THE NUMBER–FLUX DENSITY RELATION
FOR RADIO SOURCES AWAY FROM THE GALACTIC PLANE

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Summary

Two new series of observations at a frequency of 178 Mc/s have been
combined to derive the number–flux density \((N-S)\) relation for radio sources
situated at galactic latitudes \(> 20^\circ\).

In order to investigate the possible errors which might be introduced by
various instrumental selection effects, detailed investigations have been made
both of the distribution in "surface brightness" of the sources, and of the
extent to which radio sources occur in clusters. It is shown that neither effect
introduces an important error in the observed number–flux density relation.
The observations, when corrected for these effects, provide a plot of \(\log N\)
against \(\log S\) which, over the range \(100 > S > 2 \times 10^{-28}\) w.\((c/s)^{-1} \cdot m^{-2}\), may be
approximated by a straight line of slope \(-1.80\). If account is taken of the
errors in the observations and uncertainties in the analysis it is concluded
that the slope lies in the range \(-1.68\) to \(-1.93\).

1. Introduction.—Previous observations of radio sources have revealed an
almost isotropic distribution of sources having small angular diameters; in
areas near to the galactic equator there are in addition a number of extended
sources, but for \(|b| > 15^\circ - 20^\circ\) extended sources are rare and represent only
a small fraction of the total in any given range of flux density.

An analysis of the sources remote from the galactic equator showed that the
number \(N\) having a flux density greater than a given value \(S\) did not vary as
\(N \propto S^{-1.5}\) as would be expected for a uniform distribution of sources; observations
of an area of sky of \(3.2\) steradians, and flux densities between \(200\) and
\(8 \times 10^{-26}\) w.\((c/s)^{-1} \cdot m^{-2}\), indicated a relation \(N \propto S^{-2.0}\) (Edge, Shakeshaft,
McAdam, Baldwin and Archer 1959). It was also shown that, although the
partial resolution of extended sources might give rise to an effect of this kind,
the number of such sources was inadequate to account for the discrepancy.
A similar result \((N \propto S^{-1.8})\) was found by Mills, Slee and Hill (1958) but these
authors considered that the difference between their value of the exponent
and the expected value of \(-1.5\) could largely be accounted for by instrumental
effects.

A recent examination (Bennett and Smith 1961) of both surveys in the light
of new observations at Cambridge (Leslie 1961 a; Scott, Ryle and Hewish 1961)
suggests however that the survey of Mills, Slee and Hill is not suitable for this
analysis since it is probably incomplete in its coverage of the weaker sources
and since a number of the intense extended sources listed appear to be associations
of weak "point" sources. Similar reservations have been expressed by Bolton
(1960).

The new Cambridge observations, which were made with the large 178 Mc/s
interferometer (Ryle 1960), have now been used to derive a more accurate
number–flux relation density than was possible before.
The large resolving power of the instrument has permitted observations, at the level of 1 source per 30 beam areas, of sources having $S \geq 2 \times 10^{-26}$ W. (c/s)$^{-1}$ m$^{-2}$; sources of this flux density can be recorded with a signal-to-noise ratio of 25 : 1. The observations of the more intense sources ($S \geq 6 \times 10^{-26}$ W. (c/s)$^{-1}$ m$^{-2}$) were made both with a total power system and with an interferometer so that any selection effects which might arise from partial resolution of extended sources could be eliminated; at the same time, sufficient information on the distribution of sources with different "surface brightness" could be obtained to allow corrections (which are small) to be made for this selection effect in the survey of high resolution. The survey of the more intense sources covered most of the available sky in order to minimize the statistical errors arising from the small density of intense sources.

Both surveys have been used to establish the extent to which radio sources may occur in clusters (Leslie 1961 b), since a clustering tendency would also affect the apparent number–flux density relation.

The two sets of observations are described in Section 2 and the effects of extended sources, clustering and side-lobe responses in the aerial reception pattern are examined in Section 3. The results of both sets of observations in areas having $|b| > 20^\circ$ are combined in Section 4 to derive the $N$–$S$ relation for $100 > S > 2 \times 10^{-26}$ W. (c/s)$^{-1}$ m$^{-2}$.

2. The observations.

(a) The survey of sources having $S \geq 6 \times 10^{-26}$ W. (c/s)$^{-1}$ m$^{-2}$.—The observations, which have already been described in detail (Leslie 1961), were made over the whole sky between $\delta = -95^\circ$ and $\delta = +60^\circ$. Two sets of measurements were made using (i) the fixed aerial of the instrument (1450 ft $\times$ 65 ft) with a total-power receiver and (ii) both aerials as a simple interferometer of 465\,\lambda spacing.

In both systems the reception pattern to half-power was approximately 14$'$ in $\alpha$ and 4$^\circ$.6 in $\delta$.

The total-power system allowed the determination of the positions and flux densities of 910 sources having $S \geq 6 \times 10^{-26}$ W. (c/s)$^{-1}$ m$^{-2}$, 629 of which lay in areas with $|b| > 20^\circ$; at this level there is approximately 1 source per 20 beam areas. Information on the angular diameter of the sources was derived by combining these observations with those made with the interferometer; for sources having $S \geq 20 \times 10^{-26}$ W. (c/s)$^{-1}$ m$^{-2}$ angular diameters in the range 2.5$'$ to 40$'$ arc could be measured, whilst statistical information on the distribution of angular diameters of the weaker sources could also be obtained.

For sources having an angular diameter greater than 40$'$ arc the detection sensitivity which could be achieved decreased inversely as the diameter and sources whose "brightness temperature" ($T_b$) was less than 500 K could not be detected with uniform sensitivity. Observations of the most intense sources, however, showed none with brightness temperatures in the range 200–10$^4$ K, and only 2 per cent in the range 100–200 K; the latter sources have been identified with nearby normal galaxies. The remainder had $T_b > 10000$ K and 83 per cent had $T_b > 60000$ K.

The omission in these observations of any sources having very low surface brightness ($T_b < 100$ K) is of little importance, since they would be completely resolved with a 465\,\lambda interferometer and would therefore not be observed except at distances where their flux density was less than $0.07 \times 10^{-28}$ W. (c/s)$^{-1}$ m$^{-2}$. 

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Using the flux density provided by the total-power observations, the number-flux density relation for sources having \( S \geq 6 \times 10^{-26} \text{W} \cdot \text{s}^{-1} \cdot \text{m}^{-2} \) and situated more than 20° from the galactic equator is shown in Fig. 1.

(b) The high resolution observations.—The second series of observations used the method of aperture synthesis to obtain the full resolution of the instrument (Scott, Ryle and Hewish 1961). The reception pattern is that of a 465 λ interferometer with an envelope pattern 25′ arc in \( \alpha \) and 35′ arc in \( \delta \).

The present analysis is based on observations of four areas of sky covering nearly 24 hours in \( \alpha \) and \( \delta = 50°-54°, 48°-52°, 40°-44° \) and \( 30°-34°, 21°-30° \); the first two areas were chosen to overlap in order to test the overall operation of the instrument. The total area observed having \( |b| > 20° \) is approximately 0.6 steradian and includes a sufficiently large number of sources for the statistical errors to be small compared with those of the "whole-sky" counts of the more intense sources.

The positions and flux densities of all sources having \( S > 2 \times 10^{-26} \text{W} \cdot \text{s}^{-1} \cdot \text{m}^{-2} \) were obtained, a level at which there is approximately one source per 30 beam areas. The noise level corresponded to a flux density of \( 8 \times 10^{-28} \text{W} \cdot \text{s}^{-1} \cdot \text{m}^{-2} \). The errors arising from the effects of confusion with weak sources may be determined from the probability distribution, \( P(D) \), of the record amplitude \( D \) (Ryle 1958).
The form of the probability distribution depends on the actual $N$–$S$ distribution of the sources, and it may be computed for any given model (Scheuer 1957). In the present case the observations were used to determine an approximate distribution of the sources from which the theoretical $P(D)$ curve was computed. The error in the counts of sources at any given flux density will then be represented by the departure of the curve $\int_0^\infty P(D) dD$ from the assumed $N$–$S$ distribution. The error in the number of sources at the limit of the present survey ($S = 2 \times 10^{-24}$ w. (c/s)$^{-1}$ m$^{-2}$) was in this way found to be about 7 per cent.

Although this paper is concerned only with the number–flux density relation as determined from counts of individual sources, the low noise level on the records allows the use of the experimentally determined $P(D)$ distribution both to avoid this error and to extend the $N$–$S$ relation to sources of smaller flux density; this analysis has been carried out by Hewish (1961).

Fig. 2.—The number of sources per steradian ($N$) having a flux density greater than $S$, derived from four areas of sky observed with the system of high resolving power. The statistical errors shown are appropriate to a single set of observations.

The results obtained for the four areas of sky are plotted separately in Fig. 2; the statistical uncertainty of any one set of observations is shown and it can be seen that there is no significant difference in the counts obtained for the different areas of sky.
In the next section consideration is given to the corrections which may have to be applied to the results on account of source clustering, the presence of extended sources and the effects of side-lobe responses of the aerial system.

3. The effects of angular diameter and clustering of the sources, and of side-lobes of the aerial system.

(a) The presence of extended sources.—The observations of sources having $S \geq 6 \times 10^{-26} \text{ W} \cdot \text{c/s}^{-1} \cdot \text{m}^{-2}$ have provided information on their angular diameters. It is convenient to express the distribution of angular diameters in terms of an equivalent distribution of "surface brightness" so that observations in one range of flux density may easily be related to those in another.

The results obtained by Miss Leslie (1961 a) for areas of sky having $|b| > 20^\circ$ are summarized in Table I, which gives the proportion of the sources in a given range of flux density, which have brightness temperatures ($T_b$) falling in different ranges.

<table>
<thead>
<tr>
<th>$T_b$ (°K)</th>
<th>Percentages of sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>100–200</td>
<td>2 per cent</td>
</tr>
<tr>
<td>200–10⁴</td>
<td>None</td>
</tr>
<tr>
<td>$10^4$–$6 \times 10^4$</td>
<td>&gt; 15 per cent</td>
</tr>
<tr>
<td>$&gt; 6 \times 10^4$</td>
<td>&lt; 83 per cent</td>
</tr>
</tbody>
</table>

Observations of a number of sources have been made with interferometers of larger resolving power, both at Jodrell Bank and at the California Institute of Technology (Bolton 1960). Whilst these observations do not provide complete information on sources of small surface brightness, they allow closer limits to be placed on the distribution at larger values of $T_b$. The results show that 60 per cent of all sources have $T_b > 4 \times 10^5$ °K.

![Fig. 3.](https://academic.oup.com/mnras/article-abstract/122/5/389/2601340)

**Fig. 3.**—The maximum reduction in the source counts which would occur with the system of high resolving power due to sources having the derived distribution of surface brightness.

Using the results of all the observations it is now possible to determine the effect of this distribution on the counts of weaker sources ($S \geq 2 \times 10^{-26}$ W \cdot c/s$^{-1} \cdot m^{-2}$) which were made exclusively with a 465 λ interferometer. Sources whose angular extent exceeds about 6′ of arc will not be observed at all, whilst those with angular diameters in the range 2′–6′ of arc will be recorded with reduced intensity. The apparent number of sources in each range of flux density will therefore be reduced and the maximum effect which would be produced by the derived distribution of $T_b$ is shown in Fig. 3.
It can be seen that for \( S = 6 \times 10^{-26} \text{ w. (c/s)}^{-1} \text{ m}^{-2} \) the error in the apparent number of sources may be as great as 10 per cent; for smaller flux densities the error decreases to 2 per cent. The errors which would arise at \( S > 6 \times 10^{-26} \text{ w. (c/s)}^{-1} \text{ m}^{-2} \) are unimportant since measurements are here available with the total power system.

The 2 per cent reduction, which affects all sources having \( S > 0.15 \times 10^{-26} \text{ w. (c/s)}^{-1} \text{ m}^{-2} \), arises from the presence of a few sources of very low surface brightness \( (100 < T_b < 200 \text{ K}) \); it is therefore convenient to omit these sources from the analysis and to derive, with the corrections of Fig. 3, the number–flux density relation for the main class of source. Such a procedure would not be permissible if the relation were to be extended to \( S < 0.15 \times 10^{-26} \text{ w. (c/s)}^{-1} \text{ m}^{-2} \), although even at smaller flux densities the errors in source number would be only a few per cent.

(b) Source clustering.—If the distribution of the sources in space were not random, errors in deriving the number–flux density relation might arise. If, for example, a significant proportion of the sources occurred in close double or multiple systems they might be observed as individual sources when they were nearby, but as a single and possibly extended source when at a greater distance; it would then be necessary to examine separately the effects on observations made with the total-power system and with the interferometer.

An analysis of source clustering has been made by Miss Leslie (1961 b) in which angular separations of the components from \( 3'5 \) to \( 200' \) arc were investigated; these separations apply to sources having \( S \sim 5 \times 10^{-26} \text{ w. (c/s)}^{-1} \text{ m}^{-2} \). This range of angular separation is sufficient to allow a full determination of the effects of clustering on both total-power and interferometric observations.

Three different sets of observations were made, corresponding to different angular separations in \( \alpha \). The results are shown in Table II which gives, for each set, the maximum percentage of sources occurring in double or multiple systems, with the corresponding limiting flux density.

<table>
<thead>
<tr>
<th>Maximum percentage of sources</th>
<th>Angular separation (min of arc)</th>
<th>Range of flux density (( 10^{-26} \text{ w. (c/s)}^{-1} \text{ m}^{-2} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (a) ) 6 ( 0'5 )</td>
<td>15–45 15–90</td>
<td>( S &gt; 2 )</td>
</tr>
<tr>
<td>( (b) ) 15</td>
<td>2'5–5'0</td>
<td>( S &gt; 5 )</td>
</tr>
<tr>
<td>( (c) ) 4'5 2'4</td>
<td>5–15 5–60</td>
<td>( S &gt; 10 )</td>
</tr>
</tbody>
</table>

It is clear that any sources falling in category \( (a) \) in the table will not introduce errors since they may be adequately resolved for all values of \( S \). The possibility of close associations of sources \( (b) \) was derived from the measurements of angular diameter and is based on the supposition that all the sources previously recorded as being extended with \( 10^4 < T_b < 6 \times 10^4 \text{ K} \) are in fact close multiple systems. It is likely that at least half such systems represent true extended sources (Leslie 1961 b) and the number of double or multiple sources with a separation in \( \alpha \) of \( 2'5–5'0 \) of arc is therefore probably less than 7.5 per cent. It is, however, necessary to consider the alternative possibilities that the whole of the observed effect might be due to \( (a) \) extended sources or \( (b) \) close multiple sources, when
considering the limits of error in the number–flux density relation. It is any in case evident that such sources will not have an important effect, since at $S \approx 2 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$ the angular separation will be so small that the combined intensity will be measured, which is also the case when similar nearer systems ($S > 10 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$) are observed with the total-power system.

It is therefore only those sources for which limits (c) are given which might affect the results appreciably; such sources would be observed with reduced intensity with the interferometric system and the effects at different values of $S$ must be computed. The total error introduced cannot, however, exceed 4.5 per cent in the observed number of sources at a given value of $S$.

(c) The effects of aerial side-lobes.—The observed number–flux density relation would be modified if intense sources received in subsidiary responses of the aerial system were mistaken for weak sources. It can be shown that if the number of sources can be expressed as $N \propto S^{-p}$ then the existence of unrecognized spurious responses does not affect the observed law although it increases the apparent number of sources at a given flux density. Errors may however be introduced if the relation is not a power law; this is of particular importance in the case of the intense sources Cassiopeia-A and Cygnus-A, which have flux densities some 100 times greater than that expected for a power law distribution. Special attention must therefore be paid to the unintentional reception of these sources.

The observed response of the aerial system has already been given (Scott et al. 1961); except near the two principal planes the response is everywhere less than $10^{-4}$ of that in the forward direction. At the same $\delta$ the response in $\alpha$ is only appreciable within $1^\circ$ of the forward direction, the first order maxima having an amplitude of 4 per cent with the second and higher orders less than 1 per cent. At the same $\alpha$, the response in $\delta$ includes several maxima with an r.m.s. amplitude of <6 per cent within $2^\circ$ of the forward direction; elsewhere the response is <0.03 per cent except for a small number (which depends on $\delta$) of single "diffraction grating" responses each 35' in extent and <0.5 per cent in magnitude.

From these figures it is clear that observations of sources having $S \geq 2 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$ are only affected by the presence of sources having $S > 30 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$ in the immediate proximity of the area investigated, and by sources having $S \geq 400 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$ situated at nearly the same $\alpha$.

In reducing the high resolution observations, an area of sky $2^\circ \times 4^\circ$ centred on each source having $S > 25 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$ was therefore omitted, allowance being made for the small reduction (2 per cent) in the total area of sky involved in the observations of weak sources. Narrow strips were also omitted near the $\alpha$ of each of the four intense sources having $S > 400 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$.

In the total power observations the subsidiary responses were smaller, and since only sources having $S > 6 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$ were recorded, it was only necessary to omit a strip near the $\alpha$ of each of the two most intense sources.

4. The results.—The figures obtained from the two sets of observations must now be combined to give the number–flux density relation for the available range of flux density $100 > S > 2 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$. Sources having $S > 6 \times 10^{-26} \text{W} \cdot (\text{c/s})^{-1} \text{m}^{-2}$ were observed by a method which is independent of
angular diameter, and all instrumental effects are likely to be small for $S > 10 \times 10^{-26}$ w. (c/s)$^{-1}$ m$^{-2}$. Weaker sources were observed with an interferometer, and the presence of extended sources may therefore affect the apparent number of sources at a given flux density; an analysis of the distribution of surface brightness of the more intense sources allows an upper limit to be established for the effect at any given value of $S$. The observations have therefore been corrected on the basis of the best estimate of the distribution of surface brightness and limits have been determined, based on the presence of the maximum and minimum possible percentages of extended sources.

![Graph](https://academic.oup.com/mnras/article-abstract/122/5/389/2601340)

**Fig. 4.**—The combined $N$–$S$ distribution for $2 < S < 100 \times 10^{-26}$ w. (c/s)$^{-1}$ m$^{-2}$. The observed counts have been corrected for the effects of source diameter, clustering and confusion. The errors shown include the uncertainty in these effects, the statistical errors and the uncertainty in the scales of flux density used in the two sets of observations.

A similar analysis has been made of the effect of source clustering having the various angular scales considered, and limits have again been determined for the maximum effect on the number–flux density relation. The flux density scales of the two sets of observations were normalized by relating each series to the intense sources for which special observations have been made (Elsmore, Ryle and Leslie 1959) and there is an additional uncertainty of 5 per cent in this scaling.
All the observations, corrected in these ways, are shown in Fig. 4; the limits of error shown include the uncertainty arising from all the effects considered, as well as the statistical uncertainty due to the finite number of sources.

For values of flux density $2 < S < 2 \times 10^{-26}$~$\text{W} \cdot \text{m}^{-2} \cdot \text{c/s}^{-1}$ the observations are best fitted by a straight line having a slope of $-1.80$; the limits of error would allow a slope lying in the range $-1.68$ to $-1.93$.

5. Conclusion.—The new observations have confirmed the earlier conclusions that there is an apparent excess of weak sources or deficit of intense ones. Previous suggestions that the effect might be caused by the presence of extended sources or by a strong clustering tendency have been investigated and it has been shown conclusively that neither effect would be sufficient to produce a number–flux density curve approaching that observed.

The high resolution survey has been further investigated using the statistical method of record analysis (Hewish 1961) in order to obtain information on the $N–S$ relation for values of $S < 2 \times 10^{-26}$~$\text{W} \cdot \text{m}^{-2} \cdot \text{c/s}^{-1}$. The implications of the results obtained are considered in a separate paper (Ryle and Clarke 1961).

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**References**