SOME ASPECTS OF STELLAR EVOLUTION—III
 THE FORMATION OF FILAMENTS.

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Summary

There are reasons for believing that a rotating disk of gas, when it becomes gravitationally unstable, might break up into filaments or rings rather than spheres or spheroids. It is shown that filaments formed in this way would break up into fragments which might condense to form the actual stars. This hypothesis avoids the main difficulties that were encountered in our discussion of previous theories. The temperature at which the filaments might be formed is not critical and might be 1000° abs. or thereabouts; the angular momentum of a fragment of the filament is less than the angular momentum of the Sun; and it is reasonable to suppose that the break up of the filament would give rise to just that mixture of single stars and binary and multiple systems which is actually revealed to us by observation.

It may further be supposed that an embryo star would capture a certain amount of scattered material, and that this material would have a much smaller mass and a much larger endowment of angular momentum than the main body of the star. The bulk of this material might condense to form the planets, and the balance might be absorbed by the star. A theory of this type explains why the rotation of the Sun and the orbital revolution of the planets are in the same sense. It also explains, in a simple and straightforward manner, the peculiar distribution of angular momentum in the solar system.

Introduction.—In conformity with the theories already discussed, it is postulated that the first stage in the evolution of a rotating cloud of gas is the formation of a thin disk. It is further postulated that, as soon as contraction is arrested by gas pressure, the thin disk becomes gravitationally unstable and some further development must be expected. It is at this stage that the various theories diverge.

When a fall in temperature causes water vapour in a closed chamber to condense, the condensations which occur in the form of small droplets, and it was not unreasonable to assume that condensations due to gravitational instability in a cloud of gas extending in all directions would follow a similar pattern. It was therefore desirable that a trial should be made to see if a similar theory might be applicable to the case of a rotating disk.

The hypothesis that stellar evolution was a process involving the formation of spherical condensations in a rotating disk of gas was examined in Papers I and II, and it was found that attempts to develop this type of hypothesis soon became involved in apparently insurmountable difficulties. In the present paper the problem is approached from a somewhat different angle.

To begin with it may be remarked that we are called upon to discuss the question of instability in a thin sheet of material, and that this constitutes a problem in two dimensions rather than three.
If we observe the waves which are formed on the surface of a sheet of water, it is noticeable that, in general, the waves take the form, not of humps, but of long continuous lines. If a disturbance of this character is a prelude to gravitational instability, then the condensations formed may be expected to be in the shape of filaments or rings rather than in the shape of spheres or spheroids.

It will also be shown that, in a rotating disk, condensation can take place more readily and more quickly in some directions than in others, and the existence of this factor must also be taken into account.

We are led by considerations of this character to the hypothesis that a rotating disk of gas, when it becomes unstable, would probably break up into a number of long parallel filaments. The next step is to show that the filaments would contract upon themselves, and finally that they would become unstable and would break into fragments which might form the actual stars. The purpose of the present paper is to examine some of the implications of this hypothesis.

Angular Momentum in the Original Disk.—It was shown in Paper I* that the angular momentum of a point \((x, y)\) about a centre of condensation \(C\) is of the form

\[ x^2 V/r + y^2 dV/dr. \]  

Under certain conditions \(dV/dr\) may be negative, and we may then write

\[ \omega = V/r \quad \text{and} \quad \omega' = -dV/dr, \]

so that the angular momentum becomes

\[ \omega x^2 - \omega' y^2. \]

It is clear that the angular momentum is zero along the lines

\[ \omega x + \omega' y = 0. \]  

and

\[ \omega x + \omega' y = 0. \]

In the former case the flow of gas is towards \(C\), and in the latter case away from \(C\). Evidently condensation will take place most readily in the direction defined by equation (2), and particularly so if the time involved is small compared to the period of rotation of the disk. The inference is that gravitational instability will lead to the formation of filaments, and that the axis of these filaments will be at right angles to the direction in which the gas tends to condense.

There are also three other cases to be considered:

When \(dV/dr\) is zero the angular momentum in a radial direction is zero, and there will be a tendency to form rings.

When \(dV/dr\) is positive and less than \(V/r\) the angular momentum is not zero in any direction, but is less in a radial direction than in any other. There will still be a tendency to form rings, but it will be less pronounced.

Finally, if \(dV/dr\) is positive and greater than \(V/r\), the angular momentum is less in a tangential direction than in a radial direction and there will be a tendency to form radial filaments or bars.

It will thus be seen that there is a good case for the suggestion that the condensations formed in a rotating disk would be in the shape of filaments or rings rather than in the shape of spheres or spheroids.

Gravitational Instability in the Original Disk.—Let it be postulated that the temperature in the original disk becomes stabilized at about 1000° abs., and that, being unstable, the disk breaks up into filaments. The arguments which were

developed when we were discussing spherical condensations are also applicable in the present case, and it may be assumed that the diameter of a filament is of the same order as the thickness of the disk.

Let \( t \) = thickness of the disk;
\[ \sigma = \text{mass per unit area}; \]
\[ q = \text{mass of the filament per unit length}; \]
so that
\[ q = \sigma t. \]

In Paper I it was shown that \(*\)
\[ t = 2 \mathbb{R} T / \mu G \sigma; \]
therefore
\[ q = 2 \mathbb{R} T / \mu G. \quad (4) \]

Numerically, for \( T = 1000^\circ \), \( q = 2.5 \times 10^{18} \).

It will be noted that this result depends on \( T \) only and on the constants \( \mathbb{R} \), \( G \) and \( \mu \); it does not depend on \( \sigma \). It is also interesting to remark that \( T \) is apparently determined by the relative proportions of gas and dust, and may be expected to have the same value in different galaxies. It may therefore be expected that the stars in different galaxies would be of the same average mass, and this appears in fact to be the case.

*Lane's Law.*—In the case of a spherical cloud of gas in dynamical equilibrium, density, pressure and temperature can be assumed to be distributed in accordance with a fixed pattern and it is found that certain rules apply during the process of contraction, that is to say

The new radius \( r' = \alpha r; \)
the new density \( \rho' = \alpha^{-3} \rho; \)
the new pressure \( p' = \alpha^{-4} \rho; \)
the new temperature \( T' = \alpha^{-1} T. \)

Similar rules are also found to apply for cylindrical and linear contraction.

In the case of a cylinder or filament

The new radius \( r' = \alpha r; \)
the new density \( \rho' = \alpha^{-2} \rho; \)
the new pressure \( p' = \alpha^{-2} \rho; \)
the temperature \( T \) is constant, a result which will be made use of in due course.

In the case of linear contraction as in the case of a disk

The new thickness \( t' = \alpha t; \)
the new density \( \rho' = \alpha^{-1} \rho; \)
the new temperature \( T' = \alpha T. \)

The pressure \( p \) is constant, a result which is in conformity with the formulae already given in Paper I.*

*The filament: Dynamical equilibrium.*—The gravitational attraction \( g \) at a distance \( a \) due to a long filament of mass \( q \) per unit length is given by the equation
\[ g = 2Gq/a. \quad (5) \]

In the present argument it is assumed that any heat generated in the gas is free to escape, so that \( p = k_0. \)

The precise distribution of density and pressure inside a filament of gas is a complex problem which cannot be discussed here. It is sufficient to assume that the gravitational attraction inside a filament of radius \( a \) and mass \( q \) per unit

* Loc. cit.
length is approximately constant and has the value given by the above equation. The density is then given by the formula
\[ \rho = \rho_0 e^{-br}, \]
and the equation \( d\rho/dr = -g\rho \) gives
\[ kab = 2Gq, \quad (6) \]
where \( k = \frac{RT}{\mu} \).

Now
\[ q = \int_0^\infty 2\pi \rho_0 dr = \frac{2\pi \rho_0}{b^2}. \]

Putting \( \rho = \frac{1}{2} \rho_0 \) and \( q = \pi a^2 \rho \) we get \( ab = 2 \).

Therefore
\[ q = \frac{RT}{\mu G}. \quad (7) \]

It will thus be seen that there is a certain temperature for a filament of any given mass at which there is a balance between the effects of gravity and gas pressure, and that this temperature is independent of the diameter of the filament. If the actual mass of the filament is less than the value defined by this equation, the filament will expand; if it is more, it will contract.

Contraction of the Filament.—In discussing the question of stability in the original disk we obtained the formula
\[ q = 2 \frac{RT}{\mu G}, \]
which is in excess of the critical value defined by equation (7). It follows that, once formed, the filament will contract, the rate of contraction being determined by the escape of heat.

The process of contraction involves an increase in density and pressure, and the increased density may be expected to increase the transfer of heat from the gas to the dust particles, thus facilitating the escape of heat and accelerating the rate of contraction. In the early stages of its history it is to be expected that the contraction of the filament will proceed at an ever-increasing rate.

Eventually the increase of density will render the gas opaque, the flow of heat will be checked, the temperature will rise and contraction will be retarded. When this stage is reached, the filament will become longitudinally unstable and will break up into a number of separate parts.

Angular Momentum in the Filament.—There is no reason to suppose that there is any tendency for a filament of gas, formed in the manner described, to rotate about its own axis. Angular momentum is entirely in the plane of the disk, and it is of two kinds.

First, the filament as a whole tends to rotate about an axis parallel to the axis of rotation of the disk and the angular velocity is of the order \( \omega \).

Now the important point is this. In the case of a filament the process of condensation involves little or no reduction in the length of the filament, and the angular velocity remains substantially constant. Consequently, when the filament eventually breaks up, the angular velocity of rotation is still of the order \( \omega \). This result must be contrasted with the case of a spherical condensation in which the process of contraction involves an increase in the angular velocity.

After the filament breaks up the bulk of the angular momentum is in the form of relative motion of the various condensations, and only a small residue is associated with the rotation of the condensations themselves.
Again, angular momentum in the filament may arise from differences in the velocities of the various parts of the filament in a direction parallel to its axis. This relative motion can be regarded as a form of rotation, and in some cases it may be in the opposite sense to the rotation of the filament as a whole.

Whatever the direction, relative motion of different parts of the filament will be opposed by viscosity, and more especially so when the condition of the gas changes from conductive to convective equilibrium. The effect of the slowing down of the longitudinal currents in the filament is to cause a transfer of angular momentum from the system of longitudinal motion to the system of rotation. It may reasonably be assumed that the angular momentum associated with these longitudinal currents is of less importance than that due to the general rotation.

Anticipating the results of the next section and putting $a$, the radius of a condensation when the filament breaks up, equal to $2 \times 10^{14}$ and $\omega$ equal to $10^{-15}$, the angular momentum per unit mass is of the order $4 \times 10^{13}$. This is the equatorial angular momentum, and should be compared with $3 \times 10^{15}$ which is the approximate angular momentum of the Sun at the present time. It will be seen that the angular momentum of the filament accounts for only a small fraction of the total. The possible origin of the remaining angular momentum of the Sun will be discussed in due course.

Meanwhile, it may be noted that the filament theory of stellar evolution disposes of the difficulty of explaining the low value of the Sun’s angular momentum which arose in the case of the two previous theories.

Break up of the filament.—It has already been pointed out that, once the contraction of the filament has started, the effect of gravity will exceed, for a considerable time, the effect of gas pressure. Eventually, increasing density will delay the escape of heat, the rate of contraction will diminish and the question of longitudinal stability becomes significant.

It is easily shown that a filament of gas which is radially in dynamical equilibrium is longitudinally unstable, so that local condensations will make their appearance and the filament will break up into fragments. These fragments subsequently condense to form the stars.

A point of some interest may be noted in passing. In the early stages of its history, that is to say while it is still transparent, one is justified in postulating conductive equilibrium, but when the filament becomes opaque there must come a change from conductive to convective equilibrium. This change is probably of considerable significance.

If our theory were completely adequate, it should be possible to calculate the dimensions of the filament when the break up actually occurs, but unfortunately our knowledge of the properties of nebular material is not sufficient to justify a precise calculation of this character. We must be satisfied if the theory carries with it no implications which are obviously inconsistent with known facts.

Let $a$ be the radius of the filament when it becomes unstable and breaks up to form the stars. It was shown in an earlier section that the mass of the filament was probably of the order $2.5 \times 10^{18}$ gm. per cm. When the break up occurred, it may be assumed that the distance between the condensations was of the same order as the diameter of the filament, so that the mass of a condensation
would be of the order $5 \times 10^{18}$ a gm. Let us postulate that the average mass of a star is $10^{33}$ gm. Therefore

$$a = 2 \times 10^{14} \text{ cm.}$$

(8)

The opacity of the filament is closely related to its mass per cm$^2$, and the above value of $a$ gives about $10^4$ gm. per cm$^2$, equivalent to about 10 atmospheres. This appears to be a reasonable figure.

*The Planets.*—It is not to be supposed that the filaments postulated in our present theory absorbed the whole of the available material; there must have been a certain residue of scattered material which remained unabsorbed when the filaments broke up to form the stars. Some of this material would lie within the sphere of influence of the individual stars and would, in each case, become part of the stellar system.

In accordance with the arguments of Paper I *, stray material in the neighbourhood of a star is moving relatively to the star itself and its angular momentum is of the order $A^2 \omega$, where $A$ is the original radius of the cloud of material and $\omega$ is the angular velocity of the star about the centre of the galactic disk ($= 10^{-15}$ for our Galaxy). Owing to its high angular momentum, only a small proportion of this material would be absorbed by the central star and the balance would assume the form of a rotating disk and would be available for the formation of the planets.

A theory of the evolution of the solar system is called upon to answer two questions, among others; first, why the rotation of the Sun is in the same sense as the orbital revolution of the planets, and second, why the angular momentum of the Sun is such a small proportion of the whole. The difficulty is to find a theory which will give satisfactory answers to both these questions at the same time.

The filament theory appears to offer a simple and reasonable solution to the problem. It suggests that the material which now forms the solar system was condensed in two different ways. The bulk of the material, all of which went to form the Sun, was derived from a filament and possessed only a small endowment of angular momentum. The balance of the material, of much smaller mass and much higher angular momentum, was condensed at a somewhat later time from a spherical volume of scattered material which had remained unabsorbed in the formation of the filaments. It was this latter material which went to form the planets and provided the angular momentum of the Sun.

The following calculation is also of some interest. The mass of the planets is about $3 \times 10^{30}$ and their combined angular momentum about $3 \times 10^{50}$. Assuming that the material was derived from a spherical volume of gas whose equatorial angular velocity was of the order $10^{-15}$, we put

$$A = \text{radius of the sphere},$$
$$\rho = \text{density of the gas}.$$

and it is easily shown that

$$A = 5 \times 10^{17} \quad \text{and} \quad \rho = 10^{-24}.\quad (9)$$

This value of $\rho$ is quite consistent with the value of $\rho (10^{-21})$ which was taken as the original density of the gas in computing the size of the filament. Since the numerical data on which these estimates are based are quite unconnected, this agreement appears to be of considerable significance.

* Loc. cit.
The theory reviewed.—We are now in a position to review the filament theory of stellar evolution.

It will be remembered that the theory that the stars were formed by a process of direct condensation encountered the difficulty that a reasonable estimate of the temperature of the gas is inconsistent with the known masses of the actual stars. The present theory surmounts this difficulty by postulating the formation of intermediate structures in the shape of filaments. It is assumed that the filaments were formed in hydrogen gas at a temperature of about $1000^\circ$ abs., but this temperature is by no means critical and the theory would be equally effective if it should be found that the correct value of the temperature should be $100^\circ$ or $10,000^\circ$.

Both the earlier theories found difficulty in explaining the low speed of rotation of the Sun. In the present theory this difficulty completely disappears. It is an essential characteristic of the filament theory of stellar evolution that the angular momentum of the individual stars is quite low, indeed definitely less than the angular momentum of the Sun.

It is not to be assumed that the break up of a filament was an orderly process in which condensations were regularly spaced at a uniform distance apart. On the contrary, it may be assumed that a number of incipient irregularities occurred of varying magnitude and importance, and that these disturbances all influenced the final result to a greater or less extent.

It may reasonably be argued, therefore, that a break up of the type suggested would yield precisely the kind of system that is actually observed: a proportion of single stars, and a proportion of binary and multiple systems embracing a wide variety of types extending from close binaries to systems of very wide extent.

The theory that the stars were formed as a result of successive condensations of spherical shape was open to the objection that such intermediate structures would be permanent and would still be in existence if they ever existed at all. A filament, on the other hand, is not a permanent structure and would not be expected to survive for any great length of time. In so far as any trace would remain, it would be in the form of binary and multiple systems exhibiting precisely the characteristic of variety which is actually observed.

It will thus be seen that the filament theory covers, in an adequate and satisfactory manner, the main points with which a theory of stellar evolution may be expected to deal. But it goes further. It provides a satisfactory starting-point for a theory of planetary evolution and explains, without any special or artificial assumptions, how the solar system may have acquired its rather remarkable distribution of angular momentum.

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