Quasi-periodic scintillation patterns of the pulsars
PSR 1133+16 and PSR 1642−03

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Summary. A new series of interstellar scintillation measurements of the pulsars PSR 1133+16 and PSR 1642−03 at 408 and 1426 MHz made in 1982 and 1983 is presented. Quasi-periodic patterns are seen in the dynamic spectra of pulsar scintillation which change on time-scales ranging from one day to several months, but the coherence of these periodic modulations is maintained only over small portions of the time–frequency plane. These effects can be explained in terms of a theoretical model combining random diffraction from a number of small-scale electron density fluctuations in the interstellar medium with systematic refraction caused by larger irregularities. It is shown that when scintillation patterns are periodically modulated, the power-law index of the wavenumber spectrum of electron density irregularities measured in the range ~10^{11}−~10^{13} cm exceeds a value of 4.

1 Introduction

The existence of quasi-periodic patterns in the dynamic spectra of pulsar interstellar scintillation (ISS) has been known for many years (e.g. Ewing et al. 1970; Manchester & Taylor 1977). More recent work by Roberts & Ables (1982), Cordes, Weisberg & Boriakoff (1984) and in this paper, suggests that quasi-periodicity may be typical when the observations embrace a sufficiently wide range of frequency and time for the patterns to be recognized.

The conditions under which quasi-periodic scintillation could arise were considered by Hewish (1980) who discussed a model combining random diffraction from a number of small-scale irregularities with more systematic refraction caused by much larger irregularities. It was shown that quasi-periodic fringes could occur when the angles of refraction \( \theta_r \) were greater than the angles of random diffraction \( \theta_s \), and it was further suggested that the patterns could be analysed to obtain a measure of the ratio \( \theta_r / \theta_s \).

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In this paper we develop that theory and first discuss the characteristics of typical quasi-periodic patterns that should exist in dynamic spectra for different geometries of the irregular structure. It is shown how the patterns can be used to estimate the ratio $\theta_r/\theta_s$ and the method is applied to new observations of the dynamic spectra of PRS 1133+16 and 1642–03 obtained with the 100-m Effelsberg radio telescope. We then demonstrate how the ratio $\theta_r/\theta_s$ provides a comparison of the density fluctuations of the irregularities at widely separated scales and hence leads, for an assumed power-law spectrum, to an estimate of the power-law index. This method contrasts with previous investigations based on the shape of the radio frequency autocorrelation function (e.g. Armstrong & Rickett 1981) in that it samples the spectrum over a wider range of scales.

Our model of the interstellar medium is similar to one considered by Roberts & Ables (1982) but our quantitative treatment of $\theta_r/\theta_s$ allows tighter constraints to be imposed upon the power-law index. We also consider the coherence of quasi-periodic patterns in frequency and time which has not been adequately discussed before.

2 Theory

In the following analysis we show how $\theta_r$ and $\theta_s$ are related to the spectrum of the irregularities and examples will be given of different types of dynamic spectra that should be seen when $\theta_r>\theta_s$. Emphasis will be placed on physical arguments rather than on an exact treatment. Most theoretical work germane to this problem has concentrated on the solution of the equation for the fourth moment of the wave field. Such an approach is less useful here, since we are concerned more with the recognition and analysis of deterministic interference phenomena which modulate the stochastic, diffractive scattering process, than with ensemble average properties.

2.1 The Dependence of $\theta_r$ and $\theta_s$ on the Medium

We assume for simplicity that the extended medium may be regarded as a thin screen midway between the source and the observer. Let $P_N(K_x, K_y, K_z)$ be the wavenumber power spectrum of the density variations in the medium where the spatial wavelength is $2\pi K^{-1}=2\pi(K_x^2+K_y^2+K_z^2)^{-1/2}$. In what follows it is helpful to consider the spectrum as divided into two portions, although in reality the distinction becomes blurred at the interface. Let $K_F$ define a critical wavenumber such that spectral components for which $K>K_F$ cause random intensity variations at the observer due to the interference of waves scattered into an angular spectrum of width $\theta_s$, while components with $K<K_F$, considered in isolation, would give rise to tilted wavefronts on a scale such that no interference was possible. For a strongly scattering screen at distance $z$ from the observer, geometrical optics shows that irregularities causing angular tilts of magnitude $\theta_t$ on a scale $L$ generate sharply diminishing intensity variations if $L>z\theta_t$. Hence $K_F>2\pi L^{-1}$.

To estimate the magnitude of the phase gradients imposed upon wavefronts at the screen by different portions of the spectrum, consider the phase modulation $\phi(x, y)$ across a plane. By a well-known result the wavenumber spectrum of $\phi(x, y)$ is $P_\phi(K_x, K_y) = AP_N(K_x, K_y, 0)$

where $A$ is constant. Hence the mean square gradient in some direction (which can be along $x$ with loss of generality) imposed by irregularity components in the range $K_x$ to $K_x+\Delta K_x$ is

$$\left<\left(\frac{\partial \phi}{\partial x}\right)^2\right> = \frac{K_x^2}{2} \Delta K_x \int_{-\infty}^{\infty} P_\phi(K_x, K_y) dK_y. \quad (1)$$
This may be expressed in terms of the angular tilts across the wavefront by putting

$$\delta \theta_c = \frac{\lambda}{2\pi} \frac{\partial \phi}{\partial x}$$

where \(\lambda\) is the radio wavelength. The magnitudes of \(\theta_c\) and \(\theta_s\) are therefore given by

$$\langle \theta_c^2 \rangle = \frac{\lambda}{2\pi} \int_0^{K_F} \left[ \frac{K_x^2}{2} \int_{-\infty}^{\infty} P_{\phi}(K_x, K_y) \, dK_y \right] \, dK_x$$

(2)

$$\langle \theta_s^2 \rangle = \frac{\lambda}{2\pi} \int_{K_F}^{\infty} \left[ \frac{K_x^2}{2} \int_{-\infty}^{\infty} P_{\phi}(K_x, K_y) \, dK_y \right] \, dK_x$$

(3)

For power-law spectra of the type

$$P_N(K_x, K_y, K_z) \propto (K^2_x + K^2_y + K^2_z)^{-\alpha/2},$$

the evaluation of (2) will depend upon fixing some lower bound, \(K_e\), on \(K\), as \(K \to 0\), if \(\alpha > 4\). The evaluation of (3) will similarly depend upon fixing an upper bound, \(K_u\), for \(K > K_F\) if \(\alpha \leq 4\); provided that \(\alpha > 2\), so that \(\phi^2\) does not diverge at large \(K\), \(K_u\) is effectively the limit above which only weak scattering occurs. Hence

$$\int_{K_F}^{\infty} \int_{-\infty}^{\infty} P_{\phi} \, dK_y \, dK_x \geq 1 \text{ rad.}$$

The general behaviour of \(\theta_c\) and \(\theta_s\) is best illustrated by returning to expression (1). Consider, for example, a Kolmogorov spectrum for which \(\alpha = 11/3\). In this case, components in the range \(\Delta K_x\) produce angular tilts given by

$$\langle \delta \theta_c^2 \rangle \propto K_x^{-2/3} \Delta K_x.$$  

Putting \(\Delta K_x = \beta K_x\), where \(\beta < 1\) to define an appropriate range, we obtain \(\langle \delta \theta_c^2 \rangle \propto K_x^{1/3}\). This is the familiar result that angular tilts for Kolmogorov turbulence vary as \(L^{-1/6}\), where \(L\) is the scale over which tilts are considered. Clearly the smaller scales (larger \(K\)) dominate the angular tilts in a Kolmogorov spectrum so that in this case \(\theta_c < \theta_s\) for any scale. The condition \(\theta_c > \theta_s\) demands \(\alpha > 4\). In order to investigate the constraints imposed upon spectra consistent with the observed quasi-periodic structure, we now turn to a closer examination of the types of dynamic spectra expected when \(\theta_c > \theta_s\).

### 2.2 THE GENERATION OF QUASI-PERIODIC PATTERNS

When plane waves enter a uniform random medium, the diffraction pattern after propagation through a distance \(z\) is characterized by intensity fluctuations which have a correlation time \(t_c \sim \lambda / 2\pi \theta_c V\) and a correlation bandwidth \(f_c \sim c / \pi \theta_c^2\). In these relations \(\theta_c\) is a measure of the angular spectrum of waves scattered by a large number of irregularities for which \(K > K_F\), \(V\) is the speed at which the pattern is moving past the observer, and \(f_c\) is the range beyond which correlation falls to a low value (Lee 1976). In what follows we take \(z\) to be the distance of the pulsar from the observer, and assume that the same scattering occurs in a thin screen at distance \(z / 2\). The appropriate relations then become \(t_c \sim \lambda / \pi \theta_s V\), \(f_c \sim 4c / \pi \theta_s^2\). The dynamic spectrum of such a pattern is sketched in Fig. 1(a).

If, in addition to the angular scattering \(\theta_s\), the wavefront is tilted through an angle \(\theta_t\) due to the presence of large-scale irregularities for which \(K < K_F\), the dynamic spectrum is sheared by
Figure 1. Theoretical dynamic spectra of pulsar scintillation for various geometries of the scattering medium.

dispersion in the medium to produce systematic gradients of magnitude $df/dt = V_x f / z \theta_r$ where $V_x$ is the component of $V$ along the direction of refractive displacement of the pattern (Hewish 1980). This effect is shown in Fig. 1(b).

The total shear $t_s$ across the correlation bandwidth $f_c$ is

$$t_s = f_c \frac{dt}{df} = \frac{f_c z \theta_r}{V_x f},$$

so

$$t_s / t_c = 4 \theta_r / \theta_s.$$  \hfill (4)

Equation (4) may provide a useful measure of $\theta_r / \theta_s$ but the method is of limited value since the case $t_r / t_c > 4$, corresponding to $\theta_r > \theta_s$, demands good alignment of $V_x$ and $V$. When the latter does not obtain, both the correlation bandwidth and the total shear are reduced by the factor $\theta_r / \theta_s$ when $\theta_r > \theta_s$ (Hewish 1980). Hence the method underestimates $\theta_r / \theta_s$ in general. This modification of the correlation bandwidth due to the presence of large-scale irregularities shows

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that analysis of the frequency correlation function to obtain the irregularity spectrum (e.g. Armstrong & Rickett 1981) can lead to errors if \( \theta_r > \theta_s \).

When \( \theta_r > \theta_s \), the intensity pattern may result not only from the superposition of a large number of waves scattered through angles \( \theta_s \), but also from the interference of a small number of wavefronts tilted through angles \( \theta_r \). Simple geometrical optics suggests that this could occur if \( \theta_r \) is caused by refraction by irregularities of wavenumber \( K \sim 2\pi(\theta_r)^{-1} \). On the thin-screen model that we have adopted, this means that the net effect of these large-scale irregularities is to produce rough focusing at a distance somewhat less than that of the observer from the screen. The random pattern characterized by \( f_c \) and \( t_c \) will then be modulated by interference fringes on a finer scale, provided that the field incident upon the large irregularities is coherent over distances \( \sim z \theta_r \).

Some examples of dynamic spectra that could result from the interference of a few tilted wavefronts combined with small-angle scattering are sketched in Fig. 1. The spatial scale of the interference fringes is of order \( 2\lambda / \theta_r \) which gives a temporal period \( t_r \sim 2\lambda / \theta_r V_s \). The corresponding periodicity in radio frequency results from typical path differences \( \sim z \theta_r^2 / 4 \) between tilted wavefronts, which gives a separation in frequency \( f_p \sim 4c / z \theta_r^2 \). Roberts & Ables (1982) derived similar relationships; our magnitudes follow from the thin-screen assumption discussed at the beginning of this section.

An important feature of dynamic spectra such as those illustrated in Fig. 1(c)–(e) is that they are characterized by three distinct scales. The smallest scale is the fringe scale \( t_p \) and \( f_p \). Next there is the larger scale \( t_c \) and \( f_c \) which governs amplitudes and phases of the fields across the tilted wavefronts; thus the fringe groups can only maintain coherent fringe-phase and fringe visibility over scales \( \sim t_c \) and \( f_c \). This effect can produce rapid changes in the fringe patterns, while the overall arrangement of large-scale tilted wavefronts is relatively constant. Finally there is the longest time-scale \( t_c \sim 2\pi (K_0 V)^{-1} \), where \( K_0 \) is the wavenumber of the irregularities giving rise to systematic tilts; this is the time required for the largest irregularities to be replaced.

The simple considerations that we have outlined show how dynamic spectra may be used to make quantitative estimates of \( \theta_r / \theta_s \) and this theory is applied to observations of PRS 1133+16 and 1642–03 in the next section. As a simple diagnostic we note that \( t_c / t_p \sim \theta_r / 2\pi \theta_s \) and \( f_c / f_p \sim \theta_r^2 / \pi \theta_s^2 \), so that simply counting the number of fringes within a coherent group gives an approximate measure of \( \theta_r / \theta_s \). These relations also show that quasi-periodicity may be more evident along the frequency axis than along the time axis owing to the squared dependence on \( \theta_r \) and \( \theta_s \).

### 3 Observations and data analysis

#### 3.1 Observing technique

Measurements of the dynamic ISS spectra of PRS 1133+16 and 1642–03 were made with the MPIfR 100-m radio telescope at 408 and 1426 MHz during several observing sessions between 1982 November and 1983 June. Spectra for both right-hand and left-hand circularly polarized signals were obtained using the 384- and the 1024-channel correlation spectrometers simultaneously. This allowed us to observe with different bandwidths, spectral resolutions and sampling intervals at the same time. The spectrometers sampled the 192- and 512-point spectra, respectively, for each polarization, and averaged them over times equal to integral numbers of pulse periods depending on the expected ISS decorrelation time. Correlator bandwidths between 1.25 and 50 MHz and averaging times ranging from 11 to 50 s were used. All these data are listed in Table 1 for each observing session. The signal and reference spectra were obtained in a conventional way from the ‘on-pulse’ and ‘off-pulse’ intensities sampled synchronously with the apparent pulsar period (e.g. Wolszczan, Hesse & Sieber 1974). Since the spectra were integrated over a few tens of pulsar periods, frequency and time variations of intensity intrinsic to the source

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Table 1. Summary of results.

<table>
<thead>
<tr>
<th>PSR</th>
<th>Frequency (MHz)</th>
<th>Date</th>
<th>Correlator Bandwidth (MHz)</th>
<th>Frequency Resolution (kHz)</th>
<th>Integration Time (s)</th>
<th>Total Obs. Time (min)</th>
<th>$f_p$ (kHz)</th>
<th>$f_c$ (kHz)</th>
</tr>
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<tbody>
<tr>
<td>1133+16</td>
<td>408</td>
<td>1983 Feb. 18</td>
<td>25</td>
<td>48.8</td>
<td>47.4</td>
<td>168</td>
<td>1890±80</td>
<td>11000±1600</td>
</tr>
<tr>
<td>408</td>
<td>1983 Jun. 4</td>
<td>25</td>
<td>48.8</td>
<td>30.0</td>
<td>164</td>
<td>900±50</td>
<td>3000±200</td>
<td></td>
</tr>
<tr>
<td>1642+03</td>
<td>408</td>
<td>1982 Nov. 3</td>
<td>1.25</td>
<td>6.5</td>
<td>11.6</td>
<td>74</td>
<td>60±5</td>
<td>240±10</td>
</tr>
<tr>
<td>408</td>
<td>1983 Jun. 5</td>
<td>1.25</td>
<td>6.5</td>
<td>11.6</td>
<td>77</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1426</td>
<td>1983 Apr. 24</td>
<td>50</td>
<td>97.6</td>
<td>48.8</td>
<td>89</td>
<td>42</td>
<td>4300±200</td>
<td>10780±1100</td>
</tr>
<tr>
<td>1426</td>
<td>1983 May 16</td>
<td>50</td>
<td>97.6</td>
<td>20.2</td>
<td>97</td>
<td>18000±2000</td>
<td>&gt;20000</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. Dynamic spectra of PSR 1133+16 at 408 MHz. Eight grey-scale levels above the 1/e intensity threshold are displayed.
could be entirely neglected (Wolszczan, Bartel & Sieber 1981, and references therein). A series of spectra measured over a time period up to 180 min, corrected for the receiver bandpass shape and weighted according to the broadband intensity variations was used to form a dynamic ISS spectrum of a given pulsar. Examples of the spectra obtained in this way are shown in Figs 2 to 5. They display a typical ISS time–frequency structure which is, in some cases, strongly periodically modulated. Closer examination of these spectra indicates that the ISS patterns change on time-scales extending from one day to several months.
3.2 DERIVATION OF SCATTERING ANGLES $\theta_c$ AND $\theta_s$

In order to derive $f_c$ and $f_p$, a correlation analysis of the measured dynamic ISS spectra was performed. Since our observations have shown that the pulsar intensity modulation is generally more pronounced in the frequency domain than in the time domain, as expected from the theory, we have restricted ourselves to a frequency-correlation analysis of the ISS spectra. A time-averaged frequency autocorrelation function $P(\Delta f)$ has been computed from the dynamic ISS spectrum representing the intensity distribution $I(f, t)$ in time $t$ and frequency $f$, using the method applied by Armstrong & Rickett (1981) (see also Wolszczan 1983). In this method, $P(\Delta f)$
Figure 5. Dynamic spectra of PSR 1642–03 at 1426 MHz (see caption to Fig. 2).
is derived in two steps. A time-averaged power spectrum of $I(f, t)$:

$$\hat{P}(\tau) = \left| \left( \int_{-\infty}^{\infty} I(f, t) \exp(i2\pi f\tau) df \right) \right|^2,$$

(5)
is first computed and corrected for noise and slow spectral power variations. In the second step, $\hat{P}(\tau)$ is Fourier-transformed back to the frequency domain to yield $P(\Delta f)$ corrected for noise and slow variations

$$P(\Delta f) = \int_{-\infty}^{\infty} \hat{P}(\tau) \exp(-i2\pi \tau \Delta f) d(\Delta f).$$

(6)

Examples of $P(\Delta f)$ functions derived in this way are shown in Fig. 6. Evidently, a periodic intensity modulation, visible in the dynamic spectrum, reappears in the frequency correlation function as shown in Fig. 6(a). It has been demonstrated in Section 2 that in these cases the ratio of the coherence range of the periodic frequency modulation $f_c$ to the modulation period $f_p$ provides a direct measure of $\theta_C^2/\theta_P^2$. The width of $P(f)$ at zero autocorrelation has been used to define $f_c$, and the period $f_p$ has been derived directly from $I(f, t)$.  

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The values of $f_p$ and $f_c$ obtained for the periodically modulated dynamic spectra are listed in Table 1 together with the errors which were derived as explained in Roberts & Ables (1982). Given $f_p$ and $f_c$, the ratio of $\theta_r$ and $\theta_s$ can be derived from

$$\frac{\theta_r}{\theta_s} = (\sigma f_c/f_p)^{1/2}. \tag{7}$$

In the remaining spectra, no clear periodic intensity modulation was present and rough estimates of $\theta_r/\theta_s$ for these data were derived using (4).

### 3.3 Derivation of the Power-Law Index Spectrum

Having calculated $\theta_r/\theta_s$ for each pulsar and each observing session, we can estimate the index of a power-law type wavenumber spectrum of electron density fluctuations using the results of Section 2. In order to derive an isotropic power-law index $\alpha$ for $\theta_r/\theta_s > 1$, we use equations (2) and (3) to get

$$\langle \theta_r^2 \rangle \approx \int_{K_0}^{K_F} K_x^{-\alpha + 3} dK_x \propto K_F^{-\alpha + 4}$$

$$\langle \theta_s^2 \rangle \approx \int_{K_0}^{K_F} K_x^{-\alpha + 3} dK_x \propto K_0^{-\alpha + 4} \quad \text{for} \quad K_0 < K_F.$$  

Here $K_F$ and $K_0$ are the wavenumbers corresponding to the scale sizes in the medium responsible for diffractive intensity scintillation and systematic refraction-generated phase gradients, respectively. The above equations give

$$\frac{\langle \theta_r^2 \rangle}{\langle \theta_s^2 \rangle} = \left( \frac{K_F}{K_0} \right)^{-\alpha - 4}, \tag{8}$$

which can be used to derive $\alpha$ if $K_F$ and $K_0$ are known.

We identify $K_F$ with the small-scale irregularities which determine $t_c$ and $f_c$ and adopt a typical estimate $K_F \sim 2\pi/10^9$ m$^{-1}$. This is of the same order as the wavenumber corresponding to the Fresnel scale which is $2\pi (\lambda z / 2\pi)^{-1/2}$. Observational evidence in support of this value has been summarized by Armstrong, Cordes & Rickett (1981). Adopting an appropriate estimate for $K_0$ involves the greatest uncertainty in our analysis, because observations over many days are required to sample the slow time-scale $t_c$ adequately and such observations do not yet exist. The work of Roberts & Ables (1982) on PSR0628–28 shows that dynamic spectra exhibiting similar quasi-periodicity may endure for several months, whereas our data in Figs 2–5 indicate that the

### Table 2: Derived scattering parameters.

<table>
<thead>
<tr>
<th>PSR</th>
<th>Frequency (MHz)</th>
<th>Date</th>
<th>$z$ (pc)</th>
<th>$V$ (km s$^{-1}$)</th>
<th>$\theta_r/\theta_s$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1133+16</td>
<td>408</td>
<td>1983 Feb. 18</td>
<td>150</td>
<td>264</td>
<td>4.3 \pm 0.4</td>
<td>4.9 \pm 0.1</td>
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<tr>
<td></td>
<td>408</td>
<td>1983 Jun. 3</td>
<td>–</td>
<td>–</td>
<td>$\chi$ 0.5</td>
<td>$\chi$ 3.6</td>
</tr>
<tr>
<td></td>
<td>408</td>
<td>1983 Jun. 4</td>
<td>–</td>
<td>–</td>
<td>3.2 \pm 0.2</td>
<td>4.75 \pm 0.05</td>
</tr>
<tr>
<td>1642–03</td>
<td>408</td>
<td>1982 Nov. 3</td>
<td>150</td>
<td>36</td>
<td>3.5 \pm 0.2</td>
<td>5.35 \pm 0.05</td>
</tr>
<tr>
<td></td>
<td>408</td>
<td>1983 Jun. 5</td>
<td>–</td>
<td>–</td>
<td>$\chi$ 0.5</td>
<td>$\chi$ 3.3</td>
</tr>
<tr>
<td></td>
<td>1426</td>
<td>1983 Apr. 24</td>
<td>–</td>
<td>–</td>
<td>$\chi$ 0.5</td>
<td>$\chi$ 3.3</td>
</tr>
<tr>
<td></td>
<td>1426</td>
<td>1983 May 14</td>
<td>–</td>
<td>–</td>
<td>2.8 \pm 0.2</td>
<td>5.1 \pm 0.1</td>
</tr>
<tr>
<td></td>
<td>1426</td>
<td>1983 May 16</td>
<td>–</td>
<td>–</td>
<td>&gt; 2.0</td>
<td>&gt; 4.7</td>
</tr>
</tbody>
</table>
time-scale $t_c$ may be as short as one or two days. For example, the fringes seen in the dynamic spectra for PRS 1133+16 on 1983 June 4 (Fig. 3) are barely detectable on June 3. Again, the relatively fine fringe pattern seen on 1983 May 14 for PSR 1642−03 (Fig. 3) was replaced by a more dominant and coarser pattern on May 16. Thus we assume $t_c \sim 24$ hr for PSR 1133+16 and $\sim 48$ hr for PSR 1642−03. Values of $K_0$ were then obtained from the relation $K_0 = 2\pi (t_c V)^{-1}$ where $V$ is the pulsar velocity. The spectral indices derived using equation (8), or less accurate estimates using (4), are listed in Table 2.

It should be noted that the dynamic spectrum for PSR 1642−03 on 1982 November 3 (Fig. 4) shows particularly clearly the lack of coherence of the fringe phase on scales exceeding $t_c \sim 8$ min, $f_c \sim 240$ kHz, although the characteristic appearance of the pattern remains unchanged during the observations.

4 Discussion

Our new measurements of the dynamic ISS spectra of PSR 1133+16 and 1642−03 clearly show the presence of periodic time–frequency intensity modulation which exhibits significant variations on time-scales ranging from tens of hours to several months. Although these effects have been known for a long time, they have not been well understood in the past. The striking similarity of some ISS patterns, predicted by the theory outlined in Section 2, to our observations has enabled us to use our theoretical results to determine the power-law index $\alpha$ of a wavenumber spectrum of electron density fluctuations in the ISM extending over a range of scale sizes from $\sim 10^{11}$ to $\sim 10^{13}$ cm. Our results for $\alpha$ given in Table 2 are similar to the conclusions of Roberts & Ables (1982) but are more tightly constrained. The fact that refractive effects due to large-scale irregularities can vary significantly on time-scales as short as one day suggests that occasions may occur when they are absent. In such cases the Kolmogorov spectrum derived by Armstrong & Rickett (1981) and Wolszczan (1983) for the small-scale irregularities may be correct. On the other hand, the discussion of Section 2 indicates that great care is needed before it may be assumed that the correlation bandwidth is not affected by large-scale irregularities.

It is not yet clear whether the large- and small-scale structures are physically related or whether they constitute separate regimes in the interstellar medium. In the latter case the wavenumber spectrum is expected to be complex and attempts to model it by a simple power law may be misplaced. It should be mentioned that our findings appear to be consistent with the theory of slow fading presented by Rickett, Coles & Bourgeois (1984). When variations on time-scales $t_c$ and $t_p$ are averaged there should remain slow intensity fluctuations due to refractive focusing on a scale $t_c$. To produce significant variations the refractive effects must not be smeared out by random scattering, which demands $\theta_r = \theta_0$ in agreement with our conclusions.

Although our analysis has been restricted to two pulsars only, many published and unpublished data exist which provide clear evidence that quasi-periodic modulation of the ISS patterns is a common phenomenon. More systematic ISS studies must be carried out in order to improve the statistics of the parameters derived from such observations. In particular, much better knowledge of the time-scales of the changes in the ISS patterns is needed to improve our spectral index estimates. Similarly, long-term monitoring of periodicities in the ISS patterns and slow fading of pulsar intensity would be desirable in order to verify a theoretically expected relation between these phenomena.

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