Collisions between rotating interstellar clouds

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Summary. We present three-dimensional numerical hydrodynamic simulations of collisions between rotating interstellar clouds. Our results show that rotation can significantly extend the lifetimes of the resulting structures when compared to comparable cases without rotation. In many cases we find that rapidly rotating bars form, and in some cases these can form quasi-equilibrium structures which re-expand very slowly, surviving for many (>5) free-fall times. Our calculations also show the importance of spin-orbit interactions in the case of off-centre collisions. We find that, despite rapid dissipation in the collisions, the kinetic energy of each interacting pair of clouds often increases, thus providing a possible way of maintaining supersonic motions in interstellar cloud complexes. Our resultant collapsing structures often assume a nearly solid-body rotation law in the inner regions, and a Keplerian profile beyond this. This latter fact indicates that rotational support is more important in these structures than is thermal support. We provide a criterion, based on the free-fall time and the initial rotation period, for determining the outcome of a collision given various parameters pertaining to the initial configuration.

1 Introduction

Cloud–cloud collisions are thought to occur frequently in the Giant Molecular Cloud complexes of our galaxy (e.g. Turner 1984). Moreover, in the turbulent model proposed for these regions by Henriksen & Turner (1984) and by Henriksen (1986) it is essential that the cloud collision time for clouds of scale \( l_n \), namely \( t_c(l_n) \), should be about the dynamical time for clouds of the next largest scale in the hierarchy, say \( l_{n-1} \), to turn over or collapse. This is sufficiently rapid to allow

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cloud–cloud interactions to play a key rôle in the evolution of the molecular cloud complex, and yet sufficiently slow to avoid the complete dissipation of the (generally supersonic) cloud motions into heat.

In fact, we can combine the observed number–size distribution of clouds, (e.g. Tereby et al. 1986; see Henriksen 1986 for a discussion and other references)

\[ N(l_n) = \left( \frac{l_n}{l_0} \right)^{-a}, \]

where \( a \sim 2.7 \) and \( l_0 \) is the largest scale in the complex, with the observed velocity dispersion versus scale law (e.g. Scoville, Sanders & Clemens 1986; again, see Henriksen 1986 for more details)

\[ \sigma_n = \sigma_0 \left( \frac{l_n}{l_0} \right)^{1/2}, \]

where \( \sigma_0 \) is the velocity dispersion of the largest scale, to write \( t_c(l_n) \) as

\[ t_c(l_n) = \left[ \frac{N(l_{n-1}) l_{n-1}^2 \sigma_{n-1}}{N(l_n) l_n^2 \sigma_n} \right]^{-1} = \frac{l_n}{\sigma_n} \left( \frac{l_n}{l_0} \right)^{a-5/2} \approx 10^{7.2} \left( \frac{l_n}{l_0} \right)^{a-5/2} \text{ yr}. \]

Thus for ‘clouds’ of every scale the collision time is less than the dynamical time for the whole complex. When account is taken of the fact (e.g. Henriksen 1986) that the clouds may only move in a fractal dimension \( D \) and thus have cross-section \( l_n^{D-1} \), then

\[ N(l_n) = \left( \frac{l_n}{l_0} \right)^{-D} \]

and

\[ t_c(l_n) = \left( \frac{l_0}{\sigma_0} \right) \left( \frac{l_n}{l_0} \right)^{1/2} = \frac{l_n}{\sigma_n} \]

which is exactly the dynamical time on every scale.\(^1\) Also, because of the size and density variations between different scales, we expect the strongest interactions to occur between clouds on the same scale. Thus we have considered only clouds of equal masses in the numerical calculations which follow, although the analysis in Section 3 allows for different masses.

The importance of understanding the details of supersonic collisions has also been emphasized in the work of Scalo & Pumphrey (1982) and related studies (Scalo 1985 has given a comprehensive review). Recently several calculations have been dedicated to studying the persistence of supersonic motions (see e.g. Benz 1985; Monaghan & Varnas 1988). In our view this depends critically on the relevant time-scales and in any case will be an episodic phenomenon whereby a cloud on a given scale is alternately collapsing or in nearly Keplerian equilibrium (see e.g. Henriksen 1986\(^2\)). Suffice it to say here that the results we present below show cases in which, despite rapid dissipation, the cloud kinetic energy increases during the interaction.

\(^1\)One should note also that this can be derived by considering the number of clouds of scale \( n \) per supercluster of scale \( n-1 \). Then the collision time for scale \( n \) is simply

\[ t_c(l_n) = \left[ \frac{N(l_n) l_n^{D-1} \sigma_n}{N(l_{n+1}) l_{n+1}^{D-1} \sigma_n} \right]^{-1} = \frac{l_n}{\sigma_n} \]

as before.

\(^2\)From this perspective it is important to realize that it is irrelevant whether the constant \( K \) introduced by Henriksen & Turner (1984) reflects the virialization ‘pressure’ (Scalo 1985) that supports the cloud or the angular momentum loss that allows it ultimately to collapse.
The supersonic turbulent cascade picture predicts that a molecular cloud hierarchy evolves in time primarily by transporting angular momentum to larger scales by means of viscous and gravitational interactions between clouds on a given scale (Henriksen 1986). However, it can make no comment about the origin of the cloud hierarchy (initial range of the turbulence). A suggestion of long standing (Hoyle 1953) is that such a hierarchy is due to successive fragmentation of clouds [see also Zinnecker (1984), and references therein], but there might also be a 'first family' of fragments which interact and agglomerate into a Giant Molecular Cloud (GMC) complex (e.g. Henriksen 1986; Oort 1954). In the former case, a large cloud must be unstable to fragmentation, while in the latter case an enhanced density of fragments in a galactic region would lead to enhanced agglomeration. In our simulations (and those of Lattanzio et al. 1985) we find little evidence for fragmentation, but some of the rotating structures formed in this study are very long-lived, suggesting agglomeration, and lead to gravitational collapse.

In any event, it is clear that cloud–cloud interactions are of crucial importance to any model of the evolution of GMC complexes, and hence also to star formation. In this paper we conduct three-dimensional numerical simulations of rotating clouds in collision. Our purpose will be to examine the conditions under which the rotating clouds may either merge and collapse, or separate and dissipate (or possibly collapse). In this way we hope eventually to clarify the rôle of cloud–cloud collisions in both star formation and the dynamical evolution of the molecular cloud complex.

In full generality the cloud–cloud interaction is perhaps best thought of as the elementary interaction in compressible turbulence (e.g. Churilov & Shukman 1982; Henriksen & Turner 1984; Henriksen 1986). The collision is then the non-linear interaction of two compressible solitons or 'floccules' and in general the vorticity, \( \omega = \nabla \times v \), and the 'compressibility' \( d \ln q/dt = -\nabla \cdot v \) must be known everywhere at all times to describe the result. In fact we have, quite generally, the identity for any vector field sufficiently well behaved at infinity which for \( v \) takes the form:

\[
v = -\nabla \int \frac{d^3 x'}{4\pi |r-r'|} \cdot v' + \nabla \times \int \frac{d^3 x' \cdot \omega'}{4\pi |r-r'|},
\]

where, in the absence of sources on the boundaries, we have omitted the gradient of a potential function on the right. In order for the identity to have predictive power we must calculate the divergence and the vorticity from the equations of continuity and motion as

\[
\nabla \cdot v = -d \ln q/dt,
\]

and (using isentropic flow and constant shear viscosity coefficient \( \eta \) and constant bulk viscosity coefficient \( \zeta \) for illustration)

\[
\frac{\partial \omega}{\partial t} + \nabla \times (\omega \times v) = \nabla \times \left[ \frac{\eta}{Q} \nabla^2 v + \frac{\zeta + \eta/3}{Q} \nabla (\nabla \cdot v) \right].
\]

This last equation shows that the Kelvin theorem for the conserved vorticity holds only where the spatial gradients are small.\(^3\) Unfortunately neither the assumption of constant entropy nor the assumption of constant viscosity is valid in the interacting regions. However, the identity does emphasize that the compressibility and vorticity are complementary parts of the floccule–floccule interaction. If desired the \( |r-r'| \) terms can be expanded in powers of spherical harmonics, as can the vorticity and compressibility, in order to write \( v \) as a sum over various modes. Because of the non-linearity of the equations, however, these modes are generally coupled to all orders.

\(^3\) i.e. large Reynolds number. We find that \( Re \sim 10^2 \) for low-velocity collisions, and \( Re \sim 10 \) for the higher velocities considered in this paper. \( Re \) is much smaller at the interface between the clouds, of course.
Because the actual interaction is viscous (if the impact parameter $b < 2R$, where $R$ is the cloud radius), gravitational, and radiative as well as non-linear, we must appeal to numerical simulations. Ultimately we wish to study the interaction as a function of spin–orbit alignment, and the initial spatial distribution of vorticity and density, but in this paper the latter are uniform and the spin is either parallel or anti-parallel to the orbital angular momentum.

The cloud velocities are supersonic for cloud scales greater than a few parsecs, so the cloud–cloud collisions initiate strong shock waves. In this paper the gas is held isothermal throughout the interaction so that efficient cooling on a dynamical time-scale is assumed (Hausman 1981; Monaghan & Varnas 1988). This must ultimately be relaxed on the smallest spatial scales in order that stars may form. The combination of strong shocks and arbitrary geometries poses a formidable problem for simulations. For this reason most of the previous calculations of interstellar cloud collisions have been restricted to various symmetries (see the list in Lattanzio et al. (1985), hereafter LMPS).

Three-dimensional simulations were first published by Hausman (1981) who used a crude particle scheme. This suffered from a lack of resolution and, more importantly, interpenetration of the particles. More recently, LMPS presented many 3D simulations with greatly improved resolution and an artificial viscosity which prevented particle interpenetration. These calculations ignored rotation and concentrated on the effect of varying the masses and relative velocity of the clouds, plus the impact parameter. It was found that a typical collision resulted in dispersal of the clouds, rather than enhancing collapse. Collapse occurred only for cloud which were initially unstable. The resulting configurations were often highly irregular and included rotating bars.

In this paper we study the effects of collisions between clouds with an initially non-zero rotation. Section 2 will outline the numerical method used and Section 3 considers the various energy sources from the viewpoint of virial equilibrium. Section 4 presents the results for single clouds, while Sections 5 and 6 discuss the collisions with zero and non-zero impact parameter, respectively. Our conclusions are to be found in Section 7.

## 2 The numerical method

The calculations were performed with the refined smoothed particle hydrodynamics code described in Monaghan & Lattanzio (1985) and used by LMPS. Two changes have been made to the code. First, the time-step algorithm is now a modified Euler method rather than the leap-frog scheme previously used. Secondly, the gravitational force, calculated on multiple grids as described in Monaghan & Lattanzio (1985), is now solved to 4th order in the resolution length, $h$, on three grids (Monaghan 1985; Monaghan & Varnas 1988).

To elaborate on the time-step algorithm, let subscripts refer to a given time-slice and let superscripts refer to the first or second estimate of a variable. Then after making the $n$th time-step we begin by estimating the mid-point values via:

$$v_{n+1/2}^{(i)} = v_n + \frac{1}{2} \Delta t \cdot f(x_{n-1/2}^{(i)}, v_{n-1/2}^{(i)})$$

$$x_{n+1/2}^{(i)} = x_n + \frac{1}{2} \Delta t \cdot [2v_{n+1/2}^{(i)} - v_n]$$

$$= x_n + \frac{1}{2} \Delta t \cdot v_n + \frac{1}{4} (\Delta t)^2 f(x_{n-1/2}^{(i)}, v_{n-1/2}^{(i)})$$

where $x$ and $v$ are the position and the velocity (of each particle), $\Delta t$ is the time-step determined (according to specified rules) as appropriate to $x_n$ and $v_n$, and $f$ is the total force per unit mass acting on that particle. Next we calculate the new force $f(x_{n+1/2}^{(i)}, v_{n+1/2}^{(i)})$, and use this to obtain

$$v_{n+1/2}^{(2)} = v_n + \frac{1}{2} \Delta t \cdot f(x_{n+1/2}^{(2)}, v_{n+1/2}^{(2)})$$

$$x_{n+1/2}^{(2)} = x_n + \frac{1}{2} \Delta t \cdot v_{n+1/2}^{(2)}.$$
Finally we get \( x_{n+1} \) and \( v_{n+1} \) from

\[
v_{n+1} = 2v^{(2)}_{n+1/2} - v_n
\]

\[
= v_n + \delta t_n f[x^{(1)}_{n+1/2}, v^{(1)}_{n+1/2}]
\]

\[
x_{n+1} = 2x^{(2)}_{n+1/2} - x_n
\]

\[
= x_n + \delta t_n v^{(2)}_{n+1/2}.
\]

This implementation requires that the forces be calculated only once per time-step, which is advantageous because this is the longest part of the calculational procedure. A very similar algorithm has been described by Monaghan & Varnas (1988).

The success of particle schemes in handling shock calculations is critically dependent on the artificial viscosity. We have used the formulae described and tested in Lattanzio et al. (1986), which was also used successfully in LMPS. Similarly, in most respects the numerical procedure used here is that described in LMPS. In view of the difficulty associated with a realistic inclusion of the intercloud medium, we have chosen to ignore the presence of this medium. This leads to larger Jeans masses (for the same temperature and density) than when \( P_{\text{ext}} \neq 0 \), and a more rapid expansion of the very outermost layers of the clouds. We believe, however, that these effects are of secondary importance.

The scaling used in this paper is the same as that given in LMPS, although we shall quote values in physical units. In the calculations to be presented we used approximately 3500 particles per cloud.

### 3 Energy considerations

#### 3.1 SINGLE CLOUDS

For an isolated, spherical, isothermal cloud of temperature \( T \), mass \( M \), radius \( R \) and constant density \( \rho \) (with molecular weight \( \mu \)) the ratio of thermal to gravitational energy is

\[
a = \frac{E_T}{|E_G|} = \frac{5}{2} q
\]

where

\[
q = \frac{\mathcal{R} TR}{GM\mu} ;
\]

all symbols have their usual meaning, and \( \mathcal{R} \) is the gas constant. (The effective temperature might, however, be a measure of internal subscale turbulence rather than of true thermal motions.) From the virial theorem, this cloud is gravitationally unstable if \( a \leqslant 0.5 \), with corresponding Jeans radius

\[
R_j = \frac{\frac{1}{3} GM\mu}{\mathcal{R} T} = \frac{1}{5} \frac{R}{q}.
\]

In this expression we require \( R < R_j \) for instability, as \( M \) is fixed. If the cloud is rotating on cylinders (we neglect the resultant flattening, and again treat the cloud as a sphere) then we must include the rotational energy

\[
E_R = \int \frac{1}{2} \rho v_h^2 \, dV
\]
where \( v_r \) is the velocity due solely to rotation, and the integral is over the volume of the cloud. For an angular velocity \( \Omega \) we find

\[
E_R = \frac{1}{2} \Omega^2 R^2 M.
\]

In this case, gravitational instability ensues if

\[\alpha + \beta \leq 0.5\]

where

\[
\beta = \frac{E_R}{|E_G|} = \frac{\Omega^2}{4\pi G \rho}.
\]

(Note that, for a zero temperature gas, \( \beta = 1/3 \) corresponds to balance between gravity and centrifugal force at the equator.) For a cloud of fixed mass \( M \) the condition \( \alpha + \beta < 0.5 \) for gravitational instability becomes \( R < x_J R_J \) where \( R_J \) is as above and

\[x^3 + \kappa x - \kappa = 0,\]

with \( \kappa = (15/2)(\mathcal{R} T/\mu)(\Omega R_J)^{-2} \). There is one real root for \( x_J \) which for \( \kappa < 1 \) (rapid rotation) approximates \( x_J \sim \kappa^{1/3} \) and \( x_J \to 1 \) for \( \kappa > 1 \) (slow rotation). When \( R_J, x_J R_J < R \) then the cloud may be expected to form a flattened spheroidal object having strong gradients along the minor axis (\( R_J \)) and with axial ratio \( -x_J \) (i.e. with major axis \( x_J^{-1} R_J \)). (Note that the above analysis is based on virial arguments, and ignores the effects of pressure gradients at the surface. Also, pure energy balance arguments would replace all values of 0.5 by 1.0. It is thus more appropriate to say that gravitational instability results when \( \alpha + \beta \approx 1 \).)

3.2 Clouds in Collision

3.2.1 Energetics

Consider two spherical clouds of the same constant density approaching each other with relative velocity \( v_{rel} = 2v \), and just in tangential contact. Let the small cloud have a mass \( f \) times that of the larger cloud, and let both undergo solid-body rotation with angular velocity \( \Omega \) perpendicular to the direction of relative motion. Then the kinetic energy is composed of components from both the rotational and translational motion:

\[
E_K = E_{R} + E_{T}.
\]

In the centre-of-mass frame

\[
E_{T} = M v^2 \left( \frac{2f}{1+f} \right)
\]

and

\[
E_{R} = \frac{1}{2} \Omega^2 R^2 M (1 + f^{1/3}).
\]

The total gravitational potential energy is given by

\[
E_G = -\frac{3}{5} \frac{GM^2}{R} Q(f)
\]

where

\[
Q(f) = 1 + f^{5/3} + \frac{3f}{1+f^{1/3}}.
\]
The total thermal energy is

\[ E_T = \frac{3}{2} \frac{\mathcal{R} T}{\mu} (1 + f) M. \]

Thus we have

\[ \alpha = \frac{E_T}{|E_0|} = \frac{5}{2} q \frac{(1 + f)}{Q(f)} \]

\[ \beta = \frac{E_R}{|E_0|} = \frac{1}{4\pi} \frac{\Omega^2}{Gq} \frac{(1 + f^{5/3})}{Q(f)} \]

\[ \gamma = \frac{E_{\nu}}{|E_0|} = \frac{10}{3} q \left( \frac{v^2}{c^2} \right) \frac{f}{1 + f} \frac{1}{Q(f)}. \]

Again, for gravitational instability we require, from the Virial theorem,

\[ \alpha + \beta + \gamma \lesssim 1/2. \]

Using the same notation as in the case of single clouds, this condition becomes \( R < x_j R_j \) where now

\[ x_j \left[ \frac{\alpha (1 + f)}{1 + f^{5/3}} + \frac{5 v^2}{(\Omega R_j)^2 (1 + f)(1 + f^{5/3})} \right] = \frac{x Q(f)}{(1 + f^{5/3})} = 0. \]

The new effects are found essentially when \( v^2 \gg \mathcal{R} T/\mu \), whereupon

\[ x_j = \frac{3}{2} \frac{\mathcal{R} T}{\mu v^2} \frac{Q(f)(1 + f)}{2f}. \]

The flattened, merged object (when formed) may be expected to have an axial ratio \( \sim x_j \). Note, however, that this expression estimates the importance of gravity during the very earliest stages of the collision, before the spheres become deformed or show significant density variations or radiative losses. Thus we cannot predict the structure of the merged body with any confidence.

3.2.2 Angular momentum

In the centre-of-mass frame the total linear momentum vanishes, while the angular momentum has the form (when the clouds are just in tangential contact)

\[ \mathbf{L} = (\mathbf{R} - \mathbf{R}_j) \times \mathbf{P} + \mathbf{S} + \mathbf{S}_j, \]

where \( \mathbf{P} \) is the linear momentum of the larger cloud (in the centre-of-mass frame), \( \mathbf{R} \) and \( \mathbf{R}_j \) locate the centres of the clouds relative to the centre-of-mass (the \( f \) subscript refers to the smaller cloud), and \( \mathbf{S} \) and \( \mathbf{S}_j \) are the individual spin angular momenta of the two clouds. For the simple clouds discussed here, this expression becomes explicitly

\[ \mathbf{L} = M v b \mathbf{k} \pm \frac{3}{2} \Omega R^2 M (1 + f^{5/3}), \]

where the first \( \pm \) indicates whether the spin angular momenta are parallel or anti-parallel to the orbital angular momentum, and the second \( \pm \) reflects the possibility that the individual spins may be mutually parallel or anti-parallel (\( \mathbf{k} \) is a unit vector parallel to \( \Omega \)). The impact parameter \( b \) is the distance between the cloud centres, perpendicular to the direction of relative motion. During a collision we must have conservation of \( \mathbf{L} \) as given by the above equation.
4 Single cloud evolution

Before considering the collision of rotating clouds, we shall briefly discuss the fate of our single rotating clouds. The standard cloud to be used in this paper is an 8000 $M_\odot$ H\textsc{i} cloud at 74K with $R = 12.25$ pc ($n = 43$ cm$^{-3}$). This cloud has $\alpha = 0.544$ and is thus near gravitational instability. The free-fall time-scale $t_{ff} = 8 \times 10^6$ yr. The evolution of this cloud has been calculated for $\beta = 0.0$, 0.073, 0.164 and 0.292.

For $\beta = 0$ the cloud collapses to a point, exhibiting the familiar $r^{-2}$ density distribution characteristic of the similarity solution for isothermal collapse (e.g. Tohline 1982).

For $\beta = 0.073$ we see a shock form in the plane perpendicular to the rotation axis (the $z$-axis), and a slight bounce in the equatorial plane. The density increases by a factor of 11 by the time of

![Density contours](https://www.astrogeeks.com/diagrams/density_contours.png)

**Figure 1.** Density contours found during the collapse of the $\beta = 0.073$ cloud. The upper panel shows the contours through the $x$-$y$ (equatorial) plane, and the lower panel shows the contours in the $x$-$z$ plane, which contains the rotation axis. The numbers above the panels give the time, in units of $10^6$ yr, the value of the maximum-density contour, and the difference between consecutive contours (the density unit is H atom cm$^{-3}$).
the bounce ($t = 13.8 \times 10^6 \text{ yr}$) then decreases by 14 per cent before increasing again without limit. Density contours for this configuration are shown in Fig. 1. The structure is approximately that of an oblate spheroid with equidensity contours showing a flatness $\eta = z/R \approx 1.4$ where $z$ is the maximum height of any particular density level, and $R$ is its corresponding radius in the equatorial plane. Axisymmetry is maintained to a high degree about the $z$-axis.

The $\beta = 0.164$ configuration shows substantial expansion from the surface, due both to the rotation and the pressure gradient (recall that $P_{\text{ext}} = 0$). The inner regions contract until reaching a maximum density of $q_0$ ($q_0$ is the initial density) after some $10.8 \times 10^6 \text{ yr}$. This corresponds to a $z$-bounce, after which the cloud expands until $q < q_0$ everywhere, henceforth called dispersal. The cloud is again spheroidal with $\eta \approx 1.5$.

The $\beta = 0.292$ case shows copious mass loss from the surface. Again, the centre contracts slightly to a maximum density of $q = 2.6 q_0$ after $10.0 \times 10^6 \text{ yr}$, but the cloud soon disperses as a result of its high rotational energy ($\alpha + \beta = 0.84$).

One should note the apparent discrepancy between the results discussed above and those of Miyama, Hayashi & Narita (1984). These latter would imply that for a cloud of $\alpha = 0.544$ we should see fragmentation for $\beta \leq 0.22$ and a runaway collapse for $\beta \geq 0.22$. Although the details of this discrepancy will be the subject of a subsequent paper (Lattanzio & Monaghan, in preparation), we will now briefly discuss this problem.

In recent years 3D hydrodynamics codes have been used to investigate the evolution of rotating isothermal clouds [see the review by Tohline (1982) and references therein]. Details of the results depend on various artefacts of the calculations, for example, boundary conditions, external pressure, details of the code used, etc. In an attempt to clarify the various results, Stahler (1983a, b) constructed equilibrium configurations for various $\alpha$ and $\beta$ under the axisymmetric assumption. For values of $\alpha$ less than some critical value, which is a function of $\beta$, no equilibrium structures are possible. The obvious interpretation is that the cloud goes into collapse, with fragmentation as a possible outcome.

Hachisu & Eriguchi (1984, 1985) considered the evolution of (incompressible) Maclaurin spheroids, and developed the following overall scenario for cloud evolution. There are two critical values of $\alpha$, denoted by $\alpha_c$ and $\alpha_f$, both of which decrease with increasing $\beta$, and a critical value of $\beta$ denoted by $\beta_c$. For $\beta < \beta_c$ we have $\alpha < \alpha_c$, otherwise $\alpha > \alpha_c$. For $\alpha > \alpha_c$ a stable cloud results. For $\alpha < \alpha_c$, we have a central mass accreting from a flattened disc. For $\alpha < \alpha_f$, fragmentation results (see Hachisu & Eriguchi 1985, hereafter HE, especially fig. 11). The $\alpha_c(\beta)$ curve derived by HE agrees quite well with that of Stahler (1983b). Also, the 3D dynamical results of Miyama et al. seem consistent with this picture (see HE, fig. 10).

A few comments should be made. First, it is not clear to what extent the models of constant density configurations are appropriate to the dynamical studies. Fig. 1 shows density contours for the $(\alpha, \beta) = (0.544, 0.073)$ model at $t = 16.1 \times 10^6 \text{ yr}$ ($2t_\eta$). It is clear that there are substantial density, and hence pressure, gradients in this model. Secondly, it must be stressed (despite HE) that it is very difficult to actually obtain fragmentation as a natural result of a calculation. Even with an $m = 2$ mode density perturbation of 50 per cent imposed on a rotating cloud, the two density maxima seem to merge rather than form a binary [Monaghan & Lattanzio (1986), and references therein].

Finally, how are we to interpret the fragmentation found by Miyama et al. (1984)? Of direct relevance to this are the calculations of Gingold & Monaghan (1983). These authors show that spurious transport of angular momentum can lead to spiral shapes, which seem to be the precursor of the fragmentation found by Miyama et al. Compare, especially, fig. 6 of Gingold & Monaghan (1983) with figs 7 and 8 of Miyama et al. (1984). Unpublished calculations (Monaghan & Lattanzio 1987) have shown that artificial viscosity plays a crucial role in the formation of spiral shapes and subsequent fragmentation. In summary, we believe that the analysis of Maclaurin

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spheroids is of limited use in analysing the dynamical collapse of rotating clouds, and that the fragmentation found by Miyama et al. is a spurious result due to poor transport of angular momentum. Details will be published elsewhere.

5 Head-on collisions

5.1 Parallel Rotation Vectors

We consider here the resulting evolution when two of the standard clouds (discussed in the previous section) collide head-on (with impact parameter \( b = 0.0 \)). The rotation vectors \( \Omega \) are identical for each cloud, so that the rotation axes of the clouds are parallel, and are also perpendicular to the direction of relative motion. We shall take relative velocities of \( v_{rel}/c = 1, 2, 4, 6 \) and 10, and angular velocities \( \Omega/\Omega_0 = 1, 2 \) and 4 with \( \Omega_0 = 2.08 \times 10^{-15} \text{ s}^{-1} \) (which corresponds to a rotation velocity at the edge of the cloud of the local sound speed). In the centre-of-mass frame we have, initially:

\[
\alpha = 0.384, \\
\beta = 0.0512 \left( \frac{\Omega}{\Omega_0} \right)^2, \\
\gamma = 0.032 \left( \frac{v_{rel}}{c} \right)^2.
\]

We shall first look at the \((v_{rel}/c, \Omega/\Omega_0) = (1, 1)\) case. This is sequence H1 in Table 1, which lists various parameters for the head-on collisions considered. For this case \((\beta, \gamma, \alpha + \beta + \gamma) = (0.051, 0.032, 0.467)\). The evolution of these clouds is shown in Figs 2 and 3. Initially we see a shock at the interface of the clouds, resulting in strong compression of the cloud material. There is some slight expansion at the edges, but the high-density region at the interface soon initiates a collapse. The density distribution seen in Fig. 3(c) soon becomes approximately elliptical (fig. 3d) some 1.8 \times 10^5 \text{ yr} later, with eccentricity \( e = 0.34 \). After a further 1.6 \times 10^5 \text{ yr} the configuration is very nearly spherical, with only a slight flattening along the direction of the rotation axis. Although the two clouds form an elongated structure for only \( \sim 15 \) per cent of the duration of the interaction (runaway collapse results), we draw attention to the possibility of such an object acting as a source of gravitational torque (see e.g. Henriksen & Turner 1984; Larson 1984; and below).

Fig. 4 shows the time variation of the various energy sources (in the centre-of-mass frame), and the maximum density. Note that the destruction of translational kinetic energy, \( E_T \), in the collision is more than offset by the acceleration induced in each cloud by their mutual gravity, so that (even initially) \( E_T \) increases. Thus the collision is not effective at dissipating the translational kinetic energy, because of the presence of the gravitational field, which accelerates the clouds. The rapid rise in the maximum density corresponds to the onset of gravitational instability \((\approx 8 \times 10^4 \text{ yr})\) and is reflected in the rapid decrease of the gravitational potential energy (Fig. 4a). Runaway collapse ensues for \( t \approx 13 \times 10^6 \text{ yr} \). Note that the calculation is resolution limited for \( t > 8 \times 10^6 \text{ yr} \) (see LPMS for details), but this does not quantitatively affect the evolution until \( t \approx 12 \times 10^6 \text{ yr} \) when, for example, the gravitational energy \( E_G \) is not decreasing as rapidly as required. This is reflected in the increase of the total energy for \( t \approx 12 \times 10^6 \text{ yr} \) in Fig. 4(a).

Fig. 5 shows the radial variation of \( \rho \) and \( \Omega \) in the last model calculated (at \( t = 13.2 \times 10^6 \text{ yr} \)). Let \( R \) be the radial distance, in the equatorial plane, from the point of maximum density (this point is also, approximately, the new rotation axis of the resulting configuration). Then, from the figure, we have

\[
\rho \propto R^{-0.1} \text{ for } 1.2 \leq R/\text{pc} \leq 7.3 \\
\propto R^{-0.7} \text{ for } R \leq 1.2 \text{ pc}.
\]
### Table 1. Parameters for the head-on collisions.

<table>
<thead>
<tr>
<th>Name</th>
<th>$\Omega_{\infty}$</th>
<th>$\beta$</th>
<th>$4\gamma$</th>
<th>spins (^b)</th>
<th>$\dot{M}_{\text{max}}$ (^c)</th>
<th>$\Delta t$ (^d)</th>
<th>$\theta_{\text{CM}}$ (^f)</th>
<th>$\tan(i)$ (^g)</th>
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<td>0.05</td>
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</tr>
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<td>C</td>
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<td>C</td>
<td>7.8 (?)</td>
<td>1.6(^*)</td>
<td>4.0</td>
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<td>2.0</td>
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<td>C</td>
<td>9.7 (?)</td>
<td>1.2(^*)</td>
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<td>0.05</td>
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<td>2.5(^*)</td>
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<td>P</td>
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<td>H20</td>
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<td>5.0 (?)</td>
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<td>0.0</td>
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<td>A</td>
<td>C</td>
<td>1.5</td>
<td>0.0</td>
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</tr>
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</table>

\(^*\) $\Omega_{\infty} = 2.08 \times 10^{-13}$ s\(^{-1}\).

\(^b\) P or A indicate that the spins are mutually parallel or antiparallel, respectively.

\(^c\) C indicates that runaway collapse results, otherwise we quote the maximum density reached during the interaction. A question mark indicates that, although a maximum was reached, this was resolution-limited and the actual outcome is uncertain.

\(^d\) The approximate duration of the interaction, in units of the cloud's initial free-fall time. Where collapse results it is the time of the runaway. Where a bounce occurs we have quoted twice the time of the bounce. An asterisk indicates that the interaction is much longer than the quoted value, due to very slow re-expansion.

\(^f\) As defined in Section 7.

This change in the radial dependence of the density distribution near the centre is probably partially due to smoothing the density over the finite resolution length, which is 1 pc for this model. The component of the velocity in the equatorial plane which is directed toward the point of maximum density, $d R / dt$, varies roughly as $d R / dt \propto - R$ for $R \leq 3.7$ pc. To within a factor of 2 we find that $Q$ is constant for $R \leq 3.7$ pc and $Q \propto R^{-1.3}$ for $R \geq 3.7$ pc. Because we find $\Omega$ to be approximately constant for nearly four smoothing lengths, this behaviour cannot be due to the interpolation procedure. It may be due to tidal torques and/or an effective turbulent viscosity. However, it could also be an artefact of the artificial viscosity, which vanishes for rigid rotation. We intend to investigate this further in a later paper.

Now consider sequence H2 in Table 1, where $\Omega = 2 \Omega_{0}$. Here ($\beta, \gamma, \alpha + \beta + \gamma$) = (0.205, 0.032, 0.621). The evolution is qualitatively similar to case H1. The clouds are more strongly deformed, however, with the density contours in the equatorial plane again appearing roughly elliptical, but now with $\epsilon = 0.55$ (compare with the value of 0.34 for case H1). The higher rotational energy content delays the runaway collapse until $t = 16 \times 10^6$ yr.

A further doubling of $\Omega$ to $4 \Omega_{0}$ gives sequence H3. Here $\alpha + \beta + \gamma = 1.23$. Obviously individual
Figure 2. Velocity vectors in the equatorial plane for case H1. The times quoted are in 10^6 yr and the velocity unit is km s^{-1}.
clouds with such a high value of $\beta (=0.819)$ cannot survive for long, yet they are interesting for (at least) two reasons. First, if torques exist in a cloud ensemble then some clouds may be spun up to values of $\beta>1/3$, and thus the temporary existence of these clouds is possible. The subsequent evolution of these clouds is then of interest. Secondly, it is important to determine if such a high rotational energy content necessarily precludes gravitational collapse. (We shall see below that this is not the case.) For case H3 we see copious mass loss from the clouds due to their high rotation. The maximum density increases by only a factor of 4.2 in $7 \times 10^6 \text{yr}$ before decreasing. The average density of the configuration, however, decreases monotonically with time due to the cloud expansion. Soon all 'particles' in the calculation have $\varrho < \varrho_0$ and we say that the clouds have 'dispersed'. Clearly the details of this part of the evolution will depend on the intercloud medium, which was neglected in this study.

Doubling the relative velocity of sequences H1–H3 gives sequences H4–H6, respectively.
These are again qualitatively similar to those of lower relative velocity. Sequences H4 and H5 result in runaway collapse, sequence H6 in dispersal. Sequence H4 reaches an eccentricity of 0.5 in the equatorial plane before collapsing as an oblate spheroid with an oblateness $\eta=0.1$. Sequence H5 reaches $\epsilon=0.6$ before the gravitational field of the collapsing matter imposes a more nearly circular density distribution in the equatorial plane. Sequence H6 reaches $\epsilon=0.8$ although it is actually less elliptical. Fig. 6 shows the density contours for this sequence. The configuration reaches a maximum density of $5.5q_0$ after some $6.8 \times 10^4$ yr, and is quite bar-like. During the subsequent re-expansion the configuration maintains its elongated structure, but eventually disperses. The time-scale for this dispersal is roughly given by $|\dot{\rho}/\dot{\rho}| = 3 \times 10^6$ yr, or $=0.4t_\eta$.

Doubling the relative velocity to $4c$ gives sequences H7–H9, with $\Omega/\Omega_0 = 1, 2$ and 4, respectively. Again we see the formation of an elongated, approximately elliptical (in the equatorial plane) structure. Cases H7 and H8 result in gravitational collapse. Case H9 is more interesting. Here $(\beta, \gamma, \alpha + \beta + \gamma) = (0.819, 0.512, 1.72)$ so we should not expect gravitational instability. Whether this results or not depends on the efficiency with which the collision can destroy the
translational kinetic energy, and on how the rotational energy is distributed. What we see is the maximum density rising to 7.8$\rho_0$ in only $5 \times 10^6$ yr, and then decreasing very slowly. For example, the decrease from 7.8$\rho_0$ to 7.4$\rho_0$ takes $2.3 \times 10^6$ yr. (In fact, the maximum density obtained is limited by the finite resolution, and this may contribute to the re-expansion.) The resulting elongated gas cloud is clearly very near the boundary between collapse and re-expansion. During its slow re-expansion phase it has more time to rotate than the previous cases, where gravity soon reduced them to spheroids. Fig. 7 shows the density contours for selected times in this sequence. The bar rotates with $\Omega = 0.2\Omega_0$, completing one revolution in about $74\times10^6$ yr. In this case $|\theta/\dot{\theta}| = 40 \times 10^6$ yr, or $\approx 5$ $t_{\text{ff}}$, enabling the structure to rotate through $\approx 180^\circ$.

This is an interesting result. The two clouds each have $\beta = 1.15$ initially. Such a large rotational energy means that these clouds cannot survive long as discrete entities; substantial mass loss would be expected. Indeed, they can only exist as modelled if they had been recently 'spun-up' from lower values of $\beta$. The result of their collision, however, is a long-lived, relatively rapidly rotating bar which we would expect to be very efficient at generating torques. We have,
therefore, the intriguing possibility that this bar may lose angular momentum to the orbital motion of clouds in its vicinity. This increases, on average, the spin angular momentum of the clouds on the next scale. These may collide and, in turn, form rotating bars to continue the cascade of angular momentum upward (e.g. Henriksen & Turner 1984).

Cases H10–H12 have $v_{\text{rel}} = 6c$ and $\Omega/\Omega_0 = 1, 2$ and 4, respectively. The results are quite similar to the analogous cases H7–H9, with lower relative velocity. The case with $\Omega = 4\Omega_0$, sequence H12, again forms a rapidly rotating bar. The maximum density reaches $9.7\rho_0$ after $4.3 \times 10^6$ yr, and then drops very slowly to $9.5\rho_0$ after a further $1.9 \times 10^6$ yr. The bar has $\Omega = 0.5\Omega_0$ and completes one revolution in approximately $29 \times 10^6$ yr. This structure has $|q/\dot{q}| = 25t_{\text{ff}}$. Note that this bar is rotating approximately twice as rapidly as that found in sequence H9, which had half the relative velocity of case H12.

Sequences H13–H15 have $v_{\text{rel}} = 10c$, and initially behave more like the head-on collisions of
LMPS, due to the smaller fraction of the total energy contained in rotation ($\beta = 0.051$, 0.205 and 0.819, while $\gamma = 3.2$). Nevertheless, even here the rotation induces rotating bars as the outcome of the collisions. Sequence H13 shows an eccentricity of $e = 0.8$, but within $3.8 \times 10^6$ yr the structure is approximately that of an oblate spheroid, with $\eta = 0.5$, undergoing rapid collapse. This is despite the fact that $\alpha + \beta + \gamma = 3.6$, and the total energy is positive (note that a great deal of energy is lost, in the form of radiation, because of the isothermal assumption). Doubling the rotational frequency gives case H14 which is very similar again. The density contours in the equatorial plane show $e = 0.8$ changing to $\approx 0.0$ before the bar has turned through $90^\circ$. In this case the calculations were stopped just prior to what appears to be a maximum density of $300 q_0$, followed by a decrease. Further evolution would be required to be certain of the outcome, much like sequences H9 and H12. Case H15, however, is unambiguous. The high rotational energy leads to dispersal after an initial bounce producing an ellipsoid with $Q_{\text{max}} = 17 q_0$ after $3 \times 10^6$ yr.
Some trends are now clear. The flatness of the structure of the interface between the clouds depends on both the relative velocity (or $\gamma$) and the rotational energy (or $\beta$). Increasing either $\gamma$ or $\beta$ leads to more flattened (i.e. elliptical) density contours in the equatorial plane. For each $\gamma$ there is a critical $\beta$ above which the clouds disperse. Below this value the clouds show gravitational collapse, rapidly becoming oblate spheroids. These structures show density distributions of the form $\rho \propto R^{-\alpha}$ where $\alpha=2.5-3$. Similarly we typically find that $\Omega \propto R^{-1.4}$, and that the final configurations have $\beta=0.2-0.4$. The 10 per cent of the total matter which has the highest density, however, has $\beta=0.03$. The specific angular momentum of the configurations increases toward the position of the maximum density, as a result of particles falling into the gravitational potential.

The critical value of $\beta$ which divides these outcomes may decrease as $\gamma$ increases. Perhaps the most interesting result, however, is the possibility that, for some $\gamma$ and high values of $\beta$, a rotating bar will form and persist for quite long times. This may act as an efficient gravitational torque for transporting angular momentum between scales.

We note that these collisions are generally not very efficient in destroying the initial bulk motion of the clouds, because of the mutual gravitational attraction. The behaviour shown in Fig. 4(a) is typical. For $v_{\text{rel}}/c=4$ and $\Omega/\Omega_0<4$ the total kinetic energy often increases despite the shock dissipation, or decreases by only $\sim 20$ per cent. These cases ultimately collapse gravitationally, so that the kinetic energy secularly increases overall, but it is the kinetic energy on the original scale which is of interest. When $v_{\text{rel}}/c=10$ we did find examples wherein the kinetic energy was reduced to about 16 per cent of its original value, so that the mean velocity would have been reduced to $\sim 40$ per cent of its initial value.

5.2 ANTI-PARALLEL ROTATION VECTORS

We next investigate collisions between identical clouds, as in Section 5.1, but with rotation vectors antiparallel. The upper cloud in the following figures has clockwise rotation, and the lower has counter-clockwise. Sequence H16 has $v_{\text{rel}}=c$ and $\Omega = \Omega_0$. Again a shock develops at the interface and the growing density in this sheet is able to reverse the motion of the material moving away from the interface. The density contours show a core structure which is elongated along the line of relative motion but by $t=12\times10^4 \text{yr}$ most of the cloud material has rained down on the core and the structure is essentially spherical, and undergoing rapid collapse. Sequence H17 has $\Omega = 2\Omega_0$, but is otherwise identical to case H16. Again the outcome is runaway collapse, but Figs 8 and 9 show that the higher rotation in this case leads to a structure which is elongated perpendicular to the direction of relative motion. The (approximate) eccentricity is $e=0.5$, but by $t=16\times10^4 \text{yr}$ the structure is circular in the equatorial plane, with some flattening perpendicular to this plane. Note that the maximum density occurs to the left of the line joining the centres of the two clouds. This is because the (rotating) material on the right exerts a transverse ram pressure on the high-density interface. Similarly, material on the left has its rotation tending to draw it away from the highest density material, creating a lower pressure area for the ‘core’ to move into. Note, also, that there is no rotation of the resulting structure, contrary to the case of parallel rotation vectors. This is to be expected, of course, because there is no net rotation in the original configurations.

Note also that this collision produces a collimated splash moving to the left along the $x$-axis (see Fig. 8a and b). This is a low-velocity, ($\sim 2 \text{ km s}^{-1}$), low-density, unipolar flow. In the $x$–$z$ plane the outflow has the cross-section of an elliptical cone.

Doubling $\Omega$ to $4\Omega_0$ gives case H18, which results in dispersal. The maximum density is achieved after some $8\times10^4 \text{yr}$ and is only 3.5 times the initial density. The high rotation in these clouds produces a rapid expansion which removes material which was accreted onto the interfacial region in cases H16 and H17.
Case H19 has $v_{\text{rel}} = 2c$ and $\Omega = 2\Omega_0$. Again a flattened structure results at the interface between the clouds, and soon gravitational instability takes over. Doubling $\Omega$ to $4\Omega_0$ gives case H20. The high rotation leads to considerable expansion of the clouds, but the destruction of translational kinetic energy in the collision produces a flattened structure which reaches $\rho_{\text{max}} = 5\rho_0$ before re-expanding after $\approx 7.3 \times 10^6$ yr. In this case the maximum density is limited by the finite resolution, and we cannot be completely certain of the result.

Two cases were run with $v_{\text{rel}} = 4c$. Case H21 has $\Omega = \Omega_0$ and case H22 has $\Omega = 4\Omega_0$. The former has density contours in the equatorial plane with $\varepsilon \approx 0.5$ but, again, the resulting gravitational collapse soon produces an essentially spherical mass ($r \approx 8 \times 10^6$ yr). Fig. 10 shows velocity vectors in the equatorial plane and through the rotation axis, for H21. Note that, in both diagrams, we see material raining down on to the core, and a distinct shock.

Case H22 also results in gravitational instability, despite the very high rotational energy of the
clouds. This is due to the fact that, during the collision of clouds with antiparallel rotation vectors, most of the rotational energy is converted into heating the post-shock gas. Because we have used the isothermal equation of state (the cloud is assumed to be optically thin throughout) this energy is free to leave the system, and is assumed to be radiated into space. For the case where the rotation vectors are parallel, the rotational energy produces a rotating configuration. This allows case H22 to result in collapse while case H9 resulted in a rotating bar.

Sequence H23 has $v_{\text{rot}} = 10c$ and $\Omega = \Omega_0$. The evolution here closely resembles that expected for the case of no rotation (see LMPB) because $\gamma = 3.2 > \beta = 0.05$. The configuration compresses and then re-expands somewhat, with focusing along the line joining the cloud centres. The outcome is gravitational instability. Sequence H24 has the same relative velocity but twice the rotation. The flattened interfacial region becomes gravitationally unstable and the collision results in a collapse. We note that this is possible despite the initial energy being positive because of the energy lost (as radiation) through assuming isothermality. Indeed this even allows case H25 to collapse, although it has $\alpha + \beta + \gamma = 4.4$. As previously noted, with parallel rotation vectors the
rotational energy is less efficiently removed and imparts a rotation to the bar. It is worth noting that by $t = 5.3 \times 10^6$ yr the configuration is very flat, with an aspect ratio $\approx 8$. The subsequent evolution to a more spheroidal shape is initially quite slow. By $t = 9.3 \times 10^6$ yr the density contours in the equatorial plane are roughly elliptical with $\epsilon = 0.45$. The object then rapidly evolves to an oblate spheroidal shape by $t = 10.9 \times 10^6$ yr.

In summary, the head-on collision of clouds with antiparallel rotation is more likely to result in collapse than collisions between clouds with parallel rotation vectors. This is because, in the latter case, there is a net angular momentum which, due to conservation, results in a rotating configura-
Figure 11. Outcome of the head-on collisions, plotted as a function of the initial $\beta$ and $\gamma$. The parallel rotation cases are shown in the upper panel, and the antiparallel cases in the lower panel. A dotted circle represents a runaway collapse; a cross represents re-expansion and dispersal; a diamond denotes the formation of a long-lived, rotating bar; a star means that the outcome is uncertain at the time the calculations were stopped.

tion, whereas in the former case there is no net angular momentum. For a given (high!) $\beta$ there is a critical $\gamma$ below which dispersal results and above which the bar becomes gravitationally unstable. The density variation is again roughly $\rho \propto R^{-n}$ where $n \approx 2.5$. The resulting gas clouds typically have $\beta = 0.03$, although it should be stressed that this is evaluated formally by considering the tangential component of the velocity (in the equatorial plane), and that the velocity field does not show any net rotation (see, for example, Fig. 8).

For low values of $\beta$ the configurations are quite like those found for zero rotation, except that the maximum density occurs downstream of the line joining the cloud centres (see Fig. 9d) and the configuration loses reflection symmetry about this line. Finally, Fig. 11 summarizes our results for the head-on collisions in the $\beta-\gamma$ plane. The antiparallel rotation, head-on collisions
generally also do not destroy the bulk motion efficiently. The most efficient example we calculated \( v_{\text{rel}}/c = 10, \frac{\Omega}{\Omega_0} = 4 \) did reduce the kinetic energy to \( \sim 5\% \) of its initial value \( (22\% \) for the mean bulk motion) before collapsing, but this is a rather small part of our overall parameter space.

6 Off-centre collisions

In this section we discuss the results obtained when we move the upper cloud a distance \( b \) perpendicular to the line joining the cloud centres. This addition of a non-zero impact parameter greatly complicates the analysis of cloud collisions. Even with the restricting assumption that the rotation vectors are perpendicular to the direction of relative motion, we now have an effective orbital angular momentum due to the off-centre impact. It is now possible to have the spin and orbital angular momentum arranged to be either parallel (to be called SOP below) or anti-parallel (SOA), provided that the rotation vectors are parallel. If the rotation vectors are anti-parallel, then we have called the result 'spin–orbit mixed' (or SOM).

Because our two clouds are identical, so too are their spin angular momenta. If the cloud spins are mutually parallel, then the total angular momentum of the system is

\[
L = M v b k \pm \frac{3}{8} \Omega R^2 M,
\]

where \( k \) is a unit vector parallel to \( \Omega \). The two terms add when the orbital angular momentum is parallel to the spins, and subtract when anti-parallel. In both cases the relative importance of the spin and orbital angular momenta are given by

\[
\frac{\text{orbital}}{\text{spin}} = \frac{5}{8} \frac{v_{\text{rel}}}{\Omega / \Omega_0} \frac{b}{R}.
\]

In the case where the two spins are anti-parallel, the total spin angular momentum vanishes and \( L = M v b k \). These three cases are dealt with in detail below.

6.1 SPIN–ORBIT PARALLEL

In this case we move the upper cloud a distance \( b \) to the right of the line joining the cloud centres. We have taken \( b/R = 0.5 \) and 1.0, where \( R \) is the initial radius of the clouds \( (= 12.25 \text{ pc}) \). This choice was prompted by the results of LMPS which showed that the interaction is negligible for \( b/R \geq 1.5 \). Let us first deal with the \( b = 0.5R \) cases, namely SOP1–12 in Table 2. First, we discuss SOP2, with \( v_{\text{rel}} = c \) and \( \Omega = 2 \Omega_0 \). The velocity field for this case is shown in Fig. 12 and the density contours in Fig. 13, both in the equatorial plane. The area of the initial impact is where the maximum density occurs, and the resulting shock is seen clearly, especially in Fig. 12(b) and (c). As the material from the clouds continues to rain down on to the core, this maximum density in the bar increases. By \( t = 13.8 \times 10^6 \text{ yr} \) (Figs 12e and 13e) almost all of the material has become part of the bar, and the maximum density is \( 28 \Omega_0 \). The resulting gravitational instability quickly reduces the elliptical structure in Fig. 13(e) to a circular one (see Fig. 13f).

Reducing \( \Omega \) to \( \Omega_0 \) gives case SOP1. Here the shock at the cloud interface is less strong. This is because, with this initial configuration for the clouds, the rotational velocity provides the majority of the velocity difference between the clouds, which results in the shock (see Fig. 12a). A slower rotation means a weaker shock. In fact, the clouds begin collapsing individually, and three density maxima occur early in the evolution: one at the cloud–cloud interface and one at the centre of each cloud. These three all grow while converging at the interface. Soon after merging a single maxima grows as a result of collapse, and the cloud becomes roughly spherical.

Case SOP3 results in dispersal, because of its high rotation \( (\Omega = 4 \Omega_0) \). Material is spread away
Figure 12. Same as Fig. 2, but for case SOP2.
Figure 13. Same as Fig. 3, but for case SOP2.
from the clouds which, at lower rotation, would have come into contact with the high-density core and induced gravitational collapse.

Cases SOP4–6 all have $v_{\text{rel}} = 2c$ and $\Omega = \Omega_0, 2\Omega_0$ and $4\Omega_0$, respectively. Cases SOP4 and SOP5 result in gravitational collapse, and are quite similar to SOP2. The higher $v_{\text{rel}}$ leads to a more flattened bar (as was found in the head-on collisions), again oriented at about 45° to the direction of relative motion. SOP6 results in dispersal, after reaching a maximum density of $4.8\rho_0$, much like SOP3.

Increasing the relative velocity to $4c$ results in collapse for $\Omega/\Omega_0 = 1$ and 2, but dispersal for $\Omega = 4\Omega_0$. In the former two cases the resulting bar rotates somewhat, but completes only $\sim 1/8$ of a turn before the resultant oblate spheroid goes into rapid collapse.

At $v_{\text{rel}} = 8c$ the $\Omega = \Omega_0$ case results in collapse. The bar again rotates through $\sim 45°$ before becoming quite circular in the equatorial plane. With rejection of low-density particles (see Monaghan & Lattanzio 1986: we remove particle $i$ if $\rho_i < 0.005\rho_{\text{max}}$) the final object consists of 90 per cent of the initial mass. Note that this is not equivalent to saying that 10 per cent of the matter achieves escape velocity. Rejected particles represent relatively low-density fluid which may, in reality, fall back on to the core. The actual fate of such material is critically dependent on the characteristics and effect of the intercloud medium. Note that particle rejection is a purely numerical construct, implemented to improve the resolution available during the later stages of collapse, as discussed in Monaghan & Lattanzio (1986).

In the model under discussion, the rejected particles are those which form the left-most (right-most) extreme of the lower (upper) cloud, and thus represent material which is more likely to avoid the collision with the other cloud. This is why we see more mass ‘rejected’ in the high $v_{\text{rel}}$ cases, where gravity has less time to pull these particles on to the high-density core before $\rho_{\text{max}}$ becomes very large.

For $\Omega = 2\Omega_0$ a maximum density of $36\rho_0$ is reached before the density again begins to decrease. We note, however, that in this case a small, spherical core forms which may be unstable. There is copious mass ‘rejected’ (and possibly lost) from the non-overlapping parts of the clouds, and the inclusion of this expanding material inside the calculational grid significantly reduces the resolution. In fact, the lateral spread of the configuration increases so much that the resulting resolution length actually increases rather than decreases. In this case it is not clear that the resolution has been sufficient to determine correctly the subsequent behaviour of the configuration. To check this, we took a model prior to the time of maximum density, and removed all the particles outside $9\,\text{pc}$ from the position of the current maximum density. This left only material which could clearly be classified as the core, and removed almost half of the mass. The evolution of this core was continued, and showed collapse. This result may, however, depend on the removal of the outer 50 per cent of the cloud material. We cannot be certain as to the outcome of this collision, but feel that the result is most likely to be a collapsing core with extensive mass lost ($\sim 30$ per cent?) from the initial clouds. The case with $\Omega = 4\Omega_0$ resulted in dispersal.

The next sequences to be discussed have impact parameter $b = R$. Taking $v_{\text{rel}} = c$ and $\Omega/\Omega_0 = 1, 2$ and 4 gives cases SOP13–15, the first two of which result in a collapsing spheroid. For $b = R$ exactly 50 per cent of each cloud is on a trajectory which would have it not directly involved in the collision, were it not for the rotation and mutual gravitation of the clouds. The total kinetic energy of the system grows, due to the attraction between the clouds, despite the destruction of kinetic energy at the interface. For $\Omega = \Omega_0$, as in case SOP1, the slow relative velocity of the clouds enables them to begin collapsing as individual clouds during the collision. For quite some time ($t \approx 10 \times 10^6 \,\text{yr}$) the density maximum at the cloud centres exceeds that produced at the shock (see Fig. 14). The configuration is never very flat, due to the low $v_{\text{rel}}$ and $\Omega$. This also increases the time-scale of the collision, giving gravity more time to pull the material into a roughly oblate spheroid, which is its final, collapsing shape. For $\Omega = 2\Omega_0$ we again see gravitational instability.
resulting. The larger rotation leads to significant spreading of the clouds during the collision, and to a much more flattened structure at the interface. By $t = 18 \times 10^6$ yr the result is a collapsing spheroid. For $\Omega = 4 \Omega_0$ the clouds disperse, after passing through $\rho_{\text{max}} = 2.8 \rho_0$.

Increasing $v_{\text{rel}}$ to $2c$ leads to similar results. Again, at $\Omega / \Omega_0 = 1$ and 2 we see a gravitational instability develop, and dispersal for $\Omega = 4 \Omega_0$. The qualitative features of the evolution are similar to the corresponding cases with $v_{\text{rel}} = c$.

Cases SOP19–21 have $v_{\text{rel}} = 4c$. Consider first $\Omega = \Omega_0$ (case SOP19). Fig. 15 shows the velocity field, and Fig. 16 the density contours, in the equatorial plane at various times. Note how much of the rotation of the initial clouds is converted into rotation of the resulting bar, but that the subsequent gravitational instability causes the configuration to become that of an oblate spheroid.

Increasing $\Omega$ to $2 \Omega_0$ leads to a configuration which seems to result in dispersal. The rapid
Figure 15. Same as Fig. 2, but for case SOP19.
Collisions between rotating interstellar clouds

4.69, 1.6E+02, 1.8E+01

7.03, 4.7E+02, 5.2E+01

9.05, 1.1E+03, 1.2E+02

10.99, 2.1E+03, 2.3E+02

13.06, 3.1E+03, 3.5E+02

16.11, 1.4E+04, 1.6E+03

Figure 16. Same as Fig. 3, but for case SOP19.
growth of the maximum density reaches a resolution-limited value of 18.5$\rho_0$, before decreasing again. It is possible, however, that, had the resolution been adequate at all stages of the calculation, the evolution would have resulted in gravitational collapse. The outcome of this case is thus unclear, being between the obvious collapse seen for $\Omega = \Omega_0$ and the obvious dispersal seen for $\Omega = 4\Omega_0$.

The last cases we consider here have $v_{\text{rel}} = 8c$ and $\Omega = \Omega_0$ and $2\Omega_0$. As $v_{\text{rel}}$ increases the evolution resembles more closely the case with zero rotation. Hence the clouds tend to push past each other, as found by LMPS. This is clearly shown in Fig. 17 for case SOP22. Here the maximum density reached is $13.6\rho_0$ at $t = 5.9 \times 10^6$ yr (approximately Fig. 17b), which is before the resolution limit is reached. Hence we may be confident that the density peak is genuine. The two clouds now move past each other, and each individual cloud has its velocity field distorted to such an extent that each remnant disperses, as does the central, re-expanding region, which consists mainly of the post-shock gas. This was checked by considering each fragment separately, and following its subsequent evolution independent of the other fragment, thus providing adequate resolution (but removing the gravitational field of the other fragment). The dispersing fragments each had $\alpha = 1.6$ and $\beta = 0.4$. The same is true for case SOP23 with $\Omega = 2\Omega_0$. 

**Figure 17.** Same as Fig. 3, but for case SOP22.
Fig. 18 summarizes the results of this subsection. For high $\Omega$ (i.e. high rotation, or high $\beta$) the clouds spread too quickly for the collision to convert the rotational energy into heat (which is removed from the system). Increasing $v_{\text{rel}}$ decreases this critical $\Omega$ and makes collapse less likely. This is because the time-scale for the collision decreases roughly as $v_{\text{rel}}^{-1}$ and results in re-expansion before gravity can cause an instability. Collapse is less likely for larger $b$ because the amount of material (directly) involved in the collision decreases as $b$ increases. Also, in the case where the spin and orbital angular momentum are parallel, the conservation of this quantity will act as support against gravitational collapse. This is not the case when the spin and orbital rotations are anti-parallel.

These cases are generally even less efficient at destroying bulk motion than were the head-on collisions, particularly for $b = R$. In the latter case there is a class of collisions at large $v_{\text{rel}}/c$ and/or large $\Omega/\Omega_0$ which results in increasing kinetic energy during the re-expansion which drives the eventual dispersal (and thus resides on the original scale). This happens in the head-on collisions
only for $\Omega / \Omega_0 = 4$ and $v_{\text{rel}} / c = 1$, in which cases the individual clouds are not gravitationally stable. The most pronounced destruction in all of these cases occurred for SOP10 (minimum kinetic energy of 16 per cent of the initial value) and for SOP12 where the kinetic energy decreases through dispersal. The collisions with $b = R$ never achieved a minimum smaller than 60 per cent and presumably this trend continues to higher impact parameters. At some point there is a trade-off to be made between the decreasing shock/viscous dissipation and the decreasing gravitational acceleration.

6.2 Spin–Orbit Anti-Parallel

In this section the spins of the clouds are again parallel (clockwise in the figures). By moving the upper cloud a distance $b$ to the left we produce an orbital angular momentum which is in the opposite direction to the two spins. We have again chosen $b/R = 0.5$ and 1.0, and give some parameters for the runs in Table 2.

We first discuss the $b = 0.5R$ cases. With $v_{\text{rel}} = c$ and $\Omega / \Omega_0 = 1$, 2 and 4 we obtain cases SOA1–3, respectively. Fig. 19 shows the velocity field in the equatorial plane for case SOA2. Note how the material at the cloud's edges, which has the largest specific angular momentum, goes into orbit around the collapsing core. The material making up the core is the post-shock gas, which has little angular momentum initially, but carries what it possesses inwards as the collapse proceeds. Calculations were stopped when the maximum density had exceeded $10^7 \rho_0$, at which time the structure is roughly that of a prolate spheroid, with axial ratio $\approx 2$. The rotation is barely discernible, in contrast with case SOP2 (seen in Fig. 13). This reflects the difference in total angular momentum between the cases. As the evolution continues we would expect the elongation in the equatorial plane to decrease, and a collapsing oblate spheroid is predicted as the result.

Fig. 19(b) shows the formation of a bipolar outflow. The driving of this flow is terminated some six million years later by the rotation of the merged cloud (Fig. 19c). Note that the subsequent evolution of this outflow will depend on its interaction with the surrounding medium and the central gravitational pull. The important point is that this flow has a lifetime of several million years and exists prior to the formation of an embedded source (Fig. 19d).

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\( \Omega_0 = 2.08 \times 10^{-15} \text{s}^{-1} \)

P or A indicate that the spins are mutually parallel or antiparallel, respectively.

P, A or M indicate whether the spin and orbital angular momentum are parallel, antiparallel or 'mixed', respectively.

C indicates that runaway collapse results, otherwise we quote the maximum density reached during the interaction. A question mark indicates that, although a maximum was reached, this was resolution-limited and the actual outcome is uncertain.

The approximate duration of the interaction, in units of the cloud's initial free-fall time. Where collapse results it is the time of the runaway. Where a bounce occurs we have quoted twice the time of the bounce. An asterisk indicates that the interaction is much longer than the quoted value, due to very slow re-expansion.

As defined in Section 7.
For $\Omega = \Omega_0$ the runaway collapse occurs earlier, due to the lower rotational support which must be overcome, but is otherwise similar. The $\Omega = 4\Omega_0$ case results in dispersal, after reaching $q_{\text{max}} = 4q_0$ at $t = 8.3 \times 10^6$ yr.

Case SOA4 has $v_{\text{rel}} = 4c$ and $\Omega = 2\Omega_0$. The higher relative velocity leads to a more flattened structure which undergoes gravitational instability. Case SOA5 has $\Omega = 4\Omega_0$ and re-expands after passing through a maximum density of $8.9q_0$ some $7.8 \times 10^6$ yr after the clouds first make contact.

For $v_{\text{rel}}/c = 8$ we find gravitational collapse results for each case considered: $\Omega/\Omega_0 = 1, 2$ and 4, cases SOA6–8. Figs 20 and 21 show the velocity field and the density contours, respectively, in the equatorial plane for case SOA7. The post-shock gas is compressed into a thin, curved 'bar' which soon approaches a more spherical shape. The situation is qualitatively similar for cases SOA6 and SAO8, although the latter is not compressed as much because the high rotation ($4\Omega_0$) leads to significant spreading. Nevertheless, collapse results.
Again we see what could be termed a bipolar outflow, in this case due to the destruction of the overlapping regions of the clouds. Note, however, that the mass contained in this flow is much larger than seen, for example, in Fig. 19. This is not really an outflow from a collision, but is nevertheless a bipolar velocity field.

We have repeated the above set of \((v_{\text{rel}}, \Omega)\) computations with \(b = R\), and these are also listed in Table 2, as cases SOA9–16.

For \(v_{\text{rel}} = c\) there is enough time for each cloud to build-up considerable density gradients during the collision. For \(\Omega = \Omega_0\) and \(2\Omega_0\) the result is gravitational collapse, but the \(4\Omega_0\) case disperses. As \(\Omega\) increases the post-shock gas assumes a more flattened shape, but during the
Figure 21. Same as Fig. 3, but for case SOA7.

collapse it takes on the configuration of an oblate spheroid. In the collapsing models we again find the density varying as the radius to the power $\approx -2.5$.

Cases SOA12 and 13 have $v_{rel} = 4c$, and $\Omega/\Omega_0 = 2$ and 4, respectively. The former results in a collapsing, oblate spheroid. Case SOA13 reaches a resolution-limited maximum density of $8.1\phi_0$ at $t = 8 \times 10^6$ yr, before re-expanding. It is possible that, had the density maximum not been restricted by the resolution, this model may have resulted in collapse.

The final three cases of this subsection have $v_{rel} = 8c$ and $\Omega/\Omega_0 = 1, 2$ and 4. For $\Omega = \Omega_0$ the high velocity combined with the large impact parameter leads to the dispersal of the overlapping material. The non-overlapping material forms two blobs (much like case SOP22, shown in Fig. 17). Much of the initial angular momentum is now contained in the orbital motion of these fragments, but nevertheless, each has $\beta = 0.4$. The two lumps eventually disperse, due to their distorted velocity field, and the inability of the reduced mass in each fragment to hold itself together.

For $v_{rel}/c = 8$ and $\Omega/\Omega_0 = 2$ or 4 we find a collapsing structure. The case with lower rotation forms a long bar which rotates through $\sim 45^\circ$ before becoming an oblate spheroid. With $\Omega = 4\Omega_0$ the maximum density levels off at $\sim 16\phi_0$ for $\sim 6 \times 10^6$ yr while the pre-shock material rains down.
Collisions between rotating interstellar clouds

\[ T = 6.83 \times 10^0 \]
\[ V = 3.02 \]

\[ \text{Figure 22. Same as Fig. 2, but for case SOA16.} \]

on the expanding post-shock gas (see Fig. 22). Eventually the increasing mass of the core drives a runaway collapse.

Once again the destruction of bulk motion is not efficient in these collisions. The lowest minimum was \( \sim 20 \) per cent for cases SOA7 and SOA13. Cases SOA5 and SOA14 also reach 20 per cent but then increase during the following dispersal.

Fig. 23 contains a summary of the results of this subsection, and should be compared with Fig. 18. We see that, as expected, it is easier to induce collapse in the case when the spin and orbital angular momentum are antiparallel, because the total angular momentum content is reduced compared to that of parallel spin and orbital vectors. Conservation of the total angular momentum determines how much rotational support must be overcome by gravity before collapse can occur, and the lower total \( |L| \) means less of a hurdle for the gravitational field to overcome. These effects are much more noticeable at \( b = R \) than at \( b = 0.5R \), because \( L \propto b \times v \). We feel that the present calculations have merely scratched the surface of a very fruitful area for further research, and we intend to study further the interchange of spin and orbital angular momentum in cloud-cloud interactions in a subsequent paper.

6.3 Spin–Orbit Mixed

In this case the spins of the two clouds are antiparallel, which means that the orbital angular momentum is parallel to the rotation of one cloud and antiparallel to that of the other. The total angular momentum is entirely due to the orbital motion, because the spin angular momenta cancel. Because of the high degree of symmetry we have imposed (e.g. identical clouds, identical rotations) all ‘mixed’ configurations are equivalent. We now discuss the various cases, beginning with those having \( b/R = 0.5 \).

We have taken \( \Omega/\Omega_0 = 1, 2, 4 \) for each of \( v_{rel}/c = 1, 2, 4 \) and 8. These form cases SOM1–12 in Table 2 (cases SOM13–24 are equivalent, but have \( b = R \)).

Figs 24 and 25 show the evolution of case SOM2, with \( v_{rel} = c \) and \( \Omega = 2\Omega_0 \). Fig. 24 shows the
velocity field, and Fig. 25 the density contours, both through the equator. The elongated structure shows only slight rotation. Inside \( r = 8 \) pc we have \( \Omega = \pm 2.8 \Omega_0 \), and thus there is no net rotation (as expected). The velocity component directed inward (toward the central density maximum) varies roughly as \( d\Omega / dt \propto - \Omega^3 \). The density variation follows the approximate law \( \rho \propto \Omega^2 \). The structure soon exhibits circular density contours in the equatorial plane, and the final configuration is a collapsing, oblate spheroid with \( \alpha = 0.7 \) and \( \beta = 0.1 \). Note that Fig. 24 again shows a unipolar outflow.

For \( \Omega = \Omega_0 \) the results are quite similar, as would be expected because the spin angular momenta (\( \propto \Omega \)) cancel, and the total \( L \) is simply the orbital component. Nevertheless, with \( \Omega = 4 \Omega_0 \) the rapid spreading of the clouds leads to dispersal after reaching \( \rho_{\text{max}} = 3 \rho_0 \) at \( t = 7.6 \times 10^6 \) yr.

Increasing \( v_{\text{rel}} \) to 2\( c \) doubles the total angular momentum content. The rotating structures found for \( \Omega / \Omega_0 = 1 \) and 2 show \( \Omega = \pm 5.5 \Omega_0 \) inside \( r \leq 5 \) pc, which is, as expected, about twice the value found for \( v_{\text{rel}} = c \). Again, increasing \( \Omega \) to 4\( \Omega_0 \) leads to dispersal, although the peak
density obtained is limited by our finite resolution, and we cannot deny the possibility that collapse is the actual outcome.

As we go to larger $v_{\text{rel}}$ we increase both the orbital (= total) angular momentum of the resulting structure and its flatness. For $v_{\text{rel}} = 4c$ the $\Omega = 4\Omega_0$ case disperses, but persists for some $\sim 4 \times 10^7$ yr as a (slowly) rotating bar. For $\Omega/\Omega_0 = 1$ and 2 we find collapse, with the bar evolving to an oblate spheroid. At $v_{\text{rel}} = 8c$ we find all cases ($\Omega/\Omega_0 = 1, 2$ and 4, cases SOM10–12) resulting in collapsing, rotating bars. Typically the inner portion of the bar shows $\Omega = 30\Omega_0$, and the overall configuration has $\beta = 0.2–0.5$.

We have repeated the above calculations with $b = R$. This doubles the total angular momentum, and this extra rotation is clearly seen, for example, in case SOM19 shown in Fig. 26. This case has $v_{\text{rel}} = 4c$ and $\Omega = \Omega_0$. Note the ‘tail’, which extends to the lower part of the figures. The majority of the tail, in the last frame, is enclosed by the density contour at $\rho = Q_{\text{max}}/40$. The

\textbf{Figure 24. Same as Fig. 2, but for case SOM2.}
extension of this tail is the main difference between the cases with $b = R$ and those with $b = 0.5 R$. Increasing $\Omega$ increases the amount of mass in the tail (provided $\Omega \leq 4 \Omega_0$). For lower $v_{\text{rel}}$ the tail is shorter, and carries less angular momentum, eventually falling on to the collapsing spheroidal core. We note the possibility that this tail may, in some cases, be torn off from the main body and/or may contain a Jeans mass. This occurs at higher $v_{\text{rel}}$, so we discuss these runs below.

The final sequences we discuss have $v_{\text{rel}} = 8c$ and $\Omega/\Omega_0 = 1, 2$ and 4. These are cases SOM22–24 in Table 2. With $\Omega = \Omega_0$ the high relative velocity of the clouds pushes them past each other, much like was shown in Fig. 17. The tail, consisting of $\sim 25$ per cent of the total mass, breaks off from the remainder of the cloud. The main blob goes into collapse, displaying $|\tilde{q}/\tilde{\dot{q}}| = 1.5 t_{\text{ff}}$. Fig. 27(a) shows the velocity field, through the equator, relative to the velocity of the particle with the maximum density, and Fig. 27(b) shows the density distribution. Soon after splitting into two pieces the larger object is rotating with a constant $\Omega = 4 \Omega_0$, and has $(\alpha, \beta) = (1.6, 0.06)$. This

Figure 25. Same as Fig. 3, but for case SOM2.
Figure 26. Same as Fig. 2, but for case SOM19.
collapsing spheroid shows an eccentricity = 0.2. Some 10⁸ yr later we see \( \Omega \propto R^{-1} \) outside 1.2 pc (the total radius is \( \sim 5 \) pc). The smaller mass fragment disperses.

Case SOM23 (\( \Omega = 2 \Omega_0 \)) is similar – the low-mass tail separates and disperses while the more massive blob collapses. For \( \Omega = 4 \Omega_0 \) the tail stretches away without breaking off from the main body, which eventually disperses. Nevertheless, the maximum density is resolution-limited, and so the exact outcome is uncertain.

Fig. 28 summarizes these results in the (\( \beta, \gamma \)) plane. The bulk motion is once again not easily destroyed in the presence of the gravitational acceleration. Extreme cases such as SOM12 can lead to minima of \( \sim 10 \) per cent before collapse, but these are exceptional. As before, the destruction is less pronounced at the larger impact parameters.

7 Conclusions and discussion

The most important general result of this paper is the recognition of the importance of both spin and orbital angular momenta and spin–orbit coupling in determining the outcome of a cloud–
cloud collision. In particular our calculations show that the lifetime of a given merged structure (which may be terminated either by dispersion or runaway collapse) is longest when an orbital dynamical time, derived from the centre-of-mass angular frequency \( \Omega_{\text{CM}} \) below is comparable to the free-fall time. This condition is closely related to the initial virial condition of Section 3.2, but it is preferable because it remains physically relevant even after the extensive post-shock dissipation.

Thus, relative to the centre-of-mass, the \( z \)-component of angular momentum is

\[
L_{\text{CM}} = M v_{\text{rel}} \frac{b}{2} + \frac{4}{5} M \Omega R^2 k,
\]

where

\[
k = \begin{cases} 
0, & \text{for antiparallel spins;} \\
+1, & \text{for parallel spins, parallel to the orbital angular momentum;} \\
-1, & \text{for parallel spins, antiparallel to the orbital angular momentum.} 
\end{cases}
\]
Figure 29. Outcomes of the collisions as a function of tan(i) and $\Omega_{\text{CM}}/\Omega_0$. Symbols are as defined in Fig. 11. The head-on, SOP, SOA and SOM cases are shown in panels (a), (b), (c) and (d), respectively. [Note that for (d) we plot $(\Omega/\Omega_0)/(v_{\text{rel}}/c)$ against $\Omega_{\text{CM}}/\Omega_0$ since tan(i) = 0 for all the SOM cases.]

The moment of inertia, about the centre of mass, is

$$I_{\text{CM}} = 2 \left[ \frac{2}{5} MR^2 + M \left( \frac{b}{2} \right)^2 \right],$$

whence

$$\frac{\Omega_{\text{CM}}}{\Omega_0} = \frac{|L_{\text{CM}}|}{I_{\text{CM}} \Omega_0} = \frac{(\Omega/\Omega_0) k + \frac{3}{2}(v_{\text{rel}}/c) b/R}{1 + \frac{3}{2}(b/R)^2},$$

where we recall from the definition of $\alpha$ and $t_{\text{ff}}$ that

$$\Omega_0 = \sqrt{\frac{8\alpha}{15}} \sqrt{GQ} = \frac{1}{t_{\text{ff}}} \sqrt{\frac{\alpha \pi^2}{20}},$$

so that $\Omega_0 t_{\text{ff}} \approx 0.5$ for our clouds. Our results show that $\Omega_{\text{CM}}/\Omega_0 \approx 2.5$ (recall that $\Omega_{\text{CM}} \sim 1/t_{\text{ff}}$) divides dispersing collisions ($\Omega_{\text{CM}}/\Omega_0 > 2.5$) from the collapsing cases ($\Omega_{\text{CM}}/\Omega_0 < 2.5$), as shown in Fig. 29. This expression is also valid for the equal-mass cases considered by LMPS (but fails when $b/R \gg 1.5$).

In cases like SOP20 and SOP14, where $\Omega_{\text{CM}}/\Omega_0$ is close to the transition value (2.8 and 1.6) we find that the structures tend to be long-lived. This fits nicely with the arguments of Henriksen & Turner (1984) and with the observations that a virial condition should hold for each stage in a
cascade that is evolving on a time-scale larger than the free-fall time. In fact we observe that if, at every stage in the hierarchy the lifetime exceeds the free-fall time by the same factor \( \tau = \Delta t / t_f \), then the total lifetime is

\[
T = \tau t_f(0) \frac{1 - r^{N-2}}{1 - r}
\]

where there are \( N \) stages in the hierarchy, each with a scale change of factor \( r \), and \( t_f(0) \) is the free-fall time of the largest scale. For a large number of stages \( N \), and using \( \tau \approx 4 \) from the results of our long-lived cases, we find \( T \approx 10 t_f(0) \) for \( r = 1/2 \). It is not easy to change the value of \( r \) (suggesting a binary interaction; see also Henriksen 1986) unless \( r \) can be made substantially larger. We suspect that this can be the case by judicious choice of \( \Omega_{CM}/\Omega_0 \).

The head-on calculations presented here clearly pose a related question. Using the criterion suggested above, sequences H1–H15 of Table 1 should lead to collapse for \( \Omega < 2.5 \Omega_0 \). Inspection shows this to be the case, but also reveals intriguing variations in the lifetimes (\( \Delta t / t_f \)) of the merged structures. Thus H2 persists for 2.4\( t_f \) but H7 for only 0.9\( t_f \). We observe that this variation correlates (approximately) with the angle that the relative velocity makes to the head-on direction at the first point of contact (in the centre-of-mass frame). This angle is given by

\[
i = \tan^{-1} \left[ \frac{2(\Omega/\Omega_0) k \sqrt{1 - b^2/4 R^2}}{v_{rel}/c + (\Omega/\Omega_0)(b/R) k} \right]
\]

where \( k = \pm 1 \), as before. Here \( b = 0 \) and thus \( i = \tan^{-1}(4) \) for H2 and \( i = \tan^{-1}(1/2) \) for H7. Thus, the more 'tangential' the collision (as in H2) it seems the longer will a transitory structure persist. This effect is presumably related to the higher radiative losses associated with the lower \( i \) collisions, and also the tendency for higher \( i \) to lead to more rotational support in the final structures.

Sequences H16–H25 should all collapse relatively rapidly by either of the above criteria, and indeed they do except for \( \Omega/\Omega_0 = 4 \) when each of the clouds is individually rotationally unstable. Sequences H21 and H23 reveal a relatively weak dependence on the relative velocity alone.

Sequences H9 and H12 are also noteworthy in that despite being individually rotationally unstable the colliding clouds form long-lived rotating bars. In these cases \( i = \tan^{-1}(2) \) and \( i = \tan^{-1}(4/3) \) so that the collisions are rather oblique. In such cases it may be fair to say that the shear viscosity between the clouds acting throughout the interaction has transferred spin angular momentum to orbital angular momentum. Although the merged object has been left in a slowly dispersing state, it is possible that further interactions with other asymmetric structures could remove sufficient angular momentum to permit the structure to collapse to another scale. Fig. 7 shows this remarkable structure for H9. The bar is actually a sheet, due to its substantial extent parallel to the rotation axis. The rotation period is some \( 74 \times 10^6 \) yr and \( |q/\rho| = 40 \times 10^6 \) yr or \( \approx 5 t_f \). Thus the structure can rotate through about 180° before dispersing. Substantial spin–orbit exchange has occurred.

Sequence H1, as shown comprehensively in Figs 2–5, is typical of our head-on calculations. Because of the obliqueness of the collisions \( [i = \tan^{-1}(2) = 63°] \) the interaction is a relatively long-lived collapse. In this marginally supersonic sequence, relatively little rotation is communicated to the merged object. Fig. 4(a) is important in that it shows that the kinetic energy increases through the interaction, which demonstrates the possibility of gravitational driving for the supersonic motions (turbulence). This is a typical result of our calculations, and indicates a remarkable resistance to dissipation. As remarked previously, the increase is common for \( v_{rel} \leq 2c \). We find that in the initial stages of the collision the kinetic energy increases for \( v_{rel} \leq 6c \) before eventually decreasing. We stress, however, that even for high relative velocities we do not always find efficient dissipation of kinetic energy.
Figs 8–10 illustrate sequences H17 and H21. These should collapse according to our $\Omega_{CM}/\Omega_0$ criterion, and this is verified by our explicit calculations. Figs 8 and 9 display essentially zero rotation in the merged object (which again suggests strong viscous coupling, leading to intermingling of the cancelling spins) and a tendency to develop ‘upstream’ density filaments. Sequence H21, shown in Fig. 10, is remarkable in that it shows rapid collapse of a supersonic encounter. Fig. 10 shows essentially free-fall on to an isothermal core. Matter ‘accreting’ on to an isothermal core is a common result of our calculations, and is shown even more dramatically in Fig. 22. We note here that an analysis similar to that of Stahler, Shu & Taam (1980a, b, 1981) for the case of spherical symmetry would be required to determine the subsequent evolution of the structure. This is greatly complicated in the present case, however, because of the lack of geometric symmetry in the configuration.

The off-centre encounters continue to confirm our general picture based on $\Omega_{CM}$, $t_f$ and $i$. Sequence SOP2 is shown in detail in Figs 12 and 13. We observe a relatively long-lived collapsing structure as both $\Omega_{CM}/\Omega_0 = 2$ and $i = \tan^{-1}(2/3)$ suggest. Interestingly, this interaction should be longer lived than SOP14 [with $\Omega_{CM}/\Omega_0 = 1.6$ and $i = \tan^{-1}(2/3)$] but it is not. We attribute this to the difference in impact parameters ($b = 0.5 R$ for SOP2 and $1.0 R$ for SOP14). Thus the collision seems to be more effective at de-spinning the central merged object when $b$ is smaller. A similar comparison may be made between SOP5 and SOP22, etc. Sequences SOP19 (Figs 15 and 16) and SOP20 illustrate the sensitivity of the result of a supersonic collision to the various parameters. SOP19 is a moderately long-lived collapsing, rotating bar [$\Omega_{CM}/\Omega_0 = 2.15$ and $i = \tan^{-1}(\sqrt{3}/5)$] while SOP20 [$\Omega_{CM}/\Omega_0 = 2.8$ and $i = \tan^{-1}(1/3)$] disperses slowly. Sequence SOP22 (Fig. 17) has $\Omega_{CM}/\Omega_0 = 3.7$ and $i = \tan^{-1}(1/3,5)$ and disperses into two separate clouds, each of which disperses as a result of the interaction. Thus we have an example of floccule–floccule annihilation. The stronger viscous coupling existing in SOP10 and especially SOA6 seems to be enough to change this result to one of coalescence. In a terminology which is both obvious and suggestive, we can say that SOA6 is a ‘three-wave’ interaction. We have not found cases, however, where the fragments separate but collapse individually (a four-wave interaction). This points to agglomeration as the preferred mode of interactions in cloud hierarchies.

Note that this coalescence is not necessarily collapse, and may be a temporary state existing prior to re-expansion and subsequent dispersal. Nevertheless, if two clouds lose their individual identities for a collision time $t_c$ (a few free-fall times for our clouds) then a subsequent collision may occur and the original two clouds then form a transitory ‘cloud’ interacting with a third. This is consistent with our results (see $\Delta t$ in Tables 1 and 2). This does not contradict the results of LMPS for non-rotating clouds. The increased interaction time in our present results is due to the added support provided by rotation. Without rotational energy the clouds tend to be disrupted by the collisions, especially at high relative velocities (which also decreases $\Delta t$, the duration of ‘coalescence’, well below $t_c$).

We see that sequences SOA2 and SOA7 (Figs 19–21) are both three-wave interactions with SOA7 collapsing considerably faster than SOA2, as suggested by the parameters. Generally for these antiparallel spin–orbit interactions the coalescence is easier than for the parallel case as $\Omega_{CM}$ and $i$ suggest, and as Figs 18(b) and 23(b) reveal. Compare, also, case SOA8 with SOP12 and case SOA15 with SOP23. These pairs differ only in their value of $k$ and show the importance of the spin–orbit angular momentum distribution. The resulting coalescence is often intriguingly slow, however, as in SOA8. Sequence SOA16 is somewhat unexpected a priori, although $\Omega_{CM}/\Omega_0 = 8/13$ and $i = \tan^{-1}(\sqrt{3})$ predict the observed slow coalescence. Fig. 22 reveals the pronounced oblique shock driving the collapse.

The cases with ‘mixed’ spin and orbital angular momentum vectors are also specified in Table 2. Note that $i = 0$ for all of these cases. Moreover, $\Omega_{CM}$ seems to be a less precise indicator of the result of the interaction in the sense that it is generally small in these examples and, more
importantly, predicts no dependence on $\Omega$. Admittedly, this last rule is broken only when $\Omega = 4\Omega_0$, and the individual clouds are rotationally unstable. The interactions are often surprisingly slow given their small value of $\Omega_{\text{CM}}/\Omega$ and $t = 0$. Figs 25 and 26 may reveal a partial answer. The predominant new effect seems to be the distortion which tends frequently to the development of a long tail, which sometimes detaches. When it does we have found it to disperse in all calculations so far. Thus some of the total angular momentum is spun off in a fragment (an asymmetrical four-wave interaction) leading presumably to a higher tolerance for $\Omega_{\text{CM}}$ in the coalescing object. It will be of interest to continue these calculations at higher $v_{\text{rel}}/c$ and possibly for clouds of unequal masses.

Fig. 27 shows the velocity field for SOM22 at a late stage. It is remarkable that such an ordered velocity field develops in a coalescing fragment. Just after the separation this fragment is in approximate solid-body rotation. Although the core continues to rotate with $\Omega = \text{constant}$, the outer region develops an $\Omega \propto R^{-1}$ distribution. In the majority of the cases which resulted in collapse, we find the core rotating roughly as a solid-body surrounded by matter with $\Omega \propto R^n$ and $n = -1.4$. One expects $n = -1.5$ for particles in Keplerian orbit about a constant spherical mass. This appears to be roughly the case in these sequences, and thus the principal support of the merged structures is rotational rather than thermal. Such scenarios have also been studied self-similarly by Henriksen (1986).

Note that recent observations of the collapsing core of G10.6$-$0.4 (Keto, Ho & Haschick 1987) are consistent with this picture. They suggest (note that the measured errors are large) Keplerian rotation outside the high-density core, while inside the rotation law is $\Omega \propto R^{-1}$, although they believe that this may be an upper limit to the steepness of the radial dependence. The details of the rotation near the centre are dependent (in our calculations) on the strength of the turbulent gas viscosity (see below), but the decrease radial dependence of $\Omega$ in the core, together with the Keplerian profile outside the core, plus a density distribution going like $R^{-2}$ is support for the validity of the dynamical collapse scenarios discussed in this paper.

The solidly rotating central region suggests that the assumption of uniform rotation which we have made in this paper may tend to perpetuate itself. As stated earlier, it is possible that this may be due to the artificial viscosity. Numerical tests with varying strengths of the artificial viscosity (i.e. varying $\alpha$ and $\beta$; see LMPS) have shown that the deviation of $\Omega(r)$ from a constant value increases as the viscosity decreases. Nevertheless, the $\Omega(r)$ law seen near the centre of our structures is still much less steep than the Keplerian law seen outside the high-density core. Of course, some viscosity is actually present in real clouds, and we believe that, although the details clearly depend on this, the artificial viscosity we have used simulates an eddy viscosity of the correct physical magnitude. In any case, the assumption of solid-body rotation in the original clouds is one of the principal limitations in this work. It tends to underestimate the ability of rotation to halt the collapse of the inner portions of the cloud (or the same cloud at small radius). Clearly a law such as that suggested in Henriksen & Turner (1984), where $\Omega \propto R^{-1/2}$ and $\rho \propto R^{-1}$ initially, should be investigated.

Because of our isothermal assumption, much of the initial energy in the configurations is radiated into space. Nevertheless, we have found an approximate criterion for determining the outcome of the collisions which is based on $\beta$ and $\gamma$ (recall that we have considered only one value of $\alpha$). We find collapse if $\gamma < \gamma_c$ and dispersal for $\gamma > \gamma_c$, where $\gamma_c = 4 - 8\beta$.

Due to the small number of different values of $\beta$ considered, however, we must stress that this expression is only approximate. In particular, the $\beta$-dependence is very uncertain and $\gamma_c = 3.5 - 5\beta$ works equally well, and $\gamma_c = 5.5 - 20\beta$ nearly as well! No simple expression exists which accurately predicts the outcome for all collisions considered in this paper. One may set $\beta = 0$ in these

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expressions to give a condition for the outcome of collisions without any angular momentum (hence no spin and $b=0$). This results in $\gamma \leq 4$, or $v_{\text{red}}/c \leq 12$, which may be compared with Gilden’s (1984) two-dimensional calculations of head-on collisions. Allowing for the difference in the definitions of the Jeans mass, we find that (in Gilden’s notation) we predict collapse for $v_{\text{red}}/c = M < 12$ in our models, which have $(\lambda_0/d_0) = 1.7$. Gilden’s heuristic theory would predict collapse for $M < 12$ if $(\lambda_0/d_0) = 1.95$, an agreement which we consider quite satisfactory.

In many, but not all, cases the collisions generate collimated splashes which would be seen as outflows. We can produce both unipolar and bipolar flows of low-velocity (a few km s$^{-1}$~$\sim v_{\text{red}}$), post-shock gas. This may or may not be coexistent with a central luminosity source. In a complex experiencing frequent cloud collisions we would expect a wide variety of these flows. Note that collisions between non-rotating clouds do not produce these flows (see LMPS), but we find them to be a common feature of our off-centre collisions. Whether their characteristics are appropriate to observed flows will be a subject of further study.

We have received a paper from Monaghan & Varnas (1988) in which they treat directly an ensemble of 48 clouds. The clouds are started with a randomly directed supersonic velocity that is either subvirial, virial, or supervirial, in terms of the gravitational energy of the initial ensemble. Their prime objective is to test the contention of Scalo & Pumphrey (1982) that the collapse time of the supersonic ensemble can be substantially larger than the free-fall time. This they conclude is not the case in general. However, in the virial case (their sequence B) substantial structure persists on the original scale even after the formation of collapsing substructures by most of the mass. We suspect that had their original ensemble been rotating and/or had their original clouds been given some spin, longer lived structures, similar to those found in our two-body encounters, would have arisen. Recall that we found that rotational energy provides much of the support found in many of the final configurations.

It is very intriguing to see many rotating structures form in the work of Monaghan & Varnas (1988), typically with $\beta \leq 0.25$. Because their clouds initially have no rotation, this has been induced by the gravitational interaction between the clouds. It would be very useful to know the radial distribution of $\Omega$ and $q$ in these clumps. Note also that their fig. 8(b) clearly shows a head-on collision between two clouds with anti-parallel rotation vectors, a case considered in this paper (see sequences H16–H25).

It is significant that we find that rotation can lead to oblique incidence and hence to long-lived structures in accordance with the basic idea of Scalo & Pumphrey. However, we do not find consistently an effect of the size ($\sim 20 t_{\text{ff}}$) they suggest, in accordance with sequence B of Monaghan & Varnas (1988).

A related question is that of the dissipation of the bulk supersonic motion. We have found this to be remarkably difficult in the presence of the gravitational driving, although not entirely impossible. It occurs to the 30 per cent level in the mean, but only for a very supersonic combination of spin and orbital motion and at low impact parameters. Of course, if the same cloud survived to suffer many such collisions with just the right set of parameters, then the bulk motion could well become subsonic. However the most efficient destruction of kinetic energy precedes the collapse of the merged object and non-merging objects have not had their bulk motion reduced by any significant amount. Thus clouds should not be thought of as retaining their identity indefinitely, like billiard balls (see also Monaghan & Varnas 1988).

We have thus shown the influence and importance of rotation on the dynamics of interstellar cloud collisions, and provided an approximate condition for predicting the outcome of such collisions. We believe that rotation and gravitational acceleration are essential components of the dynamics of interstellar cloud complexes. Direct simulations place an understanding of these complexes within our reach for the first time.

Moreover, although our models were not chosen with such applications in mind, the collision of
rotating clouds is of importance in other contexts. A sustained attempt to understand the formation of GMC complexes, by the successive merging of smaller clouds biased by spiral arm gravitational potentials, is found in the work of Hausman (1981), Kwan & Valdes (1983), Roberts & Hausman (1984), and Tomisaka (1984, 1986). A key element in such theories is the degree of ‘inelasticity’ of cloud–cloud collisions, and indeed some previous work has suggested that merging might be improbable when there is supersonic relative motion (Hausman 1981; LMPS). Although we have not searched the whole of ‘spin-space’ in order to properly define the merger cross-section, it is already clear from our results that rotation in combination with efficient cooling markedly enhances the probability of forming merged structures, both those undergoing gravitational collapse and those experiencing a long-lived ‘bar’ mode (‘dissipative structures’ is an apt description of these objects, after Prigogine & Stengers 1984). This supports the coagulation theory of GMC formation as well as providing the internal mechanism for the turbulent cascade discussed earlier.

The origin of the primordial clouds may lie with thermal instabilities in the collapsing protogalaxy. In this case the cloud–cloud collisions may even play a decisive role in determining the ultimate shape of the galaxy (e.g. Larson 1969; Carlberg 1984a, b). Indeed, the ratio of the number of collisions causing gravitational collapse (leading ultimately to star formation) to the number forming dissipative structures has been found empirically by Carlberg to be a critical parameter. Our calculations can determine this ratio a priori, in principle.

References