THE IONIZATION STRUCTURE OF
PLANETARY NEBULAE—IX
LUMINOUS FILAMENTS

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SUMMARY

Recent observational and theoretical evidence indicates that density condensations which are optically thick to ionizing radiation exist in planetary nebulae. Such a condensation shields the gas in a column behind it from the stellar radiation field, so that the shielded gas is ionized only by the diffuse radiation from the neighbouring gas. Since stellar radiation is characterized by an effective temperature which may exceed 10⁵ K and diffuse radiation is emitted by a gas which does not differ significantly from 10⁴ K, the physical conditions in the shadowed and unshadowed regions are quite different. Although the shadow free gas can be very highly ionized, the shadow zone is composed predominantly of singly ionized atoms. Viewed in the light of lines from singly ionized atoms, the shadow appears as a luminous filament. The comet-like structures seen in NGC 7293 are suggested to be optically thick condensations (‘heads’) and the associated luminous shadows (‘tails’).

1. INTRODUCTION

The existence of density condensations in planetary nebulae has been inferred from the failure of homogeneous models to reproduce the observed intensities of emission lines from ions of low ionization potential (Williams 1970). Capriotti, Cromwell & Williams (1971) have concluded, on the basis of photographic and theoretical evidence, that small scale high density regions that are optically thick to ionizing radiation occur in planetary nebulae. If the existence of such condensations is postulated then it follows that there will be regions of a nebula which are shielded from ionizing stellar photons. The gas directly behind a condensation is therefore excited only by the diffuse radiation field of the nebula.

The spectral distribution of stellar radiation is characterized by an effective temperature $T_s$, which can range in excess of $10^5 \text{ K}$. On the other hand, diffuse radiation is emitted by the nebular gas, which typically does not differ substantially from a temperature $T_\text{e}$ of $10^4 \text{ K}$. Therefore, quite different degrees of ionization can be expected to occur in the shadowed and unshadowed regions. Observed in the light of ions with low ionization potentials, the shadows may appear brighter than the surrounding unshadowed gas which is more highly ionized. The ‘luminous shadow’ would be seen as a long straight filament extending radially away from the central star.

Comet-like structures are conspicuous in NGC 7293, each consisting of a luminous head and a straight tail pointing away from the central star. This is the closest planetary nebula, and unless it is exceptional, other nebulae have similar
features, but are too distant for them to be resolved. Vorontsov-Velyaminov (1968) has suggested that gaseous streams from the central star form the filaments, while Matthews (1968) has proposed they are due to a Rayleigh–Taylor instability where the stellar wind impacts at the inner edge of the nebula. Neither mechanism explains the bright globules at the ends of the filaments towards the star. If, however, the globules are optically thick then the cometary features in NGC 7293 find a simple explanation. A condensation forms the ‘head’ (bright globule) while its associated luminous shadow gives rise to the ‘tail’ (radially directed filament).

2. NEBULA MODEL

In order to investigate the phenomenon of luminous shadows, a simple model of a planetary nebula is employed. The nebular shell is approximated by a slab illuminated normally on one side by diluted stellar radiation. It is sufficient for our purposes to assume the gas to consist mainly of hydrogen with small amounts of C, N, O and Ne, which are the important coolants. The central star is assumed to radiate a blackbody spectrum at a temperature $T_b$ and the nebula is taken to be optically thick, i.e. $\tau_1 = \infty$, where $\tau_1$ is the optical thickness of the nebula at the Lyman edge of hydrogen. The dilution factor $W$ is held constant throughout the nebula.

The determination of the ionization structure of the nebula draws heavily on earlier papers in the present series. These will be referred to as Papers I–VIII, with Papers I, II and VI forming the background of this study. The neglect of helium in our model means that a precise comparison of the theoretical results with an actual nebula, as was done for example in Paper VIII, cannot be made. Nevertheless, the basic physical processes are included, and we expect a more rigorous calculation to affect some details but not the conclusions. Moreover, the limitation to a hydrogen nebula allows the use of functions extensively tabulated in Paper I, which simplifies the analysis.

In order to approximate the effect of diffuse photons which leave the inner boundary of the nebula, cross the hollow central region and re-enter at some other point, the boundary condition on the diffuse radiation field is taken to be

$$I_{d\nu}(\sigma, \mu) = I_{d\nu}(\sigma, -\mu) \quad (1)$$

where, as usual, $\mu = \cos \theta$. At the outer edge of the nebula there is no incident diffuse radiation, so $I_{d\nu}(\tau_1, \mu) = 0$ for $\mu < 0$. The ‘on-the-spot’ approximation is used to solve the transfer equation for the diffuse field. In this approximation a Lyman continuum photon is absorbed at the same point it was emitted. As shown in Paper I, this is equivalent to writing for the mean intensity

$$J_{\nu}^d = j_{\nu} / \kappa_{\nu} \quad (2)$$

where $j_{\nu}$ and $\kappa_{\nu}$ are the emissivity and absorptivity of the gas in the Lyman continuum. The on-the-spot approximation was compared with accurate numerical solutions of the transfer equation in Paper VI. With the symmetric boundary condition (1), the on-the-spot and the numerical results for the source function did not differ by more than 10 per cent at any point in a semi-infinite nebula. Thus, the on-the-spot approximation can be used here with some confidence as to its validity.
The shielding of the gas by an optically thick globule is taken into account by simply deleting the incident stellar flux over some area of the inner boundary of the nebula (Fig. 1). The gas in a column behind this area is irradiated only by the diffuse photons emitted by the neighbouring unshadowed gas.

![Diagram](https://academic.oup.com/mnras/article-abstract/156/1/91/2604564)

**Fig. 2.** A schematic diagram of the model used shows the shadowed gas behind an optically thick obstacle. Through most of its length, the shadow is composed predominantly of singly ionized atoms, while the neighbouring gas is more highly ionized.

The widths of the filaments in NGC 7293 are about $10^{15}$ cm if the nebula is 100 pc distant (Vorontsov-Velyanov 1968). A transverse optical thickness of unity at the Lyman edge would require a density of roughly $2 \times 10^9$ ground state hydrogen atoms per unit volume. The density of the nebula is estimated to be about ten times smaller than this (Osterbrock 1960). In this case, the shadow zones remain optically thin to photons emitted by the surrounding gas. In the present calculation, this will always be assumed. Thus, diffuse photons pass nearly unattenuated across the shadowed regions, and the photons emitted by the shielded gas do not contribute sensibly to the diffuse field. The diffuse radiation field within a shadow may thus be taken to equal that in the neighbouring shadow-free gas.

### 3. The Unshadowed Region

At any depth $\tau$, measured at the Lyman edge of hydrogen, the radiation field in the unshadowed gas is composed of an attenuated stellar component $J_{\nu}^{s}(\tau)$ and the diffuse part $J_{\nu}^{d}(\tau)$. For a pure hydrogen nebula, and in the on-the-spot approximation, the total mean intensity takes the simple form (Paper VI)

$$J_{\nu}(\tau) = WB_{\nu}(T_{0}) \exp \left(-\frac{f(\nu) \nu_{1}}{\nu} \tau\right) + D(\tau) B_{\nu}(T_{0})$$

where $f(\nu) = \nu^{-8}$ is the frequency dependence of the hydrogen photoionization cross-section in the Kramers approximation. Equation (3) shows explicitly the
contribution of two separate radiation fields proportional to blackbody functions at distinct temperatures. The function $D(\tau)$ is given by

$$D(\tau) = \frac{1 - \epsilon}{\epsilon} \int_{v_1}^{\infty} \frac{f(\nu/v_1)}{B_\nu(T_\nu)} B_{\nu}(T_\nu) \exp \left[-\frac{f(\nu/v_1)}{\nu}\right] d\nu/h \nu$$

(4)

where $\epsilon$ is the fraction of recombinations into excited states of hydrogen. In the notation of Paper I, $\epsilon = \alpha_B(T_\nu)/\alpha(T_\nu)$, and for $T_\nu = 10^4 K$, $\epsilon = \epsilon=0.625$. Equation (4) may be written in terms of the functions $u(\tau, T)$ and $F(T)$ tabulated in Paper I as

$$D(\tau) = \frac{(1 - \epsilon)}{\epsilon} W \frac{F(T_\nu)}{F(T_\nu)} u(\tau, T_\nu)$$

(5)

If $N_1$, $N_e$ and $N_+$ are the densities of ground state hydrogen atoms, electrons and protons respectively, then the ionization equation is

$$4\pi AN_1 \int_{v_1}^{\infty} f(\nu/v_1) J_{\nu} d\nu/h = N_e N_+ \alpha(T_\nu)$$

(6)

where $A = 6.3 \times 10^{-18} \text{ cm}^2$ is the photoionization cross-section at the Lyman edge. When equation (3) is used for $J_{\nu}$, this becomes

$$N_1 A Q u(\tau, T_\nu) \left[1 + \frac{1 - \epsilon}{\epsilon}\right] = N_e N_+ \alpha(T_\nu)$$

(7)

where $Q$ is the flux of photons at the inner boundary of the nebula, given by

$$Q = 4\pi W \int_{v_1}^{\infty} B_\nu(T_\nu) d\nu/h.$$  

(8)

From the definition of $\epsilon$, this may be further simplified to

$$N_1 A Q u(\tau_1, T_\nu) = N_e N_+ \alpha_B(T_\nu).$$

(9)

Since $N_e = N_+$ for a pure hydrogen nebula, and $N_1 + N_+ = N_H$, the total hydrogen number density, equation (9) becomes a simple quadratic equation in $N_e$.

Heating of the gas takes place through photoionization of hydrogen. Then the heat gain per unit volume is

$$G = 4\pi N_1 A \int_{v_1}^{\infty} f(\nu/v_1) J_{\nu} h(\nu - v_1) d\nu/h.$$  

(10)

With equation (3) for $J_{\nu}$, the heat gain may be expressed in terms of the function $u(\tau, T)$ tabulated in Paper I as

$$G = Ah v_1 Q u(\tau, T_\nu) N_1 \left[u(\tau, T_\nu) + \frac{1 - \epsilon}{\epsilon} u(\nu, T_\nu)\right].$$

(11)

From general considerations of detailed balance under conditions of thermodynamic equilibrium, it may be shown that

$$Ah v_1 Q u(\tau, T_\nu) N_1 \frac{(1 - \epsilon)}{\epsilon} u(\nu, T_\nu) = N_e N_+ k T e \beta_1(T_\nu)$$

(12)

where $N_e N_+ k T e \beta_1(T_\nu)$ is the kinetic energy per unit volume and time of electrons recombining into the ground state.
Ionization structure of planetary nebulae—IX

Energy is lost through hydrogen bound-free and free-free emission, collisional excitation and ionization of hydrogen atoms and excitation of low lying levels of the ions of C, N, O and Ne. The loss rate per unit volume is given by

$$L = N_0 N_{i+1} k T_e [\beta(T_e) + \beta_{ff}] + N_0 N_{i+1} h \nu_1 [q(T_e) + \Phi(T_e)] + L_c.$$  \hspace{1cm} (13)

The total bound-free and free-free losses are given by the first term on the right-hand side of equation (13). Collisional excitation and ionization of hydrogen are described by the second term, where $q(T_e)$ and $\Phi(T_e)$ are functions given in Paper II. The last term represents excitation of the cooling ions, and is written

$$L_c = 8.63 \times 10^{-6} N_e \sum_{i,k} N_i \frac{\Omega_{ik}}{\omega_1 T_e^{1/2}} \exp \left(-\frac{X_{ik}}{k T_e}\right).$$  \hspace{1cm} (14)

where the sum extends over all ionic species $N_i$ and the parameters $\Omega_{ik}/\omega_1$ and $X_{ik}$ are tabulated by Williams (1967) for the important collisional transitions in each ion.

It is necessary to solve the ionization equations for all the ionic species in order to evaluate the densities $N_i$ in equation (14). If $N_{i+1}$ represents the number density of ions of the same element in the next higher stage of ionization than $N_i$, then

$$N_e N_{i+1} \alpha_i(T_e) = 4\pi N_i A_i \int_{\nu_1}^{\infty} f_i(\nu/\nu_1) J_\nu d\nu/h\nu.$$  \hspace{1cm} (15)

Here, $A_i$ is the photoionization cross-section for the given ion at the edge frequency $\nu_1$, and $f_i(y)$ is its frequency dependence. Following Seaton (1958), we write this as

$$f_i(y) = \gamma_i y^{-s_i} + (1 - \gamma_i) y^{-s_i-1}.$$  \hspace{1cm} (16)

The parameters $\gamma_i$, $s_i$ and $A_i$ may be found from tables given by Seaton (1958) and Flower (1968), while the recombination coefficient $\alpha_i$ is assumed to be hydrogenic.

The integration in equation (15) is done numerically.

For thermal equilibrium, one has the equation

$$G = L$$  \hspace{1cm} (17)

which must be solved iteratively for the equilibrium value of $T_e$. With equation (12) taken into account, equation (17) becomes

$$A h \nu_1 Q(u(\tau, T_\odot) w(\tau, T_\odot) N_1 = N_0 N_i k T_e [\beta_B(T_e) + \beta_{ff}]$$

$$+ N_0 N_i h \nu_1 [q(T_e) + \Phi(T_e)] + L_c$$  \hspace{1cm} (18)

where the recombination energy loss to all excited states of hydrogen replaces the total recombination loss, i.e. $\beta_B(T_e)$ appears in place of $\beta(T_e)$.

4. THE SHADOWED REGION

Once the equilibrium temperature in the unshadowed region has been determined, the resulting diffuse radiation field is assumed to exist unmodified in the shadowed region. In order to distinguish physical parameters in the two zones, those pertaining to the shielded gas are primed. The ionization equilibrium equation for hydrogen is then

$$A Q u(\tau, T_\odot) \frac{(1 - e)}{\epsilon} N_1' = N_e' N_{i+1} \alpha(T_e').$$  \hspace{1cm} (19)

The solution of equation (19) determines $N_1'$ and $N_e'$ at a point at the shadowed
region a distance from the inner boundary of the nebula corresponding to the optical depth $\tau$ in the unshadowed gas.

The ionization equations for the ionic species follow from equation (15) with $J_\nu = D(\tau)B_\nu(T_e)$. Because $T_e$ is rarely greater than $2 \times 10^4$ K, it is possible to approximate $B_\nu(T_e)$ by the Wien expression without loss of accuracy. Then we have

$$N_{e'}N_{i+1} \alpha_i(T_e') = N_i'M_1(T_e) D(\tau). \tag{20}$$

If $x_1 = \hbar \nu_1/kT_e$ then

$$M_1(T_e) = \frac{8\pi A_1 \nu_1^3}{c^2} \int_1^{\infty} y^{(1-s_1)} \exp \left(-x_1y\right) \left[\gamma_1 y + (1-\gamma_1)\right] dy. \tag{21}$$

For all the ions of interest, except NeV, $s_1 = 1, 2$ or 3. For NeV, Seaton (1958) gives $s_1 = 2.31$. For the sake of uniformity in the computation, we take $s_1 = 2$ for NeV, which, realizing the approximate nature of the cross-sections, is certainly a minor modification. The integral in equation (21) can now be written in terms of exponential and exponential integral functions:

$$\frac{c^2}{8\pi A_1 \nu_1^3} M_1(T_e) = \begin{cases} \gamma_1 E_1(x_1) + (1-\gamma_1) E_2(x_1) & s_1 = 3 \\ \gamma_1 \exp\left(-x_1\right) + (1-\gamma_1) E_1(x_1) & s_1 = 2 \\ (1+\gamma_1 \nu_1) \exp\left(-x_1\right) & s_1 = 1. \end{cases} \tag{22}$$

Thus the relative number densities of the cooling ions can be determined without the need of numerical quadrature. With $N_{e'}', N_1'$ and $N_i'$ thus determined, $L'$ is found exactly as in the unshadowed case.

The heat gain per unit volume, $G'$, involves only the diffuse field, so that

$$G' = Ah\nu_1 Q u(\tau, T_s) \frac{1-\varepsilon}{\varepsilon} w(\varnothing, T_e) N_1'. \tag{23}$$

The equation $G' = L'$ is solved iteratively for $T_e'$. It is useful to have expressions for the functions $u(\varnothing, T_e)$, $w(\varnothing, T_e)$ and $F(T_e)$ at the temperatures found in the unshielded gas. Because the Wien approximation is valid at these temperatures, the functions may be given in analytic form if the Kramers approximation for $f(\nu/\nu_1)$ is employed. Then if $x_1 = \hbar \nu_1/kT_e = 15.7890 \times 10^4/T_e$,

$$F(T_e) = (2 + 2x_1 + x_1^2) \exp\left(-x_1\right) \tag{24}$$

$$u(\varnothing, T_e) = x_1^3 E_1(x_1)/F(T_e) \tag{25}$$

$$w(\varnothing, T_e) = E_2(x_1)/x_1 E_1(x_1). \tag{26}$$

Because of the small scale of the observed filaments in NGC 7293, these features would be dissipated in a few score years if pressure equilibrium were not established between the shadowed and the unshadowed gas. We assume pressure equilibrium to hold in the model studied, so that $P_{gas'} = P_{gas}$. Since $T_e' < T_e$, it is necessary that the total hydrogen number density in the shadow, $N_H'$, exceed that in the unshadowed gas. The equilibrium value of $N_H'$ is determined iteratively, such that the calculations of ionization equilibria and heat balance are redone for each successive $N_H'$ until $P_{gas} = P_{gas'}$.  

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5. Results

The theory developed in the previous sections will now be applied to two models. In both, the hydrogen number density is \( N_H = 1 \times 10^9 \text{ cm}^{-3} \) and the photon flux at the inner boundary is \( Q = 2 \times 10^{12} \text{ cm}^{-2} \text{ s}^{-1} \). The central star of Model 1 has an effective temperature \( T_S = 5 \times 10^4 \text{ K} \). The dilution factor corresponding to the values of \( T_S \) and \( Q \) is \( W = 8.01 \times 10^{-14} \). Abundances of the elements C, N, O and Ne are taken from Table I of Paper VIII. The degree of hydrogen ionization is represented by the quantity \( \xi = N_1/N_H \). In terms of \( \xi \), the geometrical distance \( x \) from the inner boundary is

\[
x = \frac{1}{N_H A} \int_0^x \frac{d\tau'}{\xi(\tau')}. \tag{27}
\]

![Graphs](https://example.com/graph.png)

**Fig. 2.** (a) Temperature, fraction of neutral hydrogen and optical depth in the shadow-free gas as functions of geometrical depth. (b) Temperature, fraction of neutral hydrogen and total hydrogen density in the shadowed gas as functions of geometrical depth. \( T_S = 5 \times 10^4 \text{ K} \) and \( Q = 2 \times 10^{12} \text{ cm}^{-2} \text{ s}^{-1} \).

Fig. 2(a) shows the run of \( \xi(x) \) and \( T_0(x) \) through the shadow-free regions of the nebula, while Fig. 1(b) shows \( \xi'(x) \) and \( T_0'(x) \) in the shadow. Also displayed in these figures are the behaviour of \( \tau(x) \) and \( N_H(x) \).

Ionization and thermal equilibria in a shadow zone are dependent upon the run of \( T_0 \) and \( \xi \) in the shadow-free gas, since photons from the latter are the sole source of ionizing radiation in the former. Over a sizable fraction of the nebula, \( T_0'(x) \approx T_0(x)/2 \) and \( \xi'(x) \approx \xi(x)/10 \). When the unshielded gas becomes predominantly neutral, the temperature of the shadow drops precipitously as the diffuse field diminishes. Correspondingly, \( N_H' \) rises to assure pressure equilibrium.

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The behaviour of $N_H'$ shown in Fig. 2(b) demonstrates the abruptness of the increase. Computations in the shadow zone are terminated when $N_H' > 10N_H$. At high densities, collisional de-excitation of the excited states of the cooling ions becomes important and this has not been included. Moreover, the assumption that the mean intensity of diffuse radiation is the same within and without the shadowed gas loses validity as the optical thickness across a shadow increases.

The ionization of heavy ions is greatly dissimilar in the two regions. Over nearly all the nebula, C, N, O, and Ne are doubly and triply ionized in the unshielded gas, but overwhelmingly singly ionized in the shadows. Taking into account the increased density in the shadow, there are about $2.4 \times 10^8$ more $N^+$ and $O^+$ ions per unit volume than in the unshielded H II region. The ionization fractions of N and O are shown in Fig. 3 as functions of optical depth $\tau$. Since, in our model,

![Graph showing ionization fractions of N and O](image)

**Fig. 3.** Fractional abundances of nitrogen and oxygen as functions of optical depth $\tau$ in the (a) unshadowed, and (b) shadowed gas, for $T_s = 5 \times 10^4$K and $Q = 2 \times 10^{13}$ cm$^{-2}$ s$^{-1}$. The shadowed gas is predominantly singly ionized, while the surrounding regions are doubly and triply ionized.

only hydrogen ionization attenuates the stellar radiation field, the ionization fractions change most rapidly in the H II–H I transition region. A photograph of the nebula in the light of an $N^+$ or $O^+$ line would show the shadow as a long luminous filament. It may be remarked that the photographs showing filamentary structure in NGC 7293 (Vorontsov-Velyaminov 1968) are taken in the light of H$\alpha$+[N II] $\lambda$ 6584.

The inclusion of helium should change some aspects of the behaviour discussed above. At present, the luminous shadow extends, without significant modification, across the entire H II region of the nebula. With helium present, however, photons
above the hydrogen Lyman edge are used up through ionization of He° and He+. The fractions of doubly and triply ionized components of the unshadowed gas will fall as He becomes neutral, and those of the singly ionized components will increase. At the same time, ionization in the shadow will favour formation of neutral atoms as the diffuse field is diminished. Thus, the filaments, luminous in the lines of singly ionized atoms, will disappear. It is possible, however, that a shadow, luminous in the light of neutral atoms, will reappear further out. At the present time a model containing helium is being constructed to investigate these possibilities.

The second model differs from the first only in that \( T_s = 1.5 \times 10^5 \)K. The dilution factor which gives a photon flux of \( Q = 2 \times 10^{12} \) is \( W = 1.16 \times 10^{-15} \). Figs 4 and 5 show the run of physical parameters through the nebula. Of particular

![Diagram](https://example.com/diagram.png)

**Fig. 4.** (a) Temperature, fraction of neutral hydrogen and optical depth in the shadow-free gas as functions of geometrical depth. (b) Temperature, fraction of neutral hydrogen, and total hydrogen density in the shadowed gas as functions of geometrical depth. \( T_s = 1.5 \times 10^5 \)K and \( Q = 2 \times 10^{12} \text{ cm}^{-2} \text{ s}^{-1} \).

interest is that, although the shadow free region is very highly ionized (e.g. N⁺4 and N⁺5 are major constituents), the shadowed region is still predominantly singly ionized. Thus a luminous shadow in the light of these ions is predicted. The central star of NGC 7293 is probably as hot as \( 1.5 \times 10^4 \)K and perhaps hotter (Seaton 1960).

We have thus shown that the presence of optically thick condensations in a planetary nebula can lead to filamentary structure in the form of bright streamers when viewed in the light of neutral or singly ionized atoms. There is no reason to limit this process to planetary nebulae, however, and it may play a role in
Seyfert galaxies or quasi-stellar objects, where a gas is excited by a central source. The theoretical interpretation of the origin of the optically thick globules themselves is lacking (see e.g. Capriotti 1971). It may be possible that a fluctuation causes part of the gas to become somewhat more opaque than its surroundings. The stellar radiation field is reduced, and the ionization and temperature in this region is lowered. When pressure equilibrium is established, the density of the fluctuation is enhanced, and the opacity further increased. Thus, a small effect may grow until an optically thick condensation results. This instability does not appear to have been investigated, and work is in progress to see whether it is plausible.

![Diagram](image-url)

**Fig. 5.** Fractional abundances of nitrogen and oxygen as functions of geometrical depth in the (a) unshadowed, and (b) shadowed gas for $T_e = 1.5 \times 10^5$K and $Q = 2 \times 10^{12}$ cm$^{-2}$ s$^{-1}$. In spite of the high state of ionization of the unshadowed gas, the shadow is still mostly singly ionized.

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