ROTATIONALLY EXTENDED STELLAR ENVELOPES: γ CAS

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SUMMARY

New high time- and spectral-resolution observations of γ Cas have revealed an intermittent periodicity (0.7 days) in emission line profile variations. Profiles of a number of weak spectral lines have been obtained including two He I double emission lines and some O-type absorption lines. Absorption- and emission-line profiles have been calculated for rotating stars by two independent methods. The observed profiles in the spectrum of γ Cas are fitted best by those computed for an O8 star rotating at break-up velocity and viewed at i = 55° ± 10°. A detailed model of the star and extended envelope is derived and explanations of the line profile and other spectral variations are proposed. The method is applicable to all rotating B stars.

1. INTRODUCTION

There are two types of Be star; those of luminosity Class I, whose spectral emission lines are formed in a radiatively accelerated outer envelope, and those of Class IV or V, whose emission lines are formed in a rotationally ejected outer envelope. Be stars of the latter type generally have irregular luminosity and spectral changes and in some cases show transient shell spectra. The star γ Cas (HD 5394) is one of the more active of these stars and is well known for its shell episode in the 1930’s (see e.g. Beer 1956). In recent years it has been relatively stable, and in this paper new observations are presented and used to derive a model of the star and envelope.

2. THE OBSERVATIONS

The spectrum of γ Cas (Bo IVe) is characterized by its almost complete lack of strong features. There are double emission peaks at the strong Balmer line positions, and very shallow rotationally-broadened absorption lines of the Balmer series, the strong He I lines and a few other strong lines. Except for the Hα and Hβ (occasionally Hγ) emission peaks, no spectral feature deviates more than 10 per cent from the continuum. The spectrum may be said to be dominated by the rotation of the star and is therefore of great interest in connection with rotational phenomena in general.

As the star is very well observed it is worth reporting only new observations of unusual interest, and the present work is based mainly on high spectral- and time-resolution spectrophotometric scans of line profiles and a few high resolution (2.5 A mm⁻¹) photographic spectra. The technique and results of the photoelectric observations have been reported elsewhere (Hutchings 1967, 1968, 1969), so that these will not be described in detail. Briefly, the structure of the double emission feature at Hβ and Hγ has been found to vary over periods of weeks in separation and strength, and often to vary irregularly and to a lesser extent over time intervals of a
few minutes. An intensive study was made of the Hβ and Hγ profiles during the winter of 1968–9, and from some 140 scans there is now also evidence for a period in the separation and V/R ratio of the peaks, which is less than one day.

These recent observations were made with the coudé scanner of the Victoria 48-inch telescope and have a probable error of ± 1 per cent of the continuum and a time resolution of some five minutes. Fig. 1 shows some typical Hγ profiles which represent mean profiles over periods of about 20 min. The minute-to-minute variations are therefore eliminated and the diagram illustrates the day-to-day and longer-term changes observed. The lower three profiles show the type of V/R and peak-separation changes which have been found to be periodic and have typical amplitudes in these changes. The upper profile shows how the whole feature strengthened between March and August 1969.

![Fig. 1. Mean photoelectric scans of Hγ emission profiles.](image)

The periodic fluctuation, partly hidden in the rapid irregular fluctuations, was found in all series of observations taken between November 1968 and August 1969, within some 10 days of each other. Over longer times discontinuous phase shifts were found. The period was found by means of a computer program provided by G. Hill (1969 private communication) and its value is 0.70 ± 0.03 days for both peak separation and V/R ratio. There were different amplitudes in these variables at different times. Fig. 2 shows the peak separations in some observations at Hγ and the fitted periodic relation. The mean separation of the peaks is 3.2 Å for Hβ and 3.4 Å for Hγ, with a standard deviation of 0.05 Å in each case. The mean amplitude of the periodic variation is 0.5 Å for all coherent sets of observations. The value of V/R varies more widely from month to month, but has a typical periodic amplitude of about 10 per cent.

In September 1969 the mean V/R ratio became greater than one, the emission profile became steadier and showed no further detectable periodic changes. This situation has persisted at least until mid-January 1970. Observations from 1965–67 were examined and the same general picture emerged. During its active periods (V/R ≤ 1) a period of 0.8 days was derived. This figure carries lower weight owing to the quality and timing of the observations.

In addition to the emission profiles, the Balmer lines Hβ and Hγ have a broad, shallow absorption, some 20 Å wide, and about 10 per cent deep at their interpolated central depths. Once again, photoelectric measurements gave the most
reliable profiles. The central intensities were, however, checked by photographic photometry to eliminate the possibility of a zero error. Figures from plates at 2.5 Å mm⁻¹ dispersion were in good agreement in both absorption and mean emission-line intensities. Attempts were made to observe the rapid profile changes photographically using a single long trail at moderate dispersion (15 Å mm⁻¹) but the profiles were too grainy to be of use.

The high-dispersion photographic observations revealed a further interesting point. The different peak separations observed in Hβ and Hγ continue over the whole Balmer series and two He I lines, as shown in Table I, from a spectrum taken on October 7, 1969. The two lines redward of λ4800 were measured on a plate taken in February 1968.

The observation of He I emission, particularly at λ4471, is interesting. It is possible that other He I lines are weakly in emission but the photographic observations are not good enough to show them and the lines have not yet been scanned photoelectrically. The He I λ5875 line was found to be present weakly on 1970 January 7 with a peak separation of 5·0 Å (245 km s⁻¹).

### Table I

<table>
<thead>
<tr>
<th>Line</th>
<th>A</th>
<th>km s⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hα</td>
<td>2.55</td>
<td>122 (wings extend ± 600 km s⁻¹)</td>
</tr>
<tr>
<td>He I λ5875</td>
<td>4.2</td>
<td>215</td>
</tr>
<tr>
<td>Hβ</td>
<td>3.1</td>
<td>190</td>
</tr>
<tr>
<td>Hγ</td>
<td>3.25</td>
<td>225</td>
</tr>
<tr>
<td>Hδ</td>
<td>3.59</td>
<td>256</td>
</tr>
<tr>
<td>Hε</td>
<td>3.84</td>
<td>290</td>
</tr>
<tr>
<td>H8</td>
<td>3.84</td>
<td>296</td>
</tr>
<tr>
<td>H9</td>
<td>(3.97)</td>
<td>(311)</td>
</tr>
<tr>
<td>H10</td>
<td>(3.67)</td>
<td>(290)</td>
</tr>
<tr>
<td>H11</td>
<td>(3.49)</td>
<td>(275)</td>
</tr>
<tr>
<td>He I λ4471</td>
<td>4.35</td>
<td>292</td>
</tr>
</tbody>
</table>
Other lines present in absorption are: Balmer lines to H\textsc{ii}; He\textsc{i} λ 3634, 3705, 3819, possibly 3888, 4009, 4026, 4143, 4387, 4471, possibly 4713, 4921. The He\textsc{i} lines have about half the wing spread of the Balmer lines and are deeper than any Balmer absorption at λ 3819 and 4471. There are some other lines, identified as follows: O \textsc{ii} blends at λλ 4070–76, 4317–20, 4648–51; Si \textsc{iv} λ 4089, 4116; Ca \textsc{ii} K λ 3933; He\textsc{ii} λ 4541; N \textsc{iii} λλ 4634–41, and Si \textsc{iii} blend at λλ 4552–74. Other lines, if any, are too shallow to be detected, but the above list is an interesting mixture of strong lines, being representative of both O and B spectral types. The significance of this will be discussed later. Fig. 3 shows some typical shallow absorption-line profiles.

![Fig. 3. Strong line profiles in γ Cas from high dispersion plate.](https://academic.oup.com/mnras/article-abstract/150/1/55/2602919)

Photometric observations of the star have not been made with the accuracy and time resolution required to correlate them with the spectroscopic work. Ivanova, Kupo & Mamatkazina (1969) have made broadband observations and find a slow brightening in the $B$ and $R$ over the past 20 years. They find an infra-red excess $(I, J, K, L)$ which increased by some 0.5 m in the $L$ band between 1963 and 1966. The means of visual observation by amateurs (Isles 1969, private communication) show a slow brightening in monthly averages from 2.6 to 2.4 over the years 1961–69, with irregular brightenings and fadings every six to ten months. It would be of great interest to obtain a series of closely spaced multi-colour observations over a few nights, preferably simultaneously with line scans. At present it seems clear that the stellar continuum does not correspond closely to a Bo star and a closer study of the photometric behaviour is necessary before speculating on its interpretation.

Interstellar Ca\textsc{ii} H and K and Na\textsc{i} D lines are weak in the spectrum of γ Cas. The star is not a supergiant, and is embedded in the H\textsc{ii} region S 185, 120′ in diameter. These suggest that the star is not more than some 200 pc away. The luminosity class and Schmidt-Kaler’s (1965) calibration yield a distance of 220 pcs. We may therefore estimate the absolute visual magnitude of γ Cas at $-4.2 \pm 0.2$ m.
3. THEORY

The support and behaviour of Be star envelopes has been discussed by Limber & Marlborough (1968) and a model for the shell episode of Pleione proposed by Limber (1969). It is therefore unnecessary to introduce the problem in detail. In this section we show how two model-building approaches lead to compatible results for the structure and aspect of the envelope of $\gamma$ Cas. In the process, certain physical assumptions are made and the semi-empirical approach leads to the deduction of others. The final result is compatible with existing relevant work.

It is supposed that the star is rotating at or very near rotational break-up velocity and that the emission lines are formed in an extended envelope whose origin is connected with the low or zero gravity equatorial zone. It is further assumed that the rotational velocity cannot much exceed the escape velocity so that the region of zero gravity is at most a very narrow one at the equator.

The absorption profiles

The approach of Stoeckley (1968), which makes use of observed profiles of non-rotating stars, has been followed. The stellar disc is divided into several hundred sectors as described by Hutchings (1968) and the assumed rotation and aspect of the star gives each sector a characteristic radial velocity, together with physical parameters related to the latitude, polar gravity and temperature. These are discussed in more detail below. Each sector then has a standard absorption profile which is derived from observation, and the final profile is the sum of the individual profiles, allowing for limb- and gravity-darkening effects.

The standard profiles used here are taken from H$\gamma$ and He I $\lambda 4471$ profiles from many medium- and high-dispersion spectra and scans of stars from types O5 to K0 at the Dominion Astrophysical Observatory. As the final results are primarily dependent on the stellar rotation the standard profile accuracy need be no better than about five per cent. They are described by a series of empirical equations relating core and wing depth and width with gravity and temperature.

The model input parameters are the polar (zero rotation) values of effective temperature, gravity, the stellar radius, rotational velocity, and the angle of line of sight. The variation of temperature with latitude was given by Von Zeipel's law, with a lower fixed limit as $g$ tends to zero. The value of this limit is a further free parameter. It is evident that Von Zeipel's relation breaks down near the equator as this is where the extended envelope is being formed and this method of model fitting was able to indicate by means of the temperature limit the extent of the equatorial mass-flow region.

The computations produced profiles which agreed with known rotating stars in a satisfactory way (e.g. $\eta$ U. Ma) and predicted the distortions due to axial inclination discussed by Stoeckley; and the program was then applied to $\gamma$ Cas. (The results of computations for other stars will be discussed in due course.) The rotational break-up velocity of the star is some 500 km s$^{-1}$ if it is not a supergiant, so that the radius indicated by the o-7 day period is about 7$R_\odot$. As this must refer to a region in the extended equatorial envelope the probable radius of the star is $\sim 6R_\odot$. The star itself is rotationally distorted and the polar radius must be somewhat less than this. The accuracy of the present calculations however allow this effect to be neglected. The variation of $g$ with latitude therefore is given by a spherical model.
Fig. 4 shows the best-fit model at three inclinations. It can be seen that the profile changes little for values of $i > 50^\circ$. It was also found that a high polar temperature was required to achieve the very shallow observed profile. Temperatures of the order of 25 000° expected from the observed spectral type (Bo) produced too deep a profile. The best-fit model derived from H$\gamma$ and He i $\lambda 4471$ profiles, had the following parameters:

$$T_{\text{eff}}\text{ (pole)} = 33 000^\circ \pm 10 \text{ per cent (i.e. sp. type } \sim \text{ O8)}$$

$$R^* = 6R_\odot, \quad V_{\text{rot}} = 500 \text{ km s}^{-1}, \quad \log g\text{ (pole)} = 3.75$$

$$i = 70^\circ \pm 20^\circ.$$

The Von Zeipel cut-off region extended about $10^\circ$ from the equator and the effective temperature in this region was 8000° or less.

The emission profiles

The geometry of calculation of emission-line profiles for this star has been described in an earlier publication (Hutchings 1968). Now that the mean shape and changes in the observed profiles are better known, the models can be used to greater effect and more attention can be paid to the physical situation in the stellar envelope. The emission profiles are formed in an extended low-density envelope which is quite distinct from the thin absorption-line envelope dealt with in the previous section. Furthermore, the line absorption in the light entering the bottom of the outer envelope is so weak that we may treat the two profiles completely separately. This may not be true in a star viewed closer to pole-on, or where the absorption profile depth is large.

The structure of the extended envelope is not easy to deduce by purely theoretical arguments (see e.g. Limber 1967). In the absence of a shell spectrum the mass in the envelope must be low, so that the region of zero gravity through which it is formed is probably small. In addition, the internal structure of the star would not withstand rotation at speeds greater than break-up. We also require a mechanism to support the equatorial envelope above the photosphere. Viscous or magnetic forces do not seem likely and are not obviously connected with fast rotation. It is known that radiation pressure is able to overcome gravity in non-rotating supergiants (Hutchings 1970; Lucy & Solomon 1970), and it seems to be a probable mechanism here. The luminosity of $\gamma$ Cas is some $3^m$ lower, the absorption-line strength and surface area some ten times lower, and mass-loss (see below) about 100 times lower than the hottest supergiants. The ratio of radiation pressure to surface gravity is the same at all latitudes where Von Zeipel's law holds. It is evident that the flux near the equator does not approach zero, while the gravity does. A convective region with a temperature lower limit, as suggested above, results, in which radiation
pressure can lead to mass-loss. The comparison with supergiants suggests that mass-loss of the strength observed can arise in regions whose log \( g \) is less than 2. Radiative support for the envelope is stronger at higher latitudes, however, so that the envelope may well spread out as it rises from the equator. We are therefore concerned with an envelope, contiguous with the photosphere at the equator, whose velocity field, ionization structure, angular velocity, extent, and density may vary freely within boundaries defined by the radiation temperature and conservation of mass. The structure of the envelope is strictly also determined by its internal motions other than rotation with the star. These however are found to be an order of magnitude smaller than the rotation itself (see below) and can be neglected in the crude models described here. The object of the exercise is to find the model or models which produce profiles closest to those observed and see if these are compatible with physical considerations and with the deductions from the absorption line calculations above.

The line profile used for most of the calculations was Hy. The modified Boltzmann–Saha populations of Menzel & Baker (1937) were used with the emission-line enhancement factors described in Hutchings (1968). The integration was carried out in spherical shells of thickness \( R/10 \) containing 300 sectors each out to a height of \( 5R^* \).

The observational quantities to be reproduced are the peak separation, \( V/R \) ratio, central depression, wing extent and total peak height. It became evident at an early stage that the density near the photosphere must be low everywhere, except in the equatorial belt, if the central absorption dip is to be weak, as observed. The spreading of the envelope above the equator however also has drastic effects on the profile, so that there is a strong hold on this parameter. The ionization structure of hydrogen was found to vary slowly with height in the envelope. A number of different atmospheric structures was tried and the final model had the following form: (all distances in km, \( g \) in cm s\(^{-2} \) is the effective gravity in the shell).

Surface temperature by Von Zeipel’s law down to log \( g = 2 \).

Surface density proportional to \( g/g \) (pole); log \( g < 2 \) set equal to 2.

Temperature \( = \) temp. (surf). \( (0.25 + 0.75 \exp(-0.002gX/R)) \) where \( X \) is height.

Shell density \( = \) shell dens. (surf). \( \exp(-0.02gT/R_{\text{shell}}) \) where \( T \) is the shell thickness, and shell density is the mass per shell of constant thickness.

Shell density remains constant at its previous value if log \( g < 2 \).

Angular velocity \( W \propto (R^*/R_{\text{shell}})^a \) so that variation of the free parameter \( a \) determines the spreading of the envelope by freezing the temperature and density as soon as log \( g \) falls below 2.

The above relationships, while empirical, have some physical implications. The surface temperature has been discussed above. The density law implies hydrostatic equilibrium, which should be a reasonable approximation where log \( g > 2 \). The temperature–height relation implies a scale height fall-off to a lower limit which represents the \( T_{\text{ex}} \) due to radiation in the low density upper atmosphere. Any law by which the temperature falls more rapidly in the polar regions than the equatorial produces reasonable results here and simply represents the transition from high \( g \) to low \( g \) atmospheres. Similar remarks apply to the density–height relation, with the additional proviso that mass is conserved in the zero \( g \) parts of the envelope, which
are expanding. The angular velocity relationship allows a wide range of freedom in parameter fitting. Its physical implications are discussed below.

In the profile calculations only one scattering and one electron scattering interaction per photon are considered. Some calculations including a second scattering were made and the changes to the final profiles were negligible (≤ 0.1 per cent). A full description of the calculations is given in Hutchings (1968).

Fig. 5 shows the effect of different values of a on the profile. The lower the value of a the lower the height at which log g falls below 2 and the more extensive the envelope. If W falls off rapidly (a high) there is no extended envelope and practically no profile. From this point of view the model, although semi-empirically derived,

![Fig. 5. Computed Hγ emission line profiles, showing dependence on rotational index a.](image)

![Fig. 6. Computed Hγ emission line profiles, showing dependence on g (pole).](image)

reacts in a physically meaningful way to the imposed changes. (It is unlikely however that the mechanism is physically meaningful as the line strength depends on temperatures and ionization structure which have dubious validity.) In a similar way the profile is sensitive to g (pole) as this alters the equatorial aperture and hence eventual density of the envelope. This effect is illustrated in Fig. 6.

The V/R ratio is a function of the radial movement of the envelope with respect to the star. This is an important fact which may well be the key to an understanding of many of the long-term changes in Be stars. The ratio V/R shown in Fig. 1 can be produced by a radial expansion of 20 km s⁻¹, or a velocity field from zero at the surface to 50 km s⁻¹ at 4R*, and viewed at i ≥ 50°. For lower values of i greater velocities are required and the peak separation is adversely affected. The effect can be thought of as the central absorption dip shifting with the expansion velocity of its envelope. We therefore conclude that the mean R > V, as observed from November

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1968 to August 1969, implies an expanding envelope. Conversely, from September 1969 on, when $R < V$, we have a contracting envelope. This point is discussed further below.

Table II summarizes the main effects of various parameters on the calculated profile.

<table>
<thead>
<tr>
<th>Increase in</th>
<th>Peak height</th>
<th>Peak width</th>
<th>Separation</th>
<th>Central dip (dip/peak)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle $i$</td>
<td>lower</td>
<td>(broader)</td>
<td>wider</td>
<td>lower</td>
</tr>
<tr>
<td>Temperature</td>
<td>much higher</td>
<td>narrower</td>
<td>closer</td>
<td>higher</td>
</tr>
<tr>
<td>Radial expansion vel.</td>
<td>—</td>
<td>—</td>
<td>wider</td>
<td>higher (differs with $i$)</td>
</tr>
<tr>
<td>Density</td>
<td>higher</td>
<td>broader</td>
<td>—</td>
<td>closer</td>
</tr>
<tr>
<td>$a$</td>
<td>much lower</td>
<td>broader</td>
<td>—</td>
<td>higher</td>
</tr>
</tbody>
</table>

The final model, which matched the mean observed profile (November 1968–March 1969) to within one per cent (Fig. 7), had the following structure:

$T(\text{env})$ 7000° at $4R^*$, 12 000° at surface (± 200°)

$T(\text{pole})$ 33 000° ± 3000°

Density $5 \times 10^{-11}$ g cm$^{-3}$ (shell density constant in equatorial plane)

$i = 55° \pm 10°$

$g(\text{pole}) = 5400 \pm 100$ (log $g = 3.73$)

Equatorial radius $4.5 \times 10^6$ km (6.5$R_{\odot}$)

$W(\text{surf}) = 0.0011$ rad s$^{-1}$ ($V_{\text{rot}} = 500$ km s$^{-1}$)

$a = 1°$, corresponding to conservation of angular momentum

velocity field $c$ at surface to 50 km s$^{-1}$ at $4R^*$.

Calculations of H$\beta$ and H$\delta$ profiles using this model produced very good fits to the observations and in particular matched the different peak separations observed. This is caused by a combination of two effects: (1) The stronger the line the greater the volume of envelope in which it is produced. Thus in the case of H$\beta$ more emission is seen over the pole of the star (at zero radial velocity), so that the central dip is less pronounced and the peak separation reduced, and (2) A second result of the stronger lines being formed through a greater atmospheric volume is that the emission peaks are formed in regions where the velocity of rotation is lower and thus the peak separation smaller.
The different separations of the He I emission peaks are explained in the same way. He I λ4471 is formed only in the high-temperature and low density equatorial layers and therefore has a high peak separation. He I λ5875 is a stronger line and is probably subject to non-LTE effects in the outer atmospheric layers. Thus it is formed through a large volume, and has a peak separation similar to Hβ.

We note that the model suggested by the emission-line calculations is able to reproduce all the observed mean profiles and account for all the observed features of the profiles in a physically meaningful way. The model is also compatible in every way with that derived from the absorption line profiles. There remain only the rapid profile changes to explain. As noted in Section 2, these are evident only when \( R > V \), i.e. when the envelope is expanding. We may thus suppose that they arise from density and temperature fluctuations which occur in the mass–loss region near the equator. A large region in which the density or temperature is high (or ‘condensation’) may rise from the photosphere and remain intact for several stellar revolutions before dispersing, giving rise to the \( V/R \) and peak-separation periodicity observed. The irregular occurrence of such condensations would explain the discontinuities in period and differing strength of the observed periodic phenomena as well as the irregular and non-periodic fluctuations. After September 1969 the envelope apparently started contracting and the resulting absence of ‘condensations’ explains the absence of any marked observed activity.

Some profiles were calculated with condensations about \( 0.1R^* \) in size, in which density and temperature differed by 10 per cent from the rest of the envelope, and these were sufficient to reproduce the periodic and irregular profile changes observed.

4. CONCLUSION

In summary, we conclude that γ Cas is an O8 star rotating at the limit of stability and viewed at some 55° from the poles. Gravity darkening and cooling advance the observed spectral type to B0. As pointed out in Section 2 the strong lines present represent a mixture of spectral types (e.g. O II and Si IV lines) which is explained by gravity darkening. The observed difference in width of the absorption lines of different excitation is explained by their being formed in zones of latitude of different temperature and rotational velocity. The spectral-type shift is in good agreement with that predicted by Roxburgh & Strittmatter (1965).

The extended envelope is formed by loss of matter through the low-gravity equatorial region. Mass–loss probably occurs when \( g \) falls below 100 and the support of the envelope is probably by radiation pressure. This hypothesis explains the supporting mechanism higher in the envelope, provides an equatorial band large enough to form the envelope without exceeding break-up rotation, and provides the accelerating force to produce the observed \( V/R \) emission line ratio. The angular momentum of the envelope is conserved and its effective width spreads from \( 0.1R^* \) at the surface to \( 1.0R^* \) at about three radii. Fig. 8 shows this structure schematically. Excitation-level populations are not clear, but using the Menzel–Baker theory, the ‘excitation temperature’ drops to 7000° at some \( 4R^* \), beyond which Balmer line formation rapidly becomes weak. It is evident from the observations that the envelope has phases of contraction as well as expansion. This may be caused by small changes in the radiative support by opacity changes in the envelope or luminosity changes in the star. There is evidence for both of these in the observa-
tions, although the star’s luminosity appears to be increasing slowly over a long time scale (decades). The increase in emission line strength observed in August 1969 may be a clue: the density of the envelope may build up until the opacity and radiation pressure are insufficient to support it. A period of collapse would follow until radiative support is once again able to force matter away from the star.

Finally, the star has been observed in the rocket ultra-violet by Morton (1969) who found an almost featureless spectrum. However, the C IV λ1550 resonance line was found to have a P Cygni profile. This is further indication that mass is ejected to a height where radiative excitation predominates (as in the OB supergiants) and that radiative support operates at least in the outer envelope. The mass-loss from the star is difficult to estimate as it is obviously variable and non-existent for an unknown fraction of the time. The expanding model derived which corresponds to the state of the star from November 1968 to March 1969 corresponds to a mass-loss rate of about $10^{-7} M_\odot$ per year.

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REFERENCES