Stimulated recombination emission from rapidly cooling regions in an accretion disc and its application to SS 433

L. Z. Fang* Institute of Astronomy, Madingley Road, Cambridge CB3 0HA

Received 1980 June 3; in original form 1980 March 20

Summary. The possibility of the recombination lasing mechanism working in a rapidly cooling ring-like region of an accretion disc around a black hole is proposed. Such a disc should have coherent line emissions of H and He I. This model could be used to explain consistently all the observed facts of SS 433.

1 Introduction

An accretion disc around a black hole is one of the models commonly used to explain various high energy astrophysical phenomena (Lightman, Shapiro & Rees 1978). This model has also been proposed to account for the strong emission line object SS 433 (Amitai-Milchgrub, Piran & Shaham 1979; Fang & Ruffini 1979). The most prominent kinematic feature of the ~160-day periodic variation in the red- and blue-shifted lines of Balmer and H I emission from SS 433 can be fitted very well by the model of an accretion disc with an emitting ring. But there are enormous difficulties in the energy aspect, because it needs a massive black hole of at least $10^4 M_\odot$. Such a value appears uncomfortably large to compare with that of the stellar order which is implied from the fact that the position of SS 433 lies right in the centre of the supernova remnant W50. The stimulated coherent origin of the line emission has been suggested (Ruffini 1980). Indeed, the coherent emission seems to be favoured from the following phenomena: the intensity can be changed dramatically with a short time-scale, sometimes the shifted lines are even absent, and the stochastic variation of the intensity and the wavelength for both Balmer and H I lines are independent of each other (Margon et al. 1979; Mammano, Ciatti & Vittone 1980; Bedogni et al. 1980).

However, what is the kind of lasing mechanism working in the emitting region of the Balmer and He I lines? What are the physical conditions necessary to have such mechanisms existing in SS 433? In this article we would like to propose a possible answer: that under suitable conditions, a rapidly cooling region should appear in the accretion disc so that the lasing process in a recombinating plasma probably occurs.

2 The possibility of the existence of a rapidly cooling region in an accretion disc

Almost all accretion discs so far constructed involve monotonic temperature profiles, i.e. the temperature of accreted matter increases monotonically with decreasing radius $r$ except

*Permanent address: Astrophysics Research Division, University of Science and Technology of China, Hefei, Anhwei, Peoples Republic of China.

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
in the inner boundary of the disc near the last stable circular orbit. In such a disc no cooling region exists. But this conclusion is in consequence of the assumption of the negligibility of the internal energy of accreting matter. Indeed, the internal energy of accreting matter is small compared to the total of the released gravitational energy and it can be neglected according to the energy criterion. However, if the release of internal energy could occur in some region, the temperature of the disc should drop with decreasing \( r \). As a consequence the temperature profiles should not be always monotonic; the existence of non-monotonic temperature profiles and cooling regions is possible. Therefore it is important to examine what are the conditions for the occurrence of release of internal energy of accreted matter in a disc.

2.1 PRP Model

Pringle, Rees & Pachołczyk (1973) have put forward the possibility of the existence of an accretion disc in which the outer regions are at high temperatures and may emit predominantly by the synchrotron process, while the inner region radiates at much lower temperatures. The main point of this model is that free–free emission dominates the cooling of the disc at certain radii and the internal energy is released rapidly during this cooling process. After the cooling, part of the cooling material forms a much thinner disc and another part is drained along the magnetic field which expands out of the disc plane.

2.2 Quasi-Quantitative Analyses for the \( \alpha \)-Viscosity Thin Accretion Disc

For a so-called \( \alpha \)-viscosity thin accretion disc model (Lightman et al. 1978) considering the internal energy of accreting matter, the fundamental equation for the temperature structure of an accretion disc cooled by bremsstrahlung emission in steady state is

\[
\frac{dT}{dr_*} + \frac{3}{5} \frac{T}{r_*} = A r_*^{-7/2} T^{-2} - B r_*^{-2},
\]

where \( r_* \equiv r/(GM/c^2) \), \( A = 7.2 \times 10^{33} M_*^{-1} M_* \alpha^{-2} \) and \( B = 4.6 \times 10^{12} \) K. \( M_* \equiv M/M_\odot \) and \( \dot{M}_* \equiv \dot{M}/(10^{17} \text{ g s}^{-1}) \) denote, respectively, the mass of black hole and mass accretion rate in dimensionless units. The derivation of equation (1) and its applicable conditions are given in the Appendix.

If the left side of equation (1) is set to zero, equation (1) reproduces the result applied in already developed disc models. Therefore, in mathematical language, the conditions for the correctness of neglecting the effects of internal energy are that the integral curves of the differential equation (1) in the range concerned by us are independent of the initial values (or boundary values) and approach always to the algebraic relation obtained by setting the right side of equation (1) to zero. Obviously, this point is open to question. For example, equation (1) shows that the existence of extreme values in the temperature profile is possible. If we make \( dT/dr_* \) zero in equation (1), then we find the relation of extreme values \( T_{\text{max}} \) in temperature with radius \( r_{\text{max}} \) (the second term on the left side of equation (1) can always be neglected compared with the second term on the right side if the disc is thin).

\[
T_{\text{max}} = 4.0 \times 10^{10} M_*^{-1/2} \dot{M}_*^{1/2} \alpha^{-1} r_{\text{max}}^{-3/4} \text{ K}. \tag{2}
\]

The extreme values of temperature are maxima, since we have

\[
\left( \frac{d^2 T}{dr_*^2} \right)_{r_* = r_{\text{max}}} = -\frac{3}{2} Br_{\text{max}}^{-3} < 0. \tag{3}
\]
Figure 1. Typical temperature profile with maximum, it is the solution of the equation (1) and the disc parameter $\log M_\ast^{-1} M_\ast \alpha^{-2} = -2$.

In other words, the radius $r_{\text{max}}$ at which $dT/dr_\ast = 0$ is just the position from which the release of internal energy of accreting matter should start.

Moreover, it can be obtained from equation (1) that the first term on the right hand side of equation (1) will increase more quickly than the second term inside $r_{\text{max}}$. This means that the temperature of accreting matter inside $r_{\text{max}}$ is dominated by free–free emission and the values of $dT/dr_\ast$ will increase rapidly. Hence equation (2) is, in fact, the relation between the radius of the rapidly cooling ring and the temperatures of accreting matter before this cooling. Fig. 1 is a typical temperature profile with a maximum which is one of the numerical solutions of the equation (1) with the disc parameter $\log M_\ast^{-1} M_\ast \alpha^{-2} = -2$.

In a word, the conditions for occurrence of the internal energy release region in the model developed above are equivalent to such boundary conditions from which we can find a solution of equation (1) with reasonable extreme values $T_{\text{max}}$ and $r_{\text{max}}$. There is only a narrow region of initial values satisfying this requirement. For a given radius $r_0$, this initial temperature range is $T_0$ to $T_0 + \Delta T$ where $T_0$ is the solution of $f(r_0, T_0) = A T_0^{-2} r_0^{-\gamma/2} - B r_0^2 = 0$ and $\Delta T$ may be estimated by

$$\Delta T = - (\partial f/\partial r)_0 / (\partial f/\partial T)_0^2.$$

(4)

Fig. 2 gives the region of initial values in the $\dot{M} – T$ plane. When the initial values lie in the region shown in Fig. 2, the temperature profile of the disc is highly sensitive to the parameter

Figure 2. The region of initial values $T$ and $\dot{M}_\ast$ at $\log r_\ast = 7$; in this region the temperature profiles bring maximum at significant temperature range and these are highly sensitive to the disc parameters.
on the boundary properties. As a consequence, a slight variation in the infall of matter will produce a significant change in the temperature profile and hence in the emission intensity.

3 Stimulated emissions from the rapidly cooled ring

The remarkable feature of the models mentioned in Section 2 is the rapid cooling in the ring-like region just inside the maximum temperature radius. In this ring-like region, the temperature of the incoming gas is dominated by the cooling of free–free emission. Gaseous temperature drops from a temperature at which the accreting matter is completely ionized to a much lower value at which plasma recombination starts. In the PRP model not only the temperature of the inner disc but also that of expanded out matter should be down to \( \sim 10^5 \) K.

The duration of the cooling processes can be estimated by free–free cooling rate, we obtain

\[
\Delta t \approx 3.7 \times 10^{-27} \frac{M^2}{\dot{M}_*} \alpha r_*^2 T^2 \text{ s.}
\]

On the other hand, the time-scale of free fall in the direction perpendicular to the disc is

\[
\tau = \sqrt{\frac{r^3}{GM}} = 5.0 \times 10^{-6} \frac{M_*}{r_*^{3/2}} \text{ s.}
\]

If the involved physical parameters have the values adopted in the next section, i.e. \( M_* \sim 10 \), \( \dot{M}_* \sim 10^{-1} \) and \( r_* \sim 10^2 \), both \( \Delta t \) and \( \tau \) have the same order of magnitude. This means that the cooling processes in the ring region belong to the free decay type.

Naturally, the temperature profile in the cooling region could not be calculated step-by-step according to equation (1), because it should not be consistent with condition (A13) during the increasing \( dT/dr \) over \( m_p/k \ GM/r^2 \) and, in succession, with conditions (A10), (A11) and (A12) during the decreasing \( T \). However, the reasonableness of the free decay (heat source cut-off) in this region could be accounted by an integral estimate. If we have

\[
\frac{dT}{dr} \geq \frac{m_p GM}{k r^2},
\]

then the width \( \Delta r \) of the cooling ring satisfies

\[
\Delta r \ll k T \left( \frac{G M m_p}{r^2} \right).
\]

Since the disc properties outside the cooling ring are described by the known equations, the heat added to this ring per unit time is still \( \dot{Q} = \frac{3}{2} (GM/r^2) \Delta r \dot{M} \) which is derived from the energy flux expression. On the other hand, the release rate of internal energy is \( \dot{W} = 3 M k T/m_p \). Therefore, inequality (7) implies that the heat sources are cut-off approximately, when

\[
\dot{Q} \approx \dot{W}.
\]

It has been shown in the last few years that one of the most probable ways of producing hydrogen and hydrogen-like stimulated emissions is using a rapidly cooling plasma (free decaying plasma) as the amplifying medium (Ali & Jones 1976; Jones & Ali 1978). Because the collisional-radiative recombination processes which are predominant at certain free decaying stage of plasma have considerably higher rates for higher discrete quantum levels.
than those for lower levels, cooling the plasma will result in inversion for states with lower principal quantum number.

The same conclusions have been obtained from both the time-dependent calculation and the steady-state solution. The lasing processes should occur whether the cooling plasma is optically thick or thin. This mechanism has been verified experimentally in a rapid cooling hydrogen plasma (Hoffmann & Bohn 1972) and C$^+$ plasma (Irons & Peacock 1974). Population inversions have been found for the first transitions of the Lyman, Paschen and Brackett series, respectively.

In all the analyses given above the effects of the infalling flux coming from the rest of the disc on the cooling region are not included. The cooling region receives the radiation from the other part of the disc causing the heating source to be raised to some effective value $e' = e + S$, $S$ being the contribution of the infalling flux. However, in general, we have $S \lesssim \phi_x$, where $\phi_x$ is the emission rate at maximum temperature region, i.e. X-ray emission region. Because the free–free emission rate $\phi$ in the recombinating cooling ring ($T \sim 10^5$ K) is much larger than $\phi_x$ ($T \sim 10^6$ K; see equation A9), then the infalling flux from the high temperature region could not be changing the free decay feature. Of course more detailed calculations on the lasing processes are necessary in which a more complete description of the matter behaviour and cooling in the accretion disc should be considered.

4 Application to SS 433

For fitting the kinematic properties of SS 433 we need an emitting ring with radius $r_* \sim 50$ in the accretion disc around a Kerr black hole (Fang & Ruffini 1979; Ruffini & Stella 1980). The large red- and blue-shifted lines are assumed to originate from this ring. The ring will undergo a precession around the rotation axis of the black hole as a consequence of the Lense–Thirring–Wilkins effect. The 164-day modulation in the spectral shift can thus be explained.

Let us now discuss the size of the lasing active region in the ring and the width of the emission lines. Owing to the gravitational redshift and the Doppler shift the observed wavelength of line emissions from accreting matter vary along a line-of-sight. The size, $D$, of the active regions in the direction of a line-of-sight may be found according to the following requirement: the wavelength change within the active region must be less than the corresponding spectral line broadening. Hence, we have

$$D \simeq (\Delta \lambda)_B \left( \frac{d\lambda}{dl} \right)^{-1},$$

where $(\Delta \lambda)_B$ is the spectral line broadening, $\lambda$ is the shifted wavelength in the accreting matter rotating in a gravitational field and $l$ denotes the affine parameter along a line-of-sight.

On the other hand, the relative intensity of the super-radiation or stimulated radiation may be estimated by the factor $\exp (\beta D)$, $\beta$ being the gain coefficient. For an emitting ring, the strongest emission has to originate from the opposite sides of the ring in which the $D$ is a maximum. One can call this region the maximum constant shift region (Ruffini & Stella 1980). Thus, the observed half-intensity line width $\Delta \lambda$ caused by the ring-like geometry can be obtained by

$$\Delta \lambda = \frac{\ln 2}{\beta} \left( \frac{d\lambda}{d\vartheta} \right) \left( \frac{dD}{d\vartheta} \right)^{-1},$$

where $\vartheta$ denotes the angular coordinate of the ring.
If we adopt the Doppler broadening for $(\Delta \lambda)_B$ we have from equations (8) and (9)

$$\frac{\Delta \lambda}{\lambda} \approx 6.4 \beta^{-1} M_*^{-1} T^{-1/2} r_*^{-2}.$$  

(10)

Therefore, with $T \sim 10^4$ K, $r_* \sim 10^2$ and $M_* \sim 10$, the constraint from the smallness of the width of shifted line $(\Delta \lambda / \lambda < 10^{-2}$, Bedogni et al. 1979) is equivalent to having the gain coefficient $\beta$ larger than $10^{-4}$. This is easy to satisfy in the rapidly cooling plasma (Ali & Jones 1976).

The solutions of equation (1) in the region outside $r_{\text{max}}$ can be approximately expressed by

$$r_* \approx 1.4 \times 10^{14} M_*^{-2/3} \dot{M}_*^{2/3} \alpha^{-4/3} T^{-4/3}.$$  

(11)

Therefore, there exists another optical emission region, and its radius $r_{\text{opt}}$ can be obtained by substituting $T \sim 10^6 - 2 \times 10^4$ K in equation (11). Then the ‘unshifted’ components in SS 433 spectrum could be explained by the outer optical emission region. Since the Keplerian period is given by $P = 3.1 \times 10^5 M_* r_*^{3/2}$, the ~13-day periodic variation in ‘unshifted’ components (Crampton, Cowley & Hutchings 1979) may thus be interpreted by the hotspot effect in the outer optical emission region if only we have

$$\dot{M}_* \alpha^{-2} \approx 0.2 - 0.01.$$  

(12)

The optical luminosity from outer regions with radius $r_{\text{opt}}$ and width $\Delta r_* \approx 10^{-2} r_{\text{opt}}$ is about

$$L_{\text{opt}} \approx 8.1 \times 10^{38} M_*^{2/3} \dot{M}_*^{4/3} \alpha^{-8/3}.$$  

(13)

Hence we can fit the expected luminosity $L_{\text{opt}} \sim 10^{35-36}$ erg s$^{-1}$* (Bedogni et al. 1980) if only we have

$$M_*^{2/3} \dot{M}_*^{4/3} \alpha^{-8/3} \sim 10^{-3} - 10^{-4}.$$  

(14)

The X-ray emission is generated mainly in the shell at which the temperature is near maximum. From equation (2) and $r_{\text{max}} \sim 50$, the 14.3 keV ($1.6 \times 10^8$ K) thermal bremsstrahlung spectrum of X-ray emission observed by Marshall et al. (1980) may also be fitted if only we have

$$M_*^{-1/2} \dot{M}_*^{1/2} \alpha^{-1} \approx 0.08.$$  

(15)

The X-ray luminosity is given by

$$L_x \approx 2.1 \times 10^{59} M_*^{-1} \dot{M}_*^{2} \alpha^{-2} T_{\text{max}}^{-2} r_{\text{max}}^{-5/2}.$$  

(16)

Hence the observed result, $L_x \sim (1 - 2.5 \times 10^{-2}) L_{\text{opt}}$ (Marshall et al. 1980) can be accounted if only we have

$$M_*^{-1} \dot{M}_*^{2} \alpha^{-2} \approx 3 \times 10^{-3} - 8 \times 10^{-5}.$$  

(17)

The set of equations (12), (14), (15) and (17) can be satisfied simultaneously by the reasonable parameters $M_* \sim 40, \dot{M}_* \sim 0.06$ and $\alpha \sim 0.5$.

Moreover, we have pointed out in Section 2 that such temperature profile solutions are highly sensitive to the boundary conditions. This feature may be the reason for the observed

*The luminosities of shifted components are much smaller than this value because they have a coherent and collimated emission mechanism.
fact of the stochastic variations or short time-scales in wavelengths and intensities of the shifted lines (Margon et al. 1979; Mammano et al. 1980; Bedogni et al. 1980).

5 Conclusions

The developed model can be made consistent with each of the results so far obtained by observation. The best parameters correspond to a low accretion rate \( M \sim 10^{16} \text{ g s}^{-1} \) and a high mass \( M \sim 10-100 M_\odot \). It can also be shown that the applicable conditions of equation (1), i.e. (A10)–(A13) are consistent for these parameters. That the values of mass are greater than the upper limit of mass for neutron stars provide a further support for the existence of a black hole in SS 433.

Finally, we would like to conjecture that the stimulated recombination emission mechanism might also work in other objects. Because the physical conditions necessary to have such mechanism are simple. It is only necessary that the plasma is cooled rapidly enough. For example, the existence of a region of inverse population in a rapidly expanding plasma jet has been demonstrated (Goldfarb & Lukyanov 1969).

Acknowledgment

I would like to thank Professor M. J. Rees for stimulating discussions.

References


Appendix: Derivation of equation (1)

For the \( \alpha \)-viscosity law thin accretion disc models in steady state, the fundamental equations, after averaging by integrating over the thickness of the disc, are as follows:

\[
\dot{M} = 4\pi rh \rho u, \quad \text{(mass conservation)} \tag{A1}
\]
\[
\dot{M}(GMr)^{1/2} = 4\pi r^2 h \rho p, \quad \text{(angular momentum conservation)} \tag{A2}
\]
\[
p/\rho = GMh^2/2r^3, \quad \text{(vertical pressure balance)} \tag{A3}
\]
\[
p = 2nkT, \quad \text{(equation of state)} \tag{A4}
\]
where $M$, $\dot{M}$, $p$, $\rho$, and $h$ are the mass of black hole, mass accretion rate, pressure, density and the disc half-thickness, respectively. By considering the internal energy of accreting matter, we replace the energy equation which equates the emission energy flux $\phi$ to the viscous thermal generation rate $\epsilon$ by the hydrodynamical energy equation with source and sink

$$
\rho \frac{D W}{Dt} + p \nabla \cdot \mathbf{v} = \epsilon - \phi,
$$

(A5)

where $W$ is internal energy given by $W = 3kT/m_p$. The source of heat added to the accreting matter per unit volume and unit time is given by

$$
\epsilon = \frac{3}{8\pi} \left( \frac{GM}{r^3} \right) \frac{\dot{M}}{h}
$$

(A6)

in the $\alpha$-viscosity law model. The sink term $\phi$ for an accretion disc cooled by free-free emission is given by

$$
\phi = 1.4 \times 10^{-27} n^2 T^{1/2} \text{ erg cm}^{-3} \text{ s}^{-1}.
$$

(A7)

For steady-state disc with a radial component of velocity $u$ pointing to centre, equation (A5) can be expressed by

$$
\rho u \frac{dW}{dr} + p \frac{du}{dr} + p \frac{u}{r} = \phi - \epsilon.
$$

(A8)

From equations (A1)–(A4), we can find the solution of $h$, $p$, $n$ and $u$ as a function of $M$, $\dot{M}$, $T$ and $r$. After doing this and substituting these results into equation (A8), we get equation (1)

$$
\frac{dT}{dr} + \frac{3}{5} \frac{T}{r_*} = \frac{3}{5 \alpha_*^2} \alpha^{-2} M \alpha^{-2} = 7.2 \times 10^{33} M_*^{-1} \dot{M}_* \alpha^{-2},
$$

(A9)

where

$$
A = \frac{1.4 \times 10^{-27}}{8 \pi G} \frac{m_p^{3/2} c^5}{k} \frac{\dot{M}}{\dot{M}_*^2 \alpha^{-2}} = 7.2 \times 10^{33} M_*^{-1} \dot{M}_* \alpha^{-2};
$$

$$
B = \frac{3}{5 \sqrt{2}} \frac{m_p c^2}{k} = 4.6 \times 10^{12} \text{ K}.
$$

The assumptions applied to the derivation of equation (1) are as follows.

(1) The dominant pressure source is assumed to be the gas pressure $P$: the radiation pressure $p_R$ can be neglected, i.e.

$$
p_R/p = 3.9 \times 10^{27} T^{-5/2} M_*^{-2} \dot{M}_*^2 \alpha^{-2} r_*^{-3} \ll 1.
$$

(A10)

(2) The absorption optical depth from the disc mid-plane to the surface is assumed to be thin to free-free absorption i.e.

$$
(\sigma_T \sigma_{\text{ff}})^{1/2} n h = 1.4 \times 10^{31} M_*^{-2} \dot{M}_*^{3/2} \alpha^{-3/2} r_*^{-3} T^{-7/2} \ll 1,
$$

(A11)

where $\sigma_T$ denotes Thomson scattering cross-section and $\sigma_{\text{ff}}$ is the Rosseland mean cross-section for free-free absorption.
(3) The inverse Compton cooling of the electrons is assumed to be not strong, i.e.

\[
y \equiv \frac{4kT}{m_e c^2} (\sigma_T n h)^2 = 1.0 \times 10^{16} M_\odot^{-2} \dot{M}_\odot^2 \alpha^{-2} r_\odot^{-3} T^{-1} < 1.
\]  

(A12)

(4) The dominant force which determines the rotating motion of accreting matter is gravitational, i.e.

\[
\frac{GM}{r^2} \geq \left| \frac{1}{\rho} \frac{dp}{dr} \right| = \left| \frac{6kT}{m_p r} + \frac{k}{m_p} \frac{dT}{dr} \right|.
\]  

(A13)