An ellipticity—age relation for globular clusters in the Large Magellanic Cloud — I. Measurements

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Summary. We have estimated the ellipticities of 52 globular clusters in the LMC and 93 in the Galaxy by the eye-measurement of their images on Sky Survey enlargements. These were checked against star counts in 12 clusters of the LMC sample, using a procedure developed for this purpose, and against star counts in 19 clusters of the galactic sample, using determinations reported by Kholopov. The results from the eye and count methods are in excellent agreement and a comparison of the two samples shows that the globular clusters in the LMC are significantly flatter than those in the Galaxy. As a means of investigating evolutionary effects, we have used \( UBV \) photometry to arrange the LMC clusters in a sequence of relative ages that is equivalent to the one devised by Searle, Wilkinson & Bagnuolo. Our results show that young clusters are flatter on average than old clusters and that the shapes of the oldest clusters in the LMC are similar to those of galactic globular clusters. The dependence of ellipticity on age may be due to changes in the shapes of the clusters, changes in the condition of their formation or some combination of both effects. For the clusters in our sample, there is no significant correlation between ellipticity and distance from the centre of the LMC. This restricts the degree to which changes in the conditions of formation might be responsible for the ellipticity—age relation and suggests that changes in the shapes of the clusters might play an important role.

1 Introduction

The rich star clusters of the Magellanic Clouds resemble the globular clusters of our Galaxy in structure and form. In stellar content, some of these clusters resemble the galactic component of Population II but many of them are much younger and have Population I characteristics. They were originally divided into two broad classes — ‘red globular clusters’ and ‘young populous clusters’ — on the basis of their integrated colours and preliminary colour—magnitude (CM) diagrams (Hodge 1960, 1961). Several lines of evidence now indicate that many if not most of the red clusters have intermediate ages and that the stellar contents of all the rich clusters of the Clouds can be arranged in a sequence with age as the fundamental parameter (Searle, Wilkinson & Bagnuolo 1980). This point of view complements the
impression that the young and old clusters are structurally similar objects in different phases of evolution and it encourages the search for an age-dependence in such properties as mass, metal abundance and kinematics (Woolley 1960; Freeman 1974). With this understanding, the name ‘globular cluster’ seems appropriate for the sequence as a whole and we shall use it in this sense in the discussion that follows. Our purpose in this paper is to determine the shapes of these clusters and their dependence on age. We have restricted our study to the Large Magellanic Cloud (LMC) because the Small Magellanic Cloud (SMC) does not provide a large enough sample for the statistical treatment given here.

There are good reasons for supposing that the non-spherical shapes of globular clusters in our galaxy are the result of internal rotation rather than tidal distortions or velocity anisotropies (King 1961). If this is also true of globular clusters in the Clouds, then one reason for expecting their shapes to depend on age is that rotating stellar systems tend to ‘evaporate’ angular momentum as they relax. Internal diffusion causes stars to escape preferentially in the direction of rotation because, with respect to this mean motion, the escape velocity is lower in the forward sense than it is in the backward sense. Agekian (1958) has shown that the decrease in specific angular momentum also leads to a decrease in the ellipticity of the system provided this is initially smaller than about 0.74. If the positions of globular clusters in the Clouds depend on age, then another reason for expecting their shapes to depend on age is that they formed in regions of different density and vorticity. To the extent that these quantities can be determined, the positions of the clusters can be used to disentangle the effects of initial conditions from those of later evolution on their shapes. The distribution of old clusters appears to extend farther out in the Clouds than does the distribution of young clusters but any correlation of position with age is likely to be weak in a sample confined to the inner 6° of the LMC (van den Bergh 1981). An interpretation of the results of this paper along the lines just mentioned is given in a companion paper (Fall & Frenk, in preparation, hereafter Paper II).

The shapes of globular clusters are usually expressed in terms of ellipticities, \( \epsilon \equiv 1 - \eta \), where \( \eta \) is the axial ratio of equidensity contours on the sky. This quantity can be estimated most simply by the eye-measurement of photographic images with a ruler. In this way, Shapley & Sawyer (1927) assigned ellipticities to 75 of the 93 galactic globular clusters in Shapley’s (1930) list, ranging from \( \epsilon \leq 0.1 \) for the majority up to \( \epsilon \approx 0.4 \) for M19. A more laborious but more objective method is to fit elliptical contours to the density maps of the images derived from star count data. In the most extensive study of this kind to date, Khlopov (1953) determined ellipticities at several radii near the median (i.e. half-mass) radii of 19 galactic globular clusters. His results show that the radial dependence of ellipticities is usually weak and that the measurements made by eye are often reliable. Recently, Geisler & Hodge (1980) have estimated the ellipticities of 25 red globular clusters in the LMC by fitting ellipses to microdensitometer profiles along four radial slices of each image. Their estimates are, on average, much larger than those of Shapley & Sawyer; part of this difference may reflect real differences between red LMC and galactic globular clusters but much of it appears to reflect systematic differences between the two methods of analysis. We shall return to this point after presenting our estimates of ellipticities.

A system for classifying the globular clusters of the Clouds on the basis of their spectro-photometric properties has recently been devised by Searle et al. (1980). The scheme uses two reddening-free combinations of \( u-w-g-r \) colours, \( Q(ugr) \) and \( Q(uigr) \), to monitor line-blanketing and continuum-opacity near the Balmer discontinuity. Searle et al. found that the 61 objects of their sample fell along a sequence in the \( Q-Q \) plane, which they arbitrarily segmented into seven types (hereafter called SWB types). With only a few exceptions, types I to IV comprise the blue clusters and types V to VII comprise the red
clusters of traditional nomenclature. The temporal nature of the sequence up to type III merely reflects the decreasing ultraviolet flux of stellar populations younger than $3 \times 10^8$ yr as hot stars leave the main sequence. Many of these clusters have well-established ages from CM isochrone fits but types III and IV are at the present limit of this method of dating (Gascoigne 1966, 1980; Walker 1971, 1979). For the older clusters, the $Q$ parameters are sensitive to both age and metal abundance and a correlation between these quantities is required if types IV to VII are to be a temporal sequence. Some evidence that this is so comes from the greater occurrence of RR Lyrae variables in clusters of type VII (Thackeray 1959; Graham 1973) and carbon stars in clusters of types V and VI (Mould & Aaronson 1979; Lloyd Evans 1980), as well as a progressive development of horizontal branches in types IV to VII (Hesser, Hartwick & Ugarte 1976). More quantitative estimates of the ages and chemical compositions of these clusters have recently come from Rabin's (1982) measurements of their Balmer line-strengths and Aaronson & Mould's (1982) study of their upper asymptotic giant branches. The results of these independent methods support the notion of increasing age and decreasing metal abundance along the SWB sequence and they provide a valuable means of calibrating it.

The plan for the remainder of the paper is as follows. In Section 2, we specify our sample and describe the measurements made by eye. These include the ellipticities, orientations and positions of LMC clusters and, for comparison, the ellipticities of galactic globular clusters. In Section 3, we describe in some detail a new method for determining the ellipticity of a cluster from star counts. This method is applied to a subsample of LMC clusters and the results are used to assess the reliability of our eye-estimates. In Section 4, we show that the SWB system can be supplemented by $UBV$ photometry to assign types to the clusters in our sample not classified by Searle et al. (1980). The SWB sequence is given a provisional age-scale and this is then combined with our eye-estimates to derive an ellipticity–age relation for the LMC clusters.

2 Eye-measurements

To obtain as large a sample as possible, we first searched the J prints of the SRC Sky Survey for the most conspicuous clusters lying within about 6° of the centre of the LMC. The images of these 70 or so clusters were then enlarged on polaroid film to a scale of 1.5 arcmin cm$^{-1}$. On examining the enlargements, we found it necessary to reject several clusters on the grounds that they were too contaminated by emission nebulosity or background stars for reliable measurements of ellipticity. In the process, we took great care to avoid selecting clusters on the basis of their shapes. Finally, we eliminated clusters lacking $U$, $B$ and $V$ magnitudes in van den Bergh's (1981) extensive compilation of photometric data. Our LMC sample consists of the 52 clusters in Table 1; it is similar to Hodge's (1960, 1961) survey and the two lists have 40 objects in common. Most of the clusters are brighter than $V \approx 12.5$ and larger than 1.5 arcmin but out sample is neither magnitude nor size-limited in a strict sense. We have checked that the selection procedure has not introduced a significant bias in our results by repeating the analysis described below with several subsets and extensions of the basic sample. For comparison with the LMC clusters, Dr D. A. Hanes kindly loaned his polaroid enlargements of the galactic globular clusters in Shapley's (1930) list, which were taken from the Palomar and SRC Sky Surveys to a scale of 1.7 arcmin cm$^{-1}$. At this stage, both sets of enlargements were coded in order to conceal the identities of the clusters until all the measurements were completed.

We measured ellipticities with the aid of a ruler and a graduated magnifying glass in the parts of the images between the burnt-out centres and the peripheries set by the back-
Table 1. Data for LMC clusters.

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<th>NGC</th>
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<th>$\phi$</th>
<th>R</th>
<th>type</th>
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<td>H 11</td>
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<td>10</td>
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Subsequent checks against the available surface photometry showed that, on average, our estimates were made between one and two median radii ($r_{h}$) of the clusters. These were computed from the relation $r_{h} = 0.7 (r_c r_l)^{1/2}$ where $r_c$ and $r_l$ are the core and tidal radii derived by Peterson & King (1975) and Peterson (1976) for most of the galactic sample and by Freeman (1974), Freeman & Gascoigne (1977) and Chun (1978) for six members of the LMC sample. All of the clusters were measured twice by each of us in separate sessions so as to obtain four independent sets of data. We found no systematic differences among these determinations and we therefore took the averages as our estimated ellipticities; these are given in the second column of Table 1 for the LMC sample and in Table 2 for the galactic sample. The mean internal error in $\epsilon$, computed from the four sets of measurements, is 0.05 for the first list and slightly smaller for the second. As a check on these results, we ranked the combined LMC/galactic sample on the basis of our subjective impression of shapes and found this ordering to be statistically consistent with the one obtained from our numerical estimates. Although some of these ellipticities may be superseded by the analysis of star counts, our eye-measurements should be suitable for this investigation because they were made from homogeneous photographic material in a manner free from any recognizable bias.

We also measured the position angles $\phi$ of the major axes of the LMC clusters (from north to east). The averages of our independent determinations are given in the third column of Table 1 for the clusters with $\epsilon > 0.05$; the mean internal error is 10° but the true uncertainty is likely to be about twice as large. From the orientations and positions on the sky, we computed the angle $\gamma$ between the major axis of each cluster and the direction of the bar of the
LMC as well as the angle $\beta$ between the major axis of each cluster and the direction to the centre of the LMC. In so doing, we adopted a position angle of 120° for the bar and the coordinates $\text{RA} = 5^h 23^m 5$, $\text{Dec} = -69^\circ 6$ for the dynamical centre of the system as recommended by de Vaucouleurs & Freeman (1972). The distributions of $\beta$ and $\gamma$ are shown in Fig. 1; evidently, there are no significant departures from uniformity and this is confirmed by a $\chi^2$ test. This lack of alignment with the structural features of the LMC is consistent with the notion that the shapes of the clusters are the result of internal rotation rather than tidal distortions.

To test for correlations between the shapes and the locations of clusters, some measure of their distances from the centre of the LMC is required. On the assumption that the spatial distribution of the clusters is fairly flat, their polar coordinates in the disc, $R$ and $\Theta$, are related to those on the sky, $R_0$ and $\Theta_0$, by the expressions

$$R = R_0 \sec \Theta \sec \Theta_0, \quad \tan \Theta = \tan \Theta_0 \sec i,$$

(1)

where $i$ is the inclination angle of the disc and $\Theta$ and $\Theta_0$ are both reckoned from the line of nodes. We computed positions in the disc using an angle of 27° for the inclination of the LMC and a position angle of 171° for the line of nodes (de Vaucouleurs & Freeman 1972). The radial coordinate $R$ of each member of our sample is given in the fourth column of Table 1 in degrees. Although some of the older clusters may lie outside the disc, this measure of distance to the centre of the LMC is probably better than the projected distance on the sky for the majority of clusters.

In Fig. 2, we have plotted the ellipticities determined by Kholopov (1953) from star counts against our estimates for the same galactic globular clusters (indicated by asterisks in
Figure 1. Relative frequency distributions of the angles between the major axes of clusters and (a) the direction of the centre of the LMC and (b) the direction of the bar. Only objects with $e > 0.05$ have been included.

Figure 2. Comparison of ellipticities estimated by eye and by star counts for galactic globular clusters. Left: results of this work against those of Kholopov; the latter are weighted averages over several radii and the corresponding error bars represent $\pm 1\sigma$ deviations. Right: the estimates made by Shapley & Sawyer against those made by Kholopov.

Table 2). This diagram shows that there is little or no bias in our eye-measurements and that the scatter is no worse than would be expected from the internal errors alone. Also shown in Fig. 2 is a comparison between Kholopov's ellipticities and those estimated by Shapley & Sawyer (1927) by eye; in this case, the agreement is less satisfactory and the eye-measurements are systematically high. This difference is also reflected in the mean of Shapley & Sawyer's estimates, $\bar{e} = 0.12 \pm 0.01$, as compared with the mean of our estimates, $\bar{e} = 0.08 \pm 0.01$, for the same clusters (here and below we quote standard errors). Part of this difference may be the result of the poorer photographic material in use at the time Shapley & Sawyer made their measurements. The distributions of ellipticities in our galactic and LMC samples are shown in Fig. 3 and a one-tailed U-test indicates that the LMC clusters are stochastically
flatter than the galactic globular clusters at the 99.8 per cent level of confidence. For the entire LMC sample, we find a mean of $\varepsilon = 0.11 \pm 0.01$ and for a subsample restricted to objects redder than $B - V = 0.6$, we find a similar but less certain mean ellipticity. Thus, the red globular clusters of the LMC are, on average, slightly flatter than their galactic counterparts and this result is verified by a U-test at the 96 per cent level of confidence.

Geisler & Hodge (1980) reached a similar conclusion but found a much larger difference between the ellipticities of red LMC and galactic globular clusters. Although their estimates agree with ours for the largest clusters (e.g. NGC 1783, 1835, 1978 and 2210), their mean value of $\varepsilon$ exceeds ours by 0.11 for the 18 objects in both samples. Unfortunately, we know of no definitive explanation for this discrepancy and we are therefore limited to some general remarks. As Geisler & Hodge point out, their fitting routine for the microdensitometer scans does not make full use of the two-dimensional information in an image and their quoted errors do not take into account discreteness in the distribution of stars. As their simulations show, the errors are probably under-estimated and the ellipticities are probably over-estimated because clumps of bright stars can cause spurious elongations in the fitted contours. Moreover, Geisler & Hodge find some worrisome discrepancies between the determinations made from large-scale reflector plates (1.5 and 4 m) for a few of the clusters and those made from ADH Schmidt plates (0.6 m) for the majority of the clusters in their sample. Nevertheless, their method of estimating ellipticities would, at first sight, appear to be more objective than estimates made by eye and to resolve this problem it is necessary to analyse star counts.

3 Star counts

We were fortunate to receive star count data on 12 LMC clusters from Dr K. C. Freeman and Mrs J. Fisher. These were made from prime-focus plates taken with the AAO and CTIO 4-m telescopes in the J pass-band to limiting magnitudes between 22 and 23. The format of the counts is a polar grid or reseau with 36 azimuthal sectors and 30 concentric rings that extend
to radii of 6 or 12 arcmin. As the counts were made to determine stellar luminosity functions, they are complete to nearly uniform limits across the face of each cluster and are therefore suitable for the determination of ellipticities. Our first attempt in this endeavour was to apply Kholopov’s method to the data but we soon found that it gave unsatisfactory results. His procedure is to fit smooth profiles to the counts along each sector of a cluster and then to determine the radii at which these profiles reach a set of prescribed densities. The pattern that results from connecting such points looks rather like a spiderweb and the final step is to fit to this a family of elliptical contours by least squares. Kholopov’s method is capable of detecting radial and angular changes in the shape of a cluster but it does so at the expense of statistical power in the determination of the ellipticity and position angle of each contour. For the globular clusters in our Galaxy, where the counts often include several thousand members, this is probably acceptable; but for the globular clusters in the Clouds, where the counts include only several hundred members, the method loses its effectiveness.

To overcome this problem, we have devised a method for determining the shape of a cluster by fitting a model with fewer parameters. Given Kholopov’s results for galactic globular clusters, the most natural assumption to make is that the image can be characterized by a single ellipticity $\epsilon$ and a single position angle $\phi$. In this case, the surface density $f$ of the model must depend on the polar coordinates $r$ and $\theta$ only through the elliptical coordinate $\xi$ given by

$$\xi^2 = r^2 \sin^2 (\theta - \phi) + (1 - e)^2 r^2 \cos^2 (\theta - \phi).$$  

(2)

The exact form of the function $f(\xi)$ is immaterial in the determination of $\epsilon$ and $\phi$ so long as it provides a smooth approximation to the counts. After several tests with polynomials of different orders, we adopted a cubic

$$f(\xi) = c_0 + c_1 \xi + c_2 \xi^2 + c_3 \xi^3,$$  

(3)

with coefficients $c = (c_0, c_1, c_2, c_3)$ to be determined along with $\epsilon$ and $\phi$. The ‘best-fit’ values of these parameters minimize the statistic

$$X^2 = \sum_{ij} (f_{ij} A_{ij} - N_{ij})^2 / \sigma_{ij}^2,$$  

(4)

where $f_{ij}$, $A_{ij}$, $N_{ij}$ and $\sigma_{ij}^2$ denote respectively the model density, the area, the number of stars and their variance in the $(r_i, \theta_j)$ cell of the reseau. For $r_i$, we took the area-averaged radius of the $i$th ring and for $\theta_j$ the mid-angle of the $j$th sector; $f_{ij}$ was then obtained by evaluating equations (2) and (3) at these coordinates for specific values of $\epsilon$, $\phi$, and $c$. In practice, we assumed a Poisson distribution with variance $\sigma_{ij}^2 = N_{ij}$ and minimized $X^2$ analytically with respect to the ‘uninteresting’ parameters $c$ and numerically with respect to the ‘interesting’ parameters $\epsilon$ and $\phi$.

The sampling distribution of $X^2(\epsilon, \phi, c)$ can be used to set simultaneous confidence limits on the parameters of the model. Avni (1976) has shown that, for linear models with $m$ ‘interesting’ parameters, $p = (p_1, p_2, \ldots, p_m)$, and $n$ ‘uninteresting’ parameters, $q = (q_1, q_2, \ldots, q_n)$, a level of significance $\alpha$ is obtained by excluding those values of $p$ that satisfy the inequality

$$X^2_\alpha > X^2_{\min} + \chi^2_m (\alpha),$$  

(5)

where $X^2_\alpha$ is the minimum value of $X^2(p, q)$ with respect to $q$, $X^2_{\min}$ is the minimum with respect to both $p$ and $q$ and $\chi^2_m$ is the usual chi-squared distribution with $m$ degrees of freedom. For non-linear models such as ours, however, the validity of this relation must be checked by Monte Carlo simulations. To do this, we generated artificial clusters from a
model with a realistic shape and density profile (input values of $e$, $\phi$, and $e$) by perturbing
the counts in each cell with random numbers from a Poisson distribution. For each
simulated cluster, we computed $X^2_{\text{min}}$ by minimizing the right side of equation (4) with
respect to $e$, $\phi$ and $e$ and computed $X^2_k$ by fixing $e$ and $\phi$ and minimizing with respect to $e$.
A series of 100 simulations with identical input parameters then determined the sampling
distributions of these statistics and their difference; in particular, we found $X^2_k - X^2_{\text{min}}$ to be
distributed as $\chi^2$ to within 2 per cent accuracy. (Avni obtained a similar result for his non-
linear model of X-ray spectra). The validity of inequality (5) being established for our
problem, it can now be expressed in a form that takes explicit account of the ‘goodness of
fit’:

$$X^2_k > X^2_{\text{min}} \left[ 1 + m(s - m - n)^{-1} F_{m,s-m-n}(a) \right].$$

(6)

Here, $s$ is the number of cells over which the fit is made, $F_{k,l}$ is the statistical $F$-distribution
with $k$ and $l$ degrees of freedom and the result follows from the fact that $(s - m - n)\left(\frac{X^2_k - X^2_{\text{min}}}{mX^2_{\text{min}}}\right)$ is distributed as $F_{m,s-m-n}$ (see, for example, Draper & Smith 1966).

In the application of this procedure to the LMC clusters, we thought it prudent to include
corrections for gradients in the background and crowding in the stellar images. As a first-
order approximation to the background density, we used the model

$$f^{(b)}_{ij} = a + br \cos(\theta_j - \psi),$$

(7)

and estimated the parameters $a$, $b$, and $\psi$ from the counts well outside a cluster by minimizing
a quantity analogous to $X^2$ as defined in equation (4). The full model was then taken to
be the sum of equations (3) and (7) with the background parameters fixed. As an approxima-
tion to the effects of crowding, we adjusted the actual counts $N_{ij}$ by using the empirically
derived formula of King et al. (1968); in the present notation this reads

$$\delta \log N_{ij} = 0.43 \left[ \log N_{ij} + \log (d^2/A_{ij}) + 1.41 \right] + 0.15 \log (S/16),$$

(8)

where $d$ is the diameter of the smallest stellar images and $S$ is the plate scale in arcsec $mm^{-1}$.
In practice, the background corrections were always negligible and the crowding corrections
were negligible except in the innermost ring of one cluster (NGC 2209).

To illustrate the procedure, we have chosen NGC 1831, which is one of the richest
clusters in the sample and NGC 2156, which is one of the poorest. The densities of these
clusters along $20^\circ$ sectors are plotted in Fig. 4 as examples of the noise in the counts and the
futility in trying to fit profiles over small parts of the images. The azimuthally averaged
densities, on the other hand, are much smoother and, as Fig. 5 shows, they are well
represented by the best-fitting cubic of equation (3) out to radii where the density has
dropped to within about 10 per cent of the background. We adopted this criterion through-
out the analysis so that, depending on the sizes of the clusters and reseaux, the profiles were
fitted over 4, 5 or 6 rings. The corresponding radial ranges in arcmin are listed for the 12
clusters in the second column of Table 3; in all cases, they encompass the regions where the
eye-measurements were made and, in the cases that can be checked against surface photo-
metry, they lie just outside the median radii of the clusters. In Fig. 6, we have plotted the
best-fit values and the 75 and 90 per cent confidence contours of $e$ and $\phi$ for NGC 1831 and
2156; the differences in the contours reflect differences in the richness of these clusters. As
with the other clusters, the zero-points of the position angles are not known because there
was no need to record the orientations of the reseaux in the study for which the counts were
originally made. This implies that in our analysis $\phi$ must be regarded as another ‘uninteresting’
parameter and that somewhat more restrictive confidence limits can be derived for $e$ by
Figure 4. Surface density profiles for (a) NGC 1831 and (b) NGC 2156. The different symbols refer to star counts along three representative sectors of 20° width.

Figure 5. Azimuthally averaged profiles for the two clusters of Fig. 4. The points represent the count data with ±1σ error bars and the solid lines represent the model of equation (3) with the best-fit parameter values.

taking $X^2$ to be the minimum of $X^2$ with respect to $\phi$ and $c$. This leads to values of $\log(1 - \epsilon)$ that are approximately normally distributed; the best-fit value of this quantity and the corresponding one sigma range is given in the third column of Table 3 for each of the 12 clusters.

In Fig. 7, we compare the ellipticities determined from the star counts with those determined by eye. The agreement is clearly very good and shows that there is no systematic bias in our eye-measurements over the full range of ellipticities: $0 \leq \epsilon \leq 0.3$. Moreover, the internal errors in the estimates by eye are comparable with those from the star counts and
Figure 6. Contours of the $X^2_\nu$ statistic in the $(\varepsilon, \phi)$ plane for the two clusters illustrated in Figs 4 and 5. Each contour is labelled by the significance level at which the corresponding model can be rejected and the crosses represent the best-fit parameter values. The position angle $\phi$ is measured with respect to an arbitrary direction.

Table 3. Ellipticities of LMC globular clusters from star counts.

<table>
<thead>
<tr>
<th>NGC</th>
<th>$r$/arc min</th>
<th>log $(1-\varepsilon)$</th>
<th>NGC</th>
<th>$r$/arc min</th>
<th>log $(1-\varepsilon)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1818</td>
<td>0.55–1.64</td>
<td>$-0.119\pm0.048$</td>
<td>2159</td>
<td>0.38–1.90</td>
<td>$-0.071\pm0.041$</td>
</tr>
<tr>
<td>1831</td>
<td>0.82–2.46</td>
<td>$-0.056\pm0.017$</td>
<td>2164</td>
<td>0.76–2.28</td>
<td>$-0.060\pm0.040$</td>
</tr>
<tr>
<td>1866</td>
<td>0.82–3.28</td>
<td>$-0.036\pm0.028$</td>
<td>2172</td>
<td>0.38–1.90</td>
<td>$-0.119\pm0.054$</td>
</tr>
<tr>
<td>2004</td>
<td>0.55–1.91</td>
<td>$-0.125\pm0.055$</td>
<td>2209</td>
<td>0.70–2.09</td>
<td>$-0.022\pm0.022$</td>
</tr>
<tr>
<td>2156</td>
<td>0.38–1.90</td>
<td>$-0.076\pm0.043$</td>
<td>2214</td>
<td>0.82–2.87</td>
<td>$-0.143\pm0.046$</td>
</tr>
<tr>
<td>2157</td>
<td>0.38–2.28</td>
<td>$-0.032\pm0.027$</td>
<td>H 11</td>
<td>0.62–1.64</td>
<td>$-0.041\pm0.024$</td>
</tr>
</tbody>
</table>

Figure 7. Comparison of eye-estimates of ellipticity with the values derived from star counts for 12 LMC globular clusters. The error bars are $\pm1\sigma$ deviations (internal errors in $\varepsilon_{\text{eye}}$).

The scatter about the mean relation is consistent with both sets of errors. These are somewhat larger than the errors quoted by Kholopov for galactic globular clusters, partly because he used more stars and partly because the errors he quotes do not include uncertainties in the radial smoothing of the data. Of the 12 points in Fig. 7, the most discrepant belongs to
NGC 2172, which has an image on the Sky Survey prints distorted by a few bright stars and which only marginally satisfied our selection criteria. All but one of the LMC clusters with star counts are young and we cannot make a direct comparison with the estimates made by Geisler & Hodge (1980). On the basis of the comparison shown in Fig. 7, however, we feel justified in accepting the eye-measurements as they stand. Why the human eye is so able to discriminate shapes is an interesting question for psychology, but this ability has long been exploited in astronomy (Shapley 1930; King 1980). The images of globular clusters in the Magellanic Clouds are generally smaller than their galactic counterparts and would appear to be less suitable for eye-measurement; yet we have found that their ellipticities can also be estimated reliably in this simple way.

4 Age-estimates

Although the classification scheme devised by Searle et al. (1980) makes no reference to UBV photometry, we found that their sequence of types could be reproduced in a conventional two-colour diagram. In the (U−B, B−V) plane of Fig. 8, we have plotted all the clusters in our sample and, to define the boundaries of the segments, all but three of the LMC clusters in the sample of Searle et al. (The exceptions, NGC 1751, 1841 and 2257, have uncertain or incomplete UBV data). It is remarkable that only three clusters are misclassified by this procedure (NGC 1831, 2134 and 2164) and that their types, as inferred from Fig. 8, are each one stage earlier than those assigned by Searle et al.; moreover, this shift cannot be ascribed to reddening, which would move points in the opposite direction. The explanation

![Two-colour diagram for LMC globular clusters. The filled circles and crosses represent the sample of Searle et al. (1980) and the open circles represent the remaining objects in our sample. The segments into which the sequence is divided are the UBV equivalents of the original SWB types and the three crosses represent the only clusters that are misclassified in this diagram.](https://academic.oup.com/mnras/article-abstract/199/3/565/969224)
for this correspondence with the original $Q-Q$ diagram appears to be that the clusters of the LMC, especially those of later types, are fairly free of reddening and that much of the SWB sequence runs parallel to a reddening vector in the $(U-B, B-V)$ plane. The steep slope of the sequence at the blue end and its doubling-back at the red end would also seem to reconcile the current view of reasonably continuous changes in the stellar contents of LMC clusters with the well-known dip in the distribution of their integrated colours at $B-V \approx 0.5$ (Gascoigne & Kron 1952). In any case, the correspondence between Fig. 8 and the $Q-Q$ diagram has enabled us to assign SWB types to the 20 clusters in our sample not classified by Searle et al.; these are indicated by brackets in the fifth column of Table 1. (For NGC 1831, we have adopted the type IV inferred from Fig. 8 rather than the type V assigned by Searle et al. because this object is probably too blue and too young for the later type).

To calibrate the first three SWB types, we have used the relation between age and $U-B$ colour derived by Dixon, Ford & Robertson (1972) from synthesis models of Population I clusters. Although this is not the most accurate method for dating, it can be applied to clusters without CM data and is generally consistent with isochrone fitting where the two methods can be compared (Robertson 1974; Barbaro 1981). To calibrate the later SWB types, we have relied on the ages estimated by Aaronson & Mould (1982) and Rabin (1982); many of these are uncertain and some are only upper limits but the two methods generally give consistent results. The age that divides types III and IV can be estimated from NGC 2209, which Searle et al. classified as types III–IV. According to Aaronson & Mould this cluster is younger than $2 \times 10^9\,$yr and according to Rabin its age is between 1.5 and $2.5 \times 10^9\,$yr but we have adopted an age of $1 \times 10^9\,$yr to be consistent with the isochrone fits of Gascoigne et al. (1976) and Walker (1971). The boundary between types IV and V is less certain because Aaronson & Mould found a spread of ages for clusters of type V and because several of their estimates are lower than those of Rabin; as a compromise, we have adopted an age of $3 \times 10^9\,$yr. For types V and VI, both methods suggest a dividing age of $6 \times 10^8\,$yr, as exemplified by Rabin's estimate for Kron 3 in the SMC. Finally, we adopted a range of $8-14 \times 10^9\,$yr for type VII, which is consistent with the values obtained by Aaronson & Mould. Our calibration of the SWB sequence is summarized in Table 4 and follows closely the one given by Rabin (1982). The age-scale may need revision as the methods for dating clusters improve but, even in this provisional form, it should give rough estimates of evolutionary rates along the sequence.

Table 4. Age-calibration of SWB types.

<table>
<thead>
<tr>
<th>Type</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age ($10^9,$yr)</td>
<td>0.005–0.03</td>
<td>0.03–0.1</td>
<td>0.1–1.0</td>
<td>1–3</td>
<td>3–6</td>
<td>5–10</td>
<td>8–14</td>
</tr>
</tbody>
</table>

We have combined the eye-measurements of ellipticities with the assignments of SWB types to obtain the relation depicted in Fig. 9. From this diagram, it appears that the young clusters are, on average, flatter than the old clusters and this impression is confirmed by a value of $r_s = -0.26$ for the Spearman rank correlation coefficient. For the sample of 52 objects, this correlation is significant at the 97 per cent level of confidence and is independent of the age-calibration of the SWB sequence. Clusters of types I and II, with a combined average of $\bar{e} = 0.21$, are responsible for most of the correlation. The relation has a shallow minimum at type IV and, although clusters of this type are generally the faintest in our sample, a bias in the eye-estimates seems unlikely in view of the agreement with the determinations from star counts for NGC 1831 and 2209. The relation appears to have a secondary peak at type VI but this depends sensitively on the classification of NGC 1978,
which is the flattest cluster in the sample and is more than four standard deviations away from the mean of the other type VI and VII objects. According to Rabin, the age of NGC 1978 is consistent with type VI, but the work of Aaronson & Mould suggests that it is considerably younger and the possible membership of two RR Lyrae stars (Thackeray 1959) suggests that it is older. The combined average of $\bar{e} = 0.12$ for types VI and VII is only slightly larger than the average of $\bar{e} = 0.08$ for the galactic sample, as indicated by an open circle in Fig. 9. Thus, although the globular clusters in our Galaxy are rounder on average than those in the LMC, they seem to join naturally on to the same general relation.

The correlation of ellipticity with age may reflect a combination of evolution in the shapes of clusters and variations in the conditions of formation. If the second effect were more important than the first, one might expect a dependence of ellipticity on radial coordinate in the LMC and a dependence of radial coordinate on age or SWB type. As Figs 10 and 11 show, there are no such correlations in our sample and this favours an explanation of the ellipticity–age relation in terms of evolution by evaporation or some other process. The fact that age and position are not correlated may be due in part to the exclusion of a few young clusters in the inner regions and a few old clusters in the outer regions of the LMC during the selection of our sample. This does not alter our interpretation, however, because the relations depicted in Figs 9, 10 and 11 were all derived from the same set of data. Another possibility, which is not tested by this comparison, is that the initial shapes of clusters were influenced by their distances from the equatorial plane of the LMC at the times they formed. There appears to be a general correlation of residual velocity and hence vertical scale-height with SWB type but, from the available kinematic data, it is not yet possible to determine the age that separates the disc and halo populations (Ford 1970; Andrews & Lloyd Evans 1972). If the ellipticity–age relation is to be explained in terms of this distinction, then Fig. 9 suggests that the halo phase in the history of the LMC extended to recent epochs because the biggest change in ellipticity occurs at an age of about $10^9$ yr.
Figure 10. The relation between ellipticity and distance from the centre of the LMC for our sample. Each point is the mean ellipticity of clusters in a range of distances chosen so as to obtain approximately equal numbers of objects in each bin. The error bars are ±1σ deviations and the scale at the top of the diagram is based on a distance to the LMC of 55 kpc (Gascoigne 1972).

Figure 11. The relation between distance from the centre of the LMC and SWB type for our sample. Each point is the mean distance and the error bars are ±1σ deviations. The age-scale at the top of the diagram is the same as in Table 4 and the distance scale on the right is the same as in Fig. 10.

Further work on the kinematic properties of clusters in the LMC should help to resolve this issue.

In conclusion, we have found that estimates of the ellipticities of globular clusters made by eye are in excellent agreement with those based on star counts. Our measurements show that the globular clusters in the LMC are slightly flatter on average than their galactic counterparts. We have also found a correlation of the ellipticities of clusters in the LMC with their progression along a sequence derived from $UBV$ colours. This sequence is equivalent to the one found by Searle et al. (1980) from $ugr$ photometry and can be calibrated using a variety of age-estimators. Young clusters are flatter on average than old clusters in the LMC and the oldest of these are similar to galactic globular clusters. This result assumes only that the SWB sequence is one of increasing age and does not depend on the absolute dating of the cluster types. We have also searched for correlations between the shapes of clusters and their radial coordinates in the LMC that might reflect variations in the conditions at the times of formation; none is present in our sample. Subject to a better understanding of variations in
the vertical distribution of clusters within the LMC, the ellipticity—age relation is probably explained most naturally by internal evolution in the structure of globular clusters. Further discussion of these points will be given in Paper II.

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