A size distribution of silicate grains in Eta Carinae

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Summary. Assuming a geometry for Eta Carinae a size distribution for silicate grains has been determined to account for the observed optical linear polarization. The size distribution is given as

\[ n(a) = \exp \left[ -\left( \frac{a}{0.022} \right)^{0.85} \right] \]

and grains of size 0.15 \( \mu \)m contribute most to the scattering at optical wavelengths. Using the same distribution we find that maximum emission at 10 \( \mu \)m can be expected from grains of size 0.1 \( \mu \)m.

1 Introduction

Recently Warren-Smith et al. (1979), henceforth referred to as Paper I, published a detailed optical linear polarization map of the Homunculus, the nebulosity surrounding Eta Carinae. They interpret the polarization in terms of dust scattering and invoke a novel geometry for the Homunculus. In this paper we assume the geometry of Paper I and consider the nature of the dust scattering centres.

2 Geometry of the Homunculus

Fig. 1 shows the geometry of the Homunculus as proposed in Paper I. It consists of a central star illuminating both an equatorial optically thick dust ring and the more tenuous outer regions of the nebulosity.

Radial velocity measurements of the head of the Homunculus (Thackeray 1961) show that it was ejected from Eta Carinae and is receding from us, in a direction that makes an angle of 20° with the plane of the sky. In the proposed geometry we take this angle as the tilt of the whole Homunculus with respect to the plane of the sky.

Fig. 2 shows a trace of the polarization along the major axis of the Homunculus, the trace being centred on Eta Carinae. A feature of this trace is that at large distances from the centre the polarization reaches asymptotic values (35 ± 2 per cent) to the NW; (25 ± 2 per cent) to the SE. This feature we assume arises where the nebula is optically thin and the scattering geometry is not significantly changing. Between these regions and Eta Carinae the...
300  T. F. Carty et al.

Figure 1. Illustration of the geometry of the proposed model of Warren-Smith et al. The central illuminating star is surrounded by an optically thick toroidal ring and an optically thin nebula. The measured degrees of polarization on the periphery of the nebula are given along with the proposed scattering angles.

Figure 2. Degree of linear polarization of a central strip along the major axis of the Homunculus. NW is to the left, SE to the right.

Polarization drops off considerably due in parts to the effects of increase in optical depth, changes in scattering geometry and the presence of the equatorial ring.

The scattering on the major axis at the extremities of the Homunculus will occur at scattering angles of 110° to the NW and 70° to the SE on the proposed geometry. Although these angles are symmetric about 90° a crucial observation is that the polarization of light scattered at these angles is different. The remainder of this paper is given to a discussion of the type of scattering that will give rise to an asymmetry in the polarization as observed.

3 Nature of the scattering centres

Rayleigh scattering by spherical and needle-like grains and electron scattering produce high polarizations when scattering light at angles near to 90°. However, both give polarizations which are the same at angles symmetric about 90° whereas we observe different polarizations at 70° and 110° and we therefore reject these mechanisms. They can also be rejected on other grounds (Paper I).

Scattering by grains of size $\geq \lambda$ are not necessarily symmetric in scattering angles around 90°. This type of scattering is described by the Mie formalism and the degree of polariza-
tion depends not only on the scattering angle but also on the size and nature of the scattering dust grains. Before applying Mie scattering to our model of Eta Carinae we must consider the type of grain material to be used.

Infrared observations of Eta Carinae show the 10 μm feature attributable to silicate grains (Robinson, Hyland & Thomas 1973; Gehrz et al. 1973; Aitken & Jones 1975). Mitchell & Robinson (1978) describe the spatial and spectral distribution of infrared radiation from Eta Carinae in terms of thermal emission from grains of size 0.1 μm, they use a mixture of grain species consisting of silicates, corundum and iron. Aitken et al. (1977) observed the 4.09 μm Brackett-α line from hydrogen in the central source of Eta Carinae which suggests that the intervening grains are transparent to this radiation and therefore of a size <1μm. Andriessen, Donn & Viotti (1978) using spherical geometry conclude ‘that the condensation around Eta Carinae contains disordered silicate clusters with average radius 1 μm’.

In the present paper we consider the grains to be silicate and endeavour to determine a size distribution that will describe the optical polarization results while still being compatible with the infrared studies. We are unable to consider a mixture of grain species without introducing ambiguities into our model which could not be resolved unless we had optical polarization measurements in wavebands other than the V waveband of Paper I. Following Hanner (1971) we take the refractive index of silicate in the V waveband to be 1.65–0.05i.

4 Calculation of polarization in Mie scattering

The formalism given in Wickramasinghe (1973) gives the scattering functions $S_1(\theta, x, a)$ and $S_2(\theta, x, a)$ in terms of the scattering angle $\theta$; the size of the grains $x$, where $x = 2\pi a/\lambda$, $a$ is the grain radius; and $m$ the complex refractive index of the grain material. $S_1$ and $S_2$ are the amplitudes for scattered light polarized parallel and perpendicular to the scattering plane which allow the polarization to be calculated.

It is known from studies from the interstellar medium that grains of a range of sizes exist and this is represented by a size distribution $n(a)$. A form of the size distribution can be written analytically as

$$n(a) = \exp \left[-(a/a_0)^\alpha\right]$$

where $a_0$ and $\alpha$ are constants. Greenberg (1968) introduced this size distribution for interstellar extinction studies and using ice grains he found $\alpha = 3$ and $a_0 = 0.30$. Hanner (1971) when considering silicate grains in the study of the Merope nebula took $a_0 = 0.15$. This reduction in $a_0$ compensates to some extent for the increase in refractive index between ice and silicate grains. Recently Hong & Greenberg (1978) have discussed the grain size distributions in interpretation of total to selective absorption and interstellar polarization measurements of the interstellar medium. They conclude that a grain size distribution of the form $\exp \left(-a a^3\right)$ is most appropriate. However, they are considering grains in equilibrium between growth and destruction and in Eta Carinae we are dealing with a region of grain growth.

In our calculations we consider silicate grains with a size distribution given by $n(a)$ above and determine the values of $\alpha$ and $a_0$ that describe the polarization.

In terms of $S_1$, $S_2$ and $n(a)$ the polarization $P$ for light scattered through an angle $\theta^0$ is

$$P = \frac{100\left[|f_1(\theta)|^2 - |f_2(\theta)|^2\right]}{|f_1(\theta)|^2 + |f_2(\theta)|^2}$$
where

\[ f_1(\theta) = \int_0^\infty n(a) S_1(\theta, x, m) \, da; \quad f_2(\theta) = \int_0^\infty n(a) S_2(\theta, x, m) \, da \]

with

\[ x = \frac{2 \pi a}{\lambda}. \]

As a check of our computational procedures we calculated \( S_1 \) and \( S_2 \) for a range of grain sizes and species and compared them with the same quantities given in Wickramasinghe (1973). We also check our integration routines by calculating \( f_1 \) and \( f_2 \) to give the polarizations predicted in Hanner (1971) for the parameters used in this reference. In both comparisons our results were perfectly acceptable.

The degrees of polarization for scattering through angles of 70 and 110°, the scattering angles in the head and tail of the Homunculus based on our geometry of the object, were calculated using the above expressions. We considered various values of the parameters \( a_0 \) and \( \alpha \) in the size distribution \( n(a) \) and the results are discussed in the next section.

5 Comparison of results

In Table 1 we summarize the results of our calculations to predict the observed polarization in terms of the size distribution discussed earlier. Distribution A with \( a_0 = 0.15 \) and \( \alpha = 3.0 \), values typical of the interstellar medium, does not give an acceptable fit. Distribution B with \( a_0 \) slightly modified to 0.13 gives \( P(110°) > P(70°) \) but does not describe the observations exactly. Distribution C with \( a_0 = 0.022 \) and \( \alpha = 0.85 \) gives an excellent fit to the data.

The mean sizes of distributions B and C are 0.06 and 0.03 \( \mu m \) respectively but as grains of various sizes give very different contributions to the scattering the mean size is not a very meaningful indicator of the effective size of the grains. In Fig. 3(a) we plot \( C_{sca} n(a) \) as a function of grain size for distributions B and C. \( C_{sca} \) is the total scattering cross-section for a grain of a given size. This plot shows that the maximum contribution to the scattering from both size distributions comes from grains of size 0.15 \( \mu m \). The contributions from both size

| Table 1. A comparison between the observed degree of polarization and that predicted from various size distributions of silicate grains. |
|---|---|---|
| SIZE DISTRIBUTION | OBSERVED POLARISATION | 25 ± 2 % | 35 ± 2 % |
| | \( (\theta = 70°) \) | \( (\theta = 110°) \) | |
| A: \( \exp \left \{ -\left( a/0.15 \right)^3 \right \} \) | 25 % | 20 % |
| B: \( \exp \left \{ -\left( a/0.13 \right)^3 \right \} \) | 31 % | 34 % |
| C: \( \exp \left \{ -\left( a/0.022 \right)^{0.85} \right \} \) | 26 % | 35 % |
distributions are very similar, however, the extra components of large and smaller grains from distribution C enable this distribution to give a better description of the observations.

In Fig. 3(b) we plot $C_{\text{abs}} n(a)$ as a function of grain size for radiation of wavelength 10$\mu$m using a refractive index of 1.65–0.5i (Gaustad 1963). $C_{\text{abs}}$ is the total absorption cross-section for grains of a given size. This plot gives a relative measure of the infrared emission at 10$\mu$m as a function of grain size assuming constant grain temperature. The maximum contribution comes from grains of size 0.1$\mu$m although there is a significant contribution from grains much larger than this.

6 Discussion

The grain size distribution B which is almost identical to the one used for interstellar extinction studies gives an approximate description of the observed optical polarization. This is very reassuring as presumably the grains in Eta Carinae will ultimately populate the interstellar medium and even now may be representative of at least one component of the interstellar medium. Because polarization is extremely sensitive to the size distribution we produced distribution C which appears different from distribution B but when the distributions B and C are weighted by the scattering cross-sections they are extremely similar, both showing that grains of size 0.15$\mu$m give the largest contribution to the scattering. When we look at the infrared properties of our distribution we find grains of size 0.1$\mu$m give maximum contribution to infrared emission for 10$\mu$m radiation. Mitchell & Robinson (1978) using a single grain of 0.1$\mu$m and a mixture of grain species give an adequate description of the infrared properties of Eta Carinae. It is very comforting to see that our size distribution, albeit for one grain species derived from optical studies predicts exactly the size used by Mitchell & Robinson (1978). Using size distribution C we have calculated a value of 2.4 for the ratio of the total to selective absorption. This value of the ratio is only slightly less than the typical value of the grains of the interstellar medium yet in Eta Carinae we are in a region of grain growth.
The next steps are to use our grain size distribution with a grain mixture to predict both the optical polarization and the infrared properties of Eta Carinae. The use of a grain mixture in optical polarization description will be possible when we have the accurate wavelength dependence of the polarization on the periphery of Homunculus: we have observations planned for the very near future to acquire this information.

7 Conclusion

Assuming a geometry of the Homunculus we have been able to determine grain size distributions that describe the observed optical polarization. The most effective grain size is 0.15 μm, assuming silicate as the grain species. The distribution also indicates the maximum infrared emission at a wavelength of 10 μm will come from grains of size 0.1 μm, precisely the size used by Mitchell & Robinson (1978) to describe the spectral and spatial distribution of infrared radiation from Eta Carinae. It is very pleasing to see that we can relate the optical polarization and infrared observations through the same grain size distribution.

It is obvious that optical polarization is very sensitive to the size of the grains giving rise to the scattered radiation and we see polarimetry as an accurate technique of investigating this aspect of the nature of the dust grains. Naturally calculations such as those presented in this paper require a knowledge of the geometry of the object in question. The form of the polarization map is a good indicator of the geometry and we have been fortunate in being able to incorporate also the radial velocity measurements of Thackeray (1961) to define the geometry accurately.

In the case of other ‘bipolar-type’ objects we see a combination of detailed polarization mapping at different wavelengths combined with velocity measurements of the nebulous material as an elegant method of determining the geometry of the objects and then investigating the characteristics of the size and nature of the dust grains therein.

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