N-body simulations of disc–halo galaxies: isolated systems, tidal interactions and merging

Ortwin E. Gerhard Institute of Astronomy, Madingley Road, Cambridge CB3 0HA

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Summary. Collisions of galaxies are studied through numerical simulations of 250-particle systems. Each model galaxy consists of a flat disc, an extended live halo and a core. The evolution of such systems and possible relaxation effects are investigated in detail.

Slow hyperbolic encounters between exponential disc-type galaxies give rise to strong tidal effects. These lead to asymmetric mass exchange, energy change and angular momentum loss. Resonances between particles and the relative orbit depending on the orientation of the three spins involved determine the strength of the interaction. Only a few per cent of the particles escape, but they carry away a substantial fraction of the angular momentum. Generally, the galaxy spins decrease and the disc and halo angular momentum vectors are not aligned after an encounter. The response of a galaxy to a collision with another galaxy is considerably different from that caused by a single perturber.

Merger cross-sections are spin-dependent. In the present experiments, the remnants have distinct visible cores and Hubble-type density profiles. They are triaxial, rotate slowly, and have anisotropic velocity structures. Some of them fit the observational data for elliptical galaxies reasonably well.

1 Introduction

Since Toomre & Toomre (1972) reported their extensive experiments with interacting non-self-gravitating discs, it has been accepted that ‘bridges and tails’ in galaxy pairs result from the tidal forces of the interacting systems. They also suggested that mergers should be a consequence and lead to something like elliptical galaxies. Studying this, however, needs self-gravitating systems to take into account the energy transfer from the orbit to the internal motions; and self-gravitating discs are violently unstable to bar-modes (Hohl 1971; Miller, Prendergast & Quirk 1970) unless they are stabilized by a heavy halo (Ostriker & Peebles 1973; Hohl 1976). To avoid these instabilities, previous N-body simulations of colliding and merging galaxies have used spherical systems, on the assumption that this is a good approximation if the massive haloes are very extended. The experiments by, for example, Roos &
Norman (1979) and Dekel, Lecar & Shaham (1980) show that inelastic, tidal effects are important in close encounters, causing severe damage to the systems involved. In this paper I study colliding three-component systems which consist of an exponential disc, a halo and a core. Particular attention is paid to the influence of spin orientation or equivalently plane orientation and to the influence of the halo on the strength and kind of the interaction.

There are two reasons for taking exponential discs. First, the interaction is also expected to depend on the binding energy distribution of the systems involved; so these should be chosen to be as realistic as possible, and most observed spiral galaxies follow an exponential surface brightness law (Freeman 1970). If the mass-to-light ratio is constant across the disc as indicated by the radial uniformity of the colours, this corresponds to an exponential surface mass density. Secondly, Hohl (1971, 1972) found that several computer models of initially non-exponential discs formed an azimuthally averaged exponential surface density through non-axisymmetric instabilities, and this is also true in the present $N$-body simulations.

Computer models of exponential discs using large numbers of particles have shown that these are also unstable to non-axisymmetric modes as transient spiral arms and bars unless they are surrounded by massive haloes (Hohl 1976; Hohl, Zang & Miller 1979, and references therein). These non-axisymmetries rearrange the mass and angular momentum distribution. Sellwood & James (1979) found that a galaxy model could be stabilized by an extended heavy halo so that it does not evolve and has a very slow heating rate. In the small $N$ calculations here, discs do evolve; this is shown by detailed studies of isolated systems below. The exponential disc turns out to be the most stable one.

If encounters are close and slow enough, initially unbound systems move on bound orbits after their first close passage and merge soon afterwards. This was shown for a large number of simulations using spherical systems by White (1978, 1979a). These merger remnants have an oblate structure and a Hubble-type density profile.

Although their velocity structure is anisotropic, they seem to rotate somewhat faster than elliptical galaxies do (e.g. Illingworth 1977). Thus White (1979b) suggested that the progenitors had to collide on nearly head-on orbits in order that the merger remnant resembles an elliptical galaxy. Indeed, recent cosmological $N$-body simulations (Aarseth & Fall 1980) confirmed that merging occurs mainly from marginally bound two-body orbits of low angular momentum.

When disc galaxies merge, the cross-section should be spin-dependent for two reasons. First, it is known from previous work that the strength of the interaction is mainly due to particles corotating with the relative orbit of the galaxies. Secondly, the effective half-mass radius of a disc–halo system changes with direction, so that the extent of overlapping during a close encounter depends on the relative spin orientation. These considerations suggest that the resulting parameter space will be much too large for the amount of available computer time, even if only equal galaxies are used.

Therefore, my main intention here is to answer some principal questions concerning the merger scheme. Can two exponential discs give a final Hubble profile? Is the distinction of disc and halo population still possible after merging, and how much material is lost? In addition, a comparison with the observed low rotation of elliptical galaxies and other observational parameters will be made.

For this work Aarseth's $N$-body code is used. Surely, using 250 particles per galaxy, even with a highly softened potential, is not an ideal experiment, but it is one within practical limits. The methods, initial conditions and relaxation tests are discussed in Section 2. Section 3 outlines detailed studies of isolated galaxies to give a description of how they evolve. Then, Section 4 describes specific features of tidal interactions between such systems,
and Section 5 gives the results of the merger simulations and compares them with observations. The main conclusions are listed in Section 6.

2 Initial conditions and method

2.1 A SINGLE SYSTEM

Generally, a galaxy is set up as a three-component system consisting of:

(i) a two-dimensional disc of 124 particles following a surface density \( \mu(r) = \mu_0 \exp(-\alpha r) \) (=exp disc) or \( \mu(r) = a(r_D^2 - r^2)^{1/2} \) (= sq disc), where in both cases the radius \( r_D \) is the cut-off disc radius;

(ii) a spherically symmetric halo of 125 particles following a volume density \( \rho(r) = \gamma(r^{-2} - r_H^{-2}) \), where \( r_H \) is the halo radius (\( \rho = 0 \) for \( r > r_H \));

(iii) a central core mass (one particle), typically \( \leq 10 \) per cent of the total mass.

The constants \( \mu_0, \alpha, \gamma \) are determined by the numbers of particles in the respective components and the number density is set up either by using direct random numbers, e.g. \( f(r) = \int r^2 \rho(r) dr \), or by distributing the particles in shells, depending on which is most convenient. The mass ratio of the various components is then defined by the respective particle masses \( m_D, m_H, m_c \). Notice that there is no arbitrary cut-off of the halo mass at a chosen radius, but \( \rho(r) \to 0 \) smoothly for \( r \to r_H \). Also, the mass inside \( r \), \( M(r) \propto r \) only in the inner parts of the halo. Velocity distributions are specified as follows:

(i) The disc particles were initially given a circular velocity such that they were in centripetal equilibrium with respect to the radial force component in the plane. In addition, to avoid local clumping, each disc particle was given an isotropic velocity computed by the Toomre criterion (Toomre 1964) (hereafter \( G = 1 \))

\[
\langle u^2(r) \rangle^{1/2} = h \cdot 3.36 \cdot \mu(r)/\kappa(r),
\]

where \( \mu(r) \) is the surface density, \( \kappa(r) \) the local epicyclic frequency and \( h \) a free constant typically equal to 1.2. Numerically, \( \kappa(r) \) was evaluated by computing the local radial force derivative using shells of thickness \( 1/10 \) \( r_D \) and then taking

\[
\kappa^2(r) = 3 \frac{u^2(r)}{r^2} + \frac{1}{m} \frac{\partial F(r)}{\partial r},
\]

where \( F(r) \) is the radial force in the plane at radius \( r \).

(ii) Halo particles were assumed to be in local virial equilibrium, giving \( u_j^2 = \frac{1}{2} |U_j| \), where \( U_j \) is the softened potential \( -\sum m_i/(r_{ij}^2 + \epsilon^2)^{1/2} \) at the position of particle \( j \) and the factor \( \frac{1}{2} \) comes from the fact that the total energies then satisfy approximately the virial theorem for the whole system. Note that the virial theorem in the soft potential case is \( (T, \mathcal{W} \text{ being the total kinetic and potential energy})

\[
-2\vec{T} = -\sum_j m_j u_j^2 = -\frac{1}{2} \sum_{i \neq j} \frac{m_i m_j}{r_{ij}^2 + \epsilon^2} \frac{r_{ij}^2}{(r_{ij}^2 + \epsilon^2)^{1/2}} = \mathcal{W} + \Sigma(e)
\]

giving a typical value of \( |\vec{T}/\mathcal{W}| = 0.45 \) instead of 0.5 (1/r case) in the models here. The velocities were then isotropically distributed in space.

(iii) The core mass is initially at rest.
As convenient reference quantities define the scale radius and crossing time in terms of total mass \( M \) and energy \( E \) as

\[
R_{\text{sc}} = M^2 / -4E, \quad t_{\text{cr}} = M^{5/2}(-2E)^{-3/2},
\]

(2.4)

neglecting the difference mentioned above. However, this leads to some difficulties since one can hardly speak of one scale radius in a non-spherically symmetric galaxy. The radius \( R_{\text{sc}} \) is about \( (5/4) \) times the half mass radius (from formula (2) in Spitzer & Hart 1971).

The model units are defined throughout the simulations by taking \( G = 1 \), disc particle mass \( m_D = 1 \), disc radius \( r_D = 20 \) units. Then the time units are fixed, but time will always be measured in standard crossing times.

A sq model is then defined by the set of parameters \( e, h, r_H/r_D, m_H/m_D, m_c \); for exponential discs \( \alpha \) is specified in addition. The standard sq model 4H has \( e = 2, h = 1.2, r_H/r_D = 1, m_H/m_D = 2, m_c = 25 \). Starting from this model, some of the parameters are varied to study their influence on the evolution of a galaxy.

### 2.2 Encounters

In most of the encounters a 'standard' galaxy model is taken. This consists of an exponential disc, an extended spherical halo following the volume density law \( \rho(r) \propto (r^{-2} - r_H^{-2}) \), and a heavy core particle of mass \( m_c \). Ideally, one would like (a) equal number of disc particles \( n_D \), and halo particles \( n_H \), (b) a halo extending rather further out than the disc, (c) equal particle masses, and (d) a stable disc.

The first and third requirements are due to the \( N \)-body method. One wants to keep some resolution in both disc and halo and avoid relaxation effects between particles of different masses.

As a compromise between these requirements the numbers of particles in disc and halo in the standard model are taken to be 124, 125 respectively, giving a total of 250 particles per galaxy. The halo extends 1.5 times as far as the disc and, to have the total halo-to-disc mass ratio inside the initial disc radius \( \geq 1 \), the halo particle mass is chosen \( m_H = 1.5m_D \). Finally, the core mass is \( m_c = 25m_D \), i.e. about 7.5 per cent of the total mass. An exponential disc \( \mu(r) \propto \exp(-r^2) \) with \( \varphi = 3 \) in model units (i.e. \( \varphi_D = 6.7 \)) is chosen not only on observational grounds but also because it was found to evolve only relatively slowly when stabilized by a halo of this order of mass (see Section 3). Some remaining disc evolution has to be tolerated.

The standard galaxy defined above has \( \lambda = 0.10 \), where

\[
\lambda = \frac{|L|}{L^1 E^{1/2} M^{-5/2}}
\]

(2.5)

is the dimensionless spin parameter; \( L, E, M \) the total internal angular momentum, energy and mass of the system, respectively. (Here, all the angular momentum comes from the disc, whereas mass and energy include the halo; so \( \lambda \) relates to the whole galaxy.) Arbitrary spin orientations are set up by performing a space rotation of a system around its centre of mass.

The orbital plane is chosen to be the \( xy \)-plane, with relative velocity in \( x \)-direction. Together with the space rotations this allows arbitrary encounters leaving only three free orbital parameters. These can be chosen to be the relative coordinates and velocity, \((x_0, y_0, z_0)\), or a more meaningful triplet

- \( e, p, x_0 \) (parabolic)
- \( e, p, x_0 = 0 \) at apocentre, (elliptic)
- \( \nu, p, x_0 \) (hyperbolic)
where \( p \) is the pericentre distance of the relative orbit, \( e \) the eccentricity, \( \nu \) a measure of the interaction

\[
\nu = \frac{v_p \text{(hyperbolic)}}{v_p \text{(parabolic)}},
\]

\( v_p \) pericentre velocity. Energy and angular momentum are given by (\( \mu \) reduced mass)

\[
E_0 = \frac{1}{2} \frac{L_0^2}{\mu p^2} - M^2 (p^2 + e^2)^{-1/2}, \quad L_0 = \mu v_p.
\]

Again a compromise has to be made. Ideally one would like to start the interaction at distances where the mutual interaction is really negligible. Consideration of computing costs and importance of relaxation effects before the actual collision, however, demand the opposite. Therefore the initial distances were chosen as about three times the halo radius. Finally, it should be noted that soft potential orbits are not conical orbits and that the above used eccentricity is meaningful only to a good approximation but not strictly.

Thus the list of parameters to vary now consists of

pericentre distance of orbit, \( p \),
eccentricity, \( e \),
spin orientation of \( L_1, L_2, L_{\text{orb}} \) (internal and orbital angular momentum),
relative size of the galaxies,
parameters of one single galaxy relative to those of the other as given above.

Table 1 lists the initial conditions for the experiments performed. Only non-standard systems are specifically described.

### 2.3 THE \( N \)-BODY METHOD

The program used here was developed and kindly made available by Dr S. Aarseth. It is an \( N \)-body code, integrating the equations of motion for a softened gravitational potential

\[
\Phi_{H} = -\frac{m_i m_f}{|r_i - r_f|} (1 + e^2)^{-1/2}.
\]

<table>
<thead>
<tr>
<th>Model</th>
<th>( e )</th>
<th>( \nu )</th>
<th>( p )</th>
<th>( x_0 )</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( \Delta )</th>
<th>Non-standard galaxies</th>
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<td>17.65</td>
<td>0</td>
<td>(−23, 42, −88)</td>
<td>(84, −6, −54)</td>
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<tr>
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<td>1.0</td>
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<td>100</td>
<td>as EA</td>
<td></td>
<td></td>
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<tr>
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<td>1.0</td>
<td>15</td>
<td>100</td>
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<td>(−88, 32, 34)</td>
<td>65°</td>
<td></td>
</tr>
<tr>
<td>PB</td>
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<td>1.0</td>
<td>15</td>
<td>100</td>
<td>(0, 0, −100)</td>
<td>(−7, 12, 99)</td>
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<td></td>
</tr>
<tr>
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<td>15</td>
<td>150</td>
<td>as PB</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>HA</td>
<td>(1.87)</td>
<td>1.2</td>
<td>15</td>
<td>100</td>
<td>as PB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1.2</td>
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<td>100</td>
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<td></td>
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<td>1.2</td>
<td>15</td>
<td>100</td>
<td>as HB</td>
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<td></td>
<td>( m_H \times 5/3 )</td>
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<tr>
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<td></td>
<td></td>
<td>( r_H = 10, m_H = 1.0, n_H = 63 )</td>
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</tbody>
</table>

**Notes:** Standard exponential discs with \( m_D = 1, m_H = 1.5, m_c = 25, r_D = 20, r_H = 30, \alpha = 0.33 \) are used. The internal plane (spin) vectors \( n_i \) are normalized and rounded; \( \Delta \) is their relative angle in degrees. The orbital angular momentum vector is \( (0, 0, 100) \) in all cases. The eccentricity \( e \) in the hyperbolic and elliptic cases does only approximately correspond to the soft potential orbit defined by \( x_o, y_o, \dot{x}_o \), whereas in the parabolic cases zero energy is denoted by \( e = 1 \). See Section 2 for notation.
where \( m_i \) is the mass of particle \( i \) and \( \mathbf{r}_i \) its radius vector. The softening parameter \( \epsilon \) is used to diminish two-body relaxation. The integration routine uses the Ahmad–Cohen scheme, i.e. individual time steps for each particle; separation of the total force on every particle into the irregular, strongly fluctuating part due to a few neighbour particles, and the regular, slowly changing part due to the majority of particles; and a predictor/corrector method to advance the particle coordinates and velocities in time which treats regular and irregular parts of the force differently. This saves much unnecessary effort and hence computing time while preserving sufficient accuracy. A full description of this method can be found in Ahmad & Cohen (1973) and references therein.

Considering two-body relaxation (see below) and length scales over which one wants to have some reliable information (these have to be \( > \epsilon \)) the softening was fixed by taking \( \epsilon = 2 \) units (i.e. \( \epsilon / r_D = 10 \) per cent, or about one-third to one-fourth of the half-mass radius of the standard system). The time steps were then chosen such that the total energy was conserved throughout a merger experiment to better than 1 per cent. Such an experiment then takes about 3 hr central processor time on the Cambridge IBM 370/165 with 250 particles per galaxy.

2.4 Relaxation Effects

The reference relaxation time for a spherical system of \( N \) particles is approximately

\[
t_r \approx t_d . N / 26 \log (0.4N),
\]

where the dynamical time \( t_d \) is defined by dividing the half-mass radius of the system by the rms stellar velocity (Spitzer & Hart 1971). Since this is the median relaxation time, two-body effects will be faster in the densest parts of the system.

![Figure 1. Relaxation tests. (a) Average kinetic energy per particle normalized w.r.t. the system mean for two mass groups in model 5D as a function of time in crossing times. The least squares-fitted straight lines show how little equipartition occurs. (b) Average radial distance normalized similarly. Heavy particles are shown by circles.](https://academic.oup.com/mnras/article-abstract/197/1/179/1349085)
Moreover, for systems with a flat component, such as those studied here, the situation can be quite different (Rybicki 1972).

Experimentally, Hohl (1973) proposed to use mass groups and to look for equipartition of kinetic energy and mass segregation simultaneously. This was done for the exp model 5D which has a high central density and is almost a standard galaxy ($r_H = r_D, m_H = m_D$, otherwise standard). Both the disc and the halo consist of two mass groups with mass ratio 2:1. Fig. 1 shows the average kinetic energy per particle of a mass group normalized by the mean kinetic energy over the system and the average central distance divided by the mean central distance for the two mass groups. Dots represent the heavy particles, and squares represent the light ones. There is no significant equipartition observed while mass segregation, if any, is in the wrong sense (see also Hohl 1973), after 20 crossing times. Looking at shells of thickness 1/10 $r_D$, I found no significant excess of one mass group except in the innermost disc shell after the same time. Thus, neither in the models investigating the evolution of an isolated system nor in the tidal encounters are relaxation effects important. The price one pays for this, however, is a softening parameter as large as 10 per cent of the system radius.

Since the merger remnants have not settled down until about $t = 40$ crossing times, an extra check is needed. Again, mass groups were used with mass ratio 2:1 and mass segregation was checked in the final merger remnant of model PB. Inside the three inner shells, each containing one-tenth of the total number of particles of each component, there are 44 heavy and 23 light disc particles and 38 heavy and 31 light halo particles. The outer four shells contain 31 heavy and 40 light disc and 34 heavy and 39 light halo particles. The average escaper mass is not significantly smaller than the average system particle mass and the kinetic energy fractions for the two mass-groups vary considerably from shell to shell. Thus, slight relaxation effects are present in the merger remnants and this has to be borne in mind when interpreting the results.

2.5 Definition of mass loss and symmetry plane

2.5.1 Size and mass loss from binding energy criterion

The procedure used to decide whether or not a particular particle belongs to the system was described by Dekel et al. (1980): a reference point $(r, \dot{r})$ in phase space is chosen relative to which the binding energies of all particles are computed

$$B_i = \frac{1}{2} m_i v_i^2 + \sum_{j \neq i} \Phi_{ij}.$$ (2.8)

Then the centre of mass rest-frame (CMR) of all particles with $B_i < 0$ is computed and all $B_i$ are evaluated in this frame again. Convergence is usually reached after 3–4 steps. The particles having $B_i > 0$ in the final CMR are defined as escapers. Their energy and angular momentum as well as their identity are recorded relative to the CMR-frame of the remaining bound particles. Some precautions are necessary when two binding centres are present and uncertainties arise at the borderline between the two systems. In practice, the dominating effect is that of having particles bound to the system as a whole but not to any of the individual galaxies.

2.5.2 Plane of symmetry

Clearly the symmetry plane of an initial galaxy will be the disc plane. As soon as strong interaction with another galaxy occurs the initial plane is distorted. Therefore a procedure is required to find the disc plane which is not based on initial values. Similarly, if one thinks
Figure 2. Series of projections of model 4H onto the disc-plane (xy) and the perpendicular yz-plane. Crosses represent disc particles, circles halo particles and 'O' the central core. Time is given in crossing times and the number of disc- and halo particles outside the plotted range is also given.
of a merger remnant as an ellipsoid, one wants to find the major planes. For both tasks the method of least squares can be employed, minimizing the sum of the squares of the distances of all particles to any given plane in space. A specific method was derived in my thesis (Gerhard 1980); it is a two-step linearized procedure which excludes warps and similar particularities and which gives better results than the standard two-dimensional least-square method minimizing only in one coordinate.

3 Evolution of isolated systems

In this section, the evolution of single galaxies is studied, contrasting the sq and exp models. It gives some insight into the behaviour of N-body disc–halo systems and into some of the methods used.

3.1 The formation of an exponential disc in the sq models

Due to the initial conditions just described a model galaxy is not in exact dynamical equilibrium and therefore relaxes somewhat. (This manifests in a damped oscillation in the global parameter $T/W$ with initial amplitude of 5–10 per cent.) However, global instabilities soon take over and as time goes on, all sq discs follow an exponential surface density law.

Model 4H is used as the standard sq model (see Section 2). The initial galaxy soon develops non-axisymmetric structures. After a while a thick, not very pronounced bar forms along with arms, formed by particles left behind by differential rotation (the initial rotation curve is almost flat). Fig. 2 shows some snapshots of model 4H projected on to the $xy$- and $yz$-planes. Crosses represent disc particles and circles represent halo particles. Time is given in crossing times. In the face-on views bar and arm formation can be seen. All sq models are bar-unstable (the Ostriker & Peebles (1973) parameter $t > 0.14$, except in model 4J) and the bar in Fig. 2 is fairly typical. In the edge-on views bending of the disc plane and warps are observed, especially in the later stages when the disc has expanded. Note that (i) the halo is not spherical at $t = 6.0$, (ii) because the thickness of the disc is smaller than the softening parameter, the disc is not self-gravitating in the $z$-direction. This favours warps; see Binney (1978).

By looking at individual particle orbits and global structures in a model with sufficient time-resolution two different processes changing the system properties were observed (see Gerhard 1980 for detailed plots):

(i) An orbit is suddenly changed due to a global structure in resonance at the time, e.g. the end of the bar or an arm-like structure. Usually, the particle decreases its impact parameter relative to the centre and increases its orbital velocity, thus moving to a more elongated orbit. Notice that the angular momentum of a particle does not necessarily increase during this 'transfer of eccentricity'. Instead, the disc becomes rather hot.

(ii) In a more steady interaction with the bar, a particle gains angular momentum constantly and is thus put on an elongated orbit. Such a particle never decreases its central distance. Therefore, this is a real torque effect as described by Lynden-Bell & Kalnajs (1972). There seems to be a clear preference for particles about at corotation (estimated by the end of the bar) to be affected by these processes.

Density profiles of the disc and halo in log–lin scale and the disc rotation curve are given in Fig. 3 for model 4H every three crossing times, where time increases from the upper curve (initial conditions) to the lower one.

The following method was adopted to obtain the diagrams: first, the density centre was defined by taking the core mass as a reference point and computing the centre of mass of all
particles inside a chosen radius $r$ around its position. The density centre found in this way is relatively stable against core oscillations and escaper-effects, and the biasing influence of asymmetric arm structures was minimized by choosing $r = 5$ (which is about the half-mass radius of an exponential disc/halo galaxy) inside which no arms were found to form. Then, spherical and cylindrical shells containing equal numbers of particles were determined around this density centre to get space and plane densities and rotation curves. The diagrams give the average value in each shell over its half-volume radius.

The disc surface density at $t = 6t_{cr}$ is very close to an exponential in the outer parts. In the inner parts, one might see a second, steeper exponential. At least, a central density
enhancement is superposed on the exponential disc. This result was also found by Hohl (1971, 1972). The rotation curve is essentially flat inside the halo radius (20) and falls off further out. Here, as in the graphs of Hohl et al. (1979), a wave-like structure (usually one or two oscillations) often shows in the disc density profiles and rotation curves during the intermediate stages of the evolution.

However, the evolution in the N-body experiments occurs much faster than in the large N Fourier simulations. Hohl (1971) does not find an exponential disc before $t = 2$ rotation times (one rotation time corresponding to several crossing times). This difference is probably a discreteness effect. Tests showed that decreasing the softening parameter did not change the qualitative behaviour, but everything happened on a shorter time-scale.

Since the halo forms non-spherically symmetric global structures (see Fig. 2), one would suppose that this might also have an effect on halo particle orbits. Fig. 3(c) indicates that the halo density profile evolves from the initial $\rho(r) \propto (r^2 - r_H^2)$ towards an exponential $\rho(r) \propto \exp(-\beta r)$. But since there is little difference between the two, the final density profile can still be approximated by a power law $r^{-2}$ in the inner parts. This is no longer true when the halo starts off with, say, $r^{-1}$. In the outer parts ($r > 10$), one can also fit a power law and get an exponent of about 5. Thus, over the entire halo range, the fit to an exponential density law is considerably better. Apparently, the exponential fit becomes worse if the softening is decreased (model 4B). Also, the initial dynamical conditions of the halo may have something to do with this effect. Contrary to the systematic decrease of the disc exponent $\alpha$, the halo exponent $\beta$ does not seem to evolve systematically, but rather to oscillate with irregular amplitudes. (The fits are obtained by the method of least squares, and comparisons of the respective regression errors at different times can therefore say something about the significance of the evolution, given the initial conditions here.)

Table 2 gives the time evolution of $\alpha, \beta$ in model 4H with their relative regression errors, as well as the evolution of some other interesting quantities. The numbers are fairly typical for the sq models.

(i) Mass motion, i.e. bars and arms, preferentially transforms rotational kinetic energy $\mathcal{T}_q$ into radial motion. The kinetic energy fractions in $z$ and $r$ disc motions show the heating of the disc. The radii containing 20–80 per cent of the disc particles indicate a slightly contracting centre and an expanding outer part. This is consistent with energy and angular momentum conservation during evolution. Due to the random motion imposed initially and to the increase in $\mathcal{T}_z$, the disc thickens gradually, the values given for $\langle z \rangle$ being slightly higher than typical.

(ii) Because of formation of non-axisymmetries in the disc and some analogous halo features, some angular momentum transfer from the disc to the halo is expected. After eight crossing times, 8 per cent of the total angular momentum of the system has been transferred to the halo. This value seems slightly higher than Sellwood’s (1980) 4 per cent at the

Table 2. Evolution of model 4H. Notation as in text.

| $t/t_{cr}$ | $\alpha$ | $\Delta \alpha$ (per cent) | $\beta$ | $\Delta \beta$ (per cent) | $T_{T_{z}}^{\text{Di}}/T_{\text{Di}}^{\text{Di}}$ | $R_{20}^{\text{Di}}/R_{80}^{\text{Di}}$ | $R_{20}^{\text{Di}}$ | $R_{80}^{\text{Di}}$ | $\langle |z| \rangle^{\text{Di}}$ | $L_{L} - L_{0}/L_{\text{tot}}$ | $R_{20}^{\text{Ha}}$ | $R_{80}^{\text{Ha}}$ | $n_{H}$ |
|------------|----------|----------------------------|---------|---------------------------|------------------------------------------|---------------------------------|----------------|----------------|-----------------|------------------|----------------|----------------|---------|
| 0.0        | 0.0      | -                          | -       | -                         | 0.01                                    | 0.01                            | 7.4              | 6.6            | 0.27            | 0.03             | 2.7            | 12.1          | 0.92    |
| 2.0        | 0.152    | 7.2                        | 0.339   | 6.3                       | 0.02                                    | 0.07                            | 6.3              | 17.3           | 1.03            | 0.03             | 3.3            | 11.4          | 0.93    |
| 4.0        | 0.130    | 9.0                        | 0.371   | 3.9                       | 0.03                                    | 0.12                            | 5.5              | 20.7           | 1.56            | 0.03             | 3.2            | 11.7          | 0.99    |
| 6.0        | 0.124    | 5.3                        | 0.347   | 4.6                       | 0.04                                    | 0.13                            | 5.9              | 22.7           | 1.97            | 0.05             | 3.7            | 12.3          | 0.86    |
| 8.0        | 0.113    | 6.7                        | 0.329   | 4.5                       | 0.05                                    | 0.16                            | 5.5              | 22.0           | 1.72            | 0.08             | 3.9            | 12.8          | 0.84    |
Table 3. Influence of halo mass and radius on the evolution of a sq disc—evolution of the control run.

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_H/M_D$</th>
<th>$r_H$</th>
<th>$t$</th>
<th>$T_{R}/T_{Di}$</th>
<th>$R_{20}^{Di}$</th>
<th>$R_{80}^{Di}$</th>
<th>$\pi_H$</th>
<th>$\alpha_{9.5}$</th>
<th>$\delta$</th>
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</thead>
<tbody>
<tr>
<td>4I</td>
<td>1</td>
<td>20</td>
<td>0.23</td>
<td>0.16</td>
<td>5.4</td>
<td>24.5</td>
<td>0.88</td>
<td>0.08</td>
<td>0.95</td>
</tr>
<tr>
<td>4H</td>
<td>2</td>
<td>20</td>
<td>0.16</td>
<td>0.12</td>
<td>5.5</td>
<td>20.7</td>
<td>0.99</td>
<td>0.11</td>
<td>1.10</td>
</tr>
<tr>
<td>4J</td>
<td>3</td>
<td>20</td>
<td>0.12</td>
<td>0.08</td>
<td>6.7</td>
<td>20.0</td>
<td>1.04</td>
<td>0.13</td>
<td>1.12</td>
</tr>
<tr>
<td>7H</td>
<td>2</td>
<td>40</td>
<td>0.18</td>
<td>0.16</td>
<td>5.9</td>
<td>21.6</td>
<td>0.94</td>
<td>0.13</td>
<td>0.81</td>
</tr>
<tr>
<td>CO</td>
<td>1.5</td>
<td>30</td>
<td>0.17</td>
<td>0.08</td>
<td>2.5</td>
<td>10.1</td>
<td>0.84</td>
<td>0.28</td>
<td>0.65</td>
</tr>
<tr>
<td>CO‡</td>
<td>1.5</td>
<td>30</td>
<td>0.17</td>
<td>0.14</td>
<td>2.7</td>
<td>10.1</td>
<td>0.92</td>
<td>0.23</td>
<td>0.72</td>
</tr>
</tbody>
</table>

*Notes: All values at 4 $t_{cr}$ except $\alpha_{9.5}$ at 9.5 $t_{cr}$.
* at 8 $t_{cr}$ (final data).
† at 4 $t_{cr}$.
‡ at 12 $t_{cr}$.

time he stopped his calculation. Only a small halo flattening due to the disc potential defined by $\pi_H \equiv \langle |x| \rangle / 0.5(\langle |x| \rangle + \langle |y| \rangle)$ is found compared with the initial discreteness effect.

(iii) The evolution of the radius containing 20 per cent of the halo particles shows an effect inherent in the $N$-body method: the strong softening does not allow high central densities. This affects the inner part of the density curve and may bias the overall density profile. Generally, the halo appears to be much more stable than the disc.

3.1.1 Dependence on halo mass, halo radius and core mass

Apart from discreteness or global asymmetric evolution in the halo one expects only the part of the halo inside the current disc radius to influence the disc evolution. Since the disc usually expands, the halo mass inside $2r_D$ should be important. Table 3 compares the models with varied halo-to-disc mass ratio $M_H/M_D$ and halo radius at $t = 4t_{cr}$ (the other parameters are identical to 4H). The heaviest halo model 4J has the Ostriker–Peebles parameter $t = 0.12$. From the radial kinetic energy fraction in the disc $T_r/T$ after 4$t_{cr}$ the stabilizing effect of a heavy halo is evident, although it is not as pronounced as in large $N$ simulations (Hohl et al. 1979). The inner core radius $R_{20}^{Di}$ increases with halo mass in accordance with the disappearance of the central density enhancement. Also, a heavy halo hinders the outer disc from expanding ($R_{80}^{Di}$), giving a slightly higher exponent $\alpha$ for higher $M_H$ (see Fig. 4 for the full

![Figure 4](https://academic.oup.com/mnras/article-abstract/197/1/179/1349085/1349085)

Figure 4. Evolution of the inverse scale-length with time for the exponential discs in models 4I ($a$), 4H ($\alpha$), 4J ($\phi$), 7H ($\phi$), CO ($\star$). The fitted range excludes the central enhancement.
time evolution of $\alpha$). But notice how insensitive $\alpha$ is to $M_\text{H}$ compared with the observed range of 0.1–1.0 kpc$^{-1}$.

Comparing models 4H, 4I and 7H in Fig. 4, it is seen how important it is whether or not the halo radius exceeds the disc radius after the expansion of the outer parts. Initially, the behaviour of models 4I and 7H is similar because the mass inside the initial disc radius is about the same. But after the expansion the additional halo mass seems to slow down the subsequent stretching of the disc considerably. A very heavy and large halo might even stop it completely.

A similar series of experiments varying the core mass did not show a dependence $\alpha(m_c)$. To compare the different values of $\alpha$ with observations and previous experiments, dimensionless parameters should be used, e.g. $\alpha r_m$ and $v_m/(\alpha GM_D)^{1/2}$ (Fall & Efstathiou 1980), where $v_m$ is the maximum rotation velocity of the disc and $r_m$ the radius when $v_m$ is reached. It turns out that it is not possible to estimate $r_m$ reliably from the rotation curves; however $v_m$ is well defined. For the model values of $\delta = v_m/(\alpha GM_D)^{1/2}$ see Table 3; a sample of galaxies is plotted in Fall & Efstathiou (1980), giving $0.5 < \delta < 1.2$. Comparison shows that the models and real galaxies have similar values of $\delta$ (so roughly similar $M_\text{H}/M_D$).

A typical value of the scale length $\alpha^{-1}$ in units of the initial disc radius here is about 0.4. Hohl (1972) found a much steeper exponential disc ($\alpha^{-1}/r_D\text{init} = 0.12$) from an initially Gaussian disc. This indicates that the initial density distributions rather than halo masses should give a range of scale lengths $\alpha^{-1}$ in the dynamically formed exponential disc models. If this dynamical process was at all important in real galaxies, then the observed approximate constancy of $\mu_0$ would require a rescaling of the units and consequently a mass—initial density profile—angular momentum relation (see Freeman 1970 for the pure exponential disc).

### 3.2 The Secular Evolution of Exponential Discs

The evolution of exponential discs was studied as a function of the initial $\alpha$. Three models were evolved, 5D, 5H, 5I, which have $\alpha_{\text{init}} = 0.33, 0.18$ and 0.11, respectively, and all other parameters as model 4H, except that model 5D has a half as massive halo. Model 5D shows a secular evolution in comparison with the sq models; it has increasing central density and stretching of the outer disc. This continues up to after 20 crossing times. At this time $\alpha$ has decreased to about 0.11 for the outer disc. Models 5H and 5I behave differently. An initial sharp rise in $\alpha$ is followed by the stretching effect. This stretching of exponential discs was also found in Hohl et al. (1979), but was found greatly reduced there when a heavy bulge with mass only equal to the disc mass was introduced. So it seems that the N-body discreteness amplifies their effect. But generally, exponential discs are more stable than the sq discs in the present simulations, possibly because of their high central concentration.

These results, together with the results of Section 3.1 and the requirements for an encounter galaxy, led to the set of parameters chosen in Section 2.2: $\epsilon = 2, h = 1.2, r_H/r_D = 1.5, M_4/M_D = 1.5, m_c = 25, \alpha = 0.33$. A model with this set of parameters was evolved in isolation, as control run CO, and the details of its evolution are now discussed briefly. The evolution of the density exponent $\alpha$ is shown in Fig. 4, but the fits are not as good as in e.g. 4H. As in 7H, the extended halo reduces the secular expansion and from $t = 10$ to $t = 14\tau_{\text{cr}}$, the exponent remains constant within the errors (not displayed). The calculation was stopped at $t = 14\tau_{\text{cr}}$.

Table 3 gives some interesting quantities illustrating the evolution of the control run CO. Although $t$-unstable, the galaxy evolves considerably more slowly than the comparable sq models. The thickening of the disc as opposed to the effect due to the encounters can be found in Table 4.
Figure 5. Spin dependence and asymmetry in parabolic passages. Projections of the parabolic models PA and PB onto the orbital plane immediately after the first close passage and at about greatest distance of the final bound orbit to show spin-dependent asymmetry, especially in mass flow. Spins in PB are roughly opposite; relative angle in PA 75°. See Table 1. Crosses represent disc particles, circles halo particles, and O's the two cores. Time is given in crossing times and the number of particles of the respective components outside the plotted area is shown. The three interceptions printed in the upper left hand corner define the projection plane, here the xy-plane.
The halo flattening $\pi_H$ after $12t_{cr}$ does not differ from the initial discrepancy effect. The volume density profile of the halo fits an exponential better than a power law, as measured by the regression errors. But if one were to fit an overall power law, it would be approximately $r^{-3}$. No strong evolution of the halo occurs.

4 Tidal interactions

Interactions between systems of a rotating disc and a non-rotating halo can be expected to cause different tidal patterns and changes in the galaxies, depending on the relative orientation of the spins and the relative importance of the halo. The strongest interaction occurs when the galaxies are corotating with their orbit, and the effective half-mass radius of the disc is dependent on the relative orientation of the internal spins to the orbital angular momentum. If the disc is an important fraction of the whole system in terms of extension and mass, then the total interaction will be strongly influenced by these two effects. Because of this and because of the fact that the discs have high angular momentum and the haloes have not, some decoupling of the components is also expected.

At the same time, the tidal interactions are in general asymmetric; i.e. two equal galaxies before the encounter are no longer equal afterwards. To see this, consider two objects with opposite spins just passing each other on an orbit such that the orbital plane and the two disc planes are identical. Then one is corotating and the other is counter-rotating with the relative orbit. If the encounter is close and slow enough, the resulting rate of mass loss and energy change will differ greatly. In distant and fast encounters the tidal interactions are much weaker, so the asymmetry will be smaller. Similarly, a less favourable orientation of the three spins involved will result in less asymmetry.

To illustrate this, a series of experiments have been run. First compare the two parabolic models PA, PB. They differ only by their spin vectors; in PB the described antiparallel spins with the orbital angular momentum parallel to the second spin are almost present (‘almost’ and not exactly in order to avoid possible very special effects). In PA the spins are oriented like an inverse ‘Y’, with the orbital angular momentum upwards and the two internal spins forming a relative angle of 75° both directed downwards. This gives a fairly general case to compare with the rather special one.

Fig. 5 shows projections of both PA and PB onto the orbital plane immediately after the passage and at about the greatest distance (notice the difference in the formation of tails). The third pair of plots gives model PB at identical times, but disc particles only. Clearly, one-directional mass flow has occurred for both disc and halo population much more drastically in PB than in PA. This is confirmed by Table 4 which gives relative mass and energy change, spin parameter $\lambda$, and the average distance from the galactic plane, $\langle |z| \rangle$, for the galaxies 1 and 2 (in Fig. 1 system 1 is always the lower one).

The mass loss from the galaxy pair and the fraction of angular momentum of the entire system carried away by these escapers are also given. Notice the drastic change in PB, where 5 per cent escapers carry away 25 per cent of the total angular momentum or 1.7 initial disc angular momenta. Due to the capture of many counterrotating particles and the escapers, the disc angular momentum of galaxy 1 is almost halved. Since the halo gains an equal amount of angular momentum from the orbit (oppositely directed), the final system has $\lambda \approx 0$; it is also the only galaxy in the models which gained negative binding energy. (This defines the sign convention for the energies in Table 4.) One half of the escapers are disc particles of system 2 and the rest split essentially into halo particles of both systems. Compared with this model PA shows only modest asymmetry (see Table 4 for the numbers). From Table 5 one finds another important difference. There, $\Delta$ gives the angle between the
Table 4. Tidal damage in close encounters.

| Model | $\Delta M_1/\Delta M_2$ | $\Delta M_{esc}/\Delta M_{pair}$ | $\lambda_1$ | $\lambda_2$ | $\Delta E_1/\Delta E_2$ | $L_{esc}/L_{tot}$ | $\langle|z|\rangle_1$ | $\langle|z|\rangle_2$ |
|-------|-----------------|-----------------|---------|---------|-----------------|-----------------|----------------|----------------|
| PA    | +0.4            | −4.5            | −2.0    | 0.05    | 0.09            | +5.2            | +14            | 14             | 1.8            | 5.0            |
| PB    | +13             | −24             | −4.6    | 0.00    | 0.04            | −14             | +36            | 25             | 1.9            | 1.7            |
| HE    | +0.6            | −8.9            | −4.2    | 0.05    | 0.10            | +3.9            | +24            | 14             | 1.2            | 1.3            |
| HA    | +4.5            | −8.7            | −2.1    | 0.05    | 0.09            | +6.5            | +23            | 11             | 1.8            | 1.8            |
| HB    | −0.8            | −4.3            | −2.6    | 0.08    | 0.08            | +10             | +18            | 12             | 2.0            | 3.0            |
| PHB   | −12             | (+6.2)*         | (−2.9)  | 0.08    | −              | +22             | −              | 7.6            | −              | −              |
| HC    | −3.5            | −4.1            | −3.7    | 0.05†   | 0.09            | +15             | +15            | 9              | 2.0            | 1.6            |
| HF    | −3.7            | −7.6            | −5.7    | 0.15†   | 0.17†           | +11             | +11            | 26             | 3.6            | 1.4            |
| Co    | −               | −               | −0.10   | −       | 0.2             | −               | −              | 1.2            | −              | −              |

Notes: The control run Co was followed for 14 crossing times (hyperbolic models: 12). In the hyperbolic encounters the number of escapers must be determined from the individual galaxies since the whole system is not bound and the angular momentum loss in the common CMR-frame is calculated from the orbit and internal spins at the time when the calculations were stopped. In the parabolic cases everything relates to the common CMR-frame directly.

* Mass fraction of galaxy 1 captured by perturber 2 of equal mass.
† The original galaxies have $\lambda = 0.22$ in HF and galaxy HC1 has 0.06 initially.

Table 5. Angular momentum vectors.

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>$\Delta$ (degree)</th>
<th>$\chi$ (degree)</th>
<th>Galaxy</th>
<th>$\Delta$ (degree)</th>
<th>$\chi$ (degree)</th>
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<tbody>
<tr>
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<td>29</td>
<td>PA2</td>
<td>24</td>
<td>79</td>
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<tr>
<td>PB1</td>
<td>(77)</td>
<td>(97)</td>
<td>PB2</td>
<td>9.1</td>
<td>16</td>
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<tr>
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<td>1.6</td>
<td>HA2</td>
<td>6.7</td>
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</tr>
<tr>
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<td>HB2</td>
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<td>HC2</td>
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<tr>
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<td>7</td>
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</tbody>
</table>

Notes: $\Delta$ is the angle between the initial and final angular momentum vector after the encounter, $\chi$ is the angle between final spin and final disc plane normal vectors, both in degrees.

initial and final angular momentum vector after the encounter and $\chi$ is the angle between final spin and final disc plane normal vector. Only the differences in galaxy PB2 are small (there disc and orbital plane were identical), whereas they are quite significant in PA (PB1 should not be used as an example because it has essentially no more spin). This is because significant orbital angular momentum is gained, particularly by the isothermal halo.

From this, one can draw the general conclusion that when systems of two differently rapidly rotating components undergo a close encounter of no special spin configuration, the relative orientation of the rotation axes of the components changes.

The series of encounters PB, HE, HA differ only in the pericentre passage velocity at the same (initial point-mass) pericentre, or eccentricity. As expected, the interaction becomes less violent with increasing orbital energy. Also, from the values in Tables 4 and 5 the strength of the asymmetry decreases, but is still remarkable. Notice, however, that there is no simple rule for predicting the mass change of the two galaxies involved only from orbital energy because velocity resonances are important. Model HE is just unbound after the
encounter (so one can hardly go to slower hyperbolic encounters with this spin orientation without having them on bound orbits afterwards), whereas the orbital energy of HA after the encounter is slightly larger than that of HE before it.

The next series of experiments all have the same spin configuration: essentially both galaxies corotating with the orbit, but one spin deviates by $30^\circ$ again so as not to have a special situation. This gives a violent, but rather symmetric interaction as can be seen from Table 4. Model HB can be compared directly with HA, because the two only differ in spins. Model HB is used to display tidal ‘bridges and tails’.

Fig. 6(a) shows a series of snapshots during the passage projected on to the orbital plane which coincides rather well with the two disc planes. Galaxy 2 (left at $t = 2$) appears rather round at $t = 4$, forms a somewhat oval shape just after the passage and then becomes round again. This effect is less pronounced in galaxy 1. Narrow tails persist for a few crossing times, and a cloud rather than a bridge links the two systems. This is even clearer when only disc particles are plotted (Fig. 6b). The first two projections are on to the same plane for comparison with Fig. 6(a). The last two are projected on to the plane with interceptions $-1, -1, 1$. It is interesting to notice that Arp 32 looks somewhat similar to one of them.

When comparing these hyperbolic encounters with the ones studied by Dekel et al. (1980) one striking difference is that I never get nearly as much as 30 per cent escapers from one system. Several reasons are possible, the most likely being the difference between few-body perturber and second galaxy. Therefore, model HB was repeated with system 2 replaced by an equal-mass particle (I do not think that the smaller initial distance of HB due to computing costs can be responsible for the differences below). As Table 4 shows, there is an order of magnitude difference in mass-loss from system 1, although the total relative escaper mass is not very different. This means (as was also seen by following individual particles) that mass exchange is more important than mass-loss at least in systems with circular disc and isotropic halo orbits. Seventy per cent of all escaping disc particles are bound to the perturber afterwards, but only 40 per cent of the halo particles, in agreement with the conclusion of Dekel et al. (1980) that circular orbits are more likely to be captured than radial orbits. Thus, one difference with the perturber is certainly the missing mass-flow back to the perturbed system. It is hard to estimate how much of the difference in $\Delta E/E$ comes from the fact that the perturber cannot respond to the interaction. In general, one has to be careful using point mass perturbers.

Finally, the influence of mass and size of the halo on the interaction was looked at by models HC, HF. Both were on equal orbits $(p, v, x_0)$, as was HB. Galaxy 1 in HC had a halo 1.66 times as extended and heavy as in HB. Consequently, relatively more halo than disc particles were lost ('stripping'). There seems to be some difference in the tidal interaction in the sense that (i) the connecting cloud-bridge has nearly disappeared and (ii) the tail of the heavier halo galaxy is shorter, thicker, and consists of more particles (Fig. 6c). HF had light bulges rather than haloes and was, therefore, more affected by bar instability. Most of the escapers are disc particles, the total mass loss and angular momentum loss being considerably higher than in HB. The pictures are not conclusive, but indicate that tails break up very soon in the absence of the halo (Fig. 6c). Apparently the discs have to be non-self-gravitating to some extent to give lasting tidal tails, and those become quite thick if the halo extends further out with the same $M(r)$-function as before. But no really thin bridges have been seen and this is probably due to the comparable masses and unfavourable orbits (see Toomre & Toomre 1972).

Some more common features of these close encounters are:

(i) The halo density in all objects after a hyperbolic encounter fits quite well a power law $\rho(r) \propto r^{-3}$ and is even steeper in PA and PB $(\propto r^{-3.5})$ (this depends also on the initial conditions;
Figure 6. Tidal interaction with varied halo mass. (a) Projections of the hyperbolic model HB on to its orbital plane. The spin vectors form a relative angle of about 30°. (b) Several projections for the disc component of model HB only. (c) Projections of models HF and HC. Crosses represent disc particles, circles halo particles.
Figure 7. Density profiles of the galaxy 1 in model HB initially (triangles) and after the encounter (circles). (a) disc, (b) halo. The normalization is arbitrary.

see Fig. 7). Dekel et al. (1980) found $r^{-2}$ in the inner parts; there is some indication for that here in the very centre, if one disregards the central core, but no evidence because the only small number of points in the inner parts is furthermore affected by the softening. Since the inner halo parts are well inside the disc radius, some difference is expected anyway. The disc surface density is considerably more perturbed; the only conclusive, but expected feature is the density increase in the outer parts (only bound particles are included though).
Figure 8. Series of projections of model PB on to the orbital plane during the merging process.
Figure 9. Final merger remnant EA viewed from different directions given by the interceptions of the projection planes (upper left hand corner). Disc particles only in the same projection 111 (bottom).
(ii) In all models the \( \lambda \) values are decreased and the discs lose angular momentum by escapers etc. whereas the haloes gain orbital angular momentum. This leads to an angle between the normal direction to the plane and the angular momentum unless the orbital and initial disc planes coincide (compare HB2, HC2, HF2 with HB1, HC1, HF1 and the unperturbed control run CO in Table 5). This will, of course, affect the later evolution.

(iii) The tidal interaction causes some fattening of the discs compared with the control run. Notice, however, that the strongest fattening does not correspond to the lowest \( \lambda \) values.

5 Merger remnants

Models PB, EA, and PC (initial conditions in Table 1) are followed until several crossing times after merging. After the first close passage both systems move on bound orbits and then gradually spiral into each other. From the second contact immediately before merging until the remnant has settled down, the merging process takes about 10 crossing times, or about three revolutions of the cores around each other. The process is illustrated in Fig. 8. Notice that for a long time two distinct cores can be seen in the merged object, formed by clusters of particles surrounding the heavy particles. In the north, an extended cloud forms a last fossil of the spin asymmetry. During the merging some more particles escape giving a maximum of 7.2 per cent escapers in PC, slightly less in PB, and only 1.7 per cent in EA. The angular momentum carried away by these particles is up to 30 per cent of that of the total system. In Fig. 9, the final remnant EA is viewed from several directions given by the intersections of the projection planes. One of the projections is shown without halo particles once more, and the 'disc' particles clearly form a condensed system ('disc' abbreviates the visual component).

5.1 The density structure of the remnants

Fig. 10 shows space density profiles of both visible components and haloes for the three merger remnants. They all fit a power law quite well, with exponents given in Table 6, where \( \gamma \) is the power law exponent for the luminous component and \( \beta \) for the halo. (Note that the regression errors for these fits lie within 2–5 per cent.) The density profiles were obtained using spherical shells, although the systems are triaxial and model PB is rather flattened. The haloes follow a Hubble law very closely. The small difference from \( r^{-3} \) for the luminous component may be due to (i) the spherical shells used (the exponent of the most flattened remnant PB deviates most from 3) or (ii) the observed slight mass segregation in the centre of the visual component. Thus, a Hubble profile seems to be obeyed reasonably well by the visible merger remnants, in agreement with White's (1978, 1979a) models. The accuracy of the power law fits is, however, a little surprising, because one expects (i) some influence of the core particles on the profile, (ii) perhaps an effect from the central mass segregation, and (iii) a mass deficiency in the centre since the softening length is large compared with the central interparticle distance. Apparently several of these effects cancel to give a quite accurate power law. From the values given in Table 6 for the radii containing 20/80 per cent of the bound disc/halo mass it is seen that the visible component is much more centrally condensed than the halo component, probably due to the heavy particle cores, and that the halo component extends about twice as far.

The central densities of the merger remnants are also given in units of the respective component's central density in the control run after some evolution (Table 6). But because of the large softening they are not representative. Unfortunately this method's inherent effect cannot be overcome and a comparison with observed central densities for elliptical galaxies is not possible.
As already mentioned the final objects are triaxial, although close to oblate. Major planes were determined by the method of least squares, minimizing the sum of the squares of the distances to an arbitrary plane as described in Section 2. They coincide fairly well with the orbital plane of the two galaxies, as can be seen from Table 6, which gives the angle $\Delta_{PO}$ between the two normal vectors. Except in the case of model EA the difference is smaller than the discrepancy in the control run (Table 5). The flattening was obtained by finding the principal axes coordinate system and then computing $\langle |x| \rangle$ etc. with respect to
### Table 6. Merger remnants.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Delta M/M$ (per cent)</th>
<th>$\Delta L/L$ (per cent)</th>
<th>$R_{20}^{Di}$</th>
<th>$R_{80}^{Di}$</th>
<th>$R_{20}^{Ha}$</th>
<th>$R_{80}^{Ha}$</th>
<th>$\rho_c^{Di}$</th>
<th>$\rho_c^{Ha}$</th>
<th>$\gamma$</th>
<th>$\beta$</th>
<th>$\Delta \theta_{D}$ (degree)</th>
<th>$\epsilon_{Di}$</th>
<th>$L_{Di}/L_{tot}$</th>
<th>$L_{Di}/L_{init}$</th>
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<td>PB</td>
<td>6.3</td>
<td>30</td>
<td>3.2</td>
<td>25</td>
<td>6.8</td>
<td>57</td>
<td>2</td>
<td>0.4</td>
<td>-3.4</td>
<td>-3.1</td>
<td>4</td>
<td>0.7</td>
<td>0.19</td>
<td>0.94</td>
<td>0.41</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td>EA</td>
<td>1.7</td>
<td>10</td>
<td>3.7</td>
<td>29</td>
<td>5.1</td>
<td>58</td>
<td>0.9</td>
<td>0.7</td>
<td>-3.6</td>
<td>-3.0</td>
<td>17</td>
<td>0.4</td>
<td>0.18</td>
<td>0.92</td>
<td>0.36</td>
<td>0.22</td>
<td>0.15</td>
</tr>
<tr>
<td>PC</td>
<td>7.2</td>
<td>27</td>
<td>3.6</td>
<td>26</td>
<td>4.9</td>
<td>47</td>
<td>0.9</td>
<td>0.4</td>
<td>-3.2</td>
<td>-3.0</td>
<td>6</td>
<td>0.4</td>
<td>0.38</td>
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<td>0.38</td>
<td>0.21</td>
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</table>
this coordinate system. An overall 'true ellipticity' was then defined by $\epsilon = 1 - \langle |z| \rangle / 0.5 \langle (|x|) + (|y|) \rangle$, which is also given in Table 6. The model with antiparallel spins 'fattens' during the merging to about 0.7, the largest ellipticity for real elliptical galaxies. No significant difference in the density structure between mergers from elliptic and parabolic orbits has been observed.

White (1978, 1979a) concluded from several tests that the structure of his merger remnants should not be affected greatly by two-body relaxation. Here, although slight mass segregation has been found, it is not likely to affect the overall Hubble-type power law behaviour. This can be seen by comparing White's power laws and mine for the similar haloes and noting that there is little difference in spite of the doubled softening parameter here for similar halo radius.

Finally, it is of interest that population gradients are essentially preserved, but weakened. Of the 20 innermost particles in each galaxy, respectively, about 55 per cent are found in the centre of the merger remnant, 32 per cent in the intermediate parts and only 12 per cent in the outer regions. Only 9 per cent of the 25 outermost particles in each galaxy find themselves in the centre of the remnant, whereas 23 per cent escape and 49 per cent are in its outer parts. Even particles from the intermediate regions of the initial galaxies go preferentially to the intermediate parts of the remnant. All numbers given are averages over the models PB and PC. For further discussion of mixing effects see White (1980).

5.2 Angular momenta and velocities

The final angular momentum of a merger remnant is determined by the orientation of the three spins involved. Since in a high-eccentricity or parabolic encounter the orbital angular momentum dominates, the situation with both internal spins antiparallel to the orbital spin should lead to the slowest final rotation. Antiparallel disc spins are an intermediate case, whereas all three oriented in parallel should give fast rotation. This, of course, is only a first guess because of tidal effects. The three experiments PB (the antiparallel model), EA (the 'inverse Y') and PC (a fast rotator with the three spins mutually about 70° from each other) were designed to cover some of this range.

Table 6 shows the final proportion of angular momentum in the disc component ($L_{Di}/L_{tot}$) and the value in initial disc angular momenta. Even in the expected fast rotator PC most of the angular momentum is acquired by the halo component. If one simply adds up the orbital angular momentum of the discs and their internal spins, one gets considerably more in terms of initial disc angular momenta than is observed. However, as a simple rule of thumb, the final halo component has about as much as its initial share of orbital angular momentum (remember that the haloes are isothermal, the rule should not be expected to hold too well for rotating haloes) and so subtracting the angular momentum of the escapers from the disc angular momenta gives about the right answer. These carry away about 2/3 of the orbital angular momentum of the disc component. Unless both internal spins are significantly aligned with the orbital angular momentum, the merged visual component should rotate slowly. This scenario is only approximate, however, and does not tell how pronounced a maximum there is in the rotation curve.

Table 6 also shows velocity anisotropy by the kinetic energy fractions in radial and $z$-motion where $z$ is perpendicular to the major plane. There is considerably less kinetic energy in $z$ than in $r$ (the rotation is very nearly around the minor axis or, equivalently, in the orbital plane), but no bulk motion in $r$ at this time. Model PB seems to have some counterrotation of subcomponents because the velocity dispersion is nearly as high as an initial average rotation velocity. The effect persists until the end of the simulation.
Figure 11. (a) Rotation velocity normalized by central velocity dispersion as function of radius in one projection of the visible components of models PB and EA. The points are averaged over four successive time outputs and neighbouring points in the resulting curves. Two points from both sides of the centre, respectively, form the groups of two points visible in the plots. (b) Velocity dispersion, similarly.
To compare the rotation and dispersion properties with observations of elliptical galaxies, velocity dispersions and normalized rotation velocities for the visible component are computed as functions of central distance. The system was projected onto a plane and all velocities normal to this plane were determined. The obtained dispersion and rotation curves were then averaged over various points in the curves and four successive snapshots to eliminate discreteness. One such projection of model PB and EA is shown in Fig. 11, where the pairs of points come from either side of the centre. The velocity dispersion always decreases with central distance, but the rotation curves are not always flat. Some rise and are then flattened, while others fall off again. The model values of $v_{\text{max}}/\sigma_{\text{centre}}$ versus ellipticity can be compared with observations in Fig. 12 which shows the present data for elliptical galaxies from Illingworth (1977), Davies (1978), Sargent et al. (1978), Young et al. (1978) and Schechter & Gunn (1979). Several projections giving different ellipticities and rotations have been included. There are real galaxies similar to models PB, EA, although the models do not represent the bulk of the data. Model PC seems to be somewhat outside the observed range. Thus I conclude that it is possible to get slowly rotating objects from the merging of rapidly rotating discs, but a uniform distribution of spins and orbital parameters would probably not only cover the region of observed ellipticals. Either restrictions to the orbits (see the cosmological simulations of Aarseth & Fall 1980) or to the range of allowed spin configurations would be necessary to avoid that. It has been suggested (Fall 1979) that in the hierarchical clustering picture tidal torques should cause a preference for more opposite rather than aligned spins in merging pairs.

6 Conclusions

Isolated three-component systems in the present $N$-body simulations form an exponential disc dynamically. Initially-exponential discs also evolve. Both kinds of systems were not completely stable according to the Ostriker–Peebles (1973) criterion. The evolution is driven by bars and other non-axisymmetric structures which either transport angular momentum outwards by torques (as proposed by Lynden-Bell & Kalnajs 1972) or change the eccentricity of an orbit for nearly constant angular momentum. Both processes occur preferentially at corotation. More specific results of these experiments are:

![Figure 12. Observed values of the ratio of the maximum rotational velocity to the central velocity dispersion for elliptical galaxies taken from Schechter & Gunn (1979) (○), Illingworth (1977) (●), Sargent et al. (1978) (♦), Young et al. (1978) (★) and Davies (1978) (⋆) compared with the merger remnants EA, PB, PC viewed from three different directions.](https://academic.oup.com/mnras/article-abstract/197/1/179/1349085)
(1) The inverse scale length $\alpha$ of the forming exp disc increases only slightly with the halo mass inside the disc radius and does not depend on the core mass. The initially-exponential discs show a secular effect of outward drift and decreasing $\alpha$. This effect may be considerably slowed down by a very extended, heavy halo. The exp discs are more stable and hence more suitable for collision experiments than the other discs investigated.

(2) The halo itself forms global asymmetries which lead to the formation of an exponential space density law. In the inner parts, this can still be well approximated by the initial $r^{-2}$ power law. However, the choice of initial conditions and softening effects in the centre could also contribute. Generally, the halo is much more stable than the disc. A small angular momentum transfer from the disc to the halo is observed.

Collisions between three-component galaxies prove to be strongly spin-dependent for comparable halo and disc masses and sizes. This is because of particle resonances with the orbit and possibly direction-dependent effective half-mass radii. Close encounters severely alter the participating galaxies. In particular it is found that:

(1) Spin-dependence is connected with pronounced asymmetries such as one-directional mass flow. As a result the relative sizes of the colliding galaxies change as well as the relative angular momenta and energies. The asymmetry is most pronounced in close and slow encounters with opposite galaxy spins.

(2) A few escapers carry away up to 25 per cent of the total angular momentum in the present hyperbolic encounters and first parabolic passages. Due to this and to the capture of counter-rotating particles it is found in all cases that the internal spin parameter $\lambda$ decreases.

(3) General encounters transfer significant angular momentum to the isothermal halo which is not aligned with the disc spin. As a result the total angular momentum vector of the system does not remain normal to the disc plane unless the latter coincides with the orbital plane. The angle between the normal vector and the total spin vector after an encounter can be as high as 30° in the hyperbolic case.

(4) Tidal tails form and persist for several crossing times, but no distinct bridges can be seen in the investigated cases. There is some indication that an extended heavy halo produces thicker tails which persist longer. Some fattening of the discs is observed in most experiments.

(5) Using a heavy particle rather than a second galaxy to perturb a system resulted in an order of magnitude increase in mass loss from the perturbed system and a factor of 2 in energy change. This is partly due to the missing mass-exchange, which is important in a two-galaxy encounter, and is also somewhat dependent on the type of particle orbits in the system.

Merging cross sections are spin-dependent in parabolic encounters, but no quantitative statement can be made before many more experiments are performed. The structure of the merger remnant of two exponential disc–halo galaxies can be summarized as follows:

(1) A distinct core of visible material remains. The halo component extends considerably further. Both the final luminous part and the halo are triaxial, but fairly close to oblate, and obey approximately a Hubble-type density law. Ellipticities are found to be less than 0.7.

(2) Up to 30 per cent of the total angular momentum is lost by ~7 per cent escapers in the parabolic cases. This loss is at the expense of the luminous parts which consequently rotate slowly. Two of the models agree quite well with the available $v/\sigma_c$ versus ellipticity observations of elliptical galaxies. The third one seems to rotate faster because the initial angular momenta were preferentially aligned.

(3) The velocity dispersion is anisotropic and the dispersion curve as viewed by an observer falls off with radius.
Thus two of the three merger remnants could represent observed elliptical galaxies while the third rotates somewhat too fast. Since a number of strongly interacting pairs are observed now, there should also have been some in the past. One would then expect to see some remnants; these would probably be classified as ellipticals. A crucial observational question then is whether there are subgroups of ellipticals which could be attributed to different formation mechanisms. On the other hand, either the parameter space for mergers has to be restricted (e.g. type of orbit: cosmological N-body simulations; spin orientation: hierarchical tidal torques) to prevent fast rotation or the concept of merging two similar present-day spirals must be modified.

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References