The great escape – II. Exoplanet ejection from dying multiple-star systems

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ABSTRACT

Extrasolar planets and belts of debris orbiting post-main-sequence single stars may become unbound as the evolving star loses mass. In multiple-star systems, the presence or co-evolution of the additional stars can significantly complicate the prospects for orbital excitation and escape. Here, we investigate the dynamical consequences of multi-phasic, non-linear mass loss and establish a criterion for a system of any stellar multiplicity to retain a planet whose orbit surrounds all of the parent stars. For single stars which become white dwarfs, this criterion can be combined with the Chandrasekhar Limit to establish the maximum allowable mass-loss rate for planet retention. We then apply the criterion to circumbinary planets in evolving binary systems over the entire stellar mass phase space. Through about $10^5$ stellar evolutionary track realizations, we characterize planetary ejection prospects as a function of binary separation, stellar mass and metallicity. This investigation reveals that planets residing at just a few tens of au from a central concentration of stars are susceptible to escape in a wide variety of multiple systems. Further, planets are significantly more susceptible to ejection from multiple-star systems than from single-star systems for a given system mass. For system masses greater than about $2 M_\odot$, multiple-star systems represent the greater source of free-floating planets.

Key words: planets and satellites: dynamical evolution and stability – planet–star interactions – stars: AGB and post-AGB – stars: evolution – stars: mass-loss – white dwarfs.

1 INTRODUCTION

Roughly one third of all stars in the Galactic disc are components of multiple-star systems (Lada 2006) and the majority of multiple-star systems are thought to be binary systems (Duquennoy & Mayor 1991). Further, as of 2012 February, several tens of extrasolar planets have now been detected or are suspected of existing in binary systems. Some planets have been reported in systems with even higher stellar multiplicities (e.g. Cochran et al. 1997; Raghavan et al. 2006; Guenther et al. 2009; Desidera et al. 2011). Mugrauer & Neuhauser (2009) provide a helpful list of known exoplanets in multiple-star systems, as of 2009. In binary systems, a planet may orbit one of the stars in what is sometimes called an S-type orbit (see e.g. Lowrance, Kirkpatrick & Beichman 2002; Bakos et al. 2006; Eggenberger et al. 2006; Correia et al. 2008). Alternatively the planet may orbit both stars in a P-type orbit (see e.g. Sigurdsson et al. 2003; Lee et al. 2009; Beuermann et al. 2010, 2011; Kuzuhara et al. 2011; Potter et al. 2011; Qian et al. 2011, 2012). The latter case describes circumbinary planets, whose existence has been bolstered by the recent transit-based discoveries of Kepler-16b (Doyle et al. 2011), Kepler-34b and Kepler-35b (Welsh et al. 2012). Therefore, understanding the dynamics of planets in multiple-star systems and, in particular, binary systems is becoming increasingly relevant. This understanding includes how such planets form, and how they die.

One potential avenue for planetary death is dynamical ejection as the star evolves beyond the main sequence and loses mass (Veras et al. 2011, hereafter Paper I). Evidence for free-floating planets (in Lucas & Roche 2000; Zapatero Osorio et al. 2000, 2002; Bihain et al. 2009) provides observational motivation to investigate this physical mechanism. In particular, Sumi et al. (2011) recently discovered 10 wide-orbit or free-floating bodies and calculated that 1.8+1.7−0.8 free-floating planets exist per main-sequence star. Therefore, more planets may travel between stars than orbit them, and this vast population of free-floaters cannot be explained by planet–planet scattering immediately subsequent to system formation alone (Veras & Raymond 2012). In Paper I, the authors analytically described the conditions which can lead to planetary ejection from a single star. They assumed isotropic mass loss and demonstrated that three key factors must be taken into account. These are (i) the mass-loss time-scale, (ii) the planetary orbital time-scale and (iii) the total mass of the system. That study was limited to consideration of a single phase of stellar evolution for mass loss which occurred in a linear manner.

Here, we perform a three-tiered extension to that work by considering (i) multiple stars, (ii) multiple evolutionary phases and (iii) non-linear mass loss. Our primary application is the determination of the prospects for circumbinary (P-type) planets to escape as the
parent binary evolves. Doing so provides us with the foundation to understand systems of higher stellar multiplicities, systems which often include tight binaries.

The physical evolution of a single star can, in principle, be described by just the star’s initial mass and metallicity. However, the phase space of binary star evolution is significantly broader. In addition to each star’s mass and metallicity, the separation and eccentricity of their mutual orbit (possibly including orbital variations due to massive planetary companions) are additional parameters which must be taken into account. The metallicities of both stars are often assumed to be equivalent because the stars are assumed to have formed from the same molecular cloud and not to have subsequently accreted enough planetary material to significantly alter their original [Fe/H] values. Our understanding of the physics of stars is not yet good enough to definitively link the above parameters with the wind velocity, the accretion rate on to the companion and the amount of spin angular momentum transferred between the stars. Therefore, physical properties such as these represent a further broadening of the potentially explorable phase space.

We focus our study by considering a single planet which is far from the parent binary (which may be tight or wide) and by analysing the amount of mass loss from the system as a function of time. In the absence of mass loss, the approximately elliptical orbit of the planet will be negligibly perturbed by the potentially complex interactions between both members of the binary. In addition, such complex details of this star–star interaction need only be modelled to the extent that they correctly yield the mass-loss rate from the entire system. If the binary separation is large enough so that both stellar components evolve independently, then the planet’s dynamical evolution reduces to the two-body case which was presented in Paper I, with the difference that the total system mass here is larger because of the presence of the binary companion.

In Section 2, we consider theoretically how planetary ejection is affected by multiple phases of non-linear mass loss for systems of any multiplicity and establish a criterion for retention we use throughout the rest of the paper. In Section 3, we numerically simulate binary star evolution to identify which types of systems are susceptible to planet ejection and quantify where planets must reside in these systems in order to remain bound. We compare the binary and single-star cases, and discuss the implications for higher multiplicities, other types of orbits and extensions to this work in Section 4, and conclude in Section 5.

2 CHARACTERIZING MULTI-PHASIC NON-LINEAR MASS LOSS

This section provides the analytic background which motivates the numerical simulations in Section 3 and presents formulas which may be applied to other investigations.

2.1 Phases, regimes and stages

Here we define our nomenclature. We adopt the same meaning of phases which is commonly used in stellar evolution studies. Stages are treated as subsets of phases, and regimes refer to the evolutionary environment of a planet due to stellar mass loss.

2.1.1 Phase identification

Realistic stellar evolution occurs across several phases. A sequence of phases qualitatively characterizes a star’s history. We adopt the definitions of phase in the SSE (Hurley, Pols & Tout 2000) and BSE (Hurley, Tout & Pols 2002) stellar evolutionary codes:

<table>
<thead>
<tr>
<th>Definition</th>
<th>Stages</th>
</tr>
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<tbody>
<tr>
<td>$0 = \text{low-mass} (M &lt; 0.7 M_\odot)$</td>
<td>main-sequence star</td>
</tr>
<tr>
<td>$1 = \text{high-mass} (M &gt; 0.7 M_\odot)$</td>
<td>main-sequence star</td>
</tr>
<tr>
<td>$2 = \text{Hertzsprung gap}$</td>
<td></td>
</tr>
<tr>
<td>$3 = \text{first giant branch}$</td>
<td></td>
</tr>
<tr>
<td>$4 = \text{core helium burning}$</td>
<td></td>
</tr>
<tr>
<td>$5 = \text{early asymptotic giant branch (AGB)}$</td>
<td></td>
</tr>
<tr>
<td>$6 = \text{thermally pulsing AGB}$</td>
<td></td>
</tr>
<tr>
<td>$7 = \text{naked helium star main sequence}$</td>
<td></td>
</tr>
<tr>
<td>$8 = \text{naked helium star Hertzsprung gap}$</td>
<td></td>
</tr>
<tr>
<td>$9 = \text{naked helium star giant branch}$</td>
<td></td>
</tr>
<tr>
<td>$10 = \text{helium white dwarf}$</td>
<td></td>
</tr>
<tr>
<td>$11 = \text{carbon/oxygen white dwarf}$</td>
<td></td>
</tr>
<tr>
<td>$12 = \text{oxygen/neon white dwarf}$</td>
<td></td>
</tr>
<tr>
<td>$13 = \text{neutron star}$</td>
<td></td>
</tr>
<tr>
<td>$14 = \text{black hole}$</td>
<td></td>
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<tr>
<td>$15 = \text{massless remnant}$</td>
<td></td>
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</table>

We denote phase numbers by $k$. Within each phase, the mass-loss rate can vary. Therefore, detailed modelling of a particular system necessitates fitting mass-loss rates with a piecewise non-linear model. However, doing so for a broad exploration of stellar evolution phase space is not feasible.

2.1.2 Regime identification

Instead, we seek to construct non-linear mass-loss profiles by a sequence of linear approximations. A key benefit of this approach is that we can utilize analytic results from Paper I and hence gain a better understanding of what conditions must exist for a planet to be significantly perturbed or to escape the system. Assume the stellar mass-loss rate is constant and equal to $-\alpha$, where $\alpha > 0$. In order to characterize the tendency of a system to eject a planet, we use the dimensionless mass-loss index, $\Psi$, such that

$$\Psi = \frac{\text{mass-loss time-scale}}{\text{orbital time-scale}} = \frac{\alpha}{n_\mu} = \frac{1}{2\pi} \left( \frac{a}{1 \text{ au}} \right)^2 \left( \frac{\mu}{1 \text{ M}_\odot} \right)^{-\frac{3}{2}},$$

(1)

where $a$ and $n$ represent the planetary semimajor axis and mean motion, and $\mu = \sum M_w + M_p$, where $M_w$ represents the masses of all of the stars in the system and $M_p$ represents the mass of the planet. We use the term planet to loosely describe a bound body of any mass that is not a star and does not perturb the orbits of the stars.

A planet is said to be evolving in one of two regimes depending on the value of $\Psi$. In the adiabatic regime, when $\Psi \ll 1$, a increases but the planet’s eccentricity, $e$, remains constant. In the runaway regime, when $\Psi \gg 1$, the planet’s semimajor axis continues to increase and $e$ can now vary (increase or decrease) across all possible values. Therefore, a planet may escape only if $\Psi \gtrsim 1$. This bifurcation point is not exact. It is a weak function of both $e$ and the planet’s true anomaly, $f$ (see Paper I). In fact, $\Psi$ may instead be defined as $\alpha T/\mu$, where $T$ represents the planet’s orbital period. In this case, $\Psi$ does not contain the factor of $1/(2\pi)$ that is present in equation (1).

We will henceforth refer to this factor as $\kappa$.

If the planet is evolving in the runaway regime, the eccentricity evolution is a function of the eccentricity at the start of that regime and the planet’s location along its orbit. For example, if the planet is close to pericentre on an already highly eccentric orbit, then the
planet is ejected immediately. Alternatively, if the planet is close
to apocentre on the same highly eccentric orbit, then the planet
is never ejected. If the planet is on a circular orbit, regardless of $f$, it is
ejected when the star has lost exactly half of its mass from its value
at the start of that regime.

### 2.1.3 Stage identification

Now suppose that within a given phase, the system undergoes $N$
consecutive stages of constant mass-loss evolution with mass-loss
rates of $\alpha_1, \alpha_2, \ldots, \alpha_N$. Then, based on the results from
Paper I, a planet experiencing $i$ consecutive stages of mass loss in
the adiabatic regime will have a semimajor axis and eccentricity at the end of stage $i$ of

$$ a_i = a_0 \left( \frac{\mu_i}{\mu_0} \right)^{-1} \equiv a_0 \beta_i^{-1}, \quad (2) $$

and

$$ e_i = e_0, \quad (3) $$

where $\beta_i \equiv \mu_i/\mu_0$ represents the per cent of the original system
mass that is retained by the end of stage $i$. Because eccentricity
remains constant, a planet cannot be ejected in this regime (unless
the semimajor axis evolution carries the planet out of the system),
even if almost all of one or both stars’ mass is lost. The mass-loss
index at the end of stage $i$ is

$$ \Psi_i = \kappa a_i \left( \frac{a_i \mu_0}{\mu_i^2} \right)^{\frac{3}{2}}, \quad (4) $$

where $\kappa = 1$ or $2\pi$ depending on if the orbital time-scale is defined
with respect to the orbital period or the mean motion. Equation (4)
indicates that escapability of a planet is independent of the details
of the intermediate stages of mass loss provided that both $\mu_i$ and $\alpha_i$
are known.

### 2.2 Stage-based retention criterion

Here, we describe the theory behind the critical system mass frac-
tion lost for a given semimajor axis (Sections 2.2.1–2.2.4) and the
planet’s critical semimajor axis for a given system mass-loss prescrip-
tion (Section 2.2.5). Computation of the critical semimajor axis
will be the focus of subsequent sections.

#### 2.2.2 The critical mass fraction

Equation (4) demonstrates that, at the end of the $i$th stage of evolu-
tion, the minimum fraction of the original system mass which must
be retained to guarantee that a planet with a given $a_i$ remains bound
is

$$ \beta_{\text{crit}} \equiv \left( \frac{\mu_i}{\mu_0} \right)_{\text{crit}} = \kappa^\frac{1}{3} \left( \frac{a_i}{1 \text{ au}} \right)^{\frac{1}{2}} \left( \frac{\mu_0}{1 \text{ M}_\odot} \right)^{-\frac{1}{2}}. \quad (5) $$

Note importantly how this criterion can be written in a form which
is independent of $\mu_i$ and $a_i$. If $\beta_{\text{crit}} > 1$, then the planet is not
guaranteed to remain bound, regardless of how much mass is lost
from the system. Equation (5) demonstrates that a single short but
powerful ejection of mass from a star at any time can unhinge a
planet. If this event occurs late in the life of a star, when $\mu_i$ is low,
the probability is increased. Equation (5) provides a useful way to
quickly characterize an ensemble of systems. The equation holds
for any stellar multiplicity. For single stars which become white
dwarfs, the Chandrasekhar Limit can be inserted into $\mu_i$. We may
then determine the maximum possible $\alpha_i$ that can protect a planet.

#### 2.2.2 Visualizing the criterion

We can visualize the limits on a planet’s escapability through Fig. 1,
which can be applied to systems of any stellar multiplicity in which a
planet’s orbit surrounds all of the stars. The upper and lower
panels, respectively, illustrate planet retention bounds when one
characterizes the mass-loss index with respect to the mean motion
(corresponding to $\kappa = 2\pi$ equation (1)) and the orbital period ($\kappa = 1$).
The contours are values of $\beta_{\text{crit}}$, and are a function of $\alpha_i$ and
$\mu_0$ for a planet at $a_0 = 10^4$ au. Colours denote regions separated
by adjacent contour levels in 10 per cent intervals in masses. The
upper panel of the figure demonstrates, for example, that a single
2 M$_\odot$ star which loses 60 per cent of its mass by the end of the $i$th
stage of evolution is guaranteed to retain an orbiting planet at $a_0 = 10^4$ au if $\alpha_i \lesssim 10^{-7.6}$ M$_\odot$ yr$^{-1}$.

Stars with masses up to 8 M$_\odot$ may become white dwarfs, whose
maximum mass cannot exceed 1.4 M$_\odot$. Therefore, in the single-star
limit, the thick black curve denotes the Chandrasekhar Limit. Thus,
in the upper panel, a 4 M$_\odot$ star which becomes a white dwarf has
retained at most about 35 per cent of that mass during the transition.
Planetary retention in such a system is guaranteed only if the mass-
loss rate during the transition is $\alpha_i \lesssim 10^{-7.2}$ M$_\odot$ yr$^{-1}$.

For planetary semimajor axes other than $10^4$ au, the plots are
shifted vertically but otherwise remain unaltered. For every order of
magnitude that $a_0$ is decreased, the vertical axis values are increased
by $10^{10} \approx 1.5$ units. Therefore, for $a_0 = 10^3$ au, which represents a
typical Oort cloud distance, $\alpha_i$ is rarely low enough to ever guarantee
protection of Oort cloud comets.

#### 2.2.3 Stellar multiplicity comparison

We can also extend the figure to selected multiple star situations.
Suppose that the primary has just become a white dwarf but the
secondary is still on the main sequence and is losing a negligible
amount of mass such that $M_1 = M_2 = \gamma M_{10}$. In this case, the
maximum value of $\alpha_i$ which can guarantee planetary protection becomes

$$ \alpha_i = \left[ \frac{1.4 \text{ M}_\odot + \gamma M_{10}}{a_0 \sqrt{a_0 M_{10}(1 + \gamma)}} \right]^3. \quad (6) $$

This equation describes a limiting curve which can be drawn on
Fig. 1 for a given binary system. Here, $\alpha_i$ takes on an absolute
maximum when $\gamma M_{10} = 1.4 \text{ M}_\odot$. Hence, the maximum possible
value of the ratio of the binary to single-star values of $\alpha_i$ for a given
$M_{10}$ and $\gamma$ is about 107 for the extreme bounds of $M_{10} = 8 \text{ M}_\odot$
and $\gamma = 1$. For the vast majority of realistic systems, however,
this ratio is about a few. This exercise, which could be generalized
to higher multiplicities, demonstrates that the mass-loss rate for
a main-sequence-white-dwarf binary system needs to be slightly
higher than in the single-star-white-dwarf case in order to eject a
wide-orbit planet.

#### 2.2.4 Stellar stage comparison

In order to deal with generic systems at all evolutionary stages,
reconsider equation (5). Because $\beta_{\text{crit}}$ is not explicitly dependent

on the mass-loss rates during the earlier stages of evolution, we can establish a critical value for the entire system evolution of

\[ \beta_{\text{crit}} = \text{Max} \left( \beta_{\text{crit},i} \right), \]

where \( i = 1, \ldots, N \) represents the \( i \)th stage of evolution.

Comparing \( \beta_i \) with \( \beta_{\text{crit}} \) determines the prospects for ejection. If \( \beta_i > \beta_{\text{crit}} \) for all \( \beta_i \), then ejection cannot occur. If \( \beta_i \geq \beta_{\text{crit}} \) for at least one \( \beta_i \), then ejection may occur. This is a general condition which may be applied to systems with any stellar multiplicity. In this work, we focus on binary systems, as they often represent components of higher multiplicity stellar systems and feature a representative set of physical processes that may be found in those systems.

2.2.5 The critical semimajor axis

Now suppose that \( a_0 \) is not given. Then we can determine the critical value of \( a_0, a_{\text{crit}} \), for which the planet is guaranteed to remain bound through equation (5). We obtain

\[ \frac{a_{\text{crit}}}{1\text{ au}} \approx \kappa^{-2} \left( \frac{\mu_0}{1\,M_\odot} \right) \min \left[ \beta_{\text{crit},i} \left( \frac{a_i}{1\,M_\odot \text{ yr}^{-1}} \right)^2 \right], \]

where the minimum is taken over all values of \( i \). Although Veras & Wyatt (2012) apply this criterion for the specific case of the Sun, the criterion is generally applicable to multiple-star systems. We use equation (8) to determine \( a_{\text{crit}} \) throughout the remainder of the paper.

2.3 Treating non-linear mass loss

Ideally, a non-linear mass-loss profile may be approximated with the finest possible partition into linear segments. Veras & Wyatt (2012) achieved this accuracy by considering the duration between every 88th time-step as a segment. Although effective for a detailed study of one system, this approach might not provide a consistent measure or be computationally feasible for a population of systems. Hence, we now consider alternatives.

2.3.1 The theory

We compute \( a_0 \) by dividing the change in mass with the change in time over the entire phase \( k \). However, in cases where the mass-loss rate changes non-linearly within a phase, we can model the mass-loss evolution within the phase by consecutive stages of approximately linear mass loss. We select the boundaries for these stages based on a coefficient of variation, \( C_v \), of the data. If we fit a linear model to a given time series of data, then \( C_v \) represents the residual sum of squares about that fit divided by the mean of the response variable (the mass).

An exact fit to the linear model yields \( C_v = 0 \). If we insist that each evolutionary stage satisfies \( C_v \leq C_v^* \), where \( C_v^* \) is a user-provided parameter, then the quality of our analytical treatment from Section 2 relies on the value of \( C_v^* \). If \( C_v^* \) is too low, and a phase is split into too many stages, the resulting analysis becomes computationally expensive. If \( C_v^* \) is too high, and each phase is represented by a single stage, then the analysis may represent a poor physical approximation.

Suppose a phase \( k \) is split into \( l \) stages, where \( j = 1, \ldots, l \). Then \( \text{Max}(\alpha_j) \geq \alpha_k \) because mass loss from the system is monotonic and at least one of the segments of a piecewise linear curve between both phase endpoints is steeper than the original curve. Therefore,
the limiting value \( C^* \to \infty \) does not define any additional stages and hence provides a conservative estimate for planetary ejection.

### 2.3.2 An example

Consider the binary system with \( M_{10} = 5.2 \, M_\odot \), \( M_{20} = 2.2 \, M_\odot \), \( [\text{Fe/H}] = [\text{Fe/H}]_\odot = 0.02 \), \( a_{B0} = 100 \, R_\odot \) and \( e_{B0} = 0.0 \), where the subscript \( B \) refers to the primary–secondary binary. Suppose both stars began life on the main sequence. This system undergoes nine distinct phases of evolution such that \( k_{1,2} = \{ (1, 1), (2, 1), (3, 1), (7, 1), (8, 1), (11, 1), (6, 1), (6, 15), (11, 15) \} \). This sequence illustrates that \( M_1 \) evolves off the main sequence first, on to the Hertzsprung gap and then the red giant branch. The primary’s envelope is then blown away, leaving behind a naked helium star, which again begins a main sequence phase followed by a Hertzsprung gap phase. The primary eventually becomes a white dwarf. However, at this point, the secondary is roughly three times more massive and is siphoning matter on to the primary. The primary is revitalized as a thermally pulsing AGB star, which soon after merges with the secondary. The new star continues AGB evolution until it becomes a white dwarf and dies out as such.

The mass loss for the \( k_{1,2} = (6, 15) \) phase is markedly non-linear. Fig. 2 shows the mass-loss evolution of this phase for four different values of \( C^* \). The latter three split the phase into stages. Phase and stage boundaries are given by vertical red dashed lines and brown dot–dashed lines, respectively. The total number of stages throughout the system evolution that are generated for \( C^* \to \{ \infty, 0.1, 0.01, 0.001 \} \) are \{ 9, 10, 12, 22 \}. The values of \( \beta_{\text{crit}} \) are obtained from \( \alpha \), which in turn are computed from the intersections of the mass-loss curve with the stage boundaries. Displayed in the lower-right panel are the three highest \( \beta_{\text{crit}} \) values from that stage. Note that, as \( C^* \) decreases, \( \beta_{\text{crit}} \equiv \text{Max}(\beta_i) \) increases. This must be the case because at least one of the segments of a piecewise linear curve between both phase endpoints is steeper than the original curve. The most accurate \( \beta_{\text{crit}} \) value is a factor of 1.86 greater than the least accurate value and corresponds to a factor of \( \approx 3.5 \) in \( a_0 \).

### 3 APPLICATION TO REALISTIC SYSTEMS

In this section, we attempt to sample the entire phase space of binary systems in order to determine prospects for planetary ejection. We first argue that large regions of the phase space may be treated...
in a similar manner. For the remaining regions, we utilize a proven binary stellar evolution code that can generate tracks quickly, without solving coupled differential equations. Finally, we present our results through a series of contour plots that provide information about features of the binary systems studied as well as useful parameters such as the critical semimajor axis at which a planet is guaranteed to remain bound.

3.1 Stellar evolution code

We use a slightly modified form of the BSE stellar evolution code (Hurley et al. 2002). This code utilizes empirical, algebraic formulae derived from observational sources and theoretical frameworks in order to model evolving binary stars. Although the code tracks a wide range of stellar parameters, here we are concerned solely with the mass-loss rate from the system and the physical reasons for this mass loss. We assume that the circumbinary orbit of the planet is always wider than the apocentre and common envelope radius of the binary. As shown in Paper I, for monotonic stellar mass loss, a planet’s semimajor axis must always be increasing, regardless of the shape of the orbit or the planet’s position along that orbit.

We have modified BSE with an updated form of the mass loss for stellar wind from naked helium stars. We have included a dependence on metallicity of \( M \propto \sqrt{Z} \) according to equation (22) of Nugis & Lamers (2000). The mass-loss prescription we used is also incorporated in the NBODY6 N-body code (which may be downloaded from http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm, as of 2011 December). We also modified BSE to withhold output data dumps within evolutionary phases until at least 0.01 per cent of the entire system mass has been lost. This both allows us to use a mass-dependent uniform standard for the analysis of all our simulations and enables us to cover a wide region of phase space by limiting the number of outputs (particularly for main-sequence phases) in each simulation. We always output the first instance of a new evolutionary phase.

3.2 Phase transitions

The phase and stage formalism described so far does not treat mass lost from a system during a phase transition. The BSE code computes the time-scales and mass loss for most phase transitions. In these cases, we consider the mass loss between consecutive time-steps that encompass the phase transition to be linear. However, BSE treats some transitions as instantaneous. Although this approximation may be adequate for numerous stellar applications, such as population studies, here that approximation would cause \( \alpha \rightarrow \infty \) and hence cannot be used.

In the single-star case, the only example of such an instantaneous transition occurs during a supernova. However, for binary stars, there are several more examples. In a comprehensive sampling of phase space, each of these instances must be accounted for and a time-scale established for each. We found that all such instances can be partitioned into four types of violent phenomena: (i) core-collapse supernovae, (ii) black hole core collapse, (iii) thermonuclear supernovae and (iv) common envelope evolution. In the following arguments, we assume that mass loss is isotropic and do not distinguish amongst neutrinos, ejecta and gravitational waves as separate sources of energy or mass liberation. Accurate modelling of these phenomena would likely require removing these simplifications and is beyond the scope of this study.

3.2.1 Core-collapse supernovae

Commonly referred to as a Type II, Type Ic or Type IId supernovae, a core-collapse supernova is a stellar explosion of a massive star (of \( 8-20 M_\odot \)), typically while in a giant phase (\( k \in \{ 3, 5, 6, 9 \} \)). The explosion leaves behind a neutron star (\( k = 13 \)) or a black hole (\( k = 14 \)). This event may occur in a system of any stellar multiplicity and the fraction of the exploding star’s original mass lost is typically 50–95 per cent.

If we assume the tightest-known exoplanet orbit around a main-sequence star\(^2\) (\( a \approx 0.015 \) au) and the largest known progenitor mass which results in a neutron star (\( M \approx 20 M_\odot \)), then no planet at any semimajor axis is guaranteed protection if \( \alpha \gtrsim 0.9 M_\odot \text{ h}^{-1} \). This conservative rate is comparable to the mass-loss time-scales that are predicted from supernova ejection velocities (e.g. Hamuy & Pinto 2002; Fesen et al. 2007).

Such planets are likely destroyed by the progenitor during its giant phase. For planets further away, with semimajor axes of 1 au, this critical mass-loss rate decreases by a factor of about 544, and would be enhanced by just a factor of a few in a binary system, as argued in Section 2.2.3. Hence, realistic mass-loss rates from supernova would well exceed the critical mass-loss rate for any planet that could survive the progenitor’s pre-supernova evolution.

Therefore, no planets orbiting stars of progenitor masses of between about 8 and 20 \( M_\odot \) can be guaranteed protection under the criterion of equation (7) and we can neglect sampling this region of stellar mass phase space in our simulations. There are some low-metallicity cases with \( 6 < M/M_\odot < 8 \) where the star might undergo core-collapse supernova. In these cases, which correspond to \( k_+ \in \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \} \rightarrow \{ 13, 14 \} \)

where \( w = 1 \) or \( w = 2 \), we assume that no planet is guaranteed protection.

3.2.2 Black hole core collapse

Consideration of black hole formation in this work is particularly important because all known black holes are found in binary systems (Belczynski et al. 2011). Progenitor stars with masses greater than about 20 \( M_\odot \) typically have extended layers of high density which prevent mass from escaping during an explosion. Nevertheless, the progenitor still forms a black hole through core collapse. The amount of mass loss which may accompany this core collapse is unconstrained by observations and can take on any value. A theoretical upper-bound for the time-scale for this process is 10 s (O’Connor & Ott 2011).

This mass-loss time-scale is orders of magnitude shorter than the mass-loss time-scale for core-collapse supernova. Further, no planet at any semimajor axis is guaranteed protection in a system with the largest theoretically postulated progenitor mass which might undergo core collapse and produce a black hole (\( \mu \approx 300 M_\odot \); Crowther et al. 2010) if \( \alpha \gtrsim 0.015 M_\odot \text{ s}^{-1} \). Therefore, any massive progenitor undergoing supernova-less core collapse need only lose a few tenths of a solar mass to place any orbiting planet in danger of ejection. In a binary system, this amount may be enhanced by a factor of just a few. Hence, we can claim conservatively that only

\(^1\) See the Extrasolar Planets Encyclopaedia at http://exoplanet.eu/
\(^2\) See the Exoplanet Data Explorer at http://exoplanets.org/
in exceptional circumstances is a planet in this class of systems guaranteed to remain bound. This claim allows us to neglect progenitor masses of $M \gtrsim 20 \, M_\odot$ in our simulations.

3.2.3 Thermonuclear supernovae

Commonly referred to as a Type Ia supernova, a thermonuclear supernova is an explosion and probable disintegration of a white dwarf ($k \in \{10, 11, 12\} \rightarrow 15$) owing to mass accretion from a companion. Although this companion is often a giant star, the standard model has subgiant donors. Alternatively, the companion may represent a white dwarf (Pakmor et al. 2010). The velocity of the ejecta is comparable to that from core-collapse supernovae but the total amount of mass ejected is typically less than $1 \, M_\odot$ (Mazzali et al. 2007). Despite this relatively low velocity of ejecta mass, the mass-loss rate is comparable to that from core-collapse supernovae to within an order of magnitude. Therefore, because thermonuclear supernovae afford little more protection for orbiting planets, we treat planets subjected to this type of mass loss in the same way as with core-collapse supernovae and assume that the planets cannot be guaranteed protection.

3.2.4 Common envelope evolution

Interactions between close binary stars resulting from Roche Lobe overflow may precipitate the formation and subsequent ejection of a common envelope. Details of common envelope evolution are complex (Ivanova 2011) and progress for greater understanding is being made through three-dimensional hydrodynamical simulations (e.g. Taam & Ricker 2010; Passy et al. 2012). Therefore, establishing a time-scale for common envelope evolution, $t_{\text{ce}}$, is difficult. However, this time-scale, $t_{\text{ce}}$, is thought to be within a couple of orders of magnitude of the orbital period of the binary. Even the lowest estimates for common envelope evolution time-scales guarantee that some planets remain bound. Therefore, this transition cannot be treated in the same manner as supernovae or black hole core collapse.

In cases where $\text{BSE}$ treats common envelope evolution instantaneously, we assign a value to $t_{\text{ce}}$. A value of $t_{\text{ce}} = 10^7 \, \text{yr}$ represents an upper estimate for this time-scale and with this value we can establish a conservative estimate for escape. We also sample values of $t_{\text{ce}} = 10$, 100 and $10^3 \, \text{yr}$ for relevant regions of phase space.

3.3 The lower mass extreme

We have argued that planetary retention cannot be guaranteed if at least one progenitor star has $M \gtrsim 8 \, M_\odot$. Now, we consider the lower mass extreme.

The lowest mass stars ($M < 0.8 \, M_\odot$) have a lifetime which exceeds the current age of the Universe (Parravano, McKee & Hollenbach 2011). Their mass-loss rates are constant and negligible. By themselves, their mass loss evolution does not perturb a planetary orbit. However, in a binary system, their interaction with a more massive stellar companion could become a crucial aspect of their mutual evolution. Therefore, we include masses down to ($M = 0.2 \, M_\odot$) in our simulations below. For stars with masses $M \ll 0.1 \, M_\odot$, planetary masses may no longer be comparatively negligible to the stellar masses. In that case, a less approximate treatment of the full three-body problem with mass loss would be required.

3.4 Simulation parameter choices

Having restricted the mass phase space to $0.2 \, M_\odot \lesssim M_1, M_2 \lesssim 8.0 \, M_\odot$, reduced the non-linear fitting of mass loss to one parameter, $C_\star$, and established the variable $t_{\text{ce}}$, we can now consider the binary’s initial orbit in more detail. We treat both close and wide binaries in this study, with binary separations of $a_B/R_\odot = 10$, 50, 100, 500, 10$^3$, $5 \times 10^3$, $10^4$ and $5 \times 10^4$, and binary eccentricities $e_B = 0.0, 0.5$ and 0.9. Wide binaries which do not interact with each other typically have separations greater than $10^3 \, R_\odot \approx 47 \, \text{au}$. Therefore, in the extreme case of $a_B = 5 \times 10^4 \, R_\odot$, each component of the binary evolves off of the main sequence independently. In all cases, the planet is considered to be far enough away from the binary both to not affect the orbit of the binary and to have an approximately elliptical orbit around the binary. Each stellar evolution track is simulated for $15 \, \text{Gyr}$, and we adopt $\kappa = 2 \pi$ in all our simulations.

$\text{BSE}$ contains several physical stellar parameters which may be varied. These include the accretion rate on to the secondary and mass-loss prescriptions. In order to focus our study, we use the default values for all physical stellar parameters except for metallicity. For most of our simulations, we adopt either $[\text{Fe}/H] = [\text{Fe}/H]_\odot = 0.02$ or $[\text{Fe}/H] = 0.0001$, although we also sample metallicity values corresponding to $[\text{Fe}/H] = 0.0005, 0.001, 0.005$ and 0.01.

Throughout each stellar evolution realization, the binary’s orbit evolves. The components may interact with each other in complex ways, including merging through coalescence or collision, transferring mass in dynamical, nuclear and thermal regimes, rejuvenating a companion through this mass transfer, and inciting accretion-induced collapse to achieve, for example, a supernova-less $k = 12 \rightarrow 13$ transition. In all cases, the companions remain bound or merge. Only a supernova can unhang the binary orbits. Common envelope ejection occurs beyond the binary’s orbit, and therefore should not affect the binary orbit.

3.5 Simulation results

We present our results through a series of descriptive contour plots. On each, the $x$- and $y$-axes represent the initial mass of the primary and secondary star, respectively. We sampled each initial stellar mass at intervals of $0.2 \, M_\odot$, such that $M_1 \geq M_2$. Therefore, each plot represents a grid of 800 points. We can characterize many aspects of the system evolution with these plots, such as evolutionary complexity, time-scales and mass-loss factors. Our primary interest, however, is to determine the initial critical semimajor axis, $a_{\text{crit}}$, at which a planet is guaranteed to remain bound to the system.

3.5.1 Dependence on binary separation

The initial binary separation, $a_{B_0}$, largely determines the sometimes complex manner in which binary components interact with each other when evolving off of the main sequence. Because the planet is considered to be far from the binary, if $a_{\text{crit}} < a_0$ at any point, then the planet is not guaranteed to remain bound. The actual evolution of the planet in this case is complex and would necessitate detailed modelling.

Figs 3 and 4 demonstrate how $a_{\text{crit}}$ changes as $a_{B_0}$ increases from $10 \, R_\odot$ to $5 \times 10^4 \, R_\odot$ in both the $[\text{Fe}/H] = [\text{Fe}/H]_\odot = 0.02$ and $[\text{Fe}/H] = -0.5$ cases. In both cases, $a_{\text{crit}}$ increases as $a_{B_0}$ increases.

\[ a_{\text{crit}}(a_{B_0}) = \begin{cases} 10 \, R_\odot & \text{for } [\text{Fe}/H] = [\text{Fe}/H]_\odot = 0.02 \\ 5 \times 10^4 \, R_\odot & \text{for } [\text{Fe}/H] = -0.5 \end{cases} \]
Ejection from dying multiple-star systems

Figure 3. Fiducial prospects for planetary retention. The contours on the upper two panels represent the critical semimajor axis at which a planet is guaranteed to remain bound amidst binary stellar evolution as the initial binary separation is increased from left to right. Initially, the binary has a circular orbit and solar metallicity, and we adopt $t_{\text{orb}} = 10^3$ yr and $C_v \to \infty$. The lower panel provides qualitative detail about the evolution of the systems with $a_B = 50 R_\odot$. The contour labels in the lower-left plot are: CE Only, at least one common envelope phase but no supernovae; SN Ia Only, thermonuclear supernova only; SN Ia+CE, thermonuclear supernova plus at least one common envelope phase; SN CoreCo Only, core-collapse supernova only; SN CoreCo+CE, core-collapse supernovae plus at least one common envelope phase; Both SN, both core-collapse and thermonuclear supernovae. The figure demonstrates that circumbinary planets are highly susceptible to ejection.
Figure 4. Low metallicity prospects for planetary retention. The upper two panels are the same as Fig. 3 but for $[\text{Fe/H}] = 0.0001$. The lower panel provides more details on the systems with $a_B = 500 \, R_\odot$. The total fraction of system mass loss over its lifetime is a poor indicator of $a_{\text{crit}}$, whereas the total number of phases experienced by the binary hints at the reason for the vertical green strip at $3 \, M_\odot \leq M_1 \leq 4 \, M_\odot$. Although low-metallicity progenitor masses are prone to core-collapse supernova for $M \gtrsim 7 \, M_\odot$, they afford circumbinary planets slightly more protection than higher-metallicity systems for wide binary separations.
$[\text{Fe}/\text{H}] = 0.0001$ cases. The dearth of black and blue contours on the upper panels of these plots immediately demonstrates how prone planets are to escaping these systems.

First consider the black contours: all orbiting material is protected from ejection for $M_1, M_2 < 0.8 \, M_\odot$ in all cases, and that this protected region is extended to $1.0 \, M_\odot$ for the solar metallicity case. For these lowest mass stars, their proximity to each other is unimportant. The lower panels of the figures demonstrate that in this regime, stellar evolution undergoes only a few phases; indeed, the stars never leave the main sequence. Even after 15 Gyr, the stars do not lose enough mass on the main sequence to have a noticeable effect on orbiting material. This is the only evolutionary pathway that provides safety for Oort cloud comets with $a > 10^4 \, \text{au}$, which are predicted to exist at galactocentric distances beyond the Sun’s (Brasser, Higuchi & Kaib 2010).

Next consider the blue contours, found primarily in the widest initial binary separations, with $a_{B_0} = 5000 \, R_\odot \approx 23 \, \text{au}$. Comparing these two plots in Figs 3 and 4 demonstrates that the higher metallicity case generally allows for greater planetary protection unless $M_1, M_2 > 6 \, M_\odot$, when a core-collapse supernova occurs. A circumbinary planet with $a < 1000 \, \text{au}$ is guaranteed to remain bound if $M_1, M_2 < 2 \, M_\odot$, although in many cases the planet may still be safe further away. This is the expected result from the single-star case where mass loss is treated linearly in each phase (see Paper I), because at this separation the stars each evolve independently from each other. However, this treatment is conservative; later we demonstrate how one of these plots changes when $C_{E}^*$ is decreased and higher accuracy is achieved. For $a_{B_0} = 10000 \, R_\odot \approx 4.7 \, \text{au}$, planets with $a > 1000 \, \text{au}$ may still survive if $M_1, M_2 < 2 \, M_\odot$, as long as $C_e \rightarrow \infty$. The contour plots for $a_{B_0} = 1 \times 10^4 \, R_\odot$ and $a_{B_0} = 5 \times 10^4 \, R_\odot$ are nearly indistinguishable from the $a_{B_0} = 5000 \, R_\odot$ case and are not shown.

For closer separations, the stars are more prone to interact with each other violently. The lowest-right plots in both figures hint at the complexity of the stellar evolution for $a_{B_0} = 50 \, R_\odot$ and $a_{B_0} = 500 \, R_\odot$. Hurley et al. (2002) describe the complete details of the physical mechanisms behind the variations in these contour plots. Here, we just point out some of the most important features.

When $a_{B_0} \leq 1000 \, R_\odot$, prospects for planetary protection are primarily determined by common envelope and supernovae events. The lowest-left plot of Fig. 3 characterizes where in stellar mass phase space for $a_{B_0} = 50 \, R_\odot$ these events occur. They fill almost the entire phase space. Any contours on this plot that indicate a supernova has occurred automatically yield $a_{\text{crit}} < 5 \, \text{au}$ (no protection) in the corresponding upper panel figure, as expected. Some contours which indicate that no supernova occurred also yield $a_{\text{crit}} < 5 \, \text{au}$. In these cases, BSE computes a time-scale for all phase transitions, and the mass-loss rate at some point during the evolution is great enough to prevent planetary protection.

Now consider cases where $5 < a_{\text{crit}} < 300 \, \text{au}$ for $a_{B_0} = 50 \, R_\odot$. Because we adopted $t_{\text{esc}} = 1000 \, \text{yr}$ for all these simulations, the variance in $a_{\text{crit}}$ is due to the amount of mass lost during the common envelope phase. Later, we will explore how the results change when $t_{\text{esc}}$ is altered, especially to less conservative values. The green patch found at $4 \, M_\odot \lesssim M_1, M_2 < 8 \, M_\odot$, $M_1, M_2 < 2 \, M_\odot$ indicates that the fraction of system mass lost owing to common envelope evolution here is lowest.

A conspicuous feature of the $[a_{B_0} = 1000 \, R_\odot, [\text{Fe}/\text{H}] = 0.02]$ and $[a_{B_0} = 500 \, R_\odot, [\text{Fe}/\text{H}] = 0.0001]$ contour plots is a vertical green patch surrounded by yellow and orange contours. This green patch arises because the common envelope in these systems is formed at a different phase of stellar evolution than the surrounding systems, at a phase where less of the system mass is lost.

Finally, the lower-left plot in Fig. 4 displays contours of the total fraction of system mass lost after 15 Gyr. The plot demonstrates that there is almost no correlation, except in the lowest progenitor mass cases, to the corresponding middle panel $a_{\text{crit}}$ plot. This comparison illustrates how the total system mass lost in a system is a poor proxy for $a_{\text{crit}}$ (also compare with fig. 3 of Veras & Wyatt 2012).

### 3.5.2 Dependence on binary eccentricity

Related to initial binary separation is initial binary eccentricity. The pericentre of the orbit helps determine how close to each other the stars orbit. Fig. 5 explores how the $a_{B_0} = 100 \, R_\odot$ and $a_{B_0} = 1000 \, R_\odot$ contour plots from Fig. 3 change when the binary’s initial eccentricity is moderate ($e_{B_0} = 0.5$) or high ($e_{B_0} = 0.9$). The plots reveal that $a_{\text{crit}}$ is relatively insensitive to non-zero binary eccentricities unless the eccentricity is high. In the high-eccentricity case, the plots resemble those obtained from smaller initial binary separations and circular orbits. This is consistent with the finding of Hurley et al. (2002) that it is the initial semilatus rectum, or angular orbital momentum, rather than semimajor axis and eccentricity which determine how a system evolves.

However, there are some differences. For $a_{B_0} = 100 \, R_\odot$, with the initial pericentre at $10 \, R_\odot$, a bifurcation in the plot occurs where $M_1 + M_2 < 8 \, M_\odot$, resembling the circular $10 \, R_\odot$ case. However, the behaviour at $(2 \, M_\odot \lesssim M_1, M_2 < 4 \, M_\odot, 1 \, M_\odot \lesssim M_1, M_2 < 3 \, M_\odot)$ is markedly different. The red patch bounded by this region in the circular case is caused by thermonuclear supernovae. However, in the highly eccentric case, the conditions favourable to producing these phenomena vanish. The resulting value of $a_{\text{crit}}$ is then dominated by common envelope evolution.

For $a_{B_0} = 100 \, R_\odot$, with an initial pericentre at $10 \, R_\odot$, the $a_{\text{crit}}$ distribution is lower than in the circular $a_{B_0} = 100 \, R_\odot$ case. The explanation again relates to thermonuclear supernova. However, here the high eccentricity of the binary promotes the conditions necessary for this explosion, thereby suggesting that for high values of $e_{B_0}$, the dependence of $a_{\text{crit}}$ on $e_{B_0}$ may be complex and best treated on an individual basis.

### 3.5.3 Dependence on metallicity

In Fig. 6, we explore how $a_{\text{crit}}$ changes during the transition between $[\text{Fe}/\text{H}] = 0.02$ (Fig. 3) and $[\text{Fe}/\text{H}] = 0.0001$ (Fig. 4) through the $a_{B_0} = 1000 \, R_\odot$ case. As $[\text{Fe}/\text{H}]$ decreases, primaries with $M_1 \lesssim 7 \, M_\odot$ are more likely to experience core-collapse supernovae. However, for $M_1 \lesssim 7 \, M_\odot$ and $[\text{Fe}/\text{H}] \lesssim 0.005$, the plots are nearly identical. The primary differences occur for $4.0 \, M_\odot \lesssim M_1 \lesssim 7.0 \, M_\odot$ and $M_2 \lesssim 0.6 \, M_\odot$. In this regime, no supernovae occur, and $a_{\text{crit}}$ is dominated by common envelope evolution. The amount of mass lost during this evolution is greatest for the higher metallicity stars. This is the reason for the red strip on the bottom of the upper-right plot in the figure.

### 3.5.4 Dependence on common envelope evolution

In order to demonstrate the dependence of common envelope time-scale on $a_{\text{crit}}$, we select a set of systems for study whose stellar evolution is dominated by a common envelope phase that BSE treats
as instantaneous. The upper-left plot in Fig. 7 presents our selection. The other plots in the figure demonstrate how \( a_{\text{crit}} \) changes when \( t_{\text{ce}} \) is varied from 10 to 10^3 yr (the 10^3 yr case is shown in Fig. 3).

Fig. 7 demonstrates how conservative our fiducial choice of \( t_{\text{ce}} = 10^3 \) yr is. For most of the stellar mass phase space on these plots, the initial orbital period of the binary is less than 10 yr. Therefore, if common envelope formation and destruction proceed on the order of a few dynamical time-scales, few planets would be guaranteed to survive. Those that do would have \( a_{\text{crit}} \leq a_B \), which destroys our assumptions and likely would destroy the planet as well. Additionally, note the sharp transition between the blue and red contours in the \( t_{\text{ce}} = 10^3 \) yr case, indicating the importance of the onset of common envelope evolution.
3.5.5 Dependence on non-linear mass loss

Here we relax the conservative assumption $C_v \to \infty$ for a subset of cases. Doing so for all stellar evolution realizations would be ideal but is too computationally expensive. Further, because $a_{\text{crit}}$ is determined by phase transitions for the vast majority of phase space, relaxing the assumption $C_v \to \infty$ is largely unnecessary. However, for low progenitor masses and large separations, the choice of $C_v$ affects $a_{\text{crit}}$.

Unlike for multiple-star systems, in single-star systems with $M < 8M_\odot$, $a_{\text{crit}}$ is determined by the choice of $C_v$ for the vast majority of the stellar mass phase space.

In Fig. 8, we choose a set of systems where $a_{\text{crit}}$ is dominated by $C_v$ (upper-left plot). This set features widely separated stars which evolve physically nearly independently of one another. We then computed the maximum value of $C_v$ achieved throughout each of the 800 stellar evolution realizations (upper-right plot) and used those to sample values of $C_v$ that would ensure splitting the phases with the most non-linear stellar mass-loss rates into multiple linear stages for the vast majority of simulations. The $C_v \to \infty$ case is plotted in Fig. 3.

Setting $C_v = 0.05$ (lower-right plot) eliminates the dark blue contour from Fig. 3 for $M_1 \gtrsim 2M_\odot$ and reduces $a_{\text{crit}}$ by a factor of at least a few across the entire stellar mass phase space. This result reinforces the need for detailed modelling of individual systems.
that feature highly non-linear mass loss during, for example, the last epochs of giant branch evolution.

4 DISCUSSION

Here, we discuss four relevant related topics to this work. The first compares planetary ejection rates for single and binary stars, which helps inform about their relative contributions to the free-floating planet population. The second importantly considers how the violent behaviour which often accompanies P-type planetary orbits extends to other configurations and higher multiplicities. The third discusses the limitation of our knowledge of some aspects of stellar evolution. In the fourth, we consider partitions of the runaway regime into multiple stages, and show that in some cases there exist analogues to equations (2) and (3).
Figure 8. Critical semimajor axis for planet retention as a function of $C_v$. The upper-left plot, composed of 800 points, demonstrates our motivation for choosing a set of systems with $a_R = 5000 R_{\odot}$: the vast majority of stellar evolution realizations featured in this set of systems feature well-defined and non-violent phase transitions. Therefore, the critical semimajor axis is primarily determined by mass loss within phases rather than in-between phases. The contour labels in the upper-left plot are described in the caption of Fig. 3. The upper-right plot demonstrates that our choices of $C_v^* = 0.10$ and $C_v^* = 0.05$ affect the vast majority of the systems sampled. The $C_v^* \rightarrow \infty$ case is plotted in Fig. 3. Lower values of $C_v$ provide slightly more accurate but significantly more computationally expensive results.

4.1 Escape rate comparison with single stars

Observations suggest that nearly two free-floating planets exist per main-sequence star (Sumi et al. 2011). This vast population cannot be explained by instability-induced planet–planet scattering alone (Veras & Raymond 2012). Other potential sources of free-floating planets arise from escape in evolved single-star and binary-star systems. Although a detailed ejection rate computation based on initial mass functions, binary fractions, distributions of binary separations and semimajor axis-based planetary distributions is beyond the scope of this study, we can provide some qualitative estimates here. The critical semimajor axis is given by equation (8), which is applicable for any stellar multiplicity and a planetary orbit that
surrounds and is far from the central star or stars. This equation contains the total initial system mass $\mu_0$. On the critical semimajor axes contour plots in this paper (Figs 3–8), lines of constant $\mu_0$ would be approximately diagonal from the lower right to upper left.

Therefore, the tight binaries in Figs 3–5 demonstrate that for $\mu_0 \gtrsim 2\,M_\odot$, a planet at several tens of au away from the binary stars is typically prone to escape, and a planet at several hundred au away is prone to escape in nearly all cases. In contrast, direct two-body numerical simulations from Paper I illustrate that for single stars, for no value of $\mu_0$ does a planet with a semimajor axis under 100 au escape. Further, at 500 au, planets begin to escape at $\mu_0 \gtrsim 3\,M_\odot$. Therefore, for a given system mass, planets are more prone to escape in a binary system than in a single-star system.

This conclusion is critically supported by the lack of thermonuclear supernovae (see Section 3.2.3) and common envelopes (see Section 3.2.4) in single-star systems. The bottom left contour plot of Fig. 3 illustrates that either or both of these violent phenomena occur in the vast majority of the stellar mass phase space for that set of tight ($50\,R_\odot$), solar-metallicity binary systems. In particular, the common envelope which typically forms for $2 \lesssim \mu_0/M_\odot \lesssim 6$ values is not present in single-star systems and promotes escape for all reasonable blow-off time-scales ($10^{-10}$ yr). Even in moderately wider binaries, with binary separations of several au, the stellar mass phase space is dominated by common envelope formation and ejection (see the upper-left plot of Fig. 7). If the common envelope blow-off time-scale is as short as 0.1 yr, then the bottom right plot of Fig. 7 demonstrates that nearly all planets, regardless of their semimajor axes, are prone to escape for $\mu_0 \gtrsim 4\,M_\odot$. In contrast, in the single-star case, no planet under 100 au will escape for $\mu_0 \gtrsim 4\,M_\odot$ (right panel of fig. 14 in Paper I).

For the lowest mass systems, with total masses under one solar mass, we can crudely estimate that planetary escape for single stars is negligible based on arguments from Paper I. Similarly, Figs 3–8 show that $\mu_0 \lesssim 1\,M_\odot$ binary systems protect planets within $10^5$ au. These low-mass systems represent the majority of stars in the Milky Way, based on a wide selection of initial mass functions from Parravano et al. (2011). Therefore, we conclude that these systems do not contribute to the free-floating planet population, regardless of multiplicity. For higher-mass systems, as summarized by Bastian, Covey & Meyer (2010), the binary frequency of solar-type stars is approximately 60 per cent; this value increases to nearly 100 per cent for O-type stars. Therefore, more stars appear in binaries than not for $\mu_0 \gtrsim 2\,M_\odot$. Further, as argued above, planets are more susceptible to escape from binaries than from single stars for $\mu_0 \gtrsim 2\,M_\odot$. Therefore, assuming that the number of planets which survive main-sequence evolution is approximately equal in systems of all stellar multiplicities, the contribution to the free-floating planet population is greater in multiple-star systems than in single-star systems for $\mu_0 \gtrsim 2\,M_\odot$. For the intermediate range $1 \lesssim \mu_0/M_\odot \lesssim 2$, the comparison of ejection contributions is less clear, because the binary fraction might be less than half (Bastian et al. 2010).

### 4.2 Higher multiplicities and s-type orbits

A detailed extension of these results to planets on S-type orbits or to stellar systems with more than two stars is non-trivial. However, on qualitative grounds we claim that $a_{\text{crit}}$ for both of these system types is generally lower than for the binary systems studied here.

A planet orbiting one star in a close binary is likely to be destroyed before entering a regime where it is prone to escape. Although a planet may briefly survive engulfment by a stellar envelope of a single star (Bear, Soker & Harpaz 2011), a planet residing in or near a binary common envelope would be subject to three-body forces that would likely cause a collision. Similarly, a supernova by either star might incite a three-body collision or escape if not direct destruction of the planet. A planet’s orbit that is entangled in a mass transfer stream between the two stars would likely harbour a quite interesting but destructive orbit.

If the planet orbits one star in a wide binary, then stellar evolution of the parent star would extend the planet’s orbit, possibly beyond a region of stability in the three-body problem (e.g. Holman & Wiegert 1999; Dominon 2009). If the parent star explodes, the planet is ejected, regardless of the mass of the binary companion. If instead the binary companion explodes, then the planet and its parent star together will become unbound from the companion’s remnant but could remain bound to one another. The planet’s fate then depends on single-star evolution.

Now consider P-type orbits in systems with higher multiplicities. Examples include combinations of stars and possibly other planets which are all orbited by a distant planet in an approximately elliptical orbit. Formation scenarios in which a planet can form from core accretion in a, for example, circumterrestrial disc are yet to be explored fully. However, second-generation planets may form in a variety of exotic systems (Perets 2010) and if the claim by Sumi et al. (2011) that free-floating bodies are more abundant than main-sequence stars is true, these planets may be captured by multiple-star systems. Although additional stars add to the total mass of a system, thereby inhibiting escape according to equation (6), this effect is overshadowed by the increased likelihood of repeated, violent mass-loss events. Further, binary evolution represents the simplest possible outcomes for closely interacting stars, and violent phenomena that have not yet been characterized may exist for systems of higher multiplicity. Many multiple-star systems contain binary components and thus suggests that as a minimum an orbiting planet is subject to the restrictions suggested in this work.

### 4.3 Improved physics

As researchers gain a better understanding of the underlying physical processes featured by RESE, the quantitative results of this work are likely to be modified. Particular events, such as common envelope evolution, are largely unexplored. Only recently has the dynamics of mass transfer in binaries on non-circular orbits been investigated. Sepinsky et al. (2007, 2009, 2010) developed a formalism for mass transfer in eccentric binaries that represents a step forward in understanding this physical process. van Rensbergen et al. (2008, 2010) find that the mass-transfer rate for low-mass binaries is never large enough to allow for mass loss from the system. They claim that matter can escape the binary if the kinetic energy from fast rotation plus the radiative energy of the hotspot exceeds the binding energy of the system. Mass transfer in cataclysmic variables might repeatedly vary by over an order of magnitude owing to nova outbursts (Kolb et al. 2001). Meng, Chen & Han (2008) consider the amount of mass
lost for the highest metallicity stars ([Fe/H] = 0.04–0.1), a regime not treated by BSE. Recent hydrodynamic models of mass transfer between binaries (Lajoie & Sills 2011a,b) attempt to circumvent the limitations of the Roche lobe formalism, which was developed for restricted cases. Although these simulations are just beginning to tackle the complexities of mass loss in eccentric binaries, they demonstrate that some transferred material is ejected, while other becomes loosely bound.

4.4 Multi-phasic runaway regime evolution

Investigation of the runaway regime might help assess the likelihood of planetary escape when Ψ > 1. Non-linear mass loss in this regime may be treated in a similar manner to the adiabatic regime in specific cases. By assuming that mass loss occurs in a series of consecutive linear stages, we may derive equations analogous to equations (2) and (3). However, the evolution of a and e in the runaway regime is more complex (see Paper I) and we have derived analytical evolution equations only when the planet is assumed to begin and hence remain at pericentre or apocentre.

In the first case, for $f = f(t) = 0^\circ$, despite the added complexity of the relations, they couple together to cancel out all intermediate-stage terms so that $a_i$ and $e_i$ may be expressed in terms of $a_0, e_0$ and $\beta$, only so that

$$a_i = \frac{a_0(1 - e_0)}{2 - \beta_i^{-1}(1 + e_0)}$$

and

$$e_i = \beta_i^{-1}(1 + e_0) - 1,$$

such that $a_0$ and $e_0$ are the initial values at the beginning of the runaway stage. Therefore, in this case, the prospects for planetary ejection are independent of both $a_i$ and the intermediate stages of runaway evolution, as long as $a_i$ is high enough to ensure $\Psi \gtrsim 1$. Further, as in the single-stage case, equation (11) demonstrates that the fraction of mass remaining in the star at the moment of ejection is equal to $(1 + e_0)/2$, independent of the details of the intermediate stages.

If $f_0 = 180^\circ$, then the planet’s orbit first circularizes before possibly expanding and causing ejection. The equations leading to this circularization feature the same cancellations, and

$$a_i = \frac{a_0(1 + e_0)}{2 - \beta_i^{-1}(1 - e_0)}$$

and

$$e_i = 1 - \beta_i^{-1}(1 - e_0).$$

Hence, prospects for planetary orbit circularization in this case are also independent of both $a_i$ and the intermediate stages of runaway evolution, as long as $a_i$ is high enough to ensure $\Psi \gtrsim 1$. The planet becomes circularized when the fraction of mass remaining is equal to $1 - e_0$, the same result from Paper I.

If we consider the mass loss to be instantaneous and $f_0$ is known then multi-phasic runaway evolution may be treated as a consecutive series of impulsive approximations (see section 2.7 of Paper I). We can then create relations linking $a_i, e_i$ and $f_i$ to the same variables from all previous runaway stages before ejection. However, because the transition between the adiabatic and runaway regimes is not sharp (as partially evidenced by the presence of $x_i$), attempting to link the evolution in both regimes with the above methods would likely lead to an unphysical result unless the transitional regime is bypassed.

5 CONCLUSION

Multiple stars violently interact in ways that a single evolving star cannot and the effect on orbiting material is hence greater than in the single-star case. Conservatively, planetary material residing beyond a few hundred au orbiting multiple stars each more massive than the Sun and whose minimum pairwise separation is less than 100R⊙ is unlikely to remain bound during post-main-sequence evolution. All Oort cloud analogues in post-main-sequence multiple-star systems would be disrupted and feature escape, independent of stellar separations. Planets residing at just a few tens of au from a central concentration of stars may be subject to escape in a wide range of multiple-star systems. These systems may provide a significant contribution to the free-floating planet population.

The techniques we utilized in order to obtain these results may aid in future studies of individual cases (e.g. Veras & Wyatt 2012). We have shown that the prospects for planetary escape are determined entirely by the evolutionary stage with the greatest mass-loss rate relative to the remaining system mass. Non-linear mass-loss rate profiles within a phase can be analytically treated by a partition into linear segments, where the extent of the partition determines the accuracy of the model. For single stars which become white dwarfs, specific constraints may be placed on the maximum stellar mass-loss rate given assumptions about an orbiting planet.

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