A mean density and a correlation function of rich clusters: theory and observations

A. G. Doroshkevich and S. F. Shandarin Academy of Sciences, Institute of Applied Mathematics, Miusskaya Pl. 4, Moscow 1 5047, USSR

Received 1977 April 14

Summary. In this paper we investigate a large-scale distribution of rich clusters of galaxies within the framework of the adiabatic theory of galaxy formation. It is shown that a mean density of ‘pancakes’ is close to the density of Abell’s clusters. Angular correlation functions calculated for spatial correlation functions under different assumptions about the spectrum of initial perturbations are compared with Hauser & Peebles points derived from Abell’s catalogue.

It was assumed that the initial spectrum of perturbation was

\[ b^2(\kappa) \propto \kappa^{-2} \exp(-\kappa R_c). \]

The best values of the spectrum indices are found to be \( n = 2.1, 1.6, 1.3 \) and 0.7 for \( \Omega = 0.1, 0.2, 0.3 \) and 1 respectively. The value of the amplitude of initial perturbations obtained from these arguments is close to the value based on other assumptions.

1 Introduction

An investigation of a large-scale distribution of matter in the Universe is very important for modern cosmology. At a large scale comparable with a mean distance between clusters of galaxies the distribution of matter is mainly determined by the structure of initial perturbations. For this reason a study of this distribution (along with a study of fluctuations of microwave background) can give important information about the amplitude and spectrum of initial perturbations, which is very essential for any theory of galaxy formation.

In this paper the analysis of Abell’s catalogue of rich clusters, made by Hauser & Peebles (1973) is compared with the conclusions of the adiabatic (acoustic) theory of galaxy formation. It is assumed the Friedmann’s model of the Universe dimensionless Hubble constant is \( h = H/(100 \text{ km/} \text{s Mpc}) = 0.5 \) and dimensionless mean density of matter is \( \Omega = 8\pi G\bar{\rho}_0/3H^2 \approx 0.1. \) It will be shown that: (1) the theoretical mean density of rich clusters is in a good agreement with the observational one; (2) the theoretical correlation function is close to the data of Hauser & Peebles (1973); (3) the amplitude of initial perturbations is consistent with the estimates based on different arguments (Doroshkevich & Shandarin 1976).
In this paper small-scale properties of the correlation function (Peebles 1974) are not considered at all because in the theory under consideration the small-scale structure of galaxy distribution is not primarily connected with initial perturbations but mainly with the transformation of protoclusters of galaxies into galaxies and violent relaxation in clusters. For the first time an analysis of these processes has been given in a paper by Binney (1976).

2 Mean density of rich clusters of galaxies

A difficulty of every modern theory of galaxy formation is the arbitrariness of amplitude and spectrum of initial perturbations. But in the adiabatic theory any small perturbation is smoothed out by dissipative processes. Dissipation leads to the minimal scale $R_c$ of surviving perturbations.

It can be shown that $R_c = 2/(\Omega h^2 x) \text{Mpc}$ (where $x = (1 + 2.4(40h^2)^{-4/3})^{1/2}$), which corresponds to $M_c = \frac{4}{3} \pi R_c^3 \rho_0 = 10^{13}(\Omega h^2)^{-2} x^{-3} M_\odot$. This mass is close to the mass of rich clusters of galaxies ($M_c = 2.5 \times 10^{15} M_\odot$ at $\Omega = 0.1$). This coincidence was considered as a success and an important argument for an adiabatic theory. In this section we shall estimate more exactly the dimensionless parameter connecting a number of rich clusters in unit of volume and the dissipative scale $R_c$.

The simplest approach in the spirit of a linear theory is to calculate the density of the local maximum of density perturbations which one can consider as a centre of future condensations.

The most important peculiarity of the theory of small perturbations is a factorization of the law of growth of perturbations

$$\frac{\delta \rho}{\rho} = f(q) \cdot B(t),$$

where $q$ is the Lagrangian vector of particle. For this reason the number of maxima per unit of comoving volume is constant during expansion and growth of perturbations. Besides it is necessary to take into account the different height of maxima. If we measure the height of maxima in units of dispersion of density perturbations

$$\sigma^2 = \left(\frac{\delta \rho}{\rho}\right)^2,$$

then the density of maxima with height $\mu = (\delta \rho/\rho)/\sigma_1 \gg 1$ will be

$$R_c^3 \frac{dN_\mu}{d\mu} = \phi(\mu) \approx \frac{1}{\sqrt{2\pi^2}} \mu^3 \exp(-\mu^2/2)$$

and also the density of maxima with height more than $\mu_0$ will be

$$N_\mu(\mu_0) = \frac{1}{R_c^3} \int_{\mu_0}^{\infty} \phi(\mu) \, d\mu \approx \frac{\mu_0^3 \exp(-\mu_0^2/2)}{2\sqrt{2\pi^2} R_c^3}.$$  

For a given initial spectrum of perturbations this result does not depend on the time, or the amplitude of initial perturbations. Here we assume that every maximum will grow up to $\delta \rho/\rho \gg 1$ and will form a gravitationally bound system. But these bound systems will not be able to couple into larger ones.

Another correction uses the approximate non-linear theory of gravitational instability (Zeldovich 1970). In this theory the growth of density perturbations is defined as

$$\rho = \bar{\rho}/[1 - \lambda_1 B(t)] [1 - \lambda_2 B(t)] [1 - \lambda_3 B(t)],$$

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where \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \) are functions of Lagrangian coordinates \( q \), calculated from initial perturbations \( \delta \rho/\rho \). When perturbations are small

\[
\delta \rho/\rho = B(t) (\lambda_1 + \lambda_2 + \lambda_3),
\]

which coincides with the linear theory if

\[
\lambda_1 + \lambda_2 + \lambda_3 = f(q).
\]

But the final fate of perturbations and particularly the birth of bound objects will be at \( \rho \to \infty \) with the formation of a shock wave and a 'pancake', and is determined by the condition \( 1 - \lambda_1 B(t) = 0 \). Therefore, we must now obtain the distribution of maxima \( \lambda_1(q) \) but not \( f(q) = \lambda_1 + \lambda_2 + \lambda_3 \). This value can be calculated, as above, for high maxima \( \lambda = \lambda_1/\sigma_2 \gg 1 \) (where \( \sigma_2^2 = \sigma_1^2/5 \)).

\[
R_c^3 \frac{d\mathcal{N}_\lambda}{d\lambda} = \psi(\lambda) = \frac{1}{2\pi^2} \lambda^5 \exp(-\lambda^2/2).
\]

And again as above

\[
\mathcal{N}_\lambda(\lambda_0) = \frac{1}{R_c^3} \int_{\lambda_0}^{\infty} \psi(\lambda) d\lambda = \frac{\lambda_0^6 \exp(-\lambda_0^2/2)}{4\pi^2 R_c^3}.
\]

The function \( \psi(\lambda) \) calculated numerically is presented graphically in Fig. 1.

This curve is convenient for calculation of the rate of formation of clusters of galaxies. If the amplitude of perturbation is given and the function \( B(t) \) is known then it is easy to find the value \( \lambda \) satisfying the equation \( B\lambda \sigma_2 = 1 \) at every moment of the time, and thereby to

![Figure 1. A mean density of 'pancakes' (in units \( R_c^3 \)) as a function of maximal characteristic value \( \lambda \) in the centre of a 'pancake'.](https://example.com/figure1.jpg)
obtain $dN/dt$, i.e. the rate of formation of 'pancakes' with time (Doroshkevich & Shandarin 1976).

The principal result of the work is obtaining the dimensionless number which considerably differs from it; it determines the density of maxima $\delta \rho/\rho$ and $\lambda_1$. Using for estimates of $N_\mu(\mu)$ and $N_\lambda(\lambda)$ the values $\mu = \sqrt{3}$ and $\lambda = \sqrt{5}$ at which maxima of functions $\phi(\mu)$ and $\psi(\lambda)$ are achieved we obtain $N_\mu(\sqrt{3}) = 1/50R_e^{-3}$ and $N_\lambda(\sqrt{5}) = 1/6R_e^{-3}$. It means that the volume per single maximum is considerably more than $R_e^3$. Collecting the modified estimates of and numbers of 'pancakes' in $R_e^3$ we shall obtain the density of 'pancakes' in space (using the value $N_\lambda(\sqrt{5}) = 1/6R_e^{-3}$).

$$N_{\text{pan}} = \begin{cases} 
2.5 \times 10^{-6} \text{Mpc}^{-3} & \text{at } h = 0.5, \Omega = 0.1 \\
2 \times 10^{-5} \text{Mpc}^{-3} & \text{at } h = 0.5, \Omega = 0.3.
\end{cases}$$

Comparing the above estimates with observational data we will identify a galaxies 'pancake' with a rich cluster of galaxies. According to the discussed theory a 'pancake' associated with a high maximum $\lambda_1$ and therefore formed comparatively early should fragment into individual galaxies, hypergalaxies, groups of galaxies and clusters of galaxies (Doroshkevich & Shandarin 1974; Zeldovich & Novikov 1975) conserving a flattened structure at a large scale. Such complexes may be superclusters of galaxies. However, because of difficulties of seeking such superclusters we needed to use Abell's clusters for comparing with the theory. Besides we assume that at the next evolution a 'pancake' transforms into one rich cluster of galaxies and some poorer ones. According to Abell's results (1965) there are about 4000 rich clusters in the sky at distance less than $D \approx 600 \, h^{-1} \text{Mpc}$. This value corresponds to the mean density of rich clusters $N_{\text{cl}} = 4.5 \times 10^{-6} h^3 \text{Mpc}^{-3} (6 \times 10^{-7} \text{Mpc}^{-3}$ at $h = 0.5$) which is in a good agreement with the above theoretical estimates.

3 Angular correlation function of rich clusters

More detailed information about galaxy distribution in the sky is contained in an angular correlation function defined by the relation

$$dP = \bar{n} \left[ 1 + w(\theta) \right] d\Omega,$$

where $dP$ is the probability that an object is seen in the sky within solid angle $d\Omega$ at angular distance $\theta$ from a randomly chosen object, $\bar{n}$ is the mean number density of objects. The similar relation defines the spatial correlation function

$$dP = \bar{N} \left[ 1 + \xi(r) \right] dV.$$

$w(\theta)$ is uniquely determined by $\xi(r)$ (Limber 1953; Hauser & Peebles 1973).

In this section we especially concentrate on the correlation function for rich clusters because, according to the theory under consideration, a distribution of galaxies is determined mainly by the process of relaxation and is only slightly connected with initial perturbations. Our point of view on this question is somewhat different from that of Peebles (1974).

The observational function $w(\theta)$ was derived from Abell's catalogue of rich clusters of galaxies by Hauser & Peebles (1973). They have concluded that these data are in a good agreement with the spatial correlation function.

$$\xi_{HP}(r) = A \exp(-r^2/R^2).$$

This function will be used with $R = 40 \, \text{Mpc}$. For purposes of comparison with results of Hauser & Peebles we have calculated an angular correlation function $w(\theta)$ for the spectrum
of perturbations
\[ b_k^2 \propto k^{-2} \exp(-\kappa R_c), \]
where spectrum index \( n \) was chosen to give the best fit with observational points. We considered values of dimensionless density \( \Omega = 0.1, 0.2, 0.3 \) and 1. For the spectrum under consideration the spatial correlation function is
\[ \xi_n(r) = A \frac{\sin(n \arctg r/R_c)}{n[1+(r/R_c)^2]^{n/2}}. \]

![Graph](https://example.com/graph.png)

**Figure 2.** Calculated angular correlation functions corresponding to different spatial correlation functions. Circles are Hauser & Peebles' data: (a) \( \Omega = 0.1 \); (b) \( \Omega = 0.3 \).
In Fig. 2(a) and (b) angular correlation functions are plotted for spectrum indices $n = 1$, 2 and the best index $n_{\text{opt}}$ for given values $\Omega = 0.1$ and $\Omega = 0.3$. ($n_{\text{opt}}(\Omega = 0.1) = 2.1$; $n_{\text{opt}}(\Omega = 0.3) = 1.3$). For $\Omega = 0.2$ and $\Omega = 1$ the best fits give $n = 1.6$ and $n = 0.7$ respectively. It is worth remembering that $R_c$ depends on $h$ and $\Omega$ (see above).

In Table 1 amplitudes $A_n$ and random mean square deviation $\sigma_n$ are given for the same curves.

One can see, Fig. 2(a) and (b), that at small angles $\omega(\theta)$ exceeds 1. It means that perturbations on an appropriate scale reach a non-linear stage. For this reason it is necessary to calculate a non-linear behaviour of evolution of perturbations. These calculations based on the non-linear theory of gravitational instability were made numerically for the perturbations with initial correlation function $\xi_1$ (Doroshkevich & Shandarin 1976). The main results relating to a non-linear stage of evolution of large-scale perturbations are as follows:

(i) Distortions of a spatial correlation function are comparatively small (Fig. 3).
(ii) Distribution of clusters of galaxies becomes anisotropic ($\approx$ 10 per cent); it cannot be

![Figure 3. Spatial correlation function $\xi_1 \propto 1/(1 + (r/R_c)^3)$ (a smooth curve). A histogram is a spatial correlation function of 'pancakes' on the non-linear stage (a numerical simulation).](https://academic.oup.com/mnras/article-abstract/182/1/27/1041989)
described by single spatial correlation function with accuracy better than $\approx 10$ per cent (Doroshkevich & Shandarin 1976).

(iii) Comparison of our theoretical results with the data of Hauser & Peebles (1973) shows that 'pancakes' with central value $\lambda_1 = \sigma_1$ can form today ($z = 0$). It corresponds to $\delta \rho / \rho \approx 10^{-2}$ (at $\Omega = 0.1$) at recombination. This value is in agreement with those based on other arguments.

The theory of galaxy formation based on non-linear theory of gravitational instability is assumed to lead to a conclusion that a small-scale spatial distribution of galaxies is mainly determined more by processes of transformation of 'pancakes' into galaxies than by the spectrum of initial perturbations. The violent relaxation (Lynden-Bell 1967) strongly distorts an initial correlation function and apparently leads to the same correlation function $\xi(r)$ in a scale $< 1$ Mpc as that found when using different assumptions about initial perturbations. The first example of transformation of flat disk structure into an elliptical galaxy has been considered by Binney (1976). He has shown that newly-formed distribution of points is close to the law $\rho \propto r^{-2}$. This distribution is in agreement with an angular correlation function for individual galaxies in the region of small angles by Peebles (1974). But on this question a more detailed analysis is needed.

Acknowledgments

We would like to thank Ya. B. Zeldovich for useful discussions and help.

References