THE FORMATION OF STARS WITH PARTICULAR APPLICATION TO TEMPORARY STARS AND QUASARS

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SUMMARY

It is assumed that all gas clouds have tendencies to fragment into unstable cloudlets (or floccules as McCrea (1) has termed them). A number of conditions which these cloudlets must satisfy before they play an important part in the development of a contracting gas cloud are postulated. Formulae giving the average mass of the stars that form out of a contracting gas cloud in terms of various parameters are then derived.

When numerical values lying within reasonable ranges are inserted, it is shown that a gas cloud of average stellar cluster mass, $10^{36} \text{ g}$ to $10^{38} \text{ g}$, will form stars with a mass inside the usual stellar mass range. The formation could occur in either a spherical or a flattened stage. An initial cloud of mass $10^8 M_\odot$ forms objects of mass slightly above the stellar range and these can, therefore, be classified as temporary stars. These are shown to explode at a rate of about 30 per year as is required in order to explain the quasar phenomenon, the linear dimensions of the whole group of temporary stars is of the order of a light year.

I. INTRODUCTION

The formation of stars from gas clouds has been a problem of interest to astronomers for a long time and many attempts have been made to solve it. One such theory is that by McCrea (1), where he investigates the formation of stars within the context of the problem of the origin of the solar system. He suggests that an initial gas cloud, from which a number of stars will eventually form, tends to fragment into small cloudlets, or floccules as he calls them. An individual cloudlet is not a stable condensation under its own gravitational field and so cloudlets are continually dispersing while new ones reform. In his numerical discussion McCrea assumes that the floccules move with a mean random speed of about 1 km s$^{-1}$, their mean free path for collision, $L$, being about $5 \times 10^{14}$ cm. He further assumes that about $10^5$ floccules are present in a sphere of radius $L$ and that the mean cloud density at this stage is $4 \times 10^{-12}$ g cm$^{-3}$. The mass and radius of the floccules are determinable in terms of these quantities and McCrea finds that their values are $2.1 \times 10^{28}$ g and $9.1 \times 10^{11}$ cm respectively.

When two floccules collide, the parts directly concerned in the collision will coalesce while the other parts proceed as smaller floccules. A larger than average floccule may thus result from a collision. By a series of such favourable collisions, minor condensations are continually being formed throughout the cloud. Some of these may survive, some clearly do not. A reasonably large condensation, containing 18 floccules in the case discussed by McCrea, will become gravitationally stable. It will therefore attract further material to itself, resulting in a growing condensation. McCrea argues that whatever happens inside a region of linear dimensions about
equal to \( L \) is likely to be repeated in other regions. Each such region will develop more or less independently, inside which only one main condensation is likely to form. With the numerical values adopted by McCrea, the mass of this main condensation is about one solar mass and the remnants left in this region are shown to possess mass and angular momentum consistent with the values observed in the solar system.

In this communication we attempt to generalize the concept of a floccule so that the numerical values for the floccule properties can be deduced given the initial conditions in the contracting cloud, rather than having to specify them as McCrea did. If we are successful in this then we can deduce whether the mass of the object finally formed is likely to be very dependent on the initial cloud conditions. We can also determine whether considerably larger condensations can be produced by varying the initial cloud conditions within reasonable limits. This last application has considerable interest in view of a suggestion, also coincidentally made by McCrea (2), regarding the nature of quasars.

This proposal is a modification of a theory by Burbidge (3) who suggested that both the very large energy output and the very rapid fluctuations of the optical emission which are observed in quasars could be explained in terms of the firing off in rapid succession of a number of supernovae. (This theory assumes that quasars are cosmological in origin and that the redshift observed is predominantly because of the expansion of the universe. We do not comment on this assumption but accept it as a possible basis for a theory of quasars.) A major difficulty encountered by the Burbidge theory is the lack of any suggested mechanism by which supernovae explosions could be triggered off at the rapid rate required.

In the modification proposed by McCrea, the energy output is again generated as a result of a number of stellar explosions occurring in rapid succession. However, the explosions now occur during the very early stages of stellar evolution. According to McCrea, under some unspecified conditions, proto-stars (called temporary stars by him) that are too massive to eventually exist as normal main-sequence stars, because no quasi-equilibrium states exist for such masses, begin to form. These temporary stars must, therefore, have a mass in excess of about \( 50 \, M_\odot \). The non-existence of quasi-equilibrium states will not become evident until nuclear processes become active in the stellar interior. However, when this occurs, the result is likely to be catastrophic, therefore, causing a phenomenon very similar to a supernova explosion. The total duration of such an evolutionary phase is very much shorter than the time spent by a normal star in reaching its supergiant stage and so the likely time interval between explosions will be correspondingly much shorter.

In another publication, McCrea (4) has shown that this theory is consistent with a theory based on the occultations of light sources proposed by Cannon & Penston (5) in order to explain the fluctuations of the quasar 3 C 446. He also showed that the numerical values obtained in his first paper, namely temporary stars exploding at a rate of about 10 per year for a total interval of about \( 10^5 \) years, can also be deduced from considerations of the occultations.

It is therefore of great interest to investigate whether the method of formation of stars we are proposing here is also capable of producing these massive stars in about the correct numbers, provided the initial conditions are suitably chosen.

We shall first state the postulates we make regarding the floccules and then deduce from these the conditions under which stars of any given mass will be formed.
2. POSTULATES REGARDING THE NATURE OF FLOCCULES

It is assumed that the tendency for a gas cloud to fragment into floccules always exists, putting forward no explanation for this other than that already given by McCrea. However, a floccule will only be influential in the development of a gas cloud if the following are all satisfied:

(a) If the density of the floccules is denoted by $\rho_f$ and the mean density of the whole cloud is $\rho$, then the ratio $\rho_f/\rho$ is always constant in any particular situation so that

$$\rho_f = k\rho. \quad (1)$$

If the numerical value adopted for $k$ is too large, the situation clearly becomes unreasonable, while, if the value is too close to unity no real separation into floccules will have occurred. A reasonable range for $k$ to lie in is therefore $10^2$ to $10^4$. The value used by McCrea is $1.65 \times 10^3$, well inside this range.

(b) An individual floccule, by definition, is not able to hold together under its own gravitational field but, if any eventual growth into stable condensations is to occur, some number $n$ must be stable. The virial theorem, therefore, gives

$$nm_f = \left[ \frac{5}{2} \frac{\mathcal{R}T}{G} \right]^{3/2} / \left( \frac{\mathcal{R} \pi \rho_f}{} \right)^{1/2}, \quad (2)$$

where $m_f$ is the mass of a floccule, $T$ is the mean temperature, $\mathcal{R}$ the gas constant and $G$ the gravitational constant.

The probability that a particular floccule will undergo $n$ successive favourable collisions is roughly $(\frac{1}{3})^n$. It is evident, therefore, that some upper limit of about 20 must exist for $n$ if any stable condensations are to be formed in the time available. The lower limit for $n$ is theoretically two, but any small value is really inadmissible on the grounds that a floccule then becomes virtually a condensation. The results are very insensitive to the value of $n$ adopted and so we adopt $n = 20$ throughout. The value used by McCrea is 18.

(c) We assume that the mean temperature of the cloud remains essentially constant during the interval of interest. This seems a reasonable assumption for most authors assume isothermal contraction of diffuse gas clouds. Observational evidence suggests that most gas clouds that are not heated by the proximity of a star are at about the same 'spin' temperature $125^\circ\text{K}$. However, there is some disagreement between astronomers as to whether the kinetic temperature is equal to the spin temperature. In numerical work we therefore take the temperature to be in the range $10^5$–$10^6\text{K}$. The value used by McCrea is $50^\circ\text{K}$.

(d) It is an intrinsic part of the theory that collisions between floccules occur at regular intervals. No growth will take place unless such collisions are frequent, while too rapid a number of collisions will heat the floccules considerably and so lead to assumption (c) being invalid. We, therefore, make the assumption that collisions occur at regular intervals of time, $t$. It is very difficult to give an exact numerical value to $t$. As a large number of collisions must occur in order that the proposed mechanism works, an upper limit to $t$ is about $10^3$ years. Equally, a too frequent collision will lead to overheating, and so we take $t$ to be in the range $10^1$–$10^3$ years for our numerical work. The value deducible from McCrea's work is 100 years, well inside our chosen range. In order that this collision rate be accomplished, the random velocity, $V$, of the floccules must be such that

$$Vt = L,$$
where $L$ is the collisional mean free path of the flocules. We, therefore, require that

$$V = \frac{L}{t} = \frac{m_f}{4\pi \rho t \left(\frac{4\pi \rho t}{3m_f}\right)^{2/3}},$$

(3)
on using the usual expression for $L$.

If the random velocity of the flocules is specified, then the above four postulates are sufficient to enable us to determine $\rho$, $\rho f$ and $m_f$. As the existence of a random velocity implies that some energy is tied in the floccule motion, it seems more appropriate to assume that the kinetic energy of the flocules is related to the potential energy that has been released by the large cloud in contracting from a diffuse initial stage to whatever stage it is in when flocules start accumulating into stars. We will, of course, verify that the random velocities which occur as a result of this process are realistic.

We, therefore, assume that

$$\frac{1}{2}MV^2 = \mu(\text{Potential Energy}),$$

(4)

where $M$ is the total mass of the cloud and $\mu$ a numerical constant. Conversion of potential energy into random kinetic energy is not likely to be a very effective process and most of the energy released will be radiated away. It seems reasonable to take $\mu$ as $10^{-2}$ so that about 1 per cent of the energy ends up as kinetic energy. It seems unlikely that this estimate is in error by more than an order of magnitude. Therefore, in order to keep the number of variables down and, as the dependence is not very sensitive, we take $\mu = 10^{-2}$.

These are the five assumptions we make regarding the behaviour of flocules. Before applying these to the problem of star formation in a contracting gas cloud, we investigate the effect which rotation has on such a cloud.

3. CONTRACTION OF A ROTATING GAS CLOUD

The way in which a gas cloud contracts is generally affected by the amount of angular momentum which it possesses. The main effect is that, in a plane perpendicular to the axis of rotation, centrifugal forces balance gravity so that the contraction essentially continues only along the direction of this axis. To account for this possibility, we shall investigate the behaviour of flocules when the contracting cloud is either spherical or cylindrical. The spherical cloud corresponds to a situation when no angular momentum is present. It also represents the first stages of the contraction of a cloud possessing angular momentum. We shall term this the spherical stage. The cylindrical shape corresponds to the situation where contraction in the plane of rotation has virtually ceased and the cloud is flattening into this plane. This we term the flattened stage. Most clouds will be reasonably near to one or other of these stages and so, by investigating these two cases, we shall cover most eventualities.

For the numerical work, it turns out that the only quantity of interest is the radius at which the flattened stage commences. Knowledge of this allows a determination of the density at which the spherical stage ceases to be a good approximation and the flattened stage becomes operative. In order to determine this radius, an estimate of the amount of angular momentum possessed by the cloud is required. If this is known for a particular cloud, then direct numerical substitution into the equations which follow is adequate. However we are interested in applying the
theory to the clouds from which the stars which are observed today formed and also
to hypothetical extra galactic regions where temporary stars and quasars may be
found. In neither case is there direct observational evidence available and some
method for estimating the amount of angular momentum present must be found.

Consider first clouds in our own galaxy. Until they separated from the galactic
matter these would have been rotating as part of the galaxy. At the epoch of
separation let the galactic density be \( \rho_{\text{gal}} \) and the local rotation \( \omega_{\text{gal}} \). The angular
momentum of a cloud of mass \( M \) is, therefore, proportional to

\[
KM \omega_{\text{gal}} \left( \frac{M}{\rho_{\text{gal}}} \right)^{2/3},
\]

where \( K \) is the radius of gyration squared.

However, the galaxy will be conserving angular momentum and so \( \omega_{\text{gal}}/(\rho_{\text{gal}})^{2/3} \)
will be a constant. The angular momentum of a cloud is, therefore, given by

\[
H = KCM^{5/3} = C' M^{5/3},
\]

where \( C \) and \( C' \) are related proportionality constants. Inserting values that are
relevant to the galaxy (that is present day rotation and density), we find that \( C \) lies in
the range \( 3 \times 10^{-2} \) to \( 3 \) (or \( C' \) in the range \( 10^{-2} \) to \( 1 \) for a reasonable value of \( K \)).
However, Hoyle (6) has given arguments for selecting a value for \( C' \) of \( 10^{-4} \), which,
with a reasonable value of \( \frac{1}{6} \) for \( K \) in the initial condensation, gives approximately
\( 3 \times 10^{-3} \) for \( C \).

Another approach to the problem is by considering angular momentum per unit
mass. This is not so satisfactory in that changes in the angular momentum due to
the linear dimensions of the cloud (i.e. because it is more massive) must now be
accommodated in the range of values for the angular momentum per unit mass we
adopt, thus making this range wider. McNally (7) has given the values for the
normal interstellar gas clouds as \( 10^{22} - 10^{25} \) c.g.s. units. The values for the angular
momentum calculated are then in accordance with those given by the above
procedure if the masses are typical interstellar gas cloud masses. For clouds less
than a solar mass or considerably larger than an interstellar cloud mass, they give
different results and, presumably, the range of values for the angular momentum
per unit masses would have to be widened to accommodate these extremes.

As we are interested in widely different masses, the method described gives a
more reliable estimate of the angular momentum. A typical value for \( C \) would
appear to be about \( 3 \times 10^{-2} \). It will transpire that the extension of the results to
other values of \( C \), both inside and outside the permitted range, is very obvious and,
for this reason, in the initial numerical calculations we do not take into account any
variation in the value of \( C \).

We are also interested in the formation of temporary stars in other galaxies. The
same argument can be used to derive an equation similar to (5), but now a new value
for the constant \( C(= \omega_{\text{gal}}/\rho_{\text{gal}}^{2/3}) \) appropriate to the particular galaxy under
discussion will have to be found. Because the extension to other values of \( C \) is
obvious we need not devote considerable time in obtaining values for \( C \), we use the
value of \( C \) appropriate to our galaxy (noting that most spiral galaxies have the same
rotational characteristics as our galaxy) and then discuss variations for different
values of \( C \).

When the transition from the spherical stage to the flattened stage occurs, the
gravitational field will be balanced by rotational forces, that is
\[
\frac{GM}{a^2} = a\omega^2,
\]
where \(\omega\) is the angular velocity at that time and \(a\) the radius in a plane perpendicular to the axis of rotation. But \(H = KCM^{5/3} = KMa^2\omega\), and hence we have
\[
a = \frac{C^2}{G} M^{1/3}
\]
as the critical radius when the spherical stage gives way to the flattened stage.

The density when this occurs is given by
\[
\rho = \frac{3G^3}{4\pi C^6}
\]
With the value for \(C\) suggested, the density at the transition stage is about \(10^{-14}\) g cm\(^{-3}\).

We now consider algebraically the formation of stars at the two stages of contraction.

4. FORMATION OF STARS IN THE SPHERICAL STAGE

The potential energy released by a cloud in contracting from a large radius to a radius \(R\) is given by
\[
\frac{-\frac{3}{5} GM^2}{R},
\]
on assuming that the density remains homogeneous, which seems reasonable, until the floccules become active. Hence the energy equation, equation (4), becomes
\[
V^2 = \frac{6}{5} \frac{GM}{R},
\]
while the mean density of the cloud at this stage is given by
\[
\rho = \frac{3M}{4\pi R^3}.
\]
These equations together with equations (1), (2), and (3) are five equations connecting the six variables \(\rho, \rho_f, m_f, V, R\) and \(M\). It is, therefore, possible to express all the other variables in terms of the single variable \(M\), the total mass of the contracting gas cloud. Performing the algebra, we find the following relations,
\[
\rho_f = k\rho = \frac{5}{24\pi} \left[ \frac{25}{12} \frac{\sqrt[k]{\mu^3 T^9 k^7}}{n^6 G^6 k^7} \right]^{1/4} M^{-1/2},
\]
\[
m_f = \frac{15}{2} \left( \frac{12}{25} \frac{\sqrt[k]{\mu^3 T^9}}{n^6 G^6 k^7} \right)^{1/8} M^{1/4},
\]
\[
Vt = L = \left( \frac{12}{25} \frac{\mu^3 G^2 T^6 k^7}{n^6 G^6} \right)^{1/8} \left( \frac{1}{n} \right)^{1/12} M^{1/4}.
\]
The mass of the star formed, \(\mathcal{M}\), is the mass inside a sphere of radius \(L\), we therefore have
\[
\mathcal{M} = \frac{5}{18} \left( \frac{12}{25} \frac{\mu^3 G^2 T^6 k^7}{n^6 G^6} \right)^{1/8} M^{1/4}.
\]
5. FORMATION OF STARS IN THE FLATTENED STAGE

As contraction has now ceased in the plane of rotation, the potential of the
resulting ellipsoid at any stage is

\[ \frac{3}{5} \frac{G M^2 \sin^{-1} e}{a} e, \]

where the eccentricity \( e \) is the only variable. The change in \( \sin^{-1} e/e \) as \( e \) varies from 0 to 1 is however only from 1 to \( \pi/2 \). It is, therefore, a very good approximation to assume that the energy equation for the whole of this stage can be written as

\[ \frac{1}{2} M V^2 = \frac{3}{5} \frac{G M^2}{a}, \]

the increase in \( V \) as further flattening occurs being negligible. Substituting for \( a \) from equation (6) we obtain

\[ V = \sqrt{\frac{6 \mu G}{5 C}} M^{1/3}. \]

Equations (1), (2) and (3) remain valid for this stage in the contraction and therefore, with equation (15), we have four equations connecting the five variables \( \rho, \rho_f, m_f, V \) and \( M \) and it is again possible to express all the other variables in terms of \( M \). Performing the necessary algebra, we now find

\[ m_f = \frac{15}{2} \sqrt{\frac{6 \mu \pi T k}{5 C k}} \left( \frac{M}{n^2} \right)^{1/3} \]

\[ \rho_f = k \rho = \frac{25}{144 \pi} \frac{\pi T k C}{G^3 \mu t^2} \left( \frac{M n}{n^2} \right)^{2/3} \]

and

\[ \mathcal{M} = \frac{5}{18} \sqrt{\frac{6 \mu \pi T k}{5 C}} \left( \frac{M}{n^2} \right)^{1/3}. \]

Again we note that the mass of the final star, and indeed the whole set of parameters, is only very slightly dependent on the mass of the initial contracting cloud. We can, therefore, come to the following general conclusion. The average mass of individual stars in all stellar clusters will be almost independent of cluster mass, a conclusion which is borne out by observations, all stars in fact being of about the same average mass.

6. NUMERICAL VALUES FOR CASES THAT ARE PHYSICALLY INTERESTING

In Section 2 we have selected the following numerical ranges for the parameters occurring in the foregoing equations, \( n = 20, \mu = 10^{-2}, T = 10-1000^\circ K, k = 10^2-10^4, t = 10-1000 \) years. The only quantity that remains to be specified is
the total mass of the cloud, $M$. We have already noted that the dependence on $M$ is small and so it is necessary only to choose values for $M$ that are of physical interest and not to consider any variations about those values. There are three such masses which may be considered, a galactic cluster mass with $M = 10^{38}$ g as a typical value, a globular cluster mass with $M = 10^{38}$ g as a typical value and the mass of a temporary star agglomeration for which $M = 10^{41}$ g seems appropriate.

The results of the computations are more easily shown in tabular form. To save space we shall not tabulate all the calculable quantities but show only the average mass forming into a condensation, $\mathcal{M}$, as this is of paramount importance, the mean density, $\rho$, as this determines whether formation occurs in the spherical or flattened stage and finally the random velocity, $V$, is shown as we have some intuitive idea of what a reasonable value for a velocity is. These values will be tabulated for both the spherical and the flattened stage so that we can easily see what would occur if rotation were to be different from what we have assumed and so formation actually occurs in the other stage. We also deal with each mass separately.

### Table I

<table>
<thead>
<tr>
<th>$t$ (years) $T(K)$</th>
<th>Spherical stage</th>
<th>Flattened stage</th>
</tr>
</thead>
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<td></td>
<td>$\mathcal{M}(\odot)$</td>
<td>$\rho(g,cm^{-3})$</td>
</tr>
<tr>
<td>10 10 100</td>
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</table>
(a) *Galactic cluster mass with* $M = 10^{36} \, g$

The results of the computations are shown in Table I. As the density at the transition stage is about $10^{-14} \, g \, cm^{-3}$, formation of the stars would occur in the flattened stage for all the cases considered. The average mass of the object formed would appear to be inside the mass range of stars with the exception of five cases at the extreme end of the ranges of $T$, $t$ and $k$. This extreme situation is unlikely to occur in practice as we would expect that when the temperature is high, collisions would be more frequent (small $t$) and vice-versa. The random velocity has a reasonable value throughout.

If the appropriate value of $C$ is smaller than what we have assumed by a factor of about 5, then formation would occur in the spherical stage as the transition density then becomes larger than the formation density in the spherical stage. The masses in the spherical stage are also inside the usual stellar mass range and the random velocity is reasonable.

If the correct value of $C$ is larger than what we have assumed, the mass of the object formed decreases, this mass being inversely proportional to $C$. However, an increase of order 100 is required before the majority of the cases cease to form stars with masses in the stellar mass range. Such a large increase is unlikely but equally the work shows that stars will not form in clouds with very fast rotation.

If the values of the parameters nearest to those used by McCrea are taken ($t = 100 \, \text{years}$, $T = 100^\circ \text{K}$, $k = 1000$), the mass of the object formed is $1.8 \, M_\odot$, the density is $2 \times 10^{-12} \, g \, cm^{-3}$ and the random velocity $2.4 \, \text{km} \, \text{s}^{-1}$, almost perfect agreement with the values used by him.

(b) *Globular cluster mass with* $M = 10^{38} \, g$

The results of the computations in this case are given in Table II. With the value for $C$ we have assumed it would appear that most objects would still form in the flattened stage, though a number do now form in the spherical stage. With the exception of five extreme cases, the masses are all again within the usual stellar mass range and the random velocity is reasonable. If the amount of rotation is only slightly less than what we have assumed (a decrease of only a factor of 1.4), formation will occur in the spherical stage in the majority of cases. The masses are still stellar however, and the velocities reasonable. It may well be that in regions where globular clusters are found that the rotation should in fact be less than what we have assumed as the velocity radius curve for galaxies does tend to ‘drop off’ at large radii. If this is the case, all globular clusters would be spherical. If the rotation is much more than what we have assumed, again nothing will form.

Taking values nearest to those used by McCrea, the mass of the object is now $8.3 \, M_\odot$, the density $3 \times 10^{-12} \, g \, cm^{-3}$ and the random velocity $11 \, \text{km} \, \text{s}^{-1}$.

(c) *Temporary star agglomeration, $M = 10^{41} \, g$*

The relevant results are now given in Table III. Irrespective of whether formation occurs in the spherical or flattened stage, the masses are somewhat larger than in the two previous cases, over 60 per cent of them being above $50 \, M_\odot$. What is formed here could, therefore, be termed temporary stars, unless rotation is very much more than what we have assumed in which case we may get normal stars. What is important is that it is possible to get temporary stars with reasonable values for the parameters. It is not necessary that every such cloud should produce a
### Table II

<table>
<thead>
<tr>
<th>$T$(°K) (years)</th>
<th>$k$</th>
<th>Spherical stage</th>
<th>Flattened stage</th>
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</thead>
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<td></td>
<td></td>
<td>$\mathcal{M}(\odot)$</td>
<td>$\rho$ (g cm$^{-3}$)</td>
</tr>
<tr>
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Quasar, we only need a few hundred of them to. If we take the values $T = 100$°K, $t = 100$ years, $k = 100$ again, the mass formed is either $47 \, M_\odot$ or $84 \, M_\odot$ depending on whether the cloud is spherical or flattened.

Before the temporary stars formed by the above process can be said to correspond to the temporary stars required to explain the quasar phenomenon, there are two other criteria which must be satisfied. The age spread must be short, and, as has been shown by Terrell (8) and others, the observed fluctuations in the optical emission can only be explained if the linear dimensions of the object are less than a few light years. The linear dimensions of the group of temporary stars formed in the spherical stage under the above process is $(3M/4\pi \rho)^{1/3}$, that is between $10^{17}$ cm and $10^{19}$ cm depending on $\rho$, or $C^2 M^{1/3}/G = 10^{14}$ cm if formation occurs in the flattened stage. Hence, the linear dimension is at the most a few light years.

All the condensations will not form immediately. We might expect that the spread in the formation epoch of a stable condensation is closely related, but somewhat greater than, the formation time of the condensation. We, therefore,
calculate this formation time and assume that it gives a reasonable estimate for the age dispersal of the temporary stars. If $v$ denotes the number of cloudlets in a condensation at any one time, then the rate of collision of cloudlets with this condensation (see McCrea (1)) is given by

$$
\frac{dv}{dt} = \pi V (v^{1/3} r_f)^2 \rho/m_f.
$$

The time that elapses before there are 20 cloudlets in the condensation is therefore

$$
\frac{m_f}{\pi V r_f^2} \int_0^{20} v^{-2/3} dv = \left( \frac{3m_f}{4\pi \rho_f} \right)^{1/3} \frac{10^{-8} k}{V},
$$
on using equation (1) and $m_f = \frac{1}{6} \pi \rho r_f^3$. The interesting point about this is that it is independent of $M, T, t$ and $k$ for the spherical stage and gives a value of 3000 years. The flattened stage gives a similar time interval. The age spread in the group of
temporary stars we would, therefore, expect to be about $3 \times 10^4$ years. As there are about $10^6$ stars in the agglomeration, this gives one temporary star exploding every ten days on average, a very reasonable explosion rate.

It seems, therefore, that the theory outlined above is capable of producing temporary stars to the requirement of the theory of McCrea (2) regarding quasars.

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REFERENCES