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ON THE VALIDITY OF NEWTON'S LAW AT A LONG DISTANCE

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Summary

An attempt is made to solve the longstanding problem of the stability of clusters of galaxies by assuming a law of gravitation that implies a much stronger attraction at a long distance than that predicted by the law of Newton. It is further shown that the same hypothesis could provide a solution to a number of other problems in different fields of astrophysics.

1. *Introduction.*—It has been known for a long time (1, 2) that in the great majority of clusters of galaxies the relative velocities of the member galaxies are very large and do not seem, at first, to be compatible with the stability of the clusters. As the reality of this phenomenon seems to have been established beyond doubt, we are confronted with a problem to which no really satisfactory solution has been found so far within the framework of the accepted dynamical laws.

This problem, as well as a number of other problems in different fields of astrophysics, could perhaps be solved if one were prepared to assume that at a very long distance the attraction between two masses m_1 and m_2 decreases more slowly than $1/r^2$ when the distance r increases. For the sake of clarity we shall formulate our hypothesis directly: we shall assume for the force F and potential V expressions such as

$$F=\frac{km_1m_2}{\rho^2}\left(\frac{\rho}{r}\right)^{3/2}, \quad V=-2\frac{km_1m_2}{\rho}\left(\frac{\rho}{r}\right)^{1/2}; \quad (r\gg\rho). \qquad (1)$$

Here ρ is a characteristic length which we shall take to be half a kiloparsec. Equations (1) should be considered, anyway, as merely a first guess.

It should, perhaps, be permissible to consider the possibility that Newton's law may not be valid for distances of this order, as the main experimental confirmation of the law comes from the study of the solar system, where distances smaller by some eight orders of magnitude are involved. Admittedly, we should then be forced to conclude that general relativity also does not apply when distances of the order of 1 kpc are involved. In fact, for the relatively weak fields and low velocities encountered in the study of clusters, the predictions of Newton's law and those of general relativity are basically the same. The application of general relativity to the cosmological problem would then not be justifiable.

It may perhaps be regretted that we do not give an expression for the gravitational force which is valid for every r and which fits Newton's law when r is small. This is due to the fact that we do not know of any astronomical observation from which the behaviour of the gravitational force at an intermediate distance can be

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deduced. On the other hand, we could not have deduced this behaviour from theoretical considerations; in fact, equations (1) themselves represent merely an attempt to explain some observational facts, and do not claim to have a theoretical foundation.

If the attractive forces acting between the galaxies are stronger than Newton's law would predict, the large internal velocities in the clusters could be explained. We mention now some other problems which can, perhaps, be solved by introducing the same hypothesis.

1. The estimate of the mass of the Galaxy, deduced from the motion of very distant globular clusters, (**3, 4, 5**), is about three times larger than the value deduced from the solar motion.

2. If one compares the rotational curve of M31 (the great nebula in Andromeda) obtained from radio observations (**6**) with the curve derived theoretically by Schwarzschild from the distribution of masses (**7**), one sees that at a long distance from the centre of the nebula the radio curve runs clearly higher than the theoretical curve.

Both arguments may indicate that the gravitational attraction of a galaxy decreases more slowly than $1/r^2$ at large r. The slope of the radio curve of M31 at large r seems to suggest the validity of a law such as (1). The radio curves of M33 and of M101 exhibit a similar behaviour (**8**).

3. Helium and heavy elements make up 20 per cent of the mass of the Galaxy. If we adopt Schmidt's estimate (9) of $7 \times 10^{10} M_\odot$ for the total mass, we deduce that the energy liberated in the production of these nuclei amounts to at least 2×10^{62} erg. This estimate is about four times larger than the estimate of the energy which has been radiated by the Galaxy during the 15×10^9 years of its existence. This discrepancy, which has been pointed out by several authors, can largely be corrected by adopting for the total mass of the Galaxy a value about three times smaller, which we deduce if we use equations (1) instead of Newton's law.

4. A recent estimate (**10**) gives $2{\cdot}9 \times 10^9 M_\odot$ for the total hydrogen content of the Galaxy. Taking into consideration the chemical composition (**11**), we deduce that the total amount of the gas is about $4{\cdot}5 \times 10^9 M_\odot$. If the mass of the Galaxy were $2{\cdot}5 \times 10^{10} M_\odot$ instead of $7 \times 10^{10} M_\odot$, the percentage of gas would be about 18 per cent instead of about 6·4 per cent. This higher percentage would probably be easier to reconcile with the fact that there is about 25 per cent of gas in that part of the galactic disk where the Sun is situated.

5. Using equations (1) one can deduce for the mass of M31 an approximate value of $4{\cdot}5 \times 10^{10} M_\odot$; instead of the present estimates, which are between $19 \times 10^{10} M_\odot$ and $45 \times 10^{10} M_\odot$ (**12**). The absolute photographic luminosity of M31 is, on the other hand, $1{\cdot}3 \times 10^{10} L_\odot$; the mass-luminosity ratio in solar units will therefore be 3·5 instead of between 15 and 35. The new lower value of the mass-luminosity ratio is similar to the value which one finds in the galactic disk, a very satisfactory fact in view of the close similarity between the two galaxies. Note also that the mass-luminosity ratio of the nucleus of M31 is, according to Lallemand, Duchesne and Walker (**13**), only 1·95.

6. From the observation of the [O II] λ 3727 emission line from the nucleus of the elliptical galaxy NGC 4278, Osterbrok (**14**) has deduced the approximate value 10 for the mass-luminosity ratio. This value is three or four times smaller than the value of the mass-luminosity ratio of large elliptical galaxies deduced from the velocity dispersion of stars.

This discrepancy could perhaps be accounted for in the following way. In the deduction of Osterbrok, the distances intervening are relatively short, and the application of Newton's law is certainly legitimate. On the other hand, in the derivation of the masses of elliptical galaxies from the velocity dispersion of stars, the distances intervening are of the order of a few kpc, and the use of Newton's law could lead to an over-estimation of the masses.

Clearly, the arguments mentioned above do not carry equal weight. The discrepancy between the two estimates of the mass of the Galaxy, and especially that between the two estimates of the mass-luminosity ratio of elliptical galaxies, could perhaps be accounted for by observational and statistical errors. The argument concerning the abundance of helium in the Galaxy is founded on some cosmogonical hypotheses, which may not be accepted by all astrophysicists.

Some of the arguments mentioned will now be discussed in detail.

2. *The stability of clusters of galaxies.*—In order to study the stability of a cluster of galaxies we first determine its distance from the average cosmological red shift of its members, corrected for the solar motion. For this purpose, we adopt for Hubble's constant H the value 75 km $\sec^{-1}$ per mpc. Next, we make the simplifying assumption that the mass-luminosity ratio f is the same for all the members of the cluster and we adopt therefore, for the masses M_i, the values fL_i, where L_i are the absolute photographic luminosities expressed in solar units. Using statistical methods, we can evaluate the potential energy

$$-\Omega = k\sum_{ij}\frac{M_i M_j}{r_{ij}}$$

from the examination of the cluster on a plate and the kinetic energy $E = \frac{1}{2}\sum_i M_i v_i^2$ from the residuals of the observed red shifts.

The virial theorem states that for stability

$$2\bar{E} + \bar{\Omega} = 0; \qquad (2)$$

here $\bar{E}$ and $\bar{\Omega}$ are time averages. Substituting for $\bar{E}$ and $\bar{\Omega}$ the expressions found for E and Ω, we obtain an algebraic equation from which f can be determined.

The values found in this way for f are surprisingly high; for instance it turns out that the Coma cluster, which has been particularly well studied (**7, 15**), could only be stable if $f \simeq 400$. It is not possible to accept values as high as this since in the cases in which the mass of a galaxy has been evaluated directly, there have been found for f values lower by one or two orders of magnitude.

The phenomenon has been intensively studied in recent years (**16**) and two alternative explanations have been advanced.

(*a*) V. A. Ambartsumyan has made the hypothesis that the clusters are really disrupting, so that the virial theorem does not apply. We shall mention some of the serious objections raised against this assumption.

The spatial distribution of galaxies in some clusters suggests very strongly that the clusters are in equilibrium.

If one accepts the point of view that the clusters are disrupting, then the high velocities that we observe imply that the clusters have existed for a few hundred million years only. However, some of the galaxies in the clusters seem to be very old.

Also in the local group of galaxies the motions seem to be too large for stability (17); however, the relative motion of the two most massive members, the Galaxy and M31, is one of approach.

(*b*) It has been suggested that, besides the luminous galaxies, the clusters contain a large amount of invisible matter which makes them stable. It is then not correct to apply the virial theorem to the galaxies only.

Intergalactic invisible matter could consist of ionized gas, neutral gas, stars, grains or meteorites. However, as we shall see, each of these hypotheses seems to lead to difficulties.

For most clusters the presence of sufficiently large amounts of ionized gas may not be easy to disprove. Let us consider, however, the problem of stability for supercompact clusters (27) like Stephan's Quintet and VV 166, and let us suppose, for definiteness, that the ionized gas is hydrogen. A typical supercompact cluster may have a radius of 10^{23} cm, and the amount of gas that would be required to make it stable may be of the order of $5 \times 10^{12} M_{\odot} = 10^{46}$ g. We may further take the value of 700 km sec^{-1} for the average random velocity of the galaxies in the cluster. To obtain a rough estimate, let us assume uniform density and temperature in the gas. In this way we find a particle density of about $n = 1{\cdot}5$ atoms per cm^3. To deduce the temperature we assume the system to be in equilibrium and we disregard the possible presence of magnetic fields. The kinetic energy per unit of mass of the gas, due to thermal motion, should be equal to the kinetic energy per unit of mass of matter in the galaxies. As the kinetic energy of ionized hydrogen is $2{\cdot}5 \times 10^8\, T \,\text{erg}\,\text{g}^{-1}$, we obtain the approximate value $T = 10^7$ °K for the temperature of the gas.

The rate of radiation by ionized hydrogen is (17)

$$8{\cdot}7 \times 10^{-4}\, T^{1/2} \left(1 + \frac{3{\cdot}85 \times 10^5}{T}\right) n \,\text{erg}\,\text{g}^{-1}\,\text{sec}^{-1},$$

the first term in brackets being due to Bremsstrahlung and the second to recombination. We deduce in the case under consideration a cooling time of about 2×10^7 years, which is much too short. Supercompact clusters seem to be rather rare objects, but the cooling time for the near group of M81 would not be very much longer.

The second hypothesis, namely that the intergalactic material consists of neutral gas, is ruled out without having recourse to any theoretical consideration, as it is known from radio astronomy (18) that the total amount of neutral hydrogen in the Coma cluster is less than $3 \times 10^{12} M_{\odot}$, while the amount required for stability would be about $10^{15} M_{\odot}$.

Intergalactic stars are not unknown. They could, however, account for the abnormally high mass-luminosity ratio of clusters only if their luminosity function were totally different from that of galactic stars.

Moreover, astronomical observation shows that the process of star formation is connected with the presence of gas clouds of high density, and it would therefore be difficult to assume that intergalactic stars originated from the low density intergalactic medium. It would be even more difficult to assume that the original condensation of the matter in the clusters leads directly to the formation of a small number of giant galaxies and to innumerable individual stars, with systems of stars of intermediate size playing a negligible role. For these reasons we believe (17) that intergalactic stars originated in galaxies, probably dwarf galaxies like Fornax and the Sculptor systems, whose gravitational fields were not strong enough to prevent their escape.

It is known that the bulk of visible matter in the universe is contributed by giant galaxies, and that already the contribution of galaxies of intermediate size (around $M_{pg} = -15$) is a small one. It is therefore unlikely that stars that have escaped from dwarf galaxies contribute materially to the stability of the clusters.

Finally, intergalactic matter could not consist of grains and meteorites, as the heavy elements which enter in the constitution of these bodies are formed in the interior of stars.

From the point of view which we have adopted in this paper, the explanation for the apparent violation of the virial theorem should be sought for in the fact that when one uses expression (1) for the potential V, the potential energy $-\Omega$ turns out to be much larger. For example, in the case of the Coma cluster, where the effective mean distance $\bar{r}$ between the members seems to be about 2·3 mpc (**7**), $-\Omega$ is multiplied by a factor of about

$$2\sqrt{\frac{\bar{r}}{\rho}} = 2\sqrt{4600}.$$

On the other hand, the virial theorem reads in the present case

$$4\bar{E} + \bar{\Omega} = 0, \qquad (2')$$

so that f is divided by a factor $\sqrt{\bar{r}/\rho}$. Therefore, in the case of Coma we find for f the approximate value $(400/\sqrt{4600}) = 5{\cdot}9$, which is of the correct order. The situation is basically the same for the other large clusters, like Virgo (**2**), Hercules (**19**), and Corona Borealis.

Van den Bergh (**20**) has applied the virial theorem to pairs of galaxies within the Virgo cluster, and has found in this way a mass-luminosity ratio for the pairs considerably lower than that found for the Virgo cluster as a whole. Assuming equations (1) to be valid, this fact can be satisfactorily explained by taking into account that the distances involved in the case of the pairs are much smaller than those involved in the case of the clusters.

3. *The mass of the Galaxy.*—In the discussion which follows we shall adopt the values $r_0 = 8{\cdot}2$ kpc $= 2{\cdot}53 \times 10^{22}$ cm for the radius of the galactic orbit of the Sun and $v_0 = 220$ km sec^{-1} for the orbital velocity. On the basis of Newton's law, the solar rotation can be phenomenologically accounted for by a mass

$$M = 1{\cdot}82 \times 10^{44}\,\text{gr} = 0{\cdot}91 \times 10^{11}\,M_\odot$$

placed at the centre of the Galaxy.

We know, on the other hand, that the Galaxy constitutes a flattened distribution of masses. Assuming such a distribution, it is possible to account for the solar rotation by means of a total attracting mass considerably smaller than $0{\cdot}91 \times 10^{11}\,M_\odot$. Schmidt, for instance, has built a model of the Galaxy (**9**), consisting of four non-homogeneous oblate spheroids and nine homogeneous spheroids, which reproduces rather well the rotational curve determined by radio observations for $r \leqslant r_0$ (**21**). In his model the total mass M is

$$0{\cdot}7 \times 10^{11}\,M_\odot = 1{\cdot}4 \times 10^{44}\,\text{g}.$$

An independent determination of the galactic mass is due to Kurth (**3**) and Lohmann (**4, 5**), who have used the values determined by Mayall and Kinmann of the radial velocities u_i of a large number of globular clusters, corrected for the effect of the solar motion. These authors replace the galactic field by the field of an equal mass M placed at the centre, a satisfactory approximation for the most distant clusters. If m_i, v_i and r_i are the mass, the velocity and the distance, from the galactic centre of one of the clusters, the virial theorem states that

$$\overline{m_i v_i^2} = k\overline{\frac{m_i M}{r_i}}.$$

From this equation one deduces

$$M = \frac{1}{k}\frac{\overline{v_i^2}}{\overline{\frac{1}{r_i}}} = \frac{1}{k}\frac{\overline{\sum_i v_i^2}}{\overline{\sum_i \frac{1}{r_i}}} = \frac{3}{k}\frac{\overline{\sum_i u_i^2}}{\overline{\sum_i \frac{1}{r_i}}},$$

where the sums are extended to a group of clusters. Admittedly, to derive the velocities from their radial components is less safe in the present case than in the case of the galaxies in a cluster.

Finally, one replaces the time averages by the observed values to obtain

$$M_i = \frac{3}{k}\frac{\sum_i u_i^2}{\sum_i \frac{1}{r_i}}. \tag{3}$$

Applying (3) to the group of the 28 globular clusters most distant from the galactic centre, Lohmann has found for the galactic mass the rather large value $M = 2{\cdot}3 \times 10^{11} M_\odot$.

We shall now consider the same problem, assuming that the galactic field satisfies equations (1). We shall again replace the Galaxy by a mass M placed at the centre.

From the virial theorem we deduce

$$\overline{m_i v_i^2} = \frac{k m_i M}{\rho}\overline{\left(\frac{\rho}{r}\right)^{1/2}}$$

or

$$M = \frac{\overline{v^2}\rho}{k}\overline{\left(\frac{r_i}{\rho}\right)^{1/2}}.$$

Applying this equation to the solar motion we find for M the value $2{\cdot}5 \times 10^{10} M_\odot$, which is 2·8 times smaller than that given by Schmidt. On the other hand, applying the same equation to a globular cluster that is, for example, at a distance of 27 kpc from the galactic centre, we would find a value 7·3 times smaller than the one suggested by the use of equation (3). In other words, if we apply the law of Newton instead of equations (1), we multiply the "correct" mass $2{\cdot}5 \times 10^{10} M_\odot$, by a factor which becomes larger, the farther away the object used for the determination of the galactic mass. This seems to explain the results obtained by Lohmann.

If the galactic field satisfies equations (1), the escape velocity, very far from the centre, will be twice the rotational velocity. This will make probability of escape from the Galaxy very small.

4. *The rotational curve of* M31.—Schwarzschild (7) has represented the mass distribution in M31 by means of a flat non-homogeneous disk. Taking the distance of the nebula to be 630 kpc, the disk has a radius of 18·3 kpc, which

corresponds to an angular extent of 100′. The density of matter in the disk is given by $f\sigma(r)$, where the function $\sigma(r)$ reproduces the surface brightness, as measured by Redman and Shirley, and the constant f is the mass-luminosity ratio chosen so as to fit the rotational curve to the optical measurements of Mayall in the best possible way. In this model the total mass, corrected for the different distance scale, is about $1 \cdot 9 \times 10^{11} M_{\odot}$ and the mass-luminosity ratio is about 15.

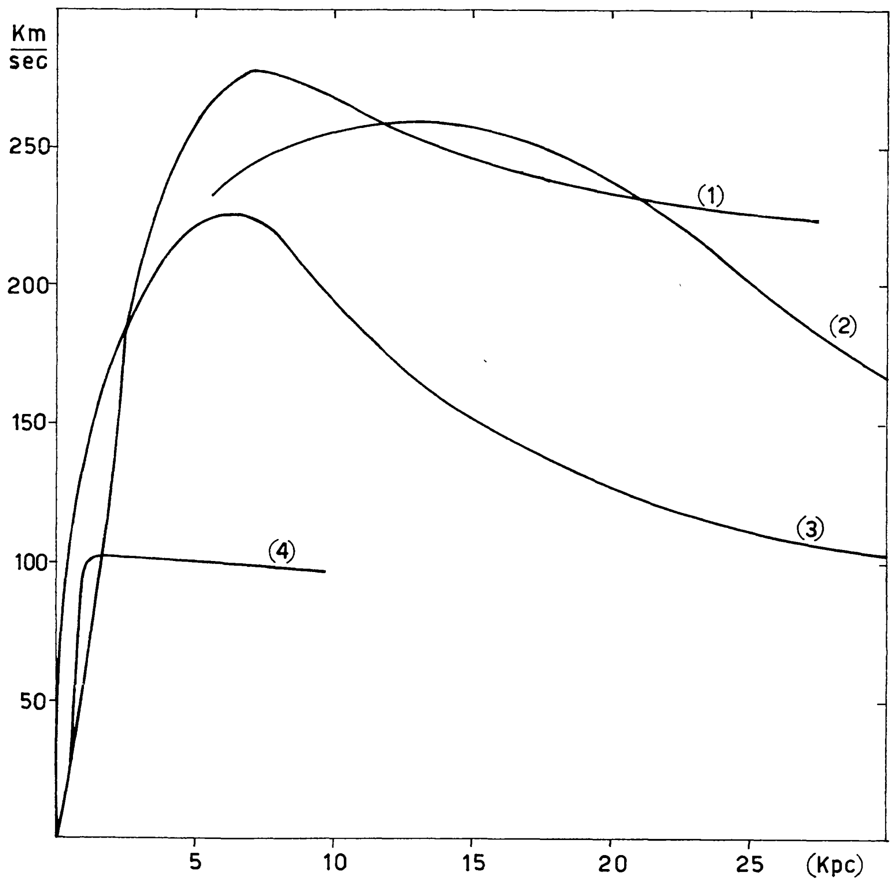

FIG. 1.

(1) M31: *radio curve* (**6**).
(2) M31: *theoretical curve* (**7**).
(3) *The Galaxy: theoretical curve* (**9**).
(4) M33: *radio curve* (**8**).

In Fig. 1 are compared the rotational curve calculated with this model and the one derived by van de Hulst, Raimond and van Woerden (**6**) from the observation of the 21 cm hydrogen line. We note two main discrepancies.

(*a*) For r about 16 kpc the theoretical curve runs higher than the radio curve. This probably means that the surface brightness, and Schwarzschild's distribution of masses, are less concentrated around the galactic axis than the real distribution of masses. This fact would not be surprising since the extreme Population I, which contributes substantially to the total luminosity but hardly anything to the total mass, is in fact less concentrated around the axis than the intermediate and old population.

(*b*) At large r the theoretical curve decreases much more steeply than the radio curve. This fact can be accounted for if one assumes that the gravitational potential of M31 at large r is given by

$$-2\frac{kM}{\rho}\left(\frac{\rho}{r}\right)^{1/2}.$$

In general, if one assumes that at large r the potential is given by

$$-\frac{1}{\alpha}\frac{kM}{\rho}\left(\frac{\rho}{r}\right)^{\alpha},$$

one will find for the rotational curve the asymptotic expression

$$\theta=\left(\frac{kM}{\rho}\right)^{1/2}\left(\frac{\rho}{r}\right)^{\alpha/2}.$$

It is an elementary property of this curve that the tangent at the point

$$\left[r,\ \nu\equiv\left(\frac{kM}{\rho}\right)^{1/2}\left(\frac{\rho}{r}\right)^{\alpha/2}\right]$$

cuts the ν-axis at

$$\nu=\left(1+\frac{\alpha}{2}\right)\left(\frac{kM}{\rho}\right)^{1/2}\left(\frac{\rho}{r}\right)^{\alpha/2}.$$

Then, an inspection of Fig. 1 shows that the choice $\alpha=\frac{1}{2}$ fits the radio curve much better than the choice $\alpha=1$. The rotational curves of M33 and M101, though more difficult to determine exactly at large r, exhibit a similar behaviour (**8**).

It should of course be possible to account for these curves, within the framework of Newton's theory, by introducing a proper distribution of masses at a great distance from the centre. One would, however, be obliged to postulate the existence of very large masses in regions from which very little light is coming (**22**).

We can obtain an approximate determination of the mass of M31 by introducing in the equation

$$\nu=\left(\frac{kM}{\rho}\right)^{1/2}\left(\frac{\rho}{r}\right)^{1/4}$$

the value $\nu=221\ \mathrm{km\,sec^{-1}}$ for $r=27{\cdot}5$ kpc, given by van de Hulst *et al.* In this way we find $M=9\times10^{43}\,\mathrm{g}=4{\cdot}5\times10^{10}\,M_{\odot}$.

A revision of the estimate of the mass of M31 would suggest also a revision of the estimate of the average density of matter in the universe. According to Oort (**23**), the density is equal to $3{\cdot}1\times10^{31}\,\mathrm{g\,cm^{-3}}$. His deduction is based, however, on the assumption that the mass-luminosity ratio is 50 for elliptical and So galaxies, 20 for Sa and Sb, 7 for Sc and irregular galaxies. We have deduced, on the other hand, the value 3·5 for the Sb galaxy M31; the above values for the mass-luminosity ratios should, perhaps, be divided by a factor of about 6; the density of the universe would then be $5\times10^{-32}\,\mathrm{g\,cm^{-3}}$.

It has often been pointed out that estimates of this kind refer to visible matter only, and therefore provide merely a lower limit for the density of matter in the universe. Although the validity of this view cannot be denied, one must realize that the remark is usually prompted by the fact that the astronomical estimate for the density is lower than would be required by relativistic cosmology. Therefore, if one accepts the ideas suggested in this paper, one has no particularly strong reason for suspecting the presence of a very large amount of invisible matter.

5. *The abundance of helium.*—We shall assume that the Galaxy in its initial composition consisted essentially of hydrogen. We shall neglect the energy emitted in the form of radio waves, cosmic rays and neutrinos. We shall then have to account for all the energy liberated in the production of helium and heavy elements, taking into account the age and luminosity of the Galaxy.

In order to evaluate the amount of helium and heavy elements which are present in the Galaxy, let us first consider that part which is contained in white dwarfs, where practically no hydrogen is to be found. 5 white dwarfs have been discovered in a sphere of radius 5 parsecs, with the Sun at its centre. Taking $0{\cdot}7\, M_\odot$ as the average mass of a white dwarf, we find an average density of $0{\cdot}007\, M_\odot$ per cubic parsec. On the other hand, the density of all other visible stars in the solar neighbourhood is $0{\cdot}044\, M_\odot$ per cubic parsec (**24**). We conclude, on the basis of these necessarily poor statistics, that helium and heavy elements contained in white dwarfs make up 14 per cent of the total mass of the Galaxy. There are in fact reasons to suspect that 14 per cent may be a low estimate.

According to a calculation of Salpeter (**25**), the percentage of helium and heavy elements in all other stars is 7 per cent, or about 6 per cent of the total mass of the Galaxy. Finally, helium and heavy elements make up more than one third of the mass of the gas clouds (**11**), or about $1{\cdot}6 \times 10^9\, M_\odot$. If we take Schmidt's estimate for the total mass of the Galaxy, this last contribution is about 2·3 per cent.

We conclude that about 22·3 per cent of the initial hydrogen has been burnt during the history of the Galaxy. We can dispense with the suspicion that by far the greatest part of this hydrogen was burnt in a very early stage. In fact, a substantial proportion of the white dwarfs are young on a galactic time scale, as we can see from their colour and sometimes from their dynamical association with early-type stars in the main sequence.

At least $6{\cdot}4 \times 10^{18}$ erg are produced when one gram of hydrogen is burnt. If 22·3 per cent of the hydrogen has been burnt, the energy produced must have been at least $1{\cdot}4 \times 10^{18}$ erg per gram of matter in the Galaxy. We conclude that the total amount of energy which has been produced is about 2×10^{62} erg. This should be approximately equal to the energy which has been radiated during the history of the Galaxy.

In order to estimate the luminosity of the Galaxy, we can compare it to M31, which belongs to the same type Sb and seems to contain about twice as much hydrogen. The photographic magnitude of M31 has been estimated at $-19{\cdot}92$ (**26**). We add half a magnitude to take into account the absorption of light within M31 itself, and again one magnitude to pass from the photographic to the bolometric magnitudes. We conclude that the energy flux radiated by M31 is equivalent to $2{\cdot}6 \times 10^{10}\, L_\odot = 10^{44}\,\mathrm{erg\,sec^{-1}}$. We assume that the present luminosity of the Galaxy is half that of M31. The luminosity was probably much greater in the past when many young stars were formed from large clouds of gas; we consider it a reasonable guess to take the average luminosity during the history of the Galaxy as twice the present value.

We conclude that $10^{44}\,\mathrm{erg\,sec^{-1}}$ must have been radiated on the average, during a period of about 15×10^9 years $= 4{\cdot}7 \times 10^{17}$ sec. The total energy radiated turns out, therefore, to have been $4{\cdot}7 \times 10^{61}$ erg, which is too small by a factor of about 4. We can, however, obtain a reasonable agreement if we adopt for the mass of the Galaxy the value about 3 times smaller, which we have deduced above using equations (1). Then the amount of helium and heavy elements contained in the Galaxy is reduced by about the same factor.

It may perhaps be argued that a discrepancy by a factor of 4 may not be meaningful in view of the very uncertain estimates involved in the deduction. In this connection, we wish to point out that the deduction is based on Schmidt's estimate of the mass of the Galaxy, which is the lowest among those based on Newton's law. Any attempt to solve the problems raised in Sections 2, 3 and 4 would inevitably require the introduction of large amounts of invisible matter in galaxies. This would lead to a much larger discrepancy in the deduction of this section.

6. *Conclusion.*—How far the arguments mentioned in this paper go towards settling the question of the validity of Newton's law at a long distance may be today a matter of personal opinion. However, relevant observational material is accumulating at a rapid pace, and a clear-cut decision may become possible in the near future.

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References

(1) F. Zwicky, *Helv. Phys. Acta*, **6,** 110, 1933.
(2) S. Smith, *Ap. J.*, **83,** 23, 1936.
(3) R. Kurth, *Zeits. f. Astrophys.*, **28,** 234, 1952.
(4) W. Lohmann, *Zeits. f. Physik*, **144,** 66, 1956.
(5) W. Lohmann, *Sitzungberichten der Österreichister Akad. der Wissenschaften*, II, **169,** 171, 1961.
(6) H. C. van de Hulst, E. Raimond and H. van Woerden, *B.A.N.*, **14,** 1, 1957.
(7) M. Schwarzschild, *Astron. J.*, **59,** 273, 1954.
(8) L. Volders, *B.A.N.*, **14,** 323, 1959.
(9) M. Schmidt, *B.A.N.*, **13,** 15, 1956.
(10) J. H. Oort, *Interstellar Matter in Galaxies*, W. A. Benjamin, Inc., New York, 1962, 237.
(11) J. S. Mathis, *Ap. J.*, **125,** 328, 1957.
(12) L. Perek, *Advances in Astronomy and Astrophysics*, Vol. 1 (p. 282), Academic Press, 1962.
(13) A. Lallemand, M. Duchesne and M. Walker, *P.A.S.P.*, **72,** 76, 1960.
(14) D. E. Osterbrok, *Interstellar Matter in Galaxies*, p. 117, W. A. Benjamin Inc., New York, 1962.
(15) S. van den Bergh, *Ap. J.*, **131,** 558, 1960.
(16) Conference on the instability of systems of galaxies, *Astron. J.*, **66,** 533, 1961.
(17) F. D. Kahn and L. Woltjer, *Ap. J.*, **130,** 705, 1959.
(18) C. A. Muller, *B.A.N.*, **14,** 339, 1959.
(19) G. R. Burbridge and M. E. Burbridge, *Ap. J.*, **130,** 629, 1956.
(20) S. van den Bergh, *M.N.*, **121,** 387, 1960.
(21) K. K. Kwee, C. A. Muller and G. Westerhout, *B.A.N.*, **12,** 211, 1954.
(22) M. Schmidt, *B.A.N.*, **14,** 17, 1957.
(23) J. H. Oort, *La structure et l'évolution de l'univers* (p. 163), Inst. de physique Solvay, 1958.
(24) W. Gliese, *Zeits. f. Astrophys.*, **39,** 1, 1956.
(25) N. Salpeter, *Ap. J.*, **129,** 608, 1959.
(26) N. Humason, N. U. Mayall and A. R. Sandage, *Astron. J.*, **61,** 97, 1956.
(27) S. van den Bergh, *Astron. J.*, **66,** 562, 1961.