MAGNETOSTATIC EQUILIBRIUM OF POLYTROPES

I. W. Roxburgh*

(Received 1965 May 17)

Summary

The equilibrium structure of magnetic fields in stars is investigated assuming the star to be a polytrope and the structure of the field is determined for values of the polytropic index \( n = 0, 1, 1.5, 2 \) and 3, using a first order perturbation theory. As the magnetic body force becomes vanishingly small in the surface layers this method is satisfactory. The first three eigen solutions are determined and it is shown that whereas for \( n \leq 1 \) the number of nodes of the field increases with an increasing ratio of toroidal to poloidal field strength, for \( n > 1 \) the field has no nodes between centre and surface, for all values of this ratio.

1. Introduction. In spite of the considerable amount of energy expended in investigations on the structure of magnetic fields in stars, no detailed results of the structure of possible fields has yet been given. These investigations were principally concerned with general mathematical theorems and were restricted to simple physics rather than being concerned with real stars (see Wentzel (1960) for references to previous work). In most of these investigations the star was assumed to be incompressible, so the results cannot be applied directly to the stellar case. Woltjer (1960) and Wentzel (1961) then considered compressible stars but unfortunately their work was not carried far enough in that they only considered very simple density distributions. In this paper we shall investigate the structure of magnetic fields in polytropic stars and calculate the structure of the field in detail for the cases \( n = 0, 1, 1.5, 2, 3 \). Moreover the investigations of Woltjer and Wentzel were confined to the first eigen solution; in this paper we shall consider the first three eigen solutions, and purely toroidal fields. The structure of a purely poloidal field has been investigated by Monaghan (1965) so we shall not consider the detailed structure of these fields.

The problem of the structure of magnetic fields in real stars has been considered by the author (Roxburgh 1963a, b) and the structure of purely toroidal fields has been calculated, but the general problem with both toroidal and poloidal components of field has not yet been solved, the difficulty arising from the non linearity of the problem. It therefore seems worthwhile to first investigate the polytropic problem in the hope that we may gain some insight into the problem which may ultimately lead to the determination of the structure of magnetic fields in real stars. A similar sort of procedure for rotating stars gave great insight into the rotational problem (Monaghan & Roxburgh 1965, Griffith & Sweet 1965).

Of course it may be argued that polytropic studies are not useful in that a polytropic equation of state implies that the curl of the magnetic body force must

* On leave of absence from King's College, University of London.
be zero, indeed it is this condition that is used to calculate the field. In a real
star the curl of the magnetic force need not be zero since we have another degree
of freedom, the temperature. A star can then have any magnetic field (Mestel
1956), provided of course, that it is not too strong. However, such a field will
in general drive circulation currents in radiative zones (Roxburgh 1963a) and
such circulation will distort the field, and hence change the circulation pattern.
A steady state will be achieved when the circulation is along the field lines of
magnetic field, or when there is no circulation. In a real star it is this condition
which determines the structure of the field rather than the condition that the
magnetic body force be curl free, and there is no reason to expect these different
conditions to give similar results. This is a valid criticism, but the problem
for real stars has so far eluded all attempts at a solution and a solution of the
polytrope problem is a first step in an attempt to solve the problem of magnetic
fields in stars.

2. Formulation of the problem. Inside a polytropic star with no internal
motion, rotation or circulation, the equation of hydrostatic support is

$$\frac{\nabla P}{\rho} = -\nabla \Phi + \frac{j \times H}{\epsilon \rho} \quad (1)$$

where $P$ is the pressure, $\rho$ the density, $\Phi$ the gravitational potential satisfying
Poisson’s equation

$$\nabla^2 \Phi = 4\pi G \rho \quad (2)$$

$G$ is the gravitational constant, $H$ is the magnetic field and $j$ the current density
related to the magnetic field by the time independent Maxwell equation

$$\text{curl } H = \frac{4\pi}{c} j. \quad (3)$$

The magnetic field $H$ must also satisfy the auxiliary equation

$$\nabla \cdot H = 0. \quad (4)$$

For a polytropic configuration the pressure and density are related by the expression

$$P = k \rho^{1+1/n} \quad (5)$$

where $k$ and $n$ are constants, $n$ being the polytropic index.

From equation (1) we then obtain the simple result

$$\text{curl} \left( \frac{H \times \text{curl } H}{\rho} \right) = 0 \quad (7)$$

and it is this equation that determines the magnetic field $H$.

We shall further confine ourselves to solutions with axial symmetry, and
introduce spherical polar coordinates $(r, \theta, \phi), r = 0$ at the centre of the star, $\theta = \pi$
the axis of symmetry ($\partial/\partial \phi$ is then zero). It follows immediately from equation
(1) that the azimuthal component of the magnetic force must be zero since there
is nothing to balance this force, hence

$$(H \times \text{curl } H)_\phi = 0. \quad (8)$$
Magnetostatic equilibrium of polytropes

This is just a special case of the more general problem including velocity that was considered by Mestel (1961) and Roxburgh (1963a). Splitting the magnetic field into two components, \( \mathbf{H}_p = (H_r, H_\theta, 0) \) and \( \mathbf{H}_\phi = (0, 0, H_\phi) \), the divergence condition in equation (4) reduces to

\[
\nabla \cdot \mathbf{H}_p = 0
\]

and equation (8) becomes

\[
\mathbf{H}_p \times \text{curl} \mathbf{H}_\phi = 0.
\]

After some manipulation this reduces to the torque free condition (see Mestel 1961).

\[
\mathbf{H}_p \cdot \nabla (r \sin \theta H_\phi) = 0
\]

which gives

\[
H_\phi = \frac{\beta}{r \sin \theta}
\]

where \( \beta \) is constant along field lines of \( \mathbf{H}_p \).

At this stage it is convenient to introduce the field line function \( S(r, \theta) \) such that

\[
H_r = \frac{i}{r^2 \sin \theta} \frac{\partial S}{\partial \theta}, \quad H_\phi = \frac{-i}{r \sin \theta} \frac{\partial S}{\partial r}
\]

such that the lines \( S = \text{constant} \) are the field lines of \( \mathbf{H}_p \). With the expressions in equation (13) for \( \mathbf{H}_p \), the condition in equation (4) is automatically satisfied. The quantity \( \beta \) is therefore a function of \( S \), \( \beta = \beta(S) \). The condition in equation (7) that determines the field can then be expressed as

\[
\frac{\partial}{\partial r} \left\{ \frac{i}{r \sin \theta} \frac{\partial S}{\partial \theta} \left( \frac{i}{r \sin \theta} \frac{\partial^2 S}{\partial r^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \frac{i}{r \sin \theta} \frac{\partial S}{\partial \theta} \right) \right) + \frac{\beta}{r^2 \sin^2 \theta} \frac{\partial \beta}{\partial \theta} \right\} - \frac{\partial}{\partial \theta} \left\{ \frac{i}{r \sin \theta} \frac{\partial S}{\partial r} \left( \frac{i}{r \sin \theta} \frac{\partial^2 S}{\partial r^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \frac{i}{r \sin \theta} \frac{\partial S}{\partial \theta} \right) \right) + \frac{\beta}{r^2 \sin \theta} \frac{\partial \beta}{\partial r} \right\} = 0.
\]

As it stands the condition in equation (14) cannot be solved for \( S \), since apart from the fact that \( \beta \) is an arbitrary function \( S \), the density \( \rho \) that appears in equation (14) is unknown. It is in fact given by equations (1–5) and itself depends upon the magnetic field \( \mathbf{H} \). To obtain the accurate solution for the magnetic field would therefore require the simultaneous solution of equations (1–5) and equation (14). This has not proved possible so we must proceed using an approximation technique. For magnetic fields that exert a force small compared to the gravitational and pressure force, we may first neglect the magnetic field in the structure equations and solve the spherically symmetric polytrope. This gives a density \( \rho_0 \). This density is then fed into equation (14) which can then be solved for \( S(r, \theta) \). We could then go back to the structure equations and determine the density perturbations \( \rho_1 \), and then replace \( \rho \) in equation (14) by \( \rho_0 + \rho_1 \) and obtain a second approximation to \( S \). We shall not proceed beyond the first determination of \( S(r, \theta) \).

It is not obvious that this approximation technique is valid since although the magnetic body force may be small compared to gravity over the bulk of the
star, in the surface regions of low density the magnetic force may become very large, depending on the structure of the field. It will be shown below that the magnetic force is always small compared to gravity, even in the low density regions.

We are here concerned with obtaining some solutions of the polytrope problem, rather than trying to evaluate all such solutions, and we shall therefore look for dipole solutions and take

\[ S(r, \theta) = A(r) \sin^2 \theta, \quad \beta = CS \]  

(15)

where \( C \) is a constant, so that the magnetic field

\[ H = (H_r, H_\theta, H_\phi) \]

\[ = \left( \frac{2A \cos \theta}{r^2}, -\frac{A' \sin \theta}{r}, \frac{CA \sin \theta}{r} \right) \]  

(16)

where the dash denotes differentiation with respect to \( r \). Equation (16) then reduces to

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{2A \sin \theta \cos \theta}{\rho r^2} \left( \frac{2A''}{r^2} - A \right) - 2C^2 \frac{\rho r^2}{A^2 \sin \theta \cos \theta} \right] \]

\[ - \frac{\partial}{\partial \theta} \left[ \frac{A' \sin^2 \theta (2A'' - A)}{\rho r^2} - C^2 \frac{A A' \sin^2 \theta}{\rho r^2} \right] = 0 \]  

(17)

with \( \rho \) replaced by the zero order spherically symmetric density \( \rho_0(r) \), this gives

\[ A d\frac{1}{\rho r^2} \left( \frac{2A}{r^2} - A^* \right) - \frac{C^2 A^2}{\rho r^2} \]  

(18)

which integrates to give

\[ \frac{2A}{r^2} - A^* - C^2 A = D \rho r^2 \]  

(19)

where \( D \) is an arbitrary constant of integration. This equation can be formally integrated in terms of spherical Bessel and Neumann functions (Woltjer 1960). However this formal representation of the solution is of little help in determining the actual details of the solution so we shall not concern ourselves with this representation.

The boundary conditions on this equation are readily determined. The magnetic field must remain finite at the centre so

\[ \frac{A}{r^2}, \frac{A'}{r} \text{ finite at } r = 0. \]  

(20)

At the surface of the star the field must be continuous with an axially symmetric curl free field. Such a field can have no azimuthal component so that \( A = 0 \) outside the star. Hence \( A' = 0 \) outside the star. Continuity of all components of magnetic field at the surface then gives the condition

\[ A = A' = 0 \text{ at the surface.} \]  

(21)

The one exception is if the constant \( C \) is zero so that \( H_\phi \) is identically zero, th
poloidal field must then be continuous with an external curl free dipole field, so that \( A \propto 1/r \) outside the star, hence,

\[
\frac{A}{r} + A' = 0 \text{ at the surface.} \tag{22}
\]

This is the case that has been considered by Monaghan (1965).

3. *Dimensionless variables.* To facilitate the evaluation of the magnetic field we introduce the following transformations

\[
r = a \xi, \quad \rho_0 = \rho_c \theta_0^n, \quad Ca = \alpha
\]

\[
A = D \rho_c a^4 \gamma, \quad a = \left[\frac{k(n+1)\rho_c^{1/n-1}}{4\pi G} \right]^{1/2}.
\tag{23}
\]

Equation (19) then reduces to

\[
\frac{d\xi'}{d\xi} - \frac{2\gamma}{\xi^2} + a^2 \gamma = -\theta_0^n \xi^2 \tag{24}
\]

where \( \theta_0 \) is given by Emden’s equation

\[
\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\theta_0}{d\xi} \right) = -\theta_0^n. \tag{25}
\]

The boundary conditions on equation (24) are

\[
\gamma = \gamma' = 0 \text{ at } \xi = 0, \quad \gamma = \gamma' = 0 \text{ at } \xi = \xi_1. \tag{26}
\]

where \( \xi_1 \) is the surface value of \( \xi \) at which \( \theta_0 = 0 \). For the special case in which \( \alpha = 0 \), no toroidal field, the surface boundary condition is

\[
\xi \frac{d\gamma}{d\xi} + \gamma = 0 \text{ at } \xi = \xi_1. \tag{27}
\]

Equation (24) is then an eigen value equation for the function \( \gamma \), with eigen value \( a^2 \).

4. *Purely poloidal field.* This is the solution of equation (24) with \( \alpha = 0 \), subject to the boundary conditions (27). For the polytropes \( n = 0, n = 1 \), we have

\[
n = 0: \quad \theta_0 = 1 - \xi^2/6
\]

\[
n = 1: \quad \theta_0 = \frac{\sin \xi}{\xi}
\tag{28}
\]

and we can integrate equation (24) to find

\[
n = 0: \quad \gamma_0 = \xi^2 \left( 1 - \frac{\xi^2}{10} \right)
\]

\[
n = 1: \quad \gamma_0 = \frac{2\xi^2}{3} - 2\xi \sin \xi + \frac{4\sin \xi}{\xi} - 4 \cos \xi.
\tag{29}
\]
5. Purely toroidal field. When $\alpha^2$ is very large we must obviously have $\gamma \to 0$ so that in its present form equation (24) is not very useful. To examine the behaviour of the solution for large $\alpha^2$ we write

$$ g = \alpha^2 \gamma $$

so that

$$ \frac{1}{\alpha^2} \left( g'' - \frac{2g'}{\alpha^2} \right) + g = -\xi^2 \theta_0^n $$

which, for $\alpha \to \infty$, has the solution

$$ g = -\xi^2 \theta_0^n $$

provided $g''$ remains finite. From equations (16) and (23) it follows that

$$ H_r = 0 \quad H_\theta = 0 \quad H_\phi = \lambda r \sin \theta $$

where $\lambda$ is a constant. (The constant $D$, introduced in equation (19) is arbitrary, and to obtain the above solution we have replaced it by $\lambda \alpha/a$. ) This solution will satisfy the boundary conditions provided

$$ \theta_0^n = \frac{d\theta_0^n}{d\xi} = 0 \text{ at } \xi = \xi_1 $$

where $\xi_1$ is the surface value of $\xi$ at which $\theta_0 = 0$. Provided $n > 1$ this condition is satisfied.

However for $n \leq 1$ the expression in equation (32) for $g$ does not satisfy the boundary conditions. This implies that for large $\alpha$, $g''$ must become very large so that the term $g''/\alpha^2$ can be of the same order as $g$. The solution of equation (30) must therefore change sign an increasing number of times with increasing $\alpha$, i.e. $g \sim \cos \alpha \xi$. This is in contrast to the solution of equation (31) which has no nodes. This different behaviour of the solution for $n \leq 1$, and $n > 1$ is brought out in the detailed eigen solutions, where for $n \leq 1$ we find nodes in the solution of equation (24), whereas for $n > 1$ there are no nodes.

The expression in equation (33) for the field is just one particular form of the class of solutions for a toroidal magnetic field in a polytrope. For a purely toroidal field $H_\phi$, equation (7) that determines the field is

$$ \frac{\partial}{\partial r} \left( \frac{H_\phi}{pr \sin \theta} \right) + \frac{\partial}{\partial \theta} \left( H_\phi r \sin \theta \right) - \frac{\partial}{\partial \theta} \left( \frac{H_\phi}{pr \sin \theta} \right) + \frac{\partial}{\partial r} \left( H_\phi r \sin \theta \right) = 0. $$

From equation (14) with $\beta = H_\phi r \sin \theta$, and no poloidal component of field we have

$$ \text{curl} (H_\phi \times \text{curl} H_\phi) = 0. $$

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
Magnetostatic equilibrium of polytropes

On manipulation this reduces to

\[
\frac{\partial}{\partial r} \left( H_\phi r^2 \sin^2 \theta \right) \times \frac{\partial}{\partial \theta} \left( \rho r^2 \sin^2 \theta \right) - \frac{\partial}{\partial \theta} \left( H_\phi r^2 \sin^2 \theta \right) \times \frac{\partial}{\partial r} \left( \rho r^2 \sin^2 \theta \right) = 0
\] (37)

which gives

\[
\nabla \left( H_\phi r^2 \sin^2 \theta \right) \times \nabla \left( \rho r^2 \sin^2 \theta \right) = 0.
\] (38)

This condition implies that the surfaces \( H_\phi r^2 \sin^2 \theta = \text{constant} \) coincide with the surfaces \( \rho r^2 \sin^2 \theta = \text{constant} \), since their normals are in the same direction. Hence

\[
H_\phi = \frac{F(\rho r^2 \sin^2 \theta)}{r^2 \sin^2 \theta}
\] (39)

where \( F \) is an arbitrary function of its argument, \( \rho r^2 \sin^2 \theta \). This is the general solution for \( H_\phi \). Clearly if \( F(x) = \lambda^2 x^3 \) we have

\[
H_\phi = \lambda \rho r \sin \theta
\] (40)

which is the solution of equation (33). This solution is identical to the steady state obtained by the author for a real star (Roxburgh 1962, 1963b). Clearly for \( n \leq 1 \) the solution (40) can only satisfy the surface boundary conditions if \( \lambda = 0 \), i.e. \( H = 0 \).

6. General solutions. For solutions with both a poloidal and toroidal component we must solve the eigen value equation (24), subject to the boundary conditions in equation (26). This was done by trial and error integrations. For a given guess at \( \alpha \) a particular integral and complementary function of equation (24) were obtained that satisfied the central boundary conditions. These were then combined to satisfy the surface condition \( \gamma = 0 \). In general the solution so obtained will not satisfy the condition \( \gamma' = 0 \) at the surface. It is this condition that determines the eigen value \( \alpha \).

In the neighbourhood of the origin \( \xi = 0 \) we solve equation (24) by series expansion, this gives a particular integral

\[
\gamma_p = \xi^2 - \frac{\xi^4}{10} (1 + \alpha^2) + \ldots
\] (41)

while the general solution of the homogeneous part of the equation is \( \lambda_0 \gamma_c \) where

\[
\gamma_c = \xi^2 - \frac{\alpha^2 \xi^4}{10} + \ldots
\] (42)

These solutions were evaluated at \( \xi = 0 \cdot 1 \) for a given \( \alpha \) and then extended by numerical integration up to the surface layers.
In the surface layers we introduce the variable

\[ \eta = \left( \frac{1}{\xi} - \frac{1}{\xi_0} \right) \]  

(43)

and Emden’s equation becomes

\[ \frac{d^2 \theta_0}{d \eta^2} = -\left( \frac{1}{\xi_0 + \eta} \right)^{-4} \theta_0^n \]  

(44)

where \( \xi_0 \) is the value of \( \xi \) at which \( \theta = 0 \). This has the solution for small \( \eta \)

\[ \theta_0 = \sigma_0 \eta + o(\eta^{n+2}) \]  

(45)

where \( \sigma_0 \) is a known constant of integration determined by matching the outer solution for \( \theta_0 \) on the interior numerical solution \( \theta_0 \). In terms of the variable \( \eta \), equation (24) for \( \gamma \) takes the form

\[ \frac{d^2 \gamma}{d \eta^2} + 2\left( \frac{1}{\xi_0 + \eta} \right)^{-1} \frac{d \gamma}{d \eta} - 2\left( \frac{1}{\xi_0 + \eta} \right)^{-2} \gamma + \alpha^2 \left( \frac{1}{\xi_0 + \eta} \right)^{-4} = -\theta_0^\alpha \left( \frac{1}{\xi_0 + \eta} \right)^{-6} \]  

(46)

where the surface boundary conditions are

\[ \gamma = 0 \quad \frac{d \gamma}{d \eta} = 0 \text{ at } \eta = 0 . \]  

(47)

Equation (46) can now be solved by series expansion in \( \eta \); this gives

\[ \gamma = -\frac{\sigma_0 \eta \xi_0^6}{(n + 1)(n + 2)} \left( \eta^{n+2} - \frac{2 \xi_0}{(n + 3)} \eta^{n+3} + \ldots \right) . \]  

(48)

This solution was evaluated at \( \xi_0 \eta = 0 \cdot 1 \). The condition of continuity of \( \gamma \) between the inner and outer solutions determines the unknown constant \( \gamma_0 \) that appeared in the inner solution. The solution for \( \gamma \) so obtained will not, in general have \( \gamma' \) continuous at the interface between the inner and outer solutions. It is this condition that determines the eigen value \( \alpha \).

Detailed computations were done for the polytropes \( n = 0, 1, 1.5, 2, 3 \), and the first three eigen values are given in Table I and the corresponding eigen solutions are illustrated in Figs. 1–5.

**Table I**

<table>
<thead>
<tr>
<th>( n )</th>
<th>0</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>2.334</td>
<td>2.356</td>
<td>2.459</td>
<td>3.732</td>
<td>6.676</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>3.678</td>
<td>3.691</td>
<td>3.831</td>
<td>5.811</td>
<td>7.116</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>4.978</td>
<td>4.398</td>
<td>4.025</td>
<td>6.171</td>
<td>7.625</td>
</tr>
</tbody>
</table>
Fig. 1. Variation of magnetic field strength along the axis of symmetry. First three eigen solutions, \( n = 0 \).

Fig. 2. Variation of magnetic field strength along the axis of symmetry. First three eigen solutions, \( n = 1 \).

Fig. 3. Variation of magnetic field strength along the axis of symmetry. First three eigen solutions, \( n = 1.5 \).
7. Discussion. The polytrope $n=0$ is the liquid star and the solution of the magnetic field in a liquid star problem has been obtained by Prendergast (1956). The first eigen solution and eigen value for the polytrope $n=1$ has been obtained by Wentzel (1961). In Table II we compare their values for the eigen value $\alpha_{2.0}$ with the value obtained above. (The values given by Prendergast and Wentzel have to be divided by $\xi_0$ to obtain the value to be compared with our results).
The agreement is very good. Moreover the inaccuracy of our method decreases with increasing polytropic index since the mass in the outer layer becomes less with increasing $n$, so that better accuracy should be obtained for $n > 1$.

The main feature of the solutions is the different behaviour for the cases $n \leq 1$, and $n > 1$. In the first case the solution has nodes between $\xi = 0$ and $\xi = \xi_1$, whereas for $n > 1$ there are no nodes. The general behaviour of the solutions can be understood when we realise that for $n \leq 1$ then as $\alpha$ increases the field must tend to zero and oscillate an increasing number of times in order that $\gamma''$ can be of the same order as $\alpha^n \gamma$. However for $n > 1$ the solution for increasing $\gamma$ gradually goes over to $-\xi^2 \theta_0 \gamma$ which has no nodes between centre and surface and $\gamma''$ is much less than $\alpha^n \gamma$.

The parameter $\alpha$ is a measure of the toroidal component of field to the poloidal component.

$$\alpha = \frac{2 \cot \theta \frac{H_\psi}{\xi}}{\frac{H_\phi}{H_r}},$$

so that increasing the ratio $H_\psi/H_r$ corresponds to increasing $\alpha$. For polytropes with $n \leq 1$ this leads to field with an ever decreasing scale due to the increasing number of nodes, and we would deduce that in stellar conditions such a field would rapidly decay. However for $n > 1$, we have no nodes and the scale of variation of the field does not change, so that in normal stellar conditions such a field would have a long decay time, no matter how large was the ratio $H_\psi/H_r$.

Having determined the solutions it remains to be shown that the magnetic force is small compared to gravity even in the low density surface regions. Equation (48) gives

$$\gamma = a \eta^{n+2} + \ldots .$$

From equation (17) we see that for small $\eta$

$$|\text{curl} \mathbf{H} \times \mathbf{H}| \propto A' A''$$

near the surface, so that the terms of lowest power in $\eta$ give

$$\frac{\text{curl} \mathbf{H} \times \mathbf{H}}{\rho} \propto \frac{\gamma' \gamma''}{\theta_0^n \gamma_{\theta}^{2n+1}} = \frac{\eta^{2n+1}}{\eta^n}$$

which tends to zero for all $n > 1$. The magnetic body force therefore tends to zero as we approach the surface so that it is always small compared to the gravitational force.
The solutions presented here are of interest in that they show how one may compute the magnetic fields inside a compressible body without much difficulty, and also show the variation of field strength within the star. It is hoped that these solutions will be of assistance in the general problem of magnetic fields in stars by giving a first guess at the structure of the field which will be necessary in order to develop a satisfactory iteration procedure to solve the stellar problem.

The solutions given here apply only to the magnetohydrostatic problem for polytropes. In a subsequent publication we shall extend this work to magnetohydrodynamic equilibria of polytropic stars.

Acknowledgments. This work was done while the author was the recipient of a National Academy of Sciences/National Research Council visiting Fellowship at the Goddard Space Flight Center, Greenbelt, Maryland, and the author would like to record his gratitude for this support. The author is also indebted to Dr J. Monaghan for making available work prior to publication.

High Altitude Observatory,
Boulder, Colorado, U.S.A.:
1965 May.

References