Pair creation in radio pulsars: the return-current mode changes and normal-null transitions

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Accepted 1986 May 13. Received 1986 May 9; in original form 1986 March 6

Summary. Density functional calculations of the ground-state energies of atoms and atomic lattices in very high magnetic fields have established the correct surface electric field boundary condition in radio pulsars. Electrons or positrons can be accelerated to energies of the order of $10^3$ GeV in the region of high parallel electric field formed by the sub-set of magnetic flux lines which intersect the light cylinder and curve towards the rotation axis. The unidirectional lateral displacement of curvature radiation pairs from the emission flux line is a corollary of the curvature. It is shown that the large displacement of lower-energy photon conversion points solves the return-current problem. Zero net charge crosses the light cylinder. The return-current kinetic energy is consistent with the measured X-ray luminosities.

The region of high parallel electric field, for positive corotational charge density, may be separated from the neutron star surface by a force-free region. Solutions are obtained for the position of the high-field region. For periods below a critical period, two or more positions exist, pair creation in one of them having properties appropriate to the null state of radio emission. Transitions between the positions provide a simple explanation for mode-changing and nulling. Above the critical period, only the null solution exists.

1 Introduction

Atomic cohesive energies for condensed matter in magnetic flux densities of $10^{12}–10^{13}$ G have been derived by density functional calculations of the ground-state energies of atoms and of rhombohedral and body-centred tetragonal atomic lattices (Jones 1985a, b, 1986). Because a neutron star surface must be a disordered state (hadronic photoabsorption reactions produce a distribution of atomic number Z), the density functional cohesive energies, which are almost independent of Z, can be assumed with confidence to be upper limits for the true cohesive energy. Both lattices have a number density of approximately $1.5\times10^{25}Z_{26}^{-1}B_1^{3/2}$ atoms cm$^{-3}$, where $B_{12}$ is the magnetic flux density in units of $10^{12}$ G and $Z_{26}=Z/26$. The cohesive energies are so small that for surface temperatures $\approx5\times10^5$ K, there is no large discontinuity in density between the lattice and the base of the gravitationally bound atmosphere of ions and electrons with which
it is in equilibrium. The calculations also confirm that the electronic work function is small. Thus there can be no reasonable doubt that for normal condensed matter surfaces, in radio pulsars with either sign of cotrotilational charge density, the free movement of ions or electrons from the atmosphere to the magnetosphere imposes the boundary condition

$$\mathbf{E} \cdot \mathbf{B} = 0$$

on the electric field at the neutron star surface $$r = R$$. The boundary condition must fail, for positive coortational charge density, if quark matter, stable at zero pressure, exists at the neutron star surface (Witten 1984; Farhi & Jaffe 1984; Baym et al. 1985). But it appears improbable that the hypothesis can be generally correct. A quark matter phase with a Coulomb barrier small enough to allow expansion to the surface would immediately dissociate normal accreted matter (kinetic energy 10–100 MeV per nucleon) into constituent quarks. The accumulation of hydrogen and helium assumed in the thermonuclear flash model of Type I X-ray burst sources (Lewin & Joss 1981) could not occur. A single strong-interaction phase throughout the volume would also be inconsistent with the period discontinuities observed in many pulsars and explained by sudden changes in the moment of inertia. [The superfluid vortex lines considered by Alpar et al. (1984a) are pinned by nuclei in the crust.]

A model of the normal and null states and of the transitions between them has been developed in a series of papers (Jones 1981, 1982, 1983). The model is incorrect inasmuch as it assumes the wrong boundary condition ($$\mathbf{E} \cdot \mathbf{B} \neq 0$$). The present paper, with the correct boundary condition, shows how pair creation solves the return-current problem and establishes that two or more states of pair creation exist at radii small compared with the light cylinder radius $$R_{LC}$$, one of them having properties appropriate to the null state of radio emission.

The electric potential in the corotating frame of reference satisfies

$$\nabla^2 \phi = -4\pi (\sigma - \sigma_0)$$

where $$\sigma$$ is the magnetospheric charge density. The corotational charge density in the inner magnetosphere ($$\Omega r < c$$) is

$$\sigma_0 = -\frac{1}{2\pi c} \Omega \cdot \mathbf{B}$$

for angular velocity $$\Omega$$. Arons & Scharlemann (1979) were the first to show that charged particles moving along the sub-set of magnetic flux lines which intersect the light cylinder and curve towards the rotation axis can reach energies of more than $$10^5$$ GeV in the inner magnetosphere. Potential differences of this magnitude exist because the ratio $$\sigma_0 \sigma^{-1}$$ increases along these flux lines for particles in relativistic motion with no pair creation. The lines are bounded by a surface $$(\text{assumed to be an equipotential, } \phi(S) = 0)$$ whose form in the inner magnetosphere is a narrow tube. The cross-sectional area and curvature of the tube are independent functions of $$r$$, both dependent on the higher multipole components believed to be present in the magnetic field. The parallel component of the electric field on the tube axis can be $$10^6$$–$$10^7$$ V cm$$^{-1}$$ in the high-field region, but is less than $$10^3$$ V cm$$^{-1}$$ in the force-free region ($$r > r_0$$) produced by the magnetic conversion of curvature radiation to electron–positron pairs and the reversal of a small fraction of the electrons. Jones (1980) showed that the position of the high-field region described by Arons & Scharlemann (for $$\sigma_0 < 0$$) could be unstable owing to the formation and growth of a force-free region ($$r < r_0$$) immediately above the neutron star surface. The disagreement between Jones (1980) and Arons (1981a) concerning the instability is more apparent than real. Its origin lies in the different assumptions made by these authors about the degree of flux-line curvature.
Although this paper is concerned in detail with the inner magnetosphere, it is necessary to form a model of the whole magnetosphere in order to specify the return current outside $S$, which maintains the charge balance of the system. It is shown that recent X-ray luminosity measurements (Helfand 1982) are of crucial importance and severely limit the form of the return current. The magnetosphere model constructed in Section 2 is consistent with the X-ray luminosity measurements, and differs in some important respects from the models of Arons (1983, fig. 9) and Mestel et al. (1985).

The properties of the curved flux-tube model are summarized in Section 3, and the factors relevant to its stability considered in Section 4. There are two or more positions for the high-field region inside $S$, each of which may be subject to fluctuation. As a pulsar evolves, with increase in period $P$ and possibly with changes in the multipole composition of the field, an epoch is reached in which transitions occur between the positions, thus explaining the nulling and mode changing observed in the radio emission. Possible transition mechanisms are considered in Section 5.

Our considerations are for neutron stars with $\sigma_0 > 0$ above the polar caps. There are some essential differences from the case $\sigma_0 < 0$. Relevant properties of the gravitationally bound atmosphere (for $\sigma_0 > 0$) are summarized in Appendix A under the assumption of local thermodynamic equilibrium. Calculated rates for bound–free transitions in relativistic ions interacting with the surface blackbody radiation field are contained in Appendix B.

2 The magnetosphere

Recent X-ray luminosity measurements for PSR 1929+10, a moderately fast rotator only 65 pc from the Earth, provide a stringent limit on the rate of dissipation of return-current kinetic energy at the neutron star surface. The unresolved source associated with this pulsar has a luminosity of $6 \times 10^{28}$ erg s$^{-1}$, with an upper limit of $2 \times 10^{28}$ erg s$^{-1}$ for a modulated component (Helfand 1982). The period is $P=0.226$ s and the inferred surface dipole field (Manchester & Taylor 1981) $5.2 \times 10^{11}$ G. The return current is of the order of $2 \pi^2 R^3 B (ecP^2)^{-1} = 1.4 \times 10^{31}$ unit charges s$^{-1}$. The small flux of reversed electrons associated with the formation of the force-free region ($r > r_d$) inside $S$ also heats the neutron star surface. Therefore, the unmodulated X-ray luminosity limits the return current kinetic energy to $\leq 3$ GeV per unit charge, provided the current heats a surface of the order of the polar cap area $2\pi^2 R^3 (cP)^{-1} = 2.9 \times 10^9$ cm$^2$. (The measured photon spectrum is not consistent with a uniform surface blackbody temperature, which would be $2 \times 10^5$ K.) Although our estimate of the return current is uncertain, the kinetic energy limit is severe and leads to the conclusion that electrons or positrons of $\geq 10^3$ GeV leaving the light cylinder do not subsequently re-enter the light cylinder and return to the star on flux lines outside $S$, as in the model of Mestel et al., unless some special dissipative process exists. Energies of this magnitude cannot be reduced to values below the limit either by interaction with collective modes of the magnetosphere plasma or by curvature radiation.

The multipole composition of the field is unknown. The form of field assumed in the inner magnetosphere near the neutron star surface (Jones 1980) has simple geometrical properties which represent the important features of any real set of magnetic flux lines curving towards the rotation axis and intersecting the light cylinder. The lines inside the surfaces $S$ and $S\prime$ (Fig. 1) are those of a dipole field whose symmetry axis $OP$ is external to $S$, and at an angle $\psi$ with $\Omega$. The axis $OP$ is fictitious, its sole purpose being the definition of the geometrical form of the lines inside $S\prime$. The radius of curvature of the central flux line is

$$\rho(x) = \frac{4}{3} R \zeta^{-1} x, \quad (4)$$

where $x = r^{1/2} R^{-1/2}$, and $\zeta$ is its polar angle at the surface $r = R$, defined with respect to $OP$. 

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The line is coplanar with $OP$ and $\Omega$. The cross-section of $S$ is assumed circular, of radius $r = r_M \eta \eta_M r_M^{3/2} R_L^{1/2}$. The corotating magnetosphere outside the surface $S$, is not a dipole field and has no symmetry axis $OP$. Curvature radiation photons from positrons in the high-field region inside $S$ convert to electron–positron pairs. Let $u(x, x')$ be the transverse component of magnetic flux density, in units of $10^{12} G$, at $x$ on the path of a photon emitted at $x'$ tangentially to a magnetic flux line. For the model assumed, $u$ has a maximum $u_m(x')$ at $x = (4/3)^{1/2} x'$. The lateral displacement of a photon at $x$ from its emission flux line is

$$d(x, x') = (x^2 - x'^2)^2 (8x^2)^{-1} 3 R \sin \xi x'.$$

(5)

(The neglect of rotation is satisfactory except for very small $P$.) The displacement of the pairs which initiate the force-free region at $x_a$ is small and can be neglected (the pairs are from the conversion of photons in the far exponential tail of the curvature radiation spectrum). But large numbers of lower-energy photons convert near the transverse field maximum $u_m$. The associated pair displacement

$$d_m(x') = \frac{1}{24} R \xi x'^3$$

(6)

for emission at $x'$, is of the order of the radius of $S$ (see the parameter list of Section 4). The non-negligible pair displacement, and the X-ray luminosity constraint on the return-current kinetic energy, are the two factors which lead to the form of the magnetosphere shown in Figs 1 and 2.

The force-free region inside $S$ above the asymmetric surface (Fig. 1) whose lowest point is at $x = x_a$ forms because pair creation cancels the variation of $\sigma_0 \sigma^{-1}$ along a flux line by one of two

![Diagram](https://example.com/diagram.png)

Figure 1. The flux lines $A$, $B$ and $C$, $D$ are longitudinal sections of the tubular surfaces $S$ and $S_e$. There is a high parallel electric field component inside $S$ in the inner magnetosphere between the surfaces shown at $x < x_0$ and $x > x_a$ bounding the regions in which curvature radiation pair-creation rates satisfy the force-free condition (21). Almost all the pairs formed above $x_a$ move out to the light cylinder (Fig. 2) without electron reversal. The positron reversal shown in Fig. 2 produces the return positron current on the force-free flux lines between $B$ and $C$. © Royal Astronomical Society • Provided by the NASA Astrophysics Data System
Figure 2. The flux lines $A-D$ are shown in the outer magnetosphere where they approach those of a true dipole, axis $OQ$. The line $OP$ is the fictitious dipole axis used to define the model inner magnetosphere flux lines and is at an angle $\psi$ with the rotation axis $\Omega$. The force-free condition is maintained by positron reversal between $S$ and $S_v (B$ and $C$) in the outer magnetosphere. A vacuum gap may exist at the upper $\Omega \cdot B = 0$ surface owing to the small rate of pair creation between $A$ and $D$.

processes:

(i) The reversal of a small fraction of the electrons.

(ii) Deceleration (without reversal) of the positrons to an average velocity lower than the electron velocity.

Both processes involve the formation of a small parallel electric field, but with opposite signs. Daugherty & Harding (1983) have shown that in the frame of reference in which the photon momentum is perpendicular to $B$, the photon energy is divided approximately equally between the positron and electron, whose longitudinal momentum components are usually small. But a very large momentum dispersion is introduced in the corotating frame by synchrotron transitions and by the wide distribution of photon energy. As a consequence, the force-free region cannot be formed by (ii) except for very high electron-positron number densities or in the cancellation of small gradients in $\sigma_0 \sigma^{-1}$. Our analysis here is in agreement with Arons (1981b). The force-free region below $x_0$ inside $S$ is formed by the curvature radiation of electrons reversed near $x_a$. In this case, process (i) involves positron reversal.

The flux lines labelled $A-D$ are common to Figs 1 and 2, which show longitudinal sections of the surfaces $S$ and $S_v$. All lines inside $S_v$ curve towards the rotation axis at $x \simeq 2$. With increasing $x$, higher multipoles become less important, and the flux lines approach those of a true dipole field.
Thus curvature towards the rotation axis does not persist to the light cylinder for all the lines inside $S$. Pairs with lateral displacement near the maximum $d_m$ are created between $S$ and $S_r$ ($B$ and $C$) and move towards the surface $\Omega \cdot \mathbf{B} = 0$. The force-free state is maintained in the vicinity of this surface by process (i) (positron reversal for the curvature concerned). The reverse flux of positrons cancels the variation of $\sigma_0 \sigma^{-1}$ on flux lines between $S$ and $S_r$ at $x \approx 2$. Thus a force-free state exists at all points on flux lines between $B$ and $C$. The surface $S$ encloses lines curving towards the rotation axis at all points in the outer magnetosphere and lines whose curvature away from the rotation axis in the outer magnetosphere is too small to require the formation of a reverse positron flux. A high-field region $x_0 < x < x_a$ can exist inside $S$. All flux lines between $S$ and $S_r$ are force-free except that on those with negligible pair creation (as between $A$ and $D$), a vacuum gap forms at the surface $\Omega \cdot \mathbf{B} = 0$. In a quantitative model, it is possible that the surfaces $S$ and $S_r$ would adjust so that the current between $S$ and $S_r$ balances the current flowing inside $S$. The positrons which flow to the neutron star surface between $S$ and $S_r$ have energies $(\approx 100\text{ MeV})$ characteristic of force-free regions and do not violate the X-ray luminosity constraint. They do not influence processes occurring inside $S$.

The magnetosphere is unchanged for $\sigma_0 < 0$ except for a reversal in sign of $\mathbf{B}$ and of all charges.

As a corollary of curvature towards the rotation axis, lower-energy curvature radiation photons convert with large displacements from the emission flux line and do not allow the formation of an Arons (1983, fig. 9) slot gap. The magnetosphere also differs from the model developed by Mestel et al. (1985) in which particle motion is non-relativistic except near the light cylinder. The crucial point is that all the flux lines in an aligned rotator curve away from the rotation axis. The aligned and oblique rotator problems are quite different. It is possible that non-relativistic motion is unstable on lines inside $S$ curving towards the rotation axis. Any fluctuation in the small force-free parallel electric field component inside $S$ tending to increase particle velocity could reduce $\sigma$ and therefore grow, so initiating a transition to relativistic motion.

3 The curved flux-tube model

The formation of potential differences of more than $10^{12}$ V on magnetic flux lines curving towards the rotation axis was first described by Arons & Scharlemann (1979). However, we shall summarize, with some small changes of notation, the basic formulae derived by Jones (1980) for the model field of Section 2.

With reference to Fig. 1, the corotational charge density on the central flux line is

$$\sigma_0(x) = \frac{B(x)}{cP} \cos \left( \psi - \frac{3}{2} x \zeta \right). \quad (7)$$

It is assumed that on other flux lines inside $S$, all current densities can be approximated by their values on the central flux line. Let $F(x)$ be the number of pairs created per unit interval of $x$, per unit positive charge crossing the surface $x = x_0$, whose electrons are reversed by the parallel electric field component. We assume a force-free region $x < x_0$. For completely relativistic motion at $x > x_0$, the deviation from $\sigma_0$ is

$$\sigma - \sigma_0 = \frac{B(x)}{cP} \left[ 2g \cos \chi - \frac{3}{2} (x - x_0) \zeta \sin \chi \right] \quad (8)$$

to first order in $(x - x_0) \zeta$, where

$$\chi(x_0) = \psi - \frac{3}{2} x_0 \zeta$$

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and
\[ g(x) = \int_{x_0}^{x} dx' F(x'). \] (9)

The potential satisfies
\[ \phi(r) = \int d^3r' (\sigma - \sigma_0) G(r, r') \frac{1}{4\pi} \int \phi(r') \nabla G \cdot dS. \] (10)

For a typical radio pulsar, \( S \) is an extremely narrow tube whose Green function \( G(r, r') \) tends to zero exponentially as \( |r - r'| \) exceeds the local tube radius. Therefore, the surface integral can be neglected and \( G \) replaced by the Green function for a uniform circular tube. With these approximations, the potential in regions of negligible pair creation (\( g = 0 \)) is
\[ \phi(x, \eta) = \frac{3}{2} (1 - \eta^2)(x - x_0) \zeta \phi_V \tan \chi, \] (11)

with
\[ -\phi_V = \frac{l}{c} = 6.6 \times 10^{12} \eta^2 M \beta_2 B_{12}(1) R_3^3 P^{-2} \cos \chi \ V, \] (12)

where \( l \) is the current flowing inside \( S \) and \( 0 < \eta < 1 \) is the dimensionless radial cylindrical polar coordinate of the uniform circular tube.

It is assumed that a curvature radiation photon of energy \( k \) (in units of the electron rest energy) emitted at \( x' \), converts at \( x \) defined by the field-dependent conditions
\[ ku(x, x') = 5.7, \quad B_{12}(x) < 2.85 \]
\[ k_\perp(x, x') = 2, \quad B_{12}(x) > 2.85. \] (13)

Comparison with the photon conversion rates calculated by Daugherty & Harding (1983) shows that the above criteria produce adequate approximations for the conversion length except for \( B_{12}(x) = 2.85 \), where the length is too small.

The number of pairs created, per unit interval of \( r \), from the curvature radiation of a positron moving along a flux line with potential difference \( \phi(x, 0) \) is, for \( B_{12}(x) < 2.85 \),
\[ N_a(r, r_0) = C_1 C_2^{1/2} \int_{x_0}^{x} dx' x'^2 (4x'^2 - 3x^2)(x'^2 - x^2)^{-3/2}(x' - x_0)^{-1/2} \exp\left[-k(x, x') k_c^{-1}(x')\right]. \] (14)

The lower limit of integration is
\[ x_c = \max\left(1, \frac{\sqrt{3}}{2} x \right), \]

and \( k_c \) is the critical energy for curvature radiation photons;
\[ k(x, x') k_c^{-1}(x') = C_2 x^8 (x'^2 - x^2)^{-1}(x' - x_0)^{-3}. \] (15)

The constants are
\[ C_1 = 7.2 \times 10^{-2} \eta M_{\odot} \beta^2 B_{12}(1) R_3^3 P^{-2} \sin \chi \ \text{cm}^{-1}, \]
\[ C_2 = 2.45 \times 10^{-5} \eta M_{\odot}^5 \beta^{-5} B_{12}^{-4}(1) R_5^{-8} P^6 (\sin \chi)^{-3}. \] (16)

The force-free region \( x > x_a \) is formed near \( x_a \) by electron reversal. More detailed calculation
of the surface at $x_a$ would be necessary to find the number of electrons reversed at $x<x_a$ (Arons 1983). However, numerical evaluation of $N_a$ as a function of $x$ near $x_a$ shows that electron reversal must occur over a distance of several tube radii above $x_a$ before the force-free state can be maintained by process (ii). Therefore, we introduce a phenomenological length, $h$, over which electron reversal occurs and derive a limit for it from the X-ray luminosity measurement referred to in Section 2.

Curvature radiation from the reversed electrons produces pairs near $x_0$: the number per unit interval of $r$, per reversed electron is, for $B_{12}(x)<2.85$,

$$N_0(r, r_a) = C_1 C_2^{1/2} \int_x^{x_a} dx' x^2 (4x'^2 - 3x^2)(x'^2 - x^2)^{-3/2}(x_a - x)^{-1/2} \exp \left[-k(x, x') k_c^{-1}(x')\right],$$

(17)

$$k(x, x') k_c^{-1}(x') = C_2 x^8 (x'^2 - x^2)^{-1} (x_a - x)^{-3},$$

or for $B_{12}(x)>2.85$,

$$N_0(r, r_a) = C_1 C_2^{1/2} \int_x^{x_a} dx' x^{-1} x'^2 (x'^2 - x^2)^{-3/2}(x_a - x)^{-1/2} \exp \left[-k(x, x') k_c^{-1}(x')\right],$$

(19)

$$k(x, x') k_c^{-1}(x') = C_3 x^2 (x'^2 - x^2)^{-1} (x_a - x)^{-3},$$

(20)

with $C_3 = 0.351 B_{12}(1) C_2$.

The condition for $\mathbf{E} \cdot \mathbf{B} = 0$, derived from equations (8) and (10), is that $(\sigma-\sigma_0) B^{-1}(x)$ should be independent of $x$. This is satisfied if the number of pairs created per unit interval of $x$, per unit positive charge crossing $x=x_0$, exceeds

$$F(x) = \frac{3}{4} \xi \tan \chi,$$

(21)

The negative of (15) has a maximum

$$-z_m = -k(x, x_m) k_c^{-1}(x_m)$$

(22)

for

$$x < \frac{4}{\sqrt{3}} x_0,$$

where $x_m$ satisfies

$$5x_m = x_0 + (x_0^2 + 15x_0^2)^{1/2}.$$

(23)

For wide ranges of the parameters contained in $C_1$ and $C_2$, the condition $z_m(x_a) = 10$ provides a useful approximation for $x_a$. The relation between $x_a$ and $P$ derived by application of this condition to equations (15), (16) and (23) is

$$P = 8.6 \eta M_5^{5/6} B_{12}^{2/3}(1) R_6^{1/3} f^{-1/6}(x_a, x_m)(\sin \chi)^{1/2} s,$$

(24)

where

$$f = x_a^8 (x_a^2 - x_m^2)^{-1} (x_m - x_0)^{-3}.$$

(25)

The maximum potential difference $-\phi_a$ obtained from equations (11) and (24) is a slowly varying function of the parameters in $C_2$ and is approximately $10^{12}$ V (see also Arons 1983).

The energy of the reversed electrons incident on the neutron star surface, per unit positive charge crossing $x=x_0$, is $-e\phi_a hF(r_a)$. This cannot exceed $L_x e l^{-1}$, where $L_x$ is the measured
X-ray luminosity. The inequality obtained from this condition and from equations (11), (12) and (21), is

$$h < 4e^2 c^{-1} \left( -e\phi_a \right)^{-2} (x_a - x_0) R L_x.$$  \hspace{1cm} (26)

The unresolved source associated with PSR 1929+10 has $L_x = 6 \times 10^{28}$ erg s$^{-1}$ (Helfand 1982). For $\phi_a = 10^{12}$ V, the inequality is

$$h < 0.8 (x_a - x_0) R,$$  \hspace{1cm} (27)

a constraint which is also valid for the case $\sigma_0 < 0$. The measured $L_x$ also constrains (for $\sigma_0 > 0$) the proton component of the current density,

$$\alpha = \alpha^e + \alpha^p + \alpha^z,$$  \hspace{1cm} (28)

in units of $\sigma_0(x_0) c$. The component $\alpha^e$ is the ion current density, including bound electrons. The protons are emitted in the decay of giant dipole states produced at the neutron star surface by the photon component of the electromagnetic showers. Approximately one proton is emitted per 20 GeV shower energy (Jones 1981) so that for 1929+10, with $Ie^{-1} = 1.4 \times 10^{31}$ s$^{-1}$, the limit is $\alpha^p \lesssim 0.15$.

4 Stability of the high-field region

The positron current density is produced by photon conversion at $x - 1 \leq \max(\theta_M, x_0 - 1)$. There are two components,

$$\alpha^e = \alpha^e_1 + \alpha^e_2,$$ \hspace{1cm} (29)

the first produced by hadronic photoabsorption reactions (Jones 1981); the second by curvature radiation photons from the electrons reversed near $x_a$,

$$\alpha^e_2 = \kappa \int_{1}^{x_0} dx N_0(x, x_a),$$ \hspace{1cm} (30)

(subject to $\alpha^e < 1$), where $\kappa$ is the reversed electron current density in units of $\sigma_0 c$. In the absence of pair creation, periodic one-dimensional solutions exist for the parallel electric field component at $x - 1 \leq \theta_M$, as shown by Mestel & Pryce (Mestel et al. 1985; Jones 1983). The onset of low-energy ($1 - 10^3$ MeV) pair creation in the periodic electric field first smoothes out the periodicity through the formation of successive concentrations of positrons and electrons, and ultimately removes it completely. The positrons are reversed and form $\alpha^e$; the electrons enter the neutron star surface. The parallel electric field producing, and being partially damped by, the reversal is that which necessarily exists near $x = 1$ as a consequence of the surface electric field boundary condition and the initial non-relativistic motion of the protons and ions.

We assume, for the purposes of calculation, that $x_0, x_a, \alpha^e$ and $\kappa$ in our solution are time-independent. Owing to the unidirectional lateral displacement of the conversion points for curvature radiation photons, time-independence is not possible (for $\sigma_0 > 0$) in the absence of $\alpha^e_1$, which introduces the necessary degree of randomness in the spatial relation between incident electron and created pair. It is known that $\alpha^e_1$ tends to zero for field strengths such that the conversion lengths for nuclear $\gamma$-rays are either much larger than $R \theta_M$ ($B \approx 5 \times 10^{11}$ G) or extremely small ($B \approx 5 \times 10^{12}$ G). Rozental & Usov (1985) have criticized the model of shower development assumed by Jones (1979, 1981) for its unsatisfactory treatment of the Landau–Pomeranchuk effect. But these authors assume that no gravitationally bound atmosphere exists. The density at a depth in the atmosphere equivalent to one radiation length is
$5 \times 10^{23} Z_{\text{p}}^2$ atoms cm$^{-3}$ (Appendix A), two orders of magnitude smaller than the density assumed by Rozental & Uslov. Thus the Landau–Pomeranchuk effect is not important in electromagnetic shower development at $10^3$ GeV. At intermediate $B$, the model relating $\alpha_I$ with the reverse electron energy flux (Jones 1979, 1981) remains valid.

The component $\alpha_I$ is of the order of $10^{-2}$. The potential $\phi(x, \eta)$ obtained from (10) can be time-dependent only as a consequence of time-dependence of a boundary condition, specifically, the position of the force-free surface $x_0$. It is prohibitively difficult to find a time-independence criterion in terms of $\alpha_I^2$ because it must involve finding three-dimensional time-dependent solutions for $\kappa$ and $\alpha^0$. We assume that the complete solution can be represented approximately by fluctuating surfaces $x_0(t)$ and $x_a(t)$ whose time-averages are the $x_0$ and $x_a$ to be obtained from our model. The fluctuation time-scale is unknown but could be as short as $10^{-4}$ s. The solutions must fluctuate for all except intermediate $B$, and may do so for all $B$.

For a given set of parameters, $x_0$ and $x_a$ can be found from equations (14), (17) or (19), (21) and (30), with the conditions,

$$ \frac{F}{2\kappa_a R} = \alpha^0 N_0(r_a, r_0), $$

$$ \frac{F}{2\kappa_0 R} = \kappa N_0(r_0, r_a). $$

The ratio $A = N_0(r_0, r_a) N_a^{-1}(r_a, r_0)$ is calculated for a trial $x_0$: the necessary condition for a time-independent solution is $A = A_c$, where

$$ A_c = x_a x_0^{-1} \left[ \kappa^{-1} \alpha_I^2 \right] + \int_1^{x_0} dx N_0(x, x_a) $$

(subject to $\alpha^2 < 1$). For $A \neq A_c$, $r_0$ moves outward with a velocity of the order of

$$ \frac{1}{2} \left( \frac{r_a - r_0}{(AA_c^{-1} - 1)} \left( -N_0^{-1} \frac{dN_0}{dr} \right) \right)^{-1} $$

in units of $c$. In a real force-free region, $\alpha = \kappa(x_0, x_a)$. We assume $\alpha = 3 \times 10^{-3}$ in equation (32) and with a posteriori justification, $\alpha_I^2 = 0$ in equation (33). The quantity $AA_c^{-1}$ is shown in Fig. 3 for $P = 1.0$ s and a standard set of parameters ($\eta = 0.01$, $\zeta = 0.466$, $B_{12} = 1$, $R_0 = 1$, $\chi(1) = \pi/3$). The arrows show the direction of motion (34) for $A \neq A_c$, from which it can be seen that one solution is stable, the other unstable. Evaluation as a function of $P$ shows that the two solutions terminate at a critical period $P_c = 1.06$ s. The calculated $\alpha^2$ increase rapidly as $P$ approaches $P_c$ (from 0.11 at $P = 0.9$ to 0.29 at $P = 1.0$). For $\alpha = 10^{-3}$, $P_c = 1.18$ s. The critical period may change slowly with time: decay of higher multipoles would reduce both $\zeta$ and $P_c$.

There is also a solution, possibly fluctuating, which does not satisfy the force-free condition (31). If the state of the system at any instant were to the right of the unstable solution in Fig. 3, $x_0$ would move outward until stabilized by the decrease in $\alpha$ which must eventually occur when pair creation inside $S$ (but not between $S$ and $S_c$) falls to negligibly low levels. In a real pulsar, the flux line radius of curvature would be expected to deviate from equation (4) with increasing $x$ and approach the much larger values found in the outer magnetosphere. We have made the realistic assumption that pair creation ceases at $x_a = 2$ and have obtained an approximate solution from equations (24) and (25). It is associated with the null state of radio emission because creation of outward moving pairs inside $S$ is negligible. Further consideration of the possible fluctuating form of the solutions, and of the ways in which transitions might occur between them, is postponed to the following section.
Although Fig. 3 has been derived for a simple model, the important properties of the normal solution \((x_0 > 1, P < P_c)\) appear to be model-independent. Their existence depends only on the general expectation that \(\sigma^e_2\) and \(A_c\) both increase rapidly with \(x_0 - 1\). Any model-dependence of \(N_0\) largely cancels in \(AA_c^{-1}\). (Note that for \(\sigma^e_0 < 0\), the normal solution is \(x_0 = 1\) because \(\sigma^e\) is independent of \(x_0 - 1\).)

For larger fields, the curvature photons produced by reversed electrons have a high probability of conversion to positronium (Shabad & Usov 1982; Herold, Ruder & Wunner 1985). Equation (19) assumes prompt photoionization, for small \(x_0 - 1\), in the blackbody radiation field of the polar cap. A finite photoionization mean free path, with positronium formation near \(x_0\) but not \(x_a\), would reduce \(A\) and thus decrease the stable \(x_0\), possibly to \(x_0 = 1\).

5 Model predictions

Measurements of the interstellar scintillation (Cordes, Weisberg & Boriakoff 1983), frequency-dependence (Kardashev et al. 1982; Davies et al. 1984) and polarization of radio pulse components (Krishnamohan & Downs 1983) are consistent with emission at \(x \approx 10\) in plasma interactions involving a neutral system of low-energy electrons and positrons. In the curved flux-tube model, the pairs are formed from the conversion of curvature radiation photons (emitted at \(x' < x_a\) or \(x' > x_a\)) where the displacement derived from (5) does not remove the conversion point from \(S\). The number of pairs formed per primary positron decreases extremely rapidly with increasing \(x_a\) (Jones 1980) and becomes negligible in the limit associated with the null state. The normal-state high-field region starts not at the neutron star surface but at \(x_0 > 1\). Thus changes in its position are possible, in principle, and should produce observable changes in pulse
properties if there is a one-to-one relation between electron–positron number density and emission frequency. The most obvious change in position is the normal to null transition shown in Fig. 3. Suppose that both states are fluctuation-free. The pulsar evolves (with increasing $P$ and possibly decreasing $\xi$ and $P_0$) until the critical period is reached. The transition is irreversible and extinction occurs with no prior nulling. Thus the fluctuation-free assumption, which we know is not always valid, is too restrictive to allow an explanation of the observed nulling. We assume that $x_0$ and $x_a$ in Fig. 3 are time-averages of fluctuating surfaces $x_0(t)$ and $x_a(t)$, and make the further obvious assumption that a transition occurs when $x_0(t)$ crosses the unstable solution curve. Unfortunately, it is not possible to make predictions of the normal and null state durations without more knowledge of their fluctuations. We do not anticipate fluctuations in the case $\alpha_0 < 0$, owing to the free flow of electrons from the gravitationally bound atmosphere. Thus nulling prior to extinction is a property of pulsars with $\alpha_0 > 0$.

An observed mode change appears consistent with a small but finite discontinuous change in the mean position or length of the high-field region. Both $\delta x_0 > 0$ and $\delta x_a < 0$ independently increase the pair density and, with the assumed mapping, increase the emission radius and angular width for a fixed frequency ($\delta x_0 > 0$ produces $\delta \alpha^e > 0$). This simple change of physical state may underlie the ordered mode changing and nulling observed in PSR 0031−07 and in PSR 2319 + 06 (Wright & Fowler 1981a, b). Transitions between three different time-averages $x_0, x_a$ would be required, apart from the null solution. It is possible but does not appear likely that the diagram, replacing Fig. 3, calculated for the real $\rho(x)$ and $B(x)$, could have the necessary structure (a total of three critical points). A more probable explanation is that the general form of Fig. 3 is valid, but with the transitions a consequence of discontinuous changes in $P_c$, possibly caused by the interaction of blackbody photons with the component $\alpha^e$. Bound–free transitions produce an additional term in equation (8) given by the replacement $g \rightarrow g + g_{bf}$. The potential at $x < x_a$ changes by

$$\bar{\phi}(x, \eta) = -2(1 - \eta^2)g_{bf}(x)\phi_{v}. \tag{35}$$

If the functions $\alpha^e(x)$ and $g_{bf}(\alpha^e)$ were known, the modified form of Fig. 3 could be found, in principle, by successive approximation. A discontinuous change could occur as follows. The current density $\alpha$ contains particles removed from the top of the LTE atmosphere and therefore having the maximum possible charge to mass ratio (Appendix A). The rates of production of protons and positrons are fixed by $\alpha$ and by equation (30), respectively. Ions are removed from the atmosphere at the rate necessary to satisfy the surface electric field boundary condition. If $\alpha^e$ approaches unity, as in the null state, a proton component develops at the position of LTE breakdown (Appendix A) if the production rate is larger than is required to satisfy $\alpha^e + \alpha^b = 1$. Depletion of the proton component during a subsequent interval of normal radio emission would cause rapid growth of $\alpha^e$ and $\bar{\phi}$ so producing a change in observed mode. The blackbody temperatures required for significant bound-free transition rates (Appendix B) are below the known limits for a number of near pulsars ($3–5 \times 10^5$ K; Helfand 1982) but are a little larger than those expected from dissipation by vortex creep in an old pulsar (Alpar et al. 1984a).

Evolution of a pulsar to the critical period must cause extinction at finite luminosity (for either sign of $\alpha_0$) if radio emission requires a low-energy neutral system of electrons and positrons. This is consistent with the survey of Dewey et al. (1985) who find no pulsars with intrinsic luminosity less than $0.3 \text{mJy kpc}^{-2}$. High blackbody surface temperatures in a very young neutron star can produce a large potential change $\phi$ cancelling $\phi$ almost completely. For some $\rho(x)$, the cancellation may prevent pair creation in the inner magnetosphere. Thus radio emission commences (for $\alpha_0 > 0$) only when the surface has cooled to temperatures less than about $3 \times 10^8$ K (Appendix B). The limit approximately equals the Vela pulsar surface temperature derived from

Our model has some unsatisfactory features. Eighteen years of intensive observation unmatched by theoretical development have made the prediction of new phenomena very difficult. Dependence on the multipole structure of the magnetic field is unavoidable for the surface electric field boundary condition (1). The fluctuating solutions must exist for some $B$ (for $\sigma_0 > 0$) but the detailed properties of the fluctuations are unknown. (The radio emission provides no evidence for steady-state pair creation.) However, we believe that the main idea involving changes in position of $x_0$ and $x_5$ is correct, although it may be difficult to establish the causative mechanism with confidence.

References


Appendix A: The gravitationally bound atmosphere

The electrons in the atmosphere are non-degenerate and occupy the lowest Landau state. At
height \( z \), where the number density is \( N(e, z) \), the chemical potential is

\[
\mu(e, z) = \beta^{-1} \ln N \Lambda \Lambda_B^2.
\]  

(A1)

The parameters are: \( \beta^{-1} = k_B T; \Lambda = \hbar m^{-1/2}(2\pi\beta)^{1/2}; \Lambda_B = (2\pi\hbar c/eB)^{1/2} \). Equation (A1) is valid for \( \Lambda_B \ll \Lambda \). The standard \( B=0 \) expressions are adequate for the ion chemical potentials \( \mu(Z-s, z) \). The ions have mass \( M \) and charge \( Z-s \). A small electrostatic potential \( V(z) \) is required to balance the local gravitational acceleration \( g \). The equilibrium conditions are,

\[
\mu(e, z) + mgz - eV(z) = \mu(e, 0), \quad \mu(Z-s, z) + Mgz + (Z-s) eV(z) = \mu(Z-s, 0),
\]

(A2)

\[
\mu(Z-s, z) - \mu(Z-s+1, z) = \mu(e, z).
\]

(A3)

With neglect of ion excited states, equation (A3) can be rewritten in terms of ion and electron number densities,

\[
N(Z-s, z) = \Lambda \Lambda_B^2 N(e, z) N(Z-s+1, z) \exp(\beta \epsilon_e).
\]

(A4)

where \( \epsilon_e \) is the ionization energy of the \( s \)th electron. Equations (A1), (A2) and (A4) have been solved numerically for \( Z=14 \) and \( B=2 \times 10^{12} \text{ G} \). The electron density scale height is almost independent of \( z \). It is

\[
\frac{z}{\beta eV(z)} \approx 10 \frac{1}{\beta Mg}.
\]

(A5)

One radiation length (the unit governing electromagnetic shower development) contains \( 6.6 \times 10^{23} \text{ [ln } 43 \text{]}^{-1} Z_{26}^2 \text{ atoms cm}^{-2} \) (Jones 1979). For a typical scale height of 0.35 cm, the ion number density is \( 5 \times 10^{23} Z_{26}^2 \text{ cm}^{-3} \) at a depth equivalent to one radiation length. Local thermodynamic equilibrium breaks down at the top of the atmosphere, with the transition to inertial acceleration.

Equation (A2) shows that the distribution with \( z \) of an ion species is determined by its charge to mass ratio. The ionization energy of the most numerous species increases slowly with \( z \) to \( \beta \epsilon_e=20 \) at LTE breakdown. If a small number of protons were introduced, their LTE distribution would be

\[
N(p, z) = N(p, 0) \exp[-\beta M_p g z - \beta eV(z)].
\]

(A6)

For \( V(z) \) satisfying (A5), the exponent in (A6) is positive: the protons form a layer at the top of the atmosphere. The proton diffusion coefficients, in a medium of ions with \( Z-s=10 \) and number density \( N \), are of the order of \( D_3=2 \times 10^{23} \text{ N}^{-1} \) and \( D_4=4 \times 10^{-26} \text{ N cm}^{2} \text{ s}^{-1} \): lateral diffusion of the protons can be neglected.

Appendix B: Bound–free transition rates

Photoionization cross-sections for hydrogenic atoms have been calculated by Wunner et al. (1982, 1983) assuming adiabatic approximation wave functions of the form \( \Phi_{ov}(\rho, \theta) f_{ov}(z) \) (\( \rho \), \( \theta \) and \( z \) are here the electron coordinates and \( \Phi_{ov} \) is the lowest Landau orbital with \( z \)-component of orbital angular momentum \( -\nu \)). However, the ground-state wave functions of multi-electron ions are unknown and the scaling laws (Wunner et al. 1982) are inapplicable. We have calculated threshold cross-sections using the analytical final-state function given by Wunner et al. (1983, equation 13) and a ground-state function \( f_{ov} = p^{1/2} \exp(-p z) \). The energy dependence above threshold is that given by the Born and dipole approximations. Transitions \( \Phi_{ov} \rightarrow \Phi_{ov} \) have a threshold at the ionization energy \( \epsilon_e \) but involve the \( z \)-component of the photon field, which is very small in the rest frame of an ion with Lorentz factor \( \gamma \approx 10^2 \). Significant transition rates occur.
only at low altitudes, $x-1 \approx 2 \theta_M$, in the polar cap radiation field with temperatures $\approx 5 \times 10^5$ K. If $\alpha^2 \neq 0$, the parallel electric field in this region is reduced until the transition probability just satisfies the force-free condition (21). The transitions $\Phi_0 \rightarrow \Phi_{1, \nu-1}$ dependent on the remaining components of the photon field have an adiabatic approximation threshold $\epsilon_\gamma + \hbar \omega_B$ (the first Landau threshold; the cyclotron energy is $\hbar \omega_B = 11.58 B_{12}$ keV). For motion with $\gamma = 10^2$ in the surface blackbody radiation field ($\approx 10^5$ K), most photon energies the ion rest frame are below the first Landau threshold. Thus the transition probability is strongly dependent on configuration mixing in the ground-state wave function, which can be substantial for $B \approx 10^{12}$ G. We assume a ground state of the form $a_0 \Phi_0 f_0 + a_1 \Phi_1 f_1$ which allows transitions $\Phi_1 \rightarrow \Phi_{0, \nu+1}$ below the first Landau threshold. The order of magnitude of the mixing coefficient is $|a_1|^2 = \frac{1}{2} (\epsilon_\gamma / \hbar \omega_B)^2$. The calculated outer electron transition probability for photons with wave-vector parallel to $\mathbf{B}$ is shown in Fig. 4 as a function of altitude and surface blackbody temperature. The surface magnetic field is $2 \times 10^{12}$ G. The ion charge is $Z-s=10$ and the ionization energy satisfies the condition $\beta e_\gamma = 20$ (Appendix A) at a polar cap temperature of $10^6$ K. The variation of $B$ with altitude is allowed for. The Lorentz factor is $\gamma = 400$ at an altitude of $2 \times 10^8$ cm. The adiabatic approximation ($a_1 = 0$) transition probability is also shown for each temperature except $10^5$ K, for which it is extremely small. We can infer that for multi-electron ions, unless $\alpha^2$ is very small, the total transition probability at a surface temperature of $3 \times 10^5$ K is of the order of $(Z-s)(\alpha^2)^{-1} F(r)$, where $F(r)$ is defined by the force-free condition (21). Pair creation should not occur until the surface has cooled to lower temperatures.