Inhomogeneous magnetic fields in extragalactic radio sources

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Received 1978 September 18; in original form 1978 April 17

Summary. A model is proposed in which concentrations of magnetic flux are maintained by the pressure of relativistic electrons within the tails of extended extragalactic radio sources. These ‘knots’ of flux originate from slight local enhancements of the field within which the electron energy density diminishes with respect to the energy density outside. Electrons diffusing into these knots are responsible for the observed non-thermal radiation. The diffusion of charged particles across a magnetic field with a turbulent component is discussed and a diffusion time-scale, \( \tau_{\text{D}} \propto L^4 B^{1/3} \gamma^{-1/3} \), derived for use in the model. \( L \) is the scale of the knot, \( B \) the magnetic flux density and \( \gamma \) the electron Lorentz factor.

1 Introduction

In discussing extended extragalactic radio sources such as Cygnus A, many authors assume equipartition of energy between a uniform magnetic field and the high-energy particles that are believed responsible for the observed non-thermal continuum radiation. However, neither theory nor observation as yet provide strong support for the idea of homogeneity and it is of interest to consider the extent to which there might be spatial variation in the magnetic field.

Consider the extreme case of a source differentiated into field and field-free regions. An important consequence of ‘cavities’ in the magnetic field is that particles may stream from the hot-spot to distant parts of the source with little energy loss, radiating only as they diffuse into local flux concentrations. This effectively increases the particle lifetime and dispenses with the need for reacceleration away from the source head.

Now if the ram pressure of large-scale fluid eddies in the field-free region is sufficient to prevent the magnetic flux relaxing, then in terms of the thermal particle density \( \rho \) and fluid velocity \( v \) we have

\[
B \leq (2 \mu_0 \rho v^2)^{1/2}.
\]

We expect \( v < v_s \) where \( v_s \), the source velocity, satisfies \( 0.02 < v_s < 0.1 c \) (Hargrave & McEllin 1975). With typical values of \( \rho \) (e.g. Hargrave & Ryle 1974), it is clear that fields greater
than the equipartition value cannot be supported, nor do we expect the thermal pressure of these particles to influence the field structure.

The purpose of this paper is to present a rudimentary discussion of the possible influence of the high-energy particles on the magnetic field distribution. A thermal instability associated with the synchrotron process has been discussed elsewhere (Simon & Axfor 1967). Here it is proposed that the magnetic field may have small inhomogeneities enhance as we now describe.

2 The model

Consider two juxtaposed volumes within a fixed boundary as shown schematically in Fig. 1. \( n \) is the number density of relativistic particles, \( P \) the total pressure and \( V \) is the volume. Initially the pressures are equal, \( P_1 = P_2 \) and \( B_1 > B_2 \). The rate of energy loss per particle by the synchrotron process is

\[
\frac{d\gamma}{dt} = -\Omega B \gamma^2,
\]

where \( \gamma = E/m_0c^2 \), \( E \) and \( m_0 \) are the particle energy and rest mass and \( \Omega \sim 0.2 \) for \( B \) in tesla. Clearly the rate of energy loss integrated over all particles per unit volume is

\[
\mathcal{L}_i = \int_{\gamma_1}^{\gamma_2} \frac{d\gamma}{dt} n_i(\gamma) \, d\gamma.
\]

If both \( n_1 \) and \( n_2 \) have the form \( n_{i0}\gamma^{-6} \) (they are supposed to have the same origin) then

\[
\mathcal{L}_i = \frac{n_{i0}B_1}{n_{20}B_2},
\]

Since

\[
P_{el} = \frac{1}{3} m_0c^2 \int_{\gamma_1}^{\gamma_2} n_i(\gamma) \, d\gamma = kn_{i0}
\]

where \( P_{el} \) is the electron pressure in region \( i \), we have, for pressure balance,

\[
\frac{B_1^2}{2\mu_0} + kn_{i0} = \frac{B_2^2}{2\mu_0} + kn_{20}.
\]

![Figure 1](https://example.com/figure1.png)

**Figure 1.** A fixed volume split into two by the boundary \( \Sigma \). The magnetic flux density (\( B \)), total pressure (\( P \)), volume (\( V \)) and relativistic particle number density (\( n \)) are as shown in the two regions.
Equation (6) with the condition $\mathcal{L}_1 / \mathcal{L}_2 > 1$ yields the two consistent conditions

$$\frac{B_1^2}{2\mu_0} < n_{20} k, \quad n_{10} k > \frac{B_2^2}{2\mu_0}.$$  \hspace{1cm} (7)

While not sufficient to ensure $\mathcal{L}_1 / \mathcal{L}_2 > 1$, they show that we can demand pressure balance and that the energy density (and so pressure) diminishes faster in region 1 than in region 2. To maintain pressure balance the boundary $\Sigma$ moves left causing $B_1$ to increase and $B_2$ to decrease. As the difference in energy loss-rates increases so the contraction of volume $V_1$ accelerates as does the increase in $B_1$. Equations (7) apply at $t = 0$ only, for the spectral forms are modified by the synchrotron process at later times. The ability of the mechanism to increase flux density is perhaps most evident in the special case $B_2 = 0$.

In the following sections we investigate the mechanism in more detail. In Section 3 we consider the diffusion of particles across the boundary $\Sigma$, and in Sections 4 and 5 a steady-state configuration and the approach to the steady state are considered.

3 The diffusion of charged particles in a magnetic field

In the model presented in the last section the flux of radiation from region 1 will diminish unless a supply of high-energy electrons can be maintained. Diffusion of particles across the boundary $\Sigma$ provides such a supply and permits the loss of accumulating low-energy particles whose pressure might otherwise disrupt the flux concentration. The particle flux across $\Sigma$ is in a sense determined by the direction of the particle-density gradient at each particular energy. Emission from short-lived knots has been considered elsewhere (Burn 1973).

We expect diffusion of charged particles across a magnetic field due to both particle–particle and wave–particle interactions. As discussed by De Young (1970) there is negligible diffusion due to the former for the plasma is essentially collisionless. The interaction that we consider is between particles and the turbulent component of the magnetic field, the power spectrum of which is taken to be of the same form as that for the velocity field. We assume a spectrum of Alfvén waves axially symmetric about the mean field with equal densities of waves of wave vectors $\mathbf{k}$ and $-\mathbf{k}$. Were the Alfvén waves produced by particle streaming (Wentzel 1968), this streaming would be limited to the Alfvén speed. However, when, as above, the Alfvén spectrum describes the turbulent field and there is no net wave propagation, particles diffuse along the field by random walking; the maximum speed of this streaming can be much below the Alfvén speed.

3.1 THE DIFFUSION TIME-SCALE

Let us use a test particle to follow a random-walking magnetic flux tube a distance $\Delta z$ along the mean field. Jokipii & Parker (1969) have found that its mean-square displacement perpendicular to this direction is

$$\langle (\Delta x)^2 \rangle = \lambda (P(0), B) \Delta z,$$  \hspace{1cm} (8)

where $\lambda$ is a known function of the zero-frequency power spectrum of turbulence and of the mean field. While the derivation of (8) requires that certain conditions be satisfied (see Appendix A) we stress that, in as much as particles are tied to field lines, (8) also gives information about the particle distribution.

Urch (1977) has discussed, at length, particle transport based on compound diffusion, while Forman & Jokipii (1978) have argued for the use of simple diffusion ($\Delta z = ur$) on the
ground that particles drift rapidly to independent field lines. To justify the use of compound diffusion, and to show that the random-walking of the field is the dominant mechanism of perpendicular particle drift, we estimate (1) the rms displacement due to microcurvature and microgradients and (2) the rms displacement due to gyroresonant interactions, the mechanism of parallel diffusion that maintains isotropy in the pitch-angle distribution.

For an electron of Lorentz factor $10^5 \gamma_3$ in a field $10^{-8} B_{-8}$ T, the gyroradius is

$$r_g = 5.4 \times 10^{-3} \gamma_3 B_{-8}^{-1} \text{ pc}. \quad (9)$$

Later we shall suggest that the turbulence correlation scale, in a situation for which $B_{-8} \sim 3$, is $L_c \sim 0.5$ pc. Thus $r_g \ll L_c$. Now we have

$$\langle (\Delta x)^2 \rangle_{\text{micro}} \approx \left( r_g c \tau \right)^{1/2} \quad (10)$$

and

$$\langle (\Delta x)^2 \rangle_{\text{GRI}} \approx \left( \tau / t_s \right)^{1/2} r_g, \quad (11)$$

where in (11) we have assumed a displacement of $\sim r_g$ in one gyroresonant interaction and $t_s$ is the collision interval. For a power spectrum $P \propto k^{-\beta}$, $\beta < 2$, $k > k_c$, Forman & Jokipii give

$$t_s \approx \frac{r_g}{c} \left( \frac{L_c}{r_g} \right)^{\beta-1} \left( \frac{B^2}{\langle \Delta B^2 \rangle} \right). \quad (12)$$

Both $t_s$ and the parallel diffusion coefficient, $\kappa_\parallel$, that we introduce later are quoted from sources which assume validity of the Fokker–Planck equation, i.e. assume $(L_c/r_g) \Delta B / B \ll 1$. This condition is clearly not satisfied here but similar results may be obtained, by a wave–particle analogy, from hard-sphere scattering theory and we believe that the Fokker–Planck condition is not relevant to their validity.

For diffusion out of a flux tube of diameter $10$ pc ($20 L_c$), we find in Section 3.2 a timescale $\tau \sim 10^7$ yr when $\gamma_3 \sim 1$. Anticipating later results we set $B^2 / \langle \Delta B^2 \rangle = 100$ and $\beta = 5/3$ to find $t_s \sim 0.3$ yr. Thus from (10) and (11) we find rms displacements in $10^7$ yr of

$$\langle (\Delta x)^2 \rangle_{\text{micro}} \sim 0.15 L_c,$$

$$\langle (\Delta x)^2 \rangle_{\text{GRI}} \sim 2 \times 10^{-5} L_c.$$
which in (8) yields
\[ \langle (\Delta x)^2 \rangle^2 = 2\lambda^2 \kappa_{\parallel} t, \]  
(14)
implying a perpendicular diffusion time-scale
\[ \tau_D \propto \frac{(\text{length scale})^4}{\kappa_{\parallel} \lambda^2} . \]  
(15)
In Appendix A we estimate the constant of proportionality as \( C \leq 10^{-3} \).

In conclusion, we believe that (15) is a plausible diffusion time-scale despite the fact that the Fokker–Planck condition is not satisfied (Urch 1977; Klimas & Sandri 1973). This is a consequence of the smallness of \( r_g \).

3.2 APPLICATION TO RADIO SOURCES

We follow De Young (1970) and use a von Karman power-spectrum which is finite at zero wavenumber and has the well-known Kolmogorov form for \( k > k_c \). For \( \kappa_{\parallel} \) we use the form given by Urch (1977 and preprint). For waves propagating along the mean field, only the fundamental mode of the gyroresonant interaction occurs. Otherwise higher harmonics are introduced but the fundamental dominates and the form given (equation (B2), Appendix B) should be a realistic approximation to \( \kappa_{\parallel} \) particularly if few waves propagate at a large angle to the mean field.

\( P(k), \kappa_{\parallel} \) and \( \lambda \) are given in Appendix B. By combining these with
\[ r_g = \frac{\gamma m_0 c}{eB} \]  
(16)
and
\[ L_c = eL, \quad e < 1, \]  
(17)
where \( L \) is the knot diameter, we find
\[ \kappa_{\parallel} = 3 \times 10^7 B^{-1/3} \left( \frac{B^2}{\langle \Delta B^2 \rangle} \right) L^{2/3} \epsilon^{2/3} \gamma^{1/3} \]  
(18)
and
\[ \tau_D = \tau_0 \gamma^{-1/3}, \]  
(19)
where
\[ \tau_0 = 5.8 \times 10^{-8} C B^{1/3} \left( \frac{B^2}{\langle \Delta B^2 \rangle} \right) L^{4/3} \epsilon^{-8/3}. \]  
(20)
All quantities are given in SI units.
It is the purpose of this section to demonstrate the possibility of particle diffusion with an appropriate magnitude and energy dependence. To this end we take the following plausible values
\[ B \sim 3 \times 10^{-8} T, \quad B^2/\langle \Delta B^2 \rangle \sim 100, \quad L \sim 10 \text{ pc}, \quad \epsilon \sim 0.05. \]
With this value of \( L \) we would not expect the field structure to have been detected by observation to date, and the implied \( L_c \sim 0.5 \text{ pc} \) is reasonable for a magnetic field that
originates in the active nucleus of a parent galaxy. With such values we have $\tau_0 \leq 10^8$ yr. The diffusion time-scale for electrons of $\gamma_3 \sim 1$ is $\sim 10^7$ yr, in which time they diffuse along the field $\sim 1$ kpc.

4 A steady-state model

We suppose that a seed field may be amplified in the active hot-spot of a radio source and, in the following, consider only the field structure of the tail. Note that the field configuration to be described is of a much smaller scale than that of the observed turbulent polarization structure. This latter structure we believe to have a scale characteristic of the beam-width, which is the scale of dominant eddies of the relativistic beam (Simon 1978).

Consider that, as a result of preferential energy-loss in regions of slightly enhanced field, the tail is now differentiated into discrete flux tubes (Fig. 2) which we call 'knots' of length $L_z$ and diameter $L_k$. These we consider to have a filamentary nature, $L_z \gg L_k$, and to be immersed in an almost field-free particle reservoir. A magnetic field adequate to allow the reservoir to be treated as fluid will be insignificant dynamically and as regards synchrotron radiation.

Consider one such knot in a reservoir of sufficient volume that the reservoir may be considered infinite and steady. We take the reservoir electron spectrum as $n_r = n_{r0} \gamma^{-6}$.

4.1 THE ELECTRON SPECTRUM OF THE KNOT

Ignoring the detailed distribution of field and particles, we simplify the transfer equation (Ginzburg & Syrovatskii 1964) to get

$$V \frac{\partial n_k}{\partial t} = (n_r - n_k) \frac{V}{\tau_D} \frac{\partial}{\partial \gamma} \left( N_k \frac{d\gamma}{dt} \right) - n_k \frac{dV}{dt}, \quad (21)$$

![Figure 2. Schematic diagram of the turbulent flux tube immersed in a reservoir from which particles diffuse at high energy.](image-url)
where $N_k$ is the total electron number in a knot of volume $V$, and $n_k = N_k/V$. In the steady state
\[
\frac{\partial n_k}{\partial t} = 0, \quad \frac{dV}{dt} = 0, \quad (22)
\]
and we write $B = B_f$, $\tau_0 = \tau_{of}$, $V = V_f$. Thus in the steady state, using (2), we have
\[
\frac{V_f n_{r0}}{\tau_{of}} \gamma^{-\delta + 1/3} - \frac{N_k}{\tau_{of}} \gamma^{1/3} + \Omega B_f^2 \frac{\partial}{\partial \gamma} (N_k \gamma^2) = 0. \quad (23)
\]
This is a linear Bernoulli equation with solution
\[
N_k(\gamma) = \frac{V_f n_{r0}}{\Omega B_f^2 \tau_{of}(\delta - \gamma^3)} \gamma^{-(\delta + 2/3)} M \left( 1, \frac{3 \delta - 2}{2}, \frac{-3 \gamma^{-2/3}}{2 \Omega B_f^2 \tau_{of}} \right). \quad (24)
\]
$M$ is a confluent hypergeometric function (Abramowitz & Stegun 1970) and we have used the boundary condition $N_k \to 0$ as $\gamma \to \infty$.

The knot spectrum is well represented by the asymptotic forms
\[
n_k(\gamma) = \frac{n_{r0}}{\Omega B_f^2 \tau_{of}(\delta - \gamma^3)} \gamma^{-(\delta + 2/3)}, \quad \gamma > \gamma_c \quad (25a)
\]
\[
n_k(\gamma) = n_{r0} \gamma^{-\delta}, \quad \gamma < \gamma_c. \quad (25b)
\]

For continuity at $\gamma_c$, where diffusion and synchrotron time-scales are approximately equal, we have
\[
\gamma_c = (\Omega B_f^2 \tau_{of}(\delta - \gamma^3))^{-3/2}. \quad (26)
\]
Note that the synchrotron-steepened spectrum has its slope increased by $2/3$.

4.2 THE ELECTRON PRESSURE
As the spectra (Fig. 3) coincide below $\gamma_c$, the knot-reservoir electron-pressure difference is
\[
\Delta P_e \approx \frac{1}{3} m_0 c^2 \int_{\gamma_c}^{\gamma_u} (n_{r0} \gamma^{-\delta} - n_{r0} \gamma_c^{2/3} \gamma^{-(\delta + 2/3)}) \gamma d\gamma. \quad (27)
\]
$\gamma_u$ is an arbitrary upper cut-off and we take $\gamma_u \gg \gamma_c$. To justify this we note that observations of Cygnus A have now been extended to 100 GHz, implying electron Lorentz factors of nearly $10^5$ (Hobbs et al. 1978). The inequality in (27) results from the use of the asymptotic forms.

The electron pressure in the reservoir is
\[
P_{er} = \frac{1}{3} m_0 c^2 n_{r0} \gamma^{-\delta + 2/3} \frac{\gamma_u}{(\delta + 2)}. \quad (28)
\]
In case (1), $\delta > 2$, and we have
\[
\Delta P_e \geq \frac{2}{3(\delta - \gamma^3)} \left( \frac{\gamma_L}{\gamma_c} \right)^{\delta - 2} P_{er}. \quad (29)
\]
In case (2), $\gamma_3 < \delta < 2$, and we have

$$\Delta P_e \geq \left(1 - \frac{(2 - \delta)}{(\delta - \frac{4}{3})}\frac{\gamma_c}{\gamma_u}\right)^{2-\delta} P_{er}. \quad (30)$$

In case (2), unless $\delta \sim \frac{4}{3}$, $\Delta P_e \sim P_{er}$. This is a consequence of the relative importance of high-energy particles in determining the total pressure when $\delta < 2$. Thus, in this case, almost the full reservoir pressure is 'available' to confine $B_f$. We shall discuss in a later paper the possibility of choosing $\delta < 2$, and achieving an index of the radiation spectrum equal to that observed by allowing for the spatial dependence of $B_f$ and $n_k$ within the knot. Here we consider case (1) with $\gamma_c \sim \gamma_L$.

### 4.3 The Low-Energy Sink

The transfer equation predicts coincidence of the reservoir and knot spectra for all $\gamma$ below $\gamma_c$. However, the knot has a finite age and we cannot assume a steady solution for arbitrarily low $\gamma$. Particles pass into the region $\gamma < \gamma_L$ at a rate $I$

$$I = n_k \frac{d\gamma}{dt} \bigg|_{\gamma_L} = \frac{n_{r0}}{\tau_{0r}(\delta - \frac{4}{3})} \gamma_L^{-\delta + 4/3}. \quad (31)$$

If we set $n_k = n_0(\gamma - \gamma_0)$, $\gamma_0 < \gamma_L$ and $n_r = 0$ for $\gamma < \gamma_L$, then $n_0$ is limited to be less than that value for which the rate of loss by diffusion is equal to $I$. This limit on the outward particle gradient at $\gamma_0$ is

$$I = \int_{1}^{\gamma_L} n_0 \frac{\delta(\gamma - \gamma_0)}{\tau_{0r} \gamma^{-1/3}} d\gamma = \frac{n_0 \gamma_0^{1/3}}{\tau_{0r}}. \quad (32)$$

(31) and (32) determine $n_0$ from which $n_{r0}$ is eliminated using (28). Then the maximum sink-pressure is

$$P_{\text{sink}} = \frac{1}{3} m_0 c^2 \int_{1}^{\gamma_L} n_0 \delta(\gamma - \gamma_0) \gamma d\gamma$$

$$= \left(\frac{\delta - 2}{\delta - \frac{4}{3}}\right) \left(\frac{\gamma_0}{\gamma_L}\right)^{2/3} P_{er}. \quad (33)$$

Thus $P_{\text{sink}} = (\gamma_0/\gamma_L)^{2/3} \Delta P_e$ and can reasonably be ignored.
4.4 Pressure Balance

Consider the example $\delta = 2.6, B_f = 3 \times 10^{-8} \text{T}$ and $\gamma_L = 10$. $\gamma_e \sim \gamma_L$ if $\tau_{of} \sim 3 \times 10^7 \text{yr}$. As discussed in Section 3, such a $\tau_{of}$ is attainable and we infer that, for $\delta > 2$, $\Delta P_e$ may take its maximum value

$$\Delta P_e \approx \frac{2}{3(\delta - \frac{4}{3})} P_{er}. \quad (34)$$

Using the standard relation $2\alpha + 1 = \delta$, we find that for high frequencies $\alpha \sim 1.15$ for an electron spectrum steepened by $2/3$. This is close to the observed high-frequency index for the tails of Cyg A for which the other values are taken to apply. Thus $\Delta P_e \approx \frac{1}{2} P_{er}$.

The crux of this result is that we can maintain a field $B_f$, where

$$\frac{B_f^2}{2\mu_0} \approx \frac{1}{2} P_{er} \approx \frac{1}{2} n_g k T_g, \quad (35)$$

for source confinement by a thermal plasma of density $n_g$ and temperature $T_g$. Most of the reservoir pressure is available to confine a field whose flux density is then only limited by the confining ability of the circumgalactic medium.

As the magnetic field is convex towards the region of greater particle pressure, the flux tube is not inherently unstable but only a thorough treatment of the problem will yield the lifetime of the knot.

4.5 The Flux of Radiation

The radiation from electrons of number density $n_0 \gamma^{-6}$ in a volume $V$ and field $B$ scales as $n_0 V B^{(\delta^* + 1)/2}$. By demanding the same flux from a uniform equipartition source with parameters $n_{E0}, V_T$ and $B_E$, as from the sum over the knots containing electrons of spectral form $n_{k0} \gamma^{-\delta^*}$, where $\delta^* = \delta + \frac{4}{3}$, we have for the filling factor $F$

$$F = \frac{\Sigma V_f}{V_T} = \frac{n_{E0}}{n_{k0}} \left( \frac{B_E}{B_f} \right)^{(\delta^* + 1)/2} \quad (36)$$

$n_{k0}$ is given directly by (25a) from which $n_{E0}$ may be eliminated using (28) and (35). We find $n_{E0}$ using the equipartition energy–density relation $u_B = \frac{1}{4} u_e$. For a spectrum of slope $\delta^*$ ($\alpha \sim 1.15$, as observed) we find

$$n_{E0} = \frac{2(\delta^* - 2) \gamma_L^{\delta^*-2} B_f^2}{3 \mu_0 m_0 c^2}, \quad (37)$$

and thus that

$$F = F_0 B_E^{(3\delta + 7)/6} B_f^{-(3\delta + 5)/6}, \quad (38)$$

where

$$F_0 = \frac{8}{27} \frac{(\delta - \frac{4}{3}) \gamma_L^{2/3} \Omega_{of}}{(\delta - 2)}.$$

(39)

For the values quoted earlier $F_0 \sim 5.5 \times 10^{14}$. Unfortunately $F$ depends strongly on $B_E$. In the tail near a hot-spot, where turbulence is most developed and the diffusion mechanism most likely to apply, a reasonable estimate of $B_E$ is $2 \times 10^{-8} \text{T}$. This value implies $F \sim 0.1$. 

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Since $B_T = B_{\text{seed}}/F$ and $\Delta B \sim B_{\text{seed}}$, we have $\Delta B/B_T \sim 0.1$ as assumed, and furthermore that $B_{\text{seed}} \sim 3 \times 10^{-9} T$, a value easily achieved by fluid convection. A smaller $F$ will result from a smaller $B_E$, but in any case the filling factor is small enough for the approximation of an infinite reservoir to be reasonable.

5 The approach to the steady state

Above $\gamma_s(t)$, defined by

$$\gamma_s = \frac{1}{\Omega B^2(t) t},$$

the spectrum has a steepened form which adjusts on the synchrotron time-scale of the highest energy electrons. We assume a quasi-equilibrium above $\gamma_s$ and use the steady-state solution with $B$ and $V$ as functions of $t$:

$$N_k(\gamma, t) = \frac{V(t) n_{\gamma_0}(\gamma^2 - \gamma_s^2)}{\Omega B^2(t) \tau_{\text{SR}}(\gamma - \gamma_s)}, \quad \gamma > \gamma_s. \quad (41)$$

This amounts to solving (21) with terms of order $N_k/t$ ignored as small compared with the term of order $N_k/\tau_{\text{SR}}$. We justify using the final value of $\tau_0$ (which depends on $B$ and thus on $t$) by noting that the term $n_r - n_k$ becomes significant only at late times when $n_r = n_k$ at $t = 0$.

For continuity at $\gamma_s$ we have

$$N_k(\gamma, t) = \frac{V(t) t^{2/3} \mu_{\gamma_0} \gamma^{-\delta}}{\Omega B^2(t) \tau_{\text{SR}}(\gamma - \gamma_s)}, \quad \gamma < \gamma_s. \quad (42)$$

For pressure balance to be maintained during contraction

$$\frac{B^2(t)}{2 \mu_0} + \frac{1}{3} m_e c^2 \left( \int_{\gamma_L}^{\gamma_u} n_k \gamma \, d\gamma + \int_{\gamma_s}^{\gamma_u} n_k \gamma \, d\gamma \right) = P_{\text{er}}. \quad (43)$$

The sink pressure builds up as the source tends to a steady state and, having ignored it in that state, we do so here. We have also ignored the initial field-pressure in the knot, being approximately one-hundredth of the final value.

We evaluate the integrals using (40), (41) and (42). We use (28) to write $n_{\gamma_0}$ in terms of $P_{\text{er}}$. $P_{\text{er}}$ is then expressed in terms of $B_T$, using (34) with $\Delta P = B_T^2/2 \mu_0$. Finally, since flux is conserved and $V \propto L^2_K$, we have $B = B_T V_t/V$. By scaling $V$ and $t$, $\tilde{V} = V_t/V$ and $T = \Omega B_T^2 \gamma_L t$, we find

$$\bar{V}^{-2/3} T^{2/3} = \frac{2 \bar{V}^{(\delta - 7/3) T^{(\delta - 4/3)}}}{3(\delta - 4/3)} = 1 - \frac{2 \bar{V}^2}{3(\delta - 4/3)}, \quad (44)$$

where we have used (26) with $\gamma_c = \gamma_L$ and let $\gamma_u \to \infty$.

Implicit in (43) is $\gamma_L < \gamma_s < \gamma_u$. As $\gamma_s$ falls below $\gamma_L$, a spurious solution is generated and (44) does not describe the system after this time.

For $\delta = \delta_o = \delta_s$ we have a solution $\tilde{V} = T$. Substituting $\tilde{V} = T + \Delta T$, $\delta = \delta_o + \Delta \delta$ and linearizing (44) we obtain, for $\delta \approx \delta_s$,

$$\tilde{V} = T + \frac{3(T^3 - T^{3(\delta - \delta_s)}}{2(T^{3(\delta - \delta_s)} - 2T^2 + 2)}. \quad (45)$$

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Note that we may write (42) as
\[ n_k = \psi n_0 \gamma^{-6}, \quad \psi = (T/\bar{V})^{2/3}. \] (46)

For \( \delta > 8/3 \) (<8/3), \( \psi \) is a monotonic decreasing (increasing) function of \( T \). This is related to whether or not synchrotron energy-loss associated with the increasing magnetic field is adequate to compensate for the increase in particle density resulting from a decrease in knot volume. This naive approach does not account for the influence of diffusion on the spectrum for \( \gamma < \gamma_s \). Diffusion will limit the inward or outward gradient and reduce it to zero in the steady state.

We can conclude from the above, however, that a volume close to the final value will be reached in approximately \( (1 - \Delta \delta)/\Omega B^2 \gamma_L \) and the final value achieved in a few \( \tau_D(\gamma_L) \). Since \( \gamma_c = \gamma_L \), these times are approximately equal.

Using values from the previous section we find that contraction occurs in a time of order \( 2 \times 10^7 \) yr. (For \( \delta = \delta_0 \), contraction occurs in a finite time, the time that \( \gamma_s \) reaches \( \gamma_L \), since we have used the asymptotic forms for all \( \gamma \).)

Using the limits on \( v_s \) quoted in the introduction, we can estimate a dynamical source age from \( \tau_s = D/v_s \), where \( D \) is the separation of the hot-spot from the parent galaxy:
\[ 3 \times 10^6 \text{yr} < \tau_s < 2 \times 10^7 \text{yr}. \]

This suggests that knots may form only late in a source’s life. Knot formation would then act as a ‘turn on’ mechanism and so is another way in which particle lifetimes may be effectively increased.

6 Conclusions

We have discussed the drift of electrons across a turbulent magnetic field and, using plausible values for the correlation length and mean field, have shown that a diffusion time-scale of an order acceptable in the radio source model may be attained.

By a suitable choice of break point in the electron spectrum, it is shown that the relativistic electron-pressure difference between two regions may sustain a magnetic field in one of them. This field is a consequence of differential synchrotron loss rates in an initially slightly inhomogeneous source. The model employs a Kolmogorov spectrum of turbulence by virtue of which the high-energy spectrum is steepened by 2/3 and not by 1 as commonly assumed.

As certain parameters (e.g. the correlation length) are, as yet, not determined by observation or theory, we have resorted to the use of likely values to build a self-consistent model. However, we believe this to be a plausible model capable of removing the need for particle acceleration and large fluid velocities away from the heads of extended extragalactic radio sources.

Acknowledgments

The author thanks Dr N. J. Holloway and Mr A. J. Allen for valuable discussion and comment. The comments of Drs Y.-M. Wang and A. S. Wilson were also much appreciated.

References

Appendix A

From Jokipii & Parker (1969) we have the probability distribution of field lines and thus of particles

\[ \phi(x, z) \, dx = (4\pi\lambda z)^{-1/2} \exp\left( -\frac{x^2}{4\lambda z} \right) \, dx, \]  

(A1)

which is valid when \( \Delta B/B \) is small and when

\[ \int_0^{\Delta z} \int_{z'}^{-z' + \Delta z} d\eta R(0, 0, \eta) \equiv \Delta z \int_{-\infty}^{\infty} R(0, 0, \eta) \, d\eta, \]  

(A2)

where \( R \) is the auto-correlation function. The latter is satisfied by choosing \( \Delta z > L_c \) and since \( \Delta B \) will be, at most, of the order of \( B \) before field compression, we can expect the former condition to be satisfied in a local flux concentration.

To effect a change of origin we replace \( x \) by \( x - x_0 \) and make the substitution \( z^2 = 2\kappa_t t \). We then integrate twice to form \( n(t) \):

\[ n(t) = \int_{-L}^{0} dx_0 \int_{0}^{\infty} dx (4\pi\lambda)^{-1/2} (2\kappa_t t)^{-1/4} \exp\left( -\frac{(x - x_0)^2}{4\lambda (2\kappa_t t)^{1/2}} \right). \]  

(A3)

This is taken to represent the total number of particles in \( x > 0 \) which were distributed uniformly in \( -L < x < 0 \) at \( t = 0 \). Since diffusion occurs both to right and left, the fraction of particles lost from \( -L < x < 0 \) by time \( t \) is

\[ F = \frac{2n(t)}{L}. \]  

(A4)

By using the substitution

\[ \beta = L(4\pi\lambda)^{-1/2} (2\kappa_t t)^{-1/4}, \]  

(A5)

we can combine (A3) and (A4) to form an equation for \( \beta \):

\[ 2(\pi^{1/4}\beta^{-1}) \int_{0}^{\infty} \int_{0}^{\beta} \exp \left[ - \frac{(x + x_0)^2}{4\lambda \beta^2} \right] dx \, dx_0 = F. \]  

(A6)
In (A5) we interpret $L$ as a length scale, the knot diameter, and $t$ as a diffusion time-scale, $\tau_D$, to get

$$\tau_D = \frac{L^4}{32\beta^4 \kappa_\parallel \lambda^2}. \tag{A7}$$

Thus the constant of proportionality in (15) is $C = G/32\beta^4$, where $\beta$ is given by (A6) and $G$ is a geometrical factor ($<1$) that allows for application of the above slab model to a cylinder. By assigning a value to $\mathcal{F}$, say 0.5, we find $C \lesssim 10^{-3}$. The inequality serves also to indicate that the time-scale is an upper limit if significant drift to independent field lines does occur as discussed in Section 3.

Appendix B

The von Karman spectrum is

$$P(k) = \frac{2\pi \Gamma(\frac{3}{2})}{\pi^{1/2} \Gamma(1/3)} \frac{L_c \langle \Delta B^2 \rangle}{(1 + k^2 L_c^2)^{5/6}} \tag{B1}$$

For the parallel diffusivity coefficient we use

$$\kappa_\parallel = \frac{1}{2} c r_g B^2 \int_0^1 \frac{d \mu \mu(1 - \mu^2)}{P(1/r_g \mu)}. \tag{B2}$$

Furthermore we have

$$\lambda(P(0), B) = \frac{P(0)}{2 B^2}. \tag{B3}$$