New results on the mechanism of magnetic flux pumping by three-dimensional convection

W. Arter and M. R. E. Proctor  Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge CB3 9EW
D. J. Galloway  Max Planck Institut für Physik und Astrophysik, Garching-bei-München, Germany

Received 1982 July 23

Summary. New calculations are presented to show that the mechanism proposed by Drobyshevski & Yuferev for pumping magnetic flux at the base of a stellar convection zone is radically altered in the case of high magnetic Reynolds number. In consequence the process as they stated it is unlikely to be operating in the Sun, though the revised mechanism may actually be more effective in maintaining magnetic fields at the bottom of the convecting layer.

1 Introduction

Drobyshevski & Yuferev (1974, hereafter DY) have pointed out a mechanism whereby three-dimensional convection confines horizontal magnetic flux to the bottom of a solar or stellar convection zone. Recently a number of arguments have been advanced for locating the Sun’s dynamo at the interface between the convective and radiative regions (Schüssler 1980; Galloway & Weiss 1981; van Ballegooijen 1982) and it is tempting to invoke this ‘topological pumping’ process to keep magnetic flux buried, in spite of its natural tendency to rise due to magnetic buoyancy. Here we show by numerical calculation that DY’s original picture is correct only for low magnetic Reynolds number $R_m$, and is thus inappropriate for the Sun.

DY give the following physical interpretation for their mechanism, which is envisaged to operate when individual cell velocities are upward at the centre and downward at the periphery, as is the case in the granulation and super granulation. The isolated ascending regions can only lift small stitches of field; the downward-moving fluid, on the other hand, forms a continuous network which can drag down whole field lines and trap them at the bottom. The effect is purely kinematic, and the final equilibrium is determined by the balance between field advection and ohmic diffusion; the picture must be modified when the field is strong enough for the Lorentz force to be non-negligible.

Analytic aspects have been discussed by Moffatt (1978) and Krause & Rädler (1980); DY’s original calculation was numerical and has been repeated with different boundary conditions by Drobyshevski, Lokesnikova & Yuferev (1980).
The present calculations (to be reported fully in Arter 1982 and Galloway & Proctor 1982) are designed to find out what actually occurs in the astrophysically relevant case of high magnetic Reynolds number \( R_m = U d / \eta \) where \( U \) is maximum flow speed, \( d \) a typical cell dimension and \( \eta \) the magnetic diffusivity. Increasing \( R_m \) corresponds to reducing the effects of diffusion (the field is more ‘frozen in’), and one would therefore expect the physical argument given earlier to be more compelling in this limit. In fact something entirely different happens. As soon as \( R_m \) is large enough for reconnection to take place (typically \( R_m \gtrsim 80 \)), the flow generates islands of opposite polarity field whose flux at the top is comparable with the amount of positive flux pumped to the bottom. As \( R_m \) is further increased, these (unsigned) fluxes become large compared with their difference, which is equal to the original input flux. Thus there is still a physical effect associated with the topological asymmetry of the flow, but it is not that envisaged in DY. Rather the effect should be thought of as an amplification mechanism for unsigned flux. Parker (1975) has argued that a combination of magnetic buoyancy and a boundary that permits the escape of flux will vitiate the effects of DY’s mechanism. However, we shall suggest in the conclusion that the new calculations show a much more effective amplification of fields at the base of the convective zone, and the mechanism may thus have a part to play in the maintenance of the solar dynamo.

2 Numerical results

In two independent numerical studies, we have investigated the effect of topologically asymmetric convection between two perfectly conducting layers at \( z = 0 \), \( d \) (in Cartesian coordinates \( x, y, z \)). Horizontal magnetic flux threads the region; its integral over \( z \) is fixed by the boundary conditions. We have considered both hexagonal cells (Christopherson 1940) and ‘square’ cells (introduced by DY). Both these geometries feature isolated rising regions of fluid as required by the hypotheses of topological pumping. The back reaction of the magnetic field \( \mathbf{B} \) on the flow \( \mathbf{u} \) is neglected, so that \( \mathbf{B} \) evolves according to the (dimensionless) induction equation

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \wedge (\mathbf{u} \wedge \mathbf{B}) + \frac{1}{R_m} \nabla^2 \mathbf{B}
\]

(2.1)

where \( \mathbf{u} \) is scaled with its maximum modulus \( U \), distances with \( d \) and time with \( d/U \). This equation is then solved numerically as an initial value problem, for fixed \( \mathbf{u} \). The square cell geometry (case A; Arter 1982) is actually identical to that studied by DY; the DAP was used to give enhanced speed, and solutions were obtained by discretizing in all three dimensions and using a leapfrog scheme similar to that used by, e.g., Weiss (1966). Properly converged results were obtained for \( R_m \) up to 200, using 24 grid points in each direction for a quarter cell. The hexagonal problem (case GP) was solved by Fourier transforming in \( z \), truncating and using a leapfrog method on the resulting set of PDE’s in \( x, y \) and \( t \). A general outline of the method is given in Galloway & Proctor (1982), and a full description will be available in Galloway & Proctor (1982). Converged results were obtained on a Cray-I machine at \( R_m < 400 \). In both cases the top and bottom boundaries are taken to be perfect electrical conductors so that the total mean field \( \mathbf{B}_0 \) in the layer is fixed. In this paper we concentrate on the ‘averaged’ flux \( \mathbf{B}(z) \), defined by

\[
\mathbf{B} = \frac{1}{|\mathbf{B}_0|} \mathbf{B}_0 \cdot \langle \mathbf{B}(x, t) dx \, dy \rangle
\]

(2.2)
and the brackets denote an average over the tesselated layer. By definition,

$$\int_0^1 \overline{B} \, dz = |B_0|.$$

We define the ‘pumping function’

$$P(R_m) = \int_0^{1/2} \overline{B} \, dz / |B_0| - \frac{1}{2},$$

(2.3)

(where $\overline{B}$ is evaluated at a steady state), as a measure of the redistribution of flux. Results for $\overline{B}(z)$ in case GP (with $B_0 \parallel \hat{y}$) are shown in Fig. 1. (It must be noted that DY only achieved values of $R_m$ of 50 or less.) In both cases $P$ becomes very large as $R_m$ increases; this is achieved by the creation of a substantial sheet of reversed flux near the top of the cell. In case GP, the initial direction of the flux does not appear to affect the results at all (this is also true trivially for case A).

Although the largest fields are to be found at the bottom of the cell (and most of the magnetic energy resides there) they are confined to tube-like structures which have less volume than the sheets near the top. In fact, for large $R_m$, the magnetic energy $\sim R_m^2$, suggesting that $P \sim R_m$ (whereas in the two-dimensional case $P = O(1)$; Proctor & Weiss 1978).

How is this reversed field achieved? Fig. 2, showing some field lines in the $y-z$ plane for case GP, shows that a tightly wound spiral structure occurs as $u$ and $B$ are almost parallel. Following field lines off the symmetry axis (see Fig. 3 for case A) shows that as $R_m$ increases many field lines wind many times round before re-emerging from the reversed flux region. It is this multiple winding that allows the unsigned flux to increase indefinitely. The topological features of the solution seem to be determined on the turnover time-scale. There then follows a diffusive relaxation which determines the steady state field. The ‘flux expulsion’ phase (Weiss 1966) that is so important in the two-dimensional case does not seem to have the same significance in three dimensions.

Figure 1. The average horizontal field $\overline{B}(z)$ in case GP with $B_0 \parallel \hat{y}$ for a series of magnetic Reynolds numbers $R_m$. Only the steady states are shown. Note the appearance of large negative fluxes near the top of the layer.
Figure 2. Field lines in planes of symmetry for case GP when $B_0 \parallel \hat{z}$, and $R_m = 400$ (steady state). (a) Horizontal field, $z = 0$; (b) Horizontal field, $z = 1$. Note that the field has reversed in direction except in a small region near $x = y = 0$. (c) Field lines in the plane $x = 0$. Note the spiral structure and the appearance of closed field lines.

3 Discussion

The phenomena outlined above show clearly that the simple ideas of pumping introduced by Drobyshevski & Yuferev are misleading in the limit of large $R_m$, since the effect of the motion is to enhance the field almost everywhere. They also lead to questions about the relevance of the mechanism for the solar convection zone, since there are no observations of reversed horizontal flux near the top of the zone (of course, most emerging flux is vertical anyway). The influence of the top boundary condition is clearly crucial. Preliminary calculations with a current free atmosphere above the convecting layer suggest that the field is initially amplified at the base of the layer, essentially by the mechanism described above, but that all the flux eventually escapes on a time-scale $R_m^{1/2}$ times the turnover time. This is probably long enough for dynamo fields to remain at the bottom of the convection zone, at least for times comparable to the solar cycle. It is not clear if convective motion on the largest scales (such as the hypothetical ‘giant cells’), which are strongly influenced by rotation and may be almost two-dimensional, has the asymmetry necessary for the effects
described here, but if so the mechanism may help resolve one of the central paradoxes of solar dynamo theory, namely how the field can avoid escape due to magnetic buoyancy on a time short compared with the 11 yr cycle. A proper resolution of the issue awaits a fully dynamical calculation in a compressible atmosphere.

References

Christopher, D. G., 1940. Quart. J. Math., 11, 63.