 yr. As expected, a clear anticorrelation is seen between the measured height of the p-mode oscillation envelope and the solar cycle.

Figure 8. Temporal variability of the amplitude of the granulation component for the model with a facular component (top) and without a facular component (bottom). Again, the solid black lines show a Gaussian running mean with a width of 0.4 yr. Some signs of a quasi-annual variability are seen, especially in the results from the model with the facular component, as was also the case in the study by Lefebvre et al. (2008).
Biennial variability is also seen in the residuals of the p-mode oscillation frequency shifts from the study by Fletcher et al. (2010). The two sets of measurements have a linear Pearson correlation coefficient of 0.34, which means that no strong correlation can be claimed between the amplitude of the facular component and the p-mode oscillation frequency shifts. The reason for this could be the relative large uncertainties on both sets of measurements and thus it cannot be ruled out that the faculae and the p-mode oscillation frequency shifts have the same origin.

The temporal variability of the amplitudes of the granulation, facular and p-mode oscillation components in the solar acoustic background have also been analysed. The amplitudes of the p-mode oscillations show a clear anticorrelation with the solar cycle and the amplitude of the granulation component shows some signs of an quasi-annual variability, which is also seen in velocity observations (Lefebvre et al. 2008), but no correlation with the solar cycle. The amplitude of the facular component does not also show any correlation with the solar cycle, but instead some signs of a quasi-biennial variability are seen. The residuals of the temporal variability of the amplitude of the facular component show a correlation with a linear Pearson correlation coefficient of 0.34 with the residuals of the p-mode oscillation frequency shifts from the study by Fletcher et al. (2010).

5 CONCLUSION

13 years of observations of the integrated brightness of the Sun from the VIRGO instrument on-board SOHO have been analysed. Three distinct features are seen in the frequency spectrum of these observations. By comparing the measured characteristic time-scale and amplitude of these phenomena to high-resolution observations of the solar surface, it is shown that the three features can convincingly be explained as granulation, faculae and p-mode oscillations.

The feature in the acoustic background related to faculae is the weakest and it is not significantly present in the average spectrum made from 119 individual one-month spectra. The feature is, on the other hand, present in 56 (47 per cent) out of the 119 one-month spectra with a significance larger than 0.99 and in 75 (63 per cent) with a significance larger than 0.95.

The temporal variability of the amplitudes of the granulation, facular and p-mode oscillation components in the solar acoustic background related to faculae is the weakest and it is not significantly present in the average spectrum made from 119 individual one-month spectra. The feature is, on the other hand, present in 56 (47 per cent) out of the 119 one-month spectra with a significance larger than 0.99 and in 75 (63 per cent) with a significance larger than 0.95.

The temporal variability of the amplitude of the facular and p-mode oscillation components in the solar acoustic background related to faculae is the weakest and it is not significantly present in the average spectrum made from 119 individual one-month spectra. The feature is, on the other hand, present in 56 (47 per cent) out of the 119 one-month spectra with a significance larger than 0.99 and in 75 (63 per cent) with a significance larger than 0.95.

In general, it has to be noted that the quasi-annual and quasi-biennial periods, seen in the granulation and facular amplitudes, are very peculiar. Varying instrumental effects such as altitude, pointing, temperature or the like could be the culprit. One of the things that were done in order to test this was to redo the analysis using only the monthly substrings where the facular signal had a significance larger than 95 and 99 per cent and this did not change any of the conclusions on the facular signal – i.e. the same kind of periodicity was still seen.

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REFERENCES


APPENDIX A: MONTE CARLO SIMULATIONS OF THE ACOUSTIC SIGNAL FROM FACULAE

In order to test the robustness of the statistical analysis developed in Section 3, a number of Monte Carlo simulations were made. In the simulations it was assumed that the acoustic spectrum followed the model given in equation (4) and an acoustic spectrum was simulated for the parameters given in Table 1. Using an inverse Fourier transform, the acoustic spectrum was converted from the frequency to the time domain, where realistic random noise were added at each time-step. Thereafter, the acoustic spectrum was converted back into the frequency domain, where it was analysed – the background parameters and the significance of the facular signal measured.

The two parameters that were changed in the simulation were the amplitude of the facular component and the noise level – all other parameters were fixed to the values given in Table 1. This means that for each set of facular amplitude and noise level, the simulation returned not only measured facular amplitude and noise level, but also measured values for all the other parameters in Table 1, which could be used to evaluate the precision of the procedure. The amplitude of the facular component was changed between 45 and 70 ppm and the noise level from 0 to 80 ppm per measurement (equivalent to the point-to-point scatter in the time series).

The main result of this Monte Carlo simulations is shown in Fig. A1, which shows the relation between the amplitude of the facular component and the logarithmic likelihood ratio between the models with and without the facular component. It is seen that there is no linear correlation between the two. This is a bit surprising as a spectrum with a high-amplitude facular component is expected to have a higher likelihood with a model with a facular component than a spectrum with a low-amplitude facular component would have. This is also the case, but the problem is that there is no linear relation between the likelihood and the amplitude of the facular component, for the model that does not include the facular component. In other words, the reason why no linear correlation seen is that, though the likelihood between the model and the observations increases linearly as a function of facular amplitude for the model with the facular component, the same (or more correctly the opposite) is not true for the model without the facular component.

This does first of all explain why some signs of variability can be seen in Fig. 9, which shows the measured facular amplitude as a function of time, but not in Fig. 4, which shows the likelihood ratio as a function of time.

It does also explain why we can measure the amplitude and characteristic time-scale of the facular component and the uncertainties on these parameters even in months where the component is not significant. The reason is that in order to measure the amplitude and characteristic time-scale, we only use the model that includes the facular component, while in order to measure the likelihood ratio we also use the model that does not include the facular component.

No trends were seen between the measured parameters and the noise level in the simulations, except for increased scatter. For some
Figure A2. Distribution of the difference between the input and output of the simulations. The figure shows a histogram of the difference between the facular amplitude that was used in the simulations and the amplitudes that were returned from the model, in terms of the uncertainty returned from the modelling of the simulated spectra (solid line). The dashed line shows a Gaussian with a variance of 1 for comparison. The fact that the distribution is not too different from a Gaussian with a variance of 1 reflects that the uncertainties returned from the modelling are realistic.

of the simulations with the highest noise levels, it was not possible to get the model to convert to the observed spectrum.

The Monte Carlo simulations have also been used to test the uncertainties on the different parameters – especially the parameters related to the facular component. This is done by plotting a histogram of the difference between the parameter values used in the simulation and the measured parameter values divided by the uncertainty on the measured parameter values \( \frac{\lambda_{\text{obs}} - \lambda_{\text{true}}}{\sigma_{\lambda}} \).

If the errors on the parameters were randomly distributed around a mean value, we would expect that these histograms would all be Gaussian function with a variance of 1. This is also the case for the parameters in Table 1 (the parameters related to the facular component, as shown in Fig. A2), though the variance might be a bit larger than 1, this is not unexpected as the uncertainties are only formal uncertainties. In order to get more reliable uncertainties, one would eventually have to perform a full Bayesian analysis as is sometimes done when modelling the p modes using Markov chain Monte Carlo (see e.g. Handberg & Campante 2011).

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