COMMENTS ON 'DISTRIBUTION OF CRATERS ON THE LUNAR SURFACE'

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Summary

In a recent study of the distribution of centres of lunar craters, Fielder found two apparent anomalies which he used as arguments against the meteoroidal impact hypothesis for the origin of lunar craters. First of all, the Poisson distribution gave a very bad fit to the number of crater centres in equiareal sectors of the continents and the maria, especially among craters smaller than 40 kilometres in diameter. The second difficulty was a systematic excess in the number density of small craters in the western (trailing) half of the Moon. We will show that these observations could also have been expected even under the impact hypothesis, since the numbers of small and of large craters in a finite region are negatively correlated. Available crater statistics therefore neither preclude nor establish the impact or volcanic hypothesis for the origin or craters.

1. Introduction. The purpose of this note is to examine a recent empirical study by G. Fielder (1965) on the diameter frequency, distribution of centres, and distribution of number density of lunar craters. His results are compared with certain theoretical studies by the author (Marcus 1966a, 1966b, 1966c). We find that the cumulative number of craters larger than diameter $\Delta$ follows an inverse power law $CA^{-s}$ with index $s=2.0$, for craters larger than 30 km in diameter: this suggests that the original distribution of crater diameter is an inverse power law with index greater than 2. The distribution of centres of craters larger than 40 km appears to be random, but the distribution of centres of craters smaller than 40 km is less random although the amount of deviation from randomness may not be incompatible with an initially random distribution. The apparent deficiency in number density of craters in the eastern (leading) half of the Moon, relative to the western (trailing) half, is partially explained by the negative correlation between the numbers of small and large craters in a region of finite size.

2. Diameter frequency of craters. The relevant data are collected in Table I, Table VI, and Table VII of Fielder's paper (1965). The statistic of interest for Table I is $N(70)(\Delta)$, the total number of craters within 70° of the centre of the lunar disk, whose diameters exceed $\Delta$. For 30 km $\leq \Delta < 300$ km the author obtains an excellent fit to Fielder's data with

\[ N(70)(\Delta) = 6.1 \times 10^5 \Delta^{-2}. \]  

See Fig. 1 of the present paper.

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For $300 \text{ km} \leq \Delta < 1 \text{ 300 km}$ the author obtains a good fit with

$$N(70)(\Delta) = 1 \cdot 4 \times 10^6 \Delta^{-2}. \tag{2}$$

Fig. 1. **Cumulative number** $N(70)(\Delta)$ **of craters larger than diameter** $\Delta$ **(after Fielder).**

The objects larger than 300 km are the circular maria. One cause of the discrepancy between equations (1) and (2) is the small number of these objects (only ten). Another is the fact that if the circular maria are in fact lava-flooded basins, their original diameters must have been smaller than their present apparent diameters (Quaide 1965). The discrepancy between equations (1) and (2) is reconciled by assuming that the present diameter of the maria is 51% larger than the original diameter, since $14/6.1 = (1.51)^2$ approximately.
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It is seen from Table VI (Fielder 1965) and our Fig. 2 that for craters between 30 and 150 km in diameter in lunarite in the eastern half of the Moon, the cumulative number of craters is again of the form $C\Delta^{-2}$. The same could be said of craters in the western half of the Moon, except for a conspicuous deficiency of large craters. Only 19 craters larger than 100 km are observed, where 28 would be expected if the crater frequency was in correct area proportion to that observed in the eastern half of the Moon (dashed line in Fig. 2 gives expected frequency). This discrepancy will be important in later work.

![Graph showing cumulative number of craters vs. diameter](https://example.com/graph.png)

**Fig. 2.** Cumulative number $N(\Delta)$ of craters larger than diameter $\Delta$, in lunarite.

Analysis of data in Table VII and our Fig. 3 shows that the cumulative number of craters larger than diameter $\Delta$ in lunabase is also of the form $C\Delta^{-2}$ approximately, for craters larger than 20 km. The same is true for craters in lunabase in the eastern half of the Moon, except for a deficiency of craters larger than 70 km (11 expected, 6 observed). Because the size distribution of craters can vary greatly from one mare to the next, (Dodd, Salisbury & Smalley 1963), it may be misleading to compare mixtures of distributions from several maria. It is our understanding that in a future paper (Fielder 1966), Fielder will use homogeneous data drawn from a single lunabase region.

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The author has shown (Marcus 1966b) that if the original size distribution of craters is characterized by a probability density of the form $p(\Delta) = a\Delta^{-\gamma-1}$, where $\gamma > 2$, then the cumulative number of craters larger than diameter $\Delta$ will be of the form $C\Delta^{-s}$. If $s = 2$ we may infer that crater-forming activity has dominated crater-filling activity, and if $s < 2$ we draw the opposite conclusion. Fielder's data support the statement $s = 2$ for craters larger than 20 or 30 km in diameter, with $s < 2$ for smaller craters. We will interpret this as the preferential filling of small craters by internally produced crater-filling mechanisms such as volcanic lava, although Fielder* believes that the interpretation of $s < 2$ for small craters as differential filling is based on incomplete data and would prefer to draw no such conclusions from his own data. Differences in the amount of crater-filling activity among various regions of the lunar surface may be a very important source of nonhomogeneity of crater counts.

* Personal communication from Fielder, 1965 November 12.
3. Distribution of crater centres. One of the major considerations in the controversy over the origin of lunar craters is the randomness or otherwise of the distribution of crater centres. If the centres of craters were not initially distributed at random over a region of the lunar surface, one would have to reject the meteoroidal impact hypothesis in favour of a hypothesis ascribing crater formation to internal causes—broadly speaking, volcanism. The main problem is that even if crater centres were initially random, the obliteration of older small craters by recent large ones would cause clumpiness in the distribution of centres of observable craters. We would expect to find a surplus of small craters in a region free of large craters, and a dearth of small craters in a region in which large craters are present. The distribution of the number of crater centres in equiareal sectors of the lunar surface will therefore not be Poisson (random), but will be of a type with larger relative variation than the Poisson distribution—perhaps approximately negative binomial (Marcus 1966c).

It is therefore not surprising that the larger the number \( n \) of craters of a given size and the number \( \nu \) of degrees of freedom, the smaller the probability that the sample is drawn from a Poisson distribution, since it is not. This tendency is conspicuous in Fielder's Table IV, which gives the probability of exceedance of the chi-squared criterion of goodness of fit of the Poisson distribution to the observed distribution of crater centres in lunarite. The smallest probability for craters of diameter 40 km or larger is 0.39, which indicates fairly good agreement with the Poisson distribution for moderately small values of \( n \), \( \nu \leq 58 \). For lunarite craters of 10–20, 20–30, and 30–40 km, we have chi-squared probabilities of 0.14, 0.01, 0.02 respectively, for \( n \) moderately large (438, 223, 105 respectively), indicating that the Poisson distribution does not give a good fit. For craters between 5 and 10 km, and \( n \) of 650, the probability is \( < 10^{-7} \), indicating a very poor fit indeed. The negative correlation between number \( n \) and probability of chi-squared criterion is evident, but is not the only explanation for the very low probabilities obtained in the four small-diameter classes.

It has been shown (Adler et al. 1964) that in certain lunarite regions such as the Cratered Plain Province, there are significantly fewer small craters than in other lunarite regions. The author's work supports their conclusion that the amount of crater-filling activity has varied considerably among various lunarite regions and must therefore be of internal, i.e. volcanic, origin (Marcus 1965b). This gross variation in the average density of small craters in lunarite will make the distribution of the number of crater centres in equiareal sectors a mixture of distributions, the mixture differing greatly from Poisson type (Marcus 1966c). The author believes that non-homogeneity of the lunar surface, which is especially noticed among craters smaller than 40 km in diameter, may explain the low probabilities in the last four rows of Fielder's Table IV. In Fielder's new paper (1966), considerable efforts are made to overcome the problem of non-homogeneity. Another possible cause of clumpiness of the distribution of centres of small craters is the existence of secondary impact craters (Shoemaker 1962) which would produce irregular clusters of small craters in some regions.

4. The distribution of number densities. Fielder has pointed out in his Table VI and Table VII that for most craters of moderate and small size, the number density is larger in the western (trailing) half of the Moon than it is in the eastern (leading) half, the converse of what one would expect if the meteoroidal impact
hypothesis were true. A possible explanation is found in two facts mentioned earlier: that there is a negative correlation between the numbers of large and small craters found in a finite region, and that there is a relative deficiency of large craters in the western half of the lunar region. One would therefore expect to find a surplus of smaller craters in the western lunar region. The amount of the surplus is not unreasonable either. The reader should consult Marcus (1966b) where it will be seen that reducing the maximum crater diameter $x_M$ will decrease the value of the function $\Lambda_1(\Delta)$ and lead to an increased value of the expected number density for craters of diameter less than $x_M$.

From this point of view, the relative deficiency of both large craters and circular maria in the western half of the Moon may have some significance as an argument in favour of the meteoroidal impact hypothesis. The converse deficiency of large craters in lunabase in the eastern half of the Moon may be explained ('explained away', perhaps) by differences in the amount of crater-filling activity in different maria—there are surely some very large ghost craters, e.g. Stadius*.

5. Impacts versus volcanism. Fielder's studies, while valuable and illuminating in their own right, neither preclude nor establish the impact hypothesis or the hypothesis of volcanism for the origin of lunar craters. It should be remembered that even if one accepts the essential randomness of crater centres, one has still not excluded the possibility of lunar volcanism, which need not be closely analogous to terrestrial volcanism.

We have not mentioned chain craters. These objects are almost certainly of internal origin, and one would be interested in knowing what proportion of lunar craters are of this type. This may actually be a most fruitful problem in the statistical study of lunar craters: several interesting aspects of the problem are discussed in Fielder's forthcoming paper.

The author is most grateful to Dr Fielder for several stimulating discussions about this paper.

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References


* Fielder regards many rings like Stadius as young features, not flooded craters. (Personal communication, 1965 November 12.) As Fielder points out, much rests on the assumptions of the nature and age of such rings as Stadius.