The interpretation of optical counts of quasars

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**Summary.** We present calculations of the number of quasars as a function of limiting $B$ magnitude for different assumptions about the evolution of the quasar population. The effect of emission lines is taken into account by using an average quasar spectrum.

The available observations are discussed and compared with our models. A good fit to the observations is obtained with an exponential luminosity evolution, at a rate very similar to that needed to explain the low-frequency radio source-counts. Pure density-evolution models, of the forms usually assumed, give a poor fit to the observations, but a mixed evolution, of both density and luminosity, is acceptable.

1 Introduction

Over the past decade or so, several groups have observed high-latitude fields in order to select a population of objects that is rich in quasars and relatively free from galactic stars (Sandage & Luyten 1967, 1969; Braccesi, Formiggini & Gandolfi 1970 (BFG); Becker 1970; Bolton & Savage 1978; Usher 1978; Steppe 1978; Schmidt 1978; Green & Schmidt 1978; Tyson & Jarvis 1979; Braccesi et al. 1980). In order to discriminate further against the latter, only those objects with an ultraviolet excess were catalogued. From spectroscopic studies of these selected objects a large number were indeed found to be quasars. One can then determine the number distribution of quasars as a function of limiting magnitude and draw some conclusions about the evolution of the quasar population with time.

Early surveys (Sandage & Luyten 1967, 1969; BFG) indicated $\log N - m$ slopes of $b = 0.75$ and 0.71 respectively, where the number–magnitude relation is approximately represented by

$$\log N(m) = a + bm$$

and $N$ is the number of sources per steradian with magnitude less than $m$. Setti & Woltjer (1973), however, from a study of both these surveys and from some smaller surveys, favoured a value of $b = 0.6$, the value expected for a uniformly populated Euclidean space. More recent work by Braccesi et al. (1980) and Green & Schmidt (1978) produced values of $b = 0.86$ and 0.93 respectively, indicating strong cosmological evolution of quasars with epoch.
Attempts have been made to model the counts by assuming an increase in the space density $\eta$ of quasars with redshift, e.g. by Schmidt (1978). He found that $\eta = \exp(10 \tau)$, where $\tau = 1 - (1 + z)^{-1}$, gives a reasonable fit to the data. Earlier (Schmidt 1972), he ruled out models of luminosity evolution on the basis that one would require a large number of quasars with redshift $z > 2.5$. We feel, however, that there is no real evidence for a cut-off in quasar redshifts, and so here we investigate models with evolution of both luminosity and density.

We have used the average optical quasar spectrum as determined by Cheney & Rowan-Robinson (1981), and the optical luminosity function calculated from a complete sample of quasars (Section 2), to predict number counts for a variety of possible evolutions (Section 3). Our results are compared with observations in Section 4, and the predicted redshift distributions as a function of magnitude are given in Section 5.

2 Determination of the optical luminosity function for QSOs

2.1 Selection of a complete sample of QSOs

In order to determine the optical luminosity function for QSOs, we need a complete sample with known redshifts down to a limiting magnitude $B_{lim}$. However, there are several problems involved in the provision of such a sample:

(i) In order to discriminate against galactic stars, the surveys which have been performed have involved searching for UV-excess objects (the definition of 'UV excess' varying from survey to survey) which are likely to be quasars. Although the mean spectrum we produced from a study of the Burbidge, Crowne & Smith (1977) catalogue (Cheney & Rowan-Robinson 1981) has colours which satisfy $U - B < -0.4$ for redshifts $< 3.5$, there is an appreciable scatter in the observed colours about this mean. Fig. 1(a) and (b) show the effect on the $UBV$ colours of $1 \sigma$ changes in the mean spectral index $\alpha$ or the equivalent width of Ly$\alpha$, $W_{\text{Ly}\alpha}$. Quasars of redshift $\sim 2.5$ may not then satisfy the criterion of UV excess which is being applied. More recent work (Smith 1975, 1976; Osmer & Smith 1976, 1977, 1980; Osmer 1977, 1978, 1980) identifies quasars by the presence of the Ly$\alpha$ line in the optical spectrum. This is effective only for redshifts $\geq 1.5$, since at lower redshifts the Ly$\alpha$ line lies in the ultraviolet. The selection effects associated with these samples make them difficult to use in the present type of study.

![Figure 1](https://academic.oup.com/mnras/article-abstract/195/3/497/1237827)

**Figure 1.** The variation in $U - B$ and $B - V$ with redshift for a quasar at the galactic pole with parameters $(\alpha, W_{\text{Ly}\alpha})$, where $\alpha$ is the optical spectral index and $W_{\text{Ly}\alpha}$ is the equivalent width of the Ly$\alpha$ line (cf. Cheney & Rowan-Robinson 1981) (a) $\alpha = 0.6, W_{\text{Ly}\alpha} = 80$ Å, (b) $\alpha = 0.5, W_{\text{Ly}\alpha} = 110$ Å.
(ii) It can be difficult to distinguish between quasars and other active galaxies such as Seyferts, particularly at high redshifts. For example, in the original Savage et al. (1978) sample, 30 per cent of the objects selected as stellar on the U, b and IIIa-O Schmidt plates were in fact found to have extended images on deep IIIa-J plates.

(iii) As the determination of the QSO redshifts involves obtaining the spectrum of each object and identifying (preferably) at least two emission lines in each case, only for the smaller samples of quasars have redshifts been determined of all the objects down to a particular limiting magnitude.

Taking these factors into consideration, the best sample still seems to be that of BFG, previously investigated by, e.g., Setti & Wolter (1973) and Fanti et al. (1973). This sample had a colour limit of $u - b < -0.35$ (equivalent to $U - B < -0.29$, using the formula given by Braccini et al. 1980). We have assumed that the sample is complete down to a limit of $B = 18.5$. This gives a sample of 26 quasars. Two more quasars may have been discovered in the area surveyed by BFG, satisfying the colour criterion (Katgert et al. 1973), so the extent of completeness of the survey is still not clear.

2.2 Calculation of the Luminosity Function

We computed the quantities $\eta_{0i} = 4\pi / (A V_{\text{max}})$ for each quasar in the sample, where $A$ is the area of the BFG survey (in square degrees) and $V_{\text{max}}$ is the maximum volume out to which the object could be observed before falling below the magnitude limit for completeness, given by

$$V_{\text{max}} = 4\pi \int_0^{z_{\text{max}}} \frac{z^2(z + 2)^2}{4(z + 1)^3} f(z, Q_D) \, dz$$

in units of $(c/H_0)^3$, where the density evolution factor is assumed to be of the form (Rowan-Robinson 1967)

$$f(z, Q_D) = \exp \{ Q_D [1 - 1/(1 + z)] \}$$

and the Milne model ($\Omega = 0$) has been assumed. Summing the $\eta_{0i}$ in bins in $\log P$, where $P$ is the monochromatic luminosity of the sources, calculated at a rest wavelength of 2500 Å, the optical luminosity function is given by:

$$\eta_0 (\log P) = \int_{\log P - \Delta/2}^{\log P + \Delta/2} \left[ \sum_{i=1}^{n} \delta(P - P_i) \eta_{0i} / \Delta \right] \, dP,$$

where $\eta_0$ is the number density per $(c/H_0)^3$ per dex in $\log P$, $\Delta$ is the bin size and $\delta$ is the Dirac delta-function.

If luminosity evolution is present, we correct the luminosity of each quasar by

$$(\log P)_{\text{corr}} = (\log P)_{\text{obs}} - \log \{ g(z, Q_L) \}$$

and assume that

$$g(z, Q_L) = \exp \{ Q_L [1 - 1/(1 + z)] \}.$$

Fig. 2 shows the luminosity function for one particular model, namely luminosity evolution with $Q_L = 6$. In this case the luminosity function is quite well represented by a power-law. Nevertheless, we use the calculated values of $\eta_0$ rather than a power-law fit in our calculation of the number counts of quasars.
3 Calculation of the number counts

From the luminosity function derived in this way we can now determine the number of quasars we expect to observe down to any particular limiting magnitude, considering different types of evolution. For a limiting magnitude $B$,

$$N(< B) = \int_{\log P_{\min}}^{\log P_{\max}} \eta_0(\log P) \left[ \int_{0}^{\tilde{z}(P, B)} dV_{\text{max}} \right] d \log P,$$

where $N$ is the number of sources per steradian brighter than $B$, $P_{\max}$ and $P_{\min}$ are the luminosities corresponding to the edges of the outermost bins in the luminosity function, and $\tilde{z}$ is the maximum redshift out to which a quasar of corrected power $P$ can be observed without falling below magnitude $B$.

4 Comparison with observations of quasar counts

In Fig. 3 are plotted counts of quasars given by several different observers, showing $\pm 1\sigma$ limits where available. The estimates given by Braccesi et al. (1980) for the BFG sample, their most recent study of the $13^h + 36^o$ region, and the SA57 sample (Steppe 1978), are maximum and minimum estimates taking into account the corrections for contamination by hot stars and losses due to selection effects.

The observed counts show a fair degree of scatter. In the case of the lowest estimates this may be due to incompleteness, possibly through using too stringent a criterion of UV excess. In other cases the counts may have been artificially increased due to Seyferts. The points of Braccesi et al. (1980) may be too high due to over-correction for selection effects. In fact the reason for making such a correction is not clear, since the selection by $U-B$ is much less severe in the SA57 sample (Becker 1970) $(U-B < -0.25)$ and the $13^h + 36^o$ sample than in previous surveys.

It should be noted that not all the observations shown are independent. Setti & Wolter (1973) used information mainly from the BFG sample and Usher used the Sandage & Luyten (1967) sample, redefined on the basis of variability studies (Usher 1978).
Interpretation of optical counts of quasars

We now discuss how our models compare with these observations:

(a) **Pure luminosity evolution.** An excellent fit to most of the observations is obtained with luminosity evolution of the form (5) with $Q_L = 6$ (Fig. 3). In particular, there is a good fit to the Green & Schmidt (1978) point at $B = 15.7$, which is important since the counts at bright magnitudes are less affected by the problems discussed in Section 2. The main discrepancy is with the values from Braccesi et al. (1980) and we believe that these may be overestimated, due either to contamination by Seyferts or to overcorrection for selection effects. Note also that this model is consistent with the upper limits derived from the work of Tyson & Jarvis (1979). If this model is correct, most of the faint stars counted by Tyson & Jarvis would be galactic in origin.

The rate of evolution required is interesting because it is almost exactly that required to explain the low-frequency radio source-counts (Rowan-Robinson 1970).

(b) **Pure density evolution.** These models are shown in Fig. 4. In order to fit the Green & Schmidt (1978) point at $B = 15.7$ we need $Q_D = 13$, but this predicts too many quasars at fainter magnitudes, violating the upper limits of Tyson & Jarvis (1979). Even the introduction of a cut-off in the number-density of quasars at $z = 2.5$ (inconsistent with the many quasars found at redshifts larger than this) does not bring this model into agreement with these upper limits.

(c) **Mixed forms of evolution.** In Fig. 5 we show some models with both density and luminosity evolution. If all active galaxies undergo luminosity evolution, as might be expected in the active galaxy scheme of Rowan-Robinson (1977), then those that satisfy the quasar condition (optical core outshines the galaxy’s starlight) will show both luminosity and density evolution. In the past, a greater fraction of active galaxies would be quasars. A reasonable fit to the observations is obtained with $Q_L = 4, Q_D = 2–4$. 

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(d) **Other forms of evolution.** We have also tried using evolutionary factors of the form \( f(z, Q_D) = (1 + z)^Q_D \), \( g(z, Q_L) = (1 + z)^Q_L \), which require a cut-off at a redshift \( z_{\text{max}} \), in contrast to the exponential forms of evolution used above. Such evolutions do not give a better fit to the observations and, in particular, we have been unable to find an acceptable pure density-evolution model.
5 Redshift distribution as a function of magnitude

In Table 1 we give the predicted redshift distribution of the quasar population as a function of magnitude, for the two evolutionary models which give the best fit to the observed optical source counts, namely pure luminosity-evolution with $Q_L = 6$, and mixed evolution with $Q_L = 4$, $Q_D = 4$. We present values for the number of quasars per steradian in redshift bins of 0.2 in log $z$ and magnitude bins of 1 mag. In order to integrate over a particular part of the luminosity function we have to use the power-law fit illustrated in Fig. 2. We have normalized the counts in each magnitude range to make them consistent with the source counts calculated in Section 3. In each case the upper figure corresponds to the pure luminosity-evolution model and the lower figure to the mixed evolution model.

Table 1. Redshift–magnitude table, indicating the number of quasars per steradian in intervals of 0.2 in log $z$ and 1.0 in $B$. The upper value corresponds to a pure luminosity-evolution ($Q_L = 6$) model and the lower value to a mixed evolution ($Q_L = 4$, $Q_D = 4$) model.

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Total  41.0  345  2100  11600  67.5  278  1560  47.5  333  2300  13500  82.5  417  1640

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6 Conclusions

We have compared theoretical optical source-counts of quasars, computed for a variety of evolutionary models for the quasar population, with observations by several different groups. Exponential luminosity evolution at a rate very similar to that used to fit the radio source-counts a decade ago ($Q_L = 6$) gives an excellent fit to most of the observations. An acceptable fit is also found with a combination of luminosity and density evolution ($Q_L = 4$, $Q_D = 2-4$), but the forms of pure density-evolution proposed to date would appear to be ruled out.

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References