A test of clustering among radio sources in 0.16 sr of the 6C survey

Colin Masson* Mullard Radio Astronomy Observatory, Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE

Received 1979 January 27; in original form 1978 December 7

Summary. Four tests for clustering of radio sources are briefly reviewed and a modified form of Covariance Function Analysis is adopted for a study of the 6C catalogue. The analysis of an area of 0.16 sr shows that the weakest sources are evenly distributed on scales of a few degrees, corresponding to linear scales of several hundred Mpc. Among the brightest sources ($S > 2.7$ Jy) there is a significant excess of close (< 12 arcmin) pairs of sources. This may be due to the detection of separate sources within clusters or superclusters of galaxies, but further observations are required to test this.

1 Introduction

The large-scale clustering of extragalactic radio sources is known to be very weak and its measurement is limited by statistical fluctuations and by spurious factors inherent in the compilation of catalogues. The most comprehensive work is that of Webster (1976a, b) who has applied a sensitive test to several radio catalogues. He concludes that there is no evidence for clustering, despite previous claims to the contrary (e.g. Maslowski, Machalski & Zieba 1973).

The study of the 6C catalogue (151 MHz) extends the domain of the tests in that it is the deepest catalogue which covers large areas of sky, and the statistical noise is correspondingly low. The only deeper catalogues which have been investigated thoroughly (Webster & Pearson 1977) are those of the 5C surveys, each of which covers only about 10 square degrees. Another advantage of studying the new catalogue is that the instrumental effects will be different from those in other surveys and the differences may enable such artefacts to be identified. Last, but not least, the statistical test has been improved since earlier studies. The usefulness of any evidence for isotropy of the radio source population depends on the strength of the test employed and, as will be shown later, the test used here has advantages over other tests previously used for the analysis of radio catalogues.

2 Tests for clustering

The four main tests which have been used are Nearest Neighbour Analysis (NNA), Binning Analysis (BA), Power Spectrum Analysis (PSA) and Covariance Function Analysis (CFA). These are described briefly below.

* Present address: California Institute of Technology, Pasadena, California 91109, USA.
NNA (e.g. Maslowski et al. 1973) is the weakest of the four tests. It involves comparison of the distances of nearest neighbours of all sources with the distribution expected for a random scatter of sources. The test is sensitive to clustering only on scales less than the mean separation of sources. In confusion-limited radio surveys, this is just the range of scales for which anticlustering (Webster 1976b) dominates, so NNA is not useful for analysing such surveys. Even in cases where anticlustering is not important, NNA is a weaker test than PSA or CFA.

BA (e.g. Machalski 1977) is performed by dividing the survey area into regions (bins) and counting the radio sources in each bin. These counts are then compared with the Poisson distribution expected for a random scatter of sources. BA is an inflexible test in practice since the results depend strongly on the sizes and positions chosen for the bins. It is possible to exaggerate a random fluctuation in the distribution of sources by an injudicious choice of bins. Conversely, it is possible to miss real evidence of clustering if the disposition of the clusters does not match that of the bins.

PSA is related to a test first used by Schuster (1897) to study tidal triggering of earthquakes and has enjoyed considerable popularity recently as a result of the work by Peebles (1973) and Webster. To compute the power spectrum of a catalogue, each source is represented by a delta function and the Fourier transform of this set of delta functions is calculated. Any clustering results in the amplitudes of the corresponding terms of the transform having larger values than those expected for a random distribution of sources.

Catalogues of optical galaxies suffer from large-scale variations in density which are due to galactic obscuration. PSA has the great advantage that these large-scale variations only affect a few terms of the power spectrum, which may easily be discarded. In the case of radio catalogues, however, larger-scale variations in sensitivity are less important than the anticlustering which results from the inability of the telescope to resolve close pairs of sources. This small-scale effect modifies all terms of the power spectrum and so cannot be dealt with simply by removing a few of them.

Formally, CFA (e.g. Peebles 1973) is a Fourier dual of PSA. The covariance function of a catalogue is derived from the autocorrelation function of the same set of delta functions used in computation of the power spectrum. The angular covariance function \( w(\theta) \) is defined by

\[
n(\theta) = \rho N [1 + w(\theta)] \, d\Omega,
\]

where \( N \) is the number of objects in the catalogue, \( \rho \) is their surface density, \( d\Omega \) is the element of solid angle in an annulus of radius \( \theta \) and width \( d\theta \), and \( n(\theta) \) is the number of catalogue members falling within the set of \( N \) such annuli, one centred on each object in the catalogue. \( w(\theta) \) is thus the fractional excess of pairs of sources with separations between \( \theta \) and \( \theta + d\theta \). For a random distribution of sources, therefore, the expectation value of the covariance function is zero at all values of angular separation. A population containing clusters of size \( A \) would give \( w(\theta) > 0 \) for \( \theta < A \) and \( w(\theta) = 0 \) for \( \theta > A \).

CFA has the considerable advantage for radio catalogues that anticlustering only affects a very small part of the covariance function. It is therefore possible to disentangle anticlustering from real clustering more efficiently by CFA than by PSA. This is the reason for choosing CFA for the present investigation.

3 CFA in practice

It is relatively expensive in computer time to calculate the true angular separation between each pair of sources in a large catalogue. Since only a small range of declinations is involved
in the present set of data, a cylindrical equal-area projection is used to reduce the computation. The use of equal-area projection for such studies has been discussed by Webster (1976a). The projection is

\[ x = a_x \alpha, \]
\[ y = a_y \sin \delta, \]

where \( x \) and \( y \) are the projected coordinates of a source having right ascension \( \alpha \) and declination \( \delta \). The scale factors \( a_x \) and \( a_y \) are chosen to equalize the projected angular scales at the centre of the projected area. The linear distortion caused by this projection is less than 20 per cent in the range of declinations treated in the present analysis of 6C data.

It is necessary next to allow for the finite size of the region, since the absence of sources beyond the edge will affect the covariance function. This could be done analytically, by computing a correction for \( w \) at each value of \( \theta \). It was found simpler to take a hint from Fourier PSA (Webster 1976a) and apply periodic boundary conditions to the region. This reduces the power of the test slightly for values of \( \theta \) which approach the width of the region, but preserves the mean and standard deviation of \( w \) expected for an infinite region. In the 6C data analysed below, this procedure affects \( w \) by less than 17 per cent.

The statistical error in each estimate of \( w \) is the reciprocal of the square root of the number of independent pairs of sources contributing to that estimate (Peebles 1973).

4 Analysis of the catalogue

4.1 The catalogue

The catalogue used for the analysis described below is the first list of 5225 sources from the 58 degree declination strip of the 6C survey (Masson 1978a; Masson et al., in preparation). Once careful cross-calibration between different areas of the survey has been performed, it will be possible to construct a uniform catalogue covering a much larger area which will permit more sensitive tests of clustering. The prospects for such improvements are discussed in the next section.

It is convenient to perform CFA on areas which are regular in shape. Consequently, only regions bounded by lines of constant right ascension and declination have been used. Such regions are rectangular on the equal-area projection. Because of attenuation by the primary beam, the declination range available is greater for brighter sources. The flux density scale of Wills (1973) was used and the calibration was based on an assumed flux density for 3C 263 of 21.2 Jy at 151 MHz.

For sources having flux densities greater than 0.5 Jy, it is possible to take the declination range from 48° to 66.5° and right ascensions between 8h 15m and 11h 45m. This is an area of 0.16 sr, containing 190 sources with \( S > 2 \) Jy, 531 with \( S > 1 \) Jy and 1212 with \( S > 0.5 \) Jy. For sources having flux densities down to 0.2 Jy, the narrower declination range of 53.5°–63°, containing 1584 sources with \( S > 0.2 \) Jy, was analysed.

4.2 Results for scales of several degrees

Fig. 1 shows the covariance functions for four samples of sources, limited at flux densities of 2, 1, 0.5 and 0.2 Jy. Separations of less than 6arcmin have been omitted because the telescope cannot resolve close pairs of sources, and the normalization has been altered to allow for this. The covariance function for the subsample with \( S > 0.2 \) Jy is only drawn for \( \theta < 3° \) because the width of the region analysed is small.
The effects of anticlustering and holes should be negligible on these angular scales for the brighter two samples. There is a hint of clustering in the $S > 2$ Jy sample, although none of the plotted points deviates from 0 by as much as two standard deviations. Adding together all the points for separations less than $3^\circ$, the resulting values of $w$ are

\[
w = + 0.065 \pm 0.032 \quad 0^\circ < \theta < 3^\circ \quad S > 2$ Jy\]

\[
w = + 0.011 \pm 0.011 \quad 0^\circ < \theta < 3^\circ \quad S > 1$ Jy\]

There is a marginally significant excess ($p < 0.05$) in this range for the $S > 2$ Jy sample; it cannot be ascribed to variations in the sensitivity of the survey since the values of $w$ for the weaker subsamples are much closer to zero. The lack of evidence for clustering among 4C sources at a similar flux level (Masson 1978b) is not in conflict with the present result, since anticlustering affects the values of $w$ for the 4C catalogue on scales of a few degrees.

The status of the result for the $S > 2$ Jy sample will become clearer when larger areas or the 6C catalogue are analysed. The data from the North Polar region (Waggett 1978) were examined and showed no evidence for clustering on scales of a few degrees, but the area covered was comparatively small.

As discussed by Seldner & Peebles (1978), a sharp increase of $w$ with $S$ indicates that the clustering is attributable to relatively weak, nearby objects. This suggests a study of the cross-correlation between radio sources and galaxies, which would give a measure of the proportion of these radio sources which are in nearby clusters.
The weaker two samples are isotropically distributed to the limit set by statistical fluctuations. Combining the results in the range $1^\circ < \theta < 3^\circ$ for the weakest sample, we find $w = -0.0012 \pm 0.0029$.

Anticlustering and holes have a negligible effect on scales greater than $1^\circ$ for these data so, at the 90 per cent confidence level, $w$ is less than 0.005. This is the strongest limit yet placed on anisotropies in radio source distributions on these angular scales. Since any irregularities in the coverage of the survey would have increased the observed value of $w$, this result permits some confidence to be placed in the construction of the catalogue.

### 4.3 The Covariance Function on Small Angular Scales

Fig. 2 gives the covariance functions for the same samples as before, but this time plotted at intervals of 6 arcmin. The points for separations less than 6 arcmin are again omitted. The values of $w$ for separations less than 30 arcmin are listed in Table 1.

![Covariance functions for four samples of 6C sources, limited at 0.2, 0.5, 1, and 2 Jy.](https://academic.oup.com/mnras/article-abstract/188/2/261/991583)

In this range of angles there is a tendency for $w$ to increase at small $\theta$. Again this is most pronounced at the highest flux level. The probability of the 6–12 arcmin value for the $S > 2$ Jy sample being due to chance is less than 0.02. The value of $w$ for the smallest separations decreases with flux level. Part of this decrease is undoubtedly caused by anticlustering, particularly in the case of the $S > 0.2$ Jy sample.

In assessing the significance of this result, it must be borne in mind that many different values of $w$ have been calculated and that, even with a purely random distribution of
Table 1. Values of $w$ for separations less than 30 arcmin.

<table>
<thead>
<tr>
<th>Separation</th>
<th>$S &gt; 0.2$ Jy</th>
<th>$S &gt; 0.5$ Jy</th>
<th>$S &gt; 1.0$ Jy</th>
<th>$S &gt; 2.0$ Jy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6' - 12'$</td>
<td>$-0.03 \pm 0.05$</td>
<td>$0.15 \pm 0.12$</td>
<td>$0.42 \pm 0.20$</td>
<td>$1.47 \pm 0.56$</td>
</tr>
<tr>
<td>$12' - 18'$</td>
<td>$0.07 \pm 0.04$</td>
<td>$0.17 \pm 0.09$</td>
<td>$0.25 \pm 0.15$</td>
<td>$0.30 \pm 0.36$</td>
</tr>
<tr>
<td>$18' - 24'$</td>
<td>$-0.01 \pm 0.03$</td>
<td>$-0.08 \pm 0.08$</td>
<td>$-0.15 \pm 0.13$</td>
<td>$-0.20 \pm 0.32$</td>
</tr>
<tr>
<td>$24' - 30'$</td>
<td>$0.01 \pm 0.03$</td>
<td>$0.03 \pm 0.07$</td>
<td>$0.09 \pm 0.11$</td>
<td>$0.07 \pm 0.29$</td>
</tr>
</tbody>
</table>

sources, 5 per cent of these will be significant at the $p < 0.05$ level. The value of $w$ in the range 60–66 arcmin for the 2 Jy sample differs from zero by 2.6 standard deviations, which is just as significant as the 6–12 arcmin value. Even so, the 6–12 arcmin result deserves consideration because it conforms with the predictions of most clustering models that $w$ varies inversely with $\theta$.

It is worth pointing out that these values of $w$ could not be produced by holes in the survey coverage (Masson 1978b). If a small fraction $f$ of the sources are removed by holes in the survey coverage, the maximum value of $w$ is approximately equal to $f$. In practice it is unlikely that holes will increase $w$ by more than 0.02, even for the weakest sample considered here.

Figure 3. Contour maps of the nine closest pairs of sources in the $S > 2$ Jy sample. The contour intervals are 0.4 Jy/beam area for 4C 52.23, 62.13, 62.14, 63.14 and 4CP 64.10A, and 0.8 Jy/beam area for the rest. The first contour is drawn at one-half of this level. The beamwidth is $4 \times 4 \cosec \delta$ arcmin.
A test of radio source clustering

Figure 3 – continued

It should be possible in this case, as in the 4C case discussed earlier, to test the reality of this result by looking more closely at the actual pairs of sources involved. In the \( S > 2 \) Jy sample there are nine pairs of sources with separations less than 12 arcmin, as opposed to three to four expected. Contour maps of these sources at 151 MHz are presented in Fig. 3. When the Palomar Sky Survey prints were examined at the positions of these sources, no bright galaxies \( (m_\rho < 17) \) were found either coincident with the sources or on the line joining them. Further observations of these nine pairs are in progress to determine their radio structures and any possible relationships between the members of the pairs.
4.4 A SEARCH FOR RESOLVED DOUBLE SOURCES

One of the features of the 6C survey maps is an apparently excessive number of pairs of sources with roughly equal intensities, separated by only a few arcmin. It is very tempting to suppose that these are parts of resolved double sources. A few undoubtedly are, but an attempt has been made to test the idea quantitatively by computing the covariance function for sources with a ratio of flux densities less than 2. Since many double sources have components with approximately equal intensities, this test should select such pairs, while rejecting chance superpositions of unrelated sources. The efficacy of this procedure is limited by the steepness of the source count curve, since a large fraction of the sources in any radio catalogue have flux densities within a factor of 2 of the limiting flux density. When the test was carried out for a sample limited at 0.25 Jy, the covariance function for \( \theta < 30 \) arcmin actually dropped slightly, although the sensitivity to genuine double sources was increased by 50 per cent.

5 Discussion

Apart from artefacts, there are three possible causes of a significant increase of \( w \): (1) a large-scale fluctuation (>100 Mpc) in the space density of radio sources, (2) detection of nearby sources which are clustered in the same way as optically observed galaxies, (3) resolution of individual sources with large angular sizes, which then appear in the catalogue as two separate sources. We now discuss these in turn.

5.1 LARGE-SCALE VARIATIONS

The first possibility, of clustering on scales greater than 100 Mpc, is the most interesting since this range of distance is not well covered by catalogues of galaxies. Studies of radio source counts (e.g. Robertson 1978) demonstrate that, for most plausible models, the mean redshift of a sample of radio sources increases as their flux density decreases. Essentially, this is because the slope of the radio luminosity function is less than the slope of the radio source count curve. Only in the most extreme model considered by Robertson was a sharp decrease of mean redshift with decreasing flux density found, and this applied only at flux densities \( S < 0.3 \) Jy at 408 MHz. Thus it is reasonable to assume that the samples containing the weakest 6C sources have the largest proportion of high-redshift objects.

Clustering on slightly smaller scales has been studied using catalogues of galaxies and clusters of galaxies (e.g., Peebles 1978 and references therein). Since these catalogues reach to redshifts of only 0.1–0.2, corresponding to distances of 600–1200 Mpc if \( H_0 = 50 \) km \( s^{-1} \) Mpc\(^{-1}\), no reliable information can be obtained from them about clustering on scales of a few hundred Mpc or more. Radio catalogues reach a great deal deeper, with typical sources having redshifts of 1–2. In addition they are not affected by the galactic obscuration which complicates optical studies.

It is very difficult to relate an assumed model of large-scale clustering to its observable effects on a catalogue of radio sources. To do this properly it would be necessary to know, or assume, at least the following:

(1) a cosmological model;
(2) a description of the nature of the clustering (e.g. discrete clusters or power-law correlation) and whether the clusters are gravitationally bound or expanding with the Hubble flow;
(3) a knowledge of the luminosity function of the sources;
(4) miscellaneous data, such as the fraction of radio sources which are clustered, any correlation between clustering and luminosity, etc. Such factors would arise if, for example, the propensity of a galaxy to produce radio emission depended on whether it was a member of a cluster.

It is not worth fitting models with so many free parameters to what is essentially a null result. If a measurable value of $w$ had been obtained, then its magnitude and form would have permitted more detailed interpretation. It is worthwhile, however, to estimate the magnitude of clustering which could have been detected and what improvements might be possible when better catalogues become available.

The major limitation on the detection of clustering is the superposition of unrelated clusters at different distances. Consider two independent samples with identical covariance functions and containing equal numbers of members. If the two samples are combined, the new covariance function will have the same form as before but one-half of the previous amplitude, because the excess of true pairs at any separation is doubled while the number of random pairs is quadrupled. If $n$ such samples are superposed, the amplitude of the covariance function is reduced by a factor of $n$.

If we are investigating clustering of physical size $L$ and the members of the catalogue have a typical distance $D$, with $D > L$, then the observed two-dimensional covariance function $w$ is related to the three-dimensional covariance function $\xi$ by

$$w(\theta) \sim (L/D) \xi(L),$$

where $\theta$ is the angle subtended by a length $L$ at distance $D$.

The value of $w$ found for the $s > 0.2$ Jy sample on scales $1-3^\circ$ was $-0.0012 \pm 0.0029$. Since $\theta \sim 0.05$, the relation between $\xi$ and $w$ is

$$\xi(L) \sim w(1 + z)/0.05,$$

where $z$ is the typical redshift of the sources and $L$ is the length which subtends an angle of 0.05 rad at this redshift. The typical redshift was estimated from Robertson's (1978) 'standard' source-count model as $z \approx 1.5$. In this case, the value of $\xi$ is

$$\xi(L) = -0.06 \pm 0.015,$$

where the quoted error reflects only the statistical error in the estimate of $w$. Taking $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$ and $q_0 = 0$, 0.05 rad corresponds to a length, $L$, of 130 Mpc at a redshift of 1.5. For unbound clusters, expanding with the Hubble flow, it would correspond to a length of 300 Mpc at the present epoch. The value of $\xi$ is altered by a factor of order unity if account is taken of the fact that $\theta$ corresponds to different physical sizes for objects at different distances.

The values both of $\xi$ and $L$ depend on the redshift distribution of the radio sources, which is known only indirectly, as a result of models of radio-source luminosity functions and evolution. Some idea of the possible range of $z$ may be gained by reference to the work of Robertson (1978) and references therein. Until such time as the redshift distributions of samples of weak radio sources are measured directly, the values of $\xi$ and $L$ calculated above will remain uncertain by a factor of $\sim 3$.

The limits on $\xi$ for optically selected galaxies have been obtained by fitting smooth $\xi$ functions to the observed values of $w$. This approach may also be useful for radio sources if it proves possible to measure the $\xi$ function at physical separations of a few Mpc. The shape of this function may then indicate that $\xi$ falls below my $1\sigma$ upper limit of 0.08 at some
distance less than 320 Mpc. This would be consistent with the somewhat stronger limits which have been set on the clustering of galaxies (Peebles & Hauser 1974) and of clusters of galaxies (Hauser & Peebles 1973). These studies indicate that the covariance function for Abell clusters drops to unity at a distance of 60 Mpc and that the covariance function for galaxies, $\xi_g(r)$, is

$$\xi_g(100 \text{ Mpc}) \leq 0.025,$$

for $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

In order to calculate the possible improvement on this figure which might be achieved by future surveys, consider a catalogue of $M$ sources covering a solid angle $\Omega$. The expected number of random source pairs having angular separations less than some value $\theta$ is $\sim \theta^2 M^2/\Omega$ for $\theta < 1$. Since the distribution is Poissonian, the standard deviation of the covariance estimate is $\sim \Omega^{1/2}/M \theta$.

The increase of $M$ gained by the use of a larger fraction of the 6C catalogue will reduce the standard deviation by $M$ since the source density is unchanged. If 1 sr is available instead of the 0.08 used here, the sensitivity of the test will be increased by a factor of 3.5, and it will also be possible to measure the values of $w$ for larger angular separations.

5.2 CLUSTERING ON SCALES LESS THAN 100 Mpc

Because the 151-MHz telescope cannot resolve sources which are separated by less than a few arcmin, separate components of individual sources of clustering on the scale of single clusters of galaxies can only be detected amongst relatively nearby objects. The physical separations involved are unlikely to be much more than 1 Mpc and therefore could not be resolved at redshifts greater than about 0.1, where the corresponding angular separation is 7 arcmin.

Only a fraction of the objects in the catalogue have such small redshifts, so the detection of any clustering will be hindered by the noise arising from the isotropically distributed background objects. Since the fraction of nearby objects increases with flux density, any clustering should show up more clearly amongst the members of the stronger subsamples.

Such a variation with flux density is indeed found in the covariance function for small angular scales (Section 4.3 above), where the greatest value of $w$ was found for the strongest sample ($S > 2 \text{ Jy}$), and there is also a hint of a variation on scales of 1–3° (Section 4.2). Although it is not possible to determine the nature of the clustering from the present data alone, the study by Seldner & Peebles (1978) has shown that such a variation is consistent with the detection of clustering amongst relatively nearby radio galaxies.

The absence of bright galaxies associated with the closest pairs indicates that the increase in $w$ cannot be ascribed to the resolution of large radio sources into separate components. This agrees with the finding by Masson (1978a) that the number of radio sources of large angular size in the 6C catalogue is consistent with previous expectations.

Further progress can be made by observing the closest pairs of sources at high resolution. Another approach, which should prove fruitful, is to combine the results presented above with those of a cross-correlation analysis of radio catalogues and galaxy catalogues, of the type made by Seldner & Peebles. A study of this sort will be very valuable for elucidating the relationship between radio sources and clusters of galaxies. It would extend the work which has been done on radio sources identified with Abell clusters (e.g. McHardy 1977), by offering improved sensitivity and resolution and by including sources which are not associated with rich clusters.
6 Conclusions

A form of CFA which is particularly suitable for studying radio source catalogues has been described. Application of this to the 6C catalogue has produced the following results.

(1) The weakest and most distant radio sources have a distribution which is uniform to the limit of statistical errors. This confirms the uniformity of the 6C survey and indicates a lack of clustering on scales of several hundred Mpc.

(2) There is marginally significant clustering detectable amongst the stronger sources. This is attributable to resolution of nearby clusters and/or large radio sources, and further observations are in progress to investigate this.

(3) The observed covariance functions, together with measurements of the cross-correlation between galaxies and radio sources, will form a powerful tool for studying the connection between radio sources and clusters of galaxies.

References