The origin and internal structure of the magnetic fields of the CP stars

David Moss Mathematics Department, The University, Manchester M13 9PL

Accepted 1988 August 18. Received 1988 August 18; in original form 1988 June 13

Summary. The dynamo and fossil theories for the origin of the magnetic fields of the CP stars are critically examined. It is deduced that the observed fields are unlikely to be the product of a contemporary dynamo situated in the convective core. The consequences for the fossil theory of an inhomogeneous, rope-like, internal field structure are investigated with the aid of self-consistent, time-dependent numerical models. It is found that models with inhomogeneous internal fields have relatively homogeneous surface fields. The observable effective and surface fields calculated for the ‘one-rope’ models are quite similar to those of the conventional displaced-dipole/oblique-rotator model. However, models in which the internal field is concentrated into two or more ropes of comparable strength seem unlikely to be consistent with the body of observational evidence, although the existence of such structures cannot be excluded in all circumstances.

1 Introduction

The study of stellar magnetism, in common (until recently) with all problems of stellar structure, faces the task of deducing information about the properties of the stellar interior from observations relating to a very small fraction of the stellar mass. As far as problems of stellar structure on or near the main sequence are concerned, the observed stellar properties, basically luminosity and radius as a function of mass and age, can be expected to be fairly closely correlated with the interior properties. Moreover, in the last decade or so it has become possible to probe the solar interior fairly directly by observing (or not) neutrinos, and by studying short-period oscillations, which in principle can provide some verification to theoretical models (lack of solar neutrinos not withstanding!). In contrast, magnetic fields, for nearly all stars, cause at most a minor perturbation to the stellar structure. Fields of the lower main-sequence stars are believed to be generated in the outer convection zone and to depend fairly sensitively on the depth of the convection zone and the angular velocity (e.g. Noyes et al. 1984). In contrast, the fields of the magnetic chemically peculiar stars are more deep-rooted and, prima-facie, it seems unnecessary that there should be a strong correlation between surface and interior fields.

None the less, it is of interest to determine how much freedom the observations of the surface field allow in the internal field structure. This question cannot be separated from a
discussion of the origin of the fields of the CP stars. Three explanations for the strong magnetic fields seen at the surface of these stars have been discussed in some detail in the last few years—the dynamo, fossil and battery theories.

Biermann’s original (1950) battery theory has been developed in a somewhat different form by Dolginov (1977). Such a mechanism as this may well play a role in maintaining toroidal fields near the surface in regions of chemical inhomogeneity in the presence of a background poloidal field generated by an alternative mechanism (e.g. Mestel & Moss 1983), but it seems unlikely to be able to explain the complete field distribution of the magnetic CP stars. In its present state of development this theory does not appear to make any detailed testable predictions, and thus it will not be discussed further in this paper.

In the simplest terms the dynamo theory proposes that the observed magnetic fields of the CP stars are surface manifestations of a hydromagnetic dynamo operating in the convective core of these middle/upper main-sequence stars (e.g. Krause 1983), whereas the fossil theory claims that the fields are the slowly decaying relics of either the interstellar field swept up during star formation, or of a dynamo field built during a convective phase of pre-main-sequence evolution. Moss (1986a) gives a more detailed review of these rival theories. Sections 2 and 3 of this paper address the problem of discriminating between the fossil and dynamo theories. The basic feasibility of each mechanism is examined, some difficulties identified, and results of model building are compared with observations.

In the remaining sections of the paper a form of the fossil theory is assumed to be valid and models are constructed, following the general scheme of Moss (1984, 1987a), to investigate how closely the interior field structure is constrained by observations of the surface field. For example, Preston (1971) states that the surface field is not concentrated in a few, restricted, regions where the field strength is very much greater than the mean strength measured. If valid, what does this piece of evidence tell us about the interior field structure? In particular, in this work, the interior field at age zero is assumed to take the form of one or two non-axisymmetric flux ropes, idealizations of a field that might remain after a largely convective period of pre-main-sequence evolution, or as the relic of a ‘turbulent dynamo’ which produced a field of spatially intermittent structure. Surface and effective field variations are constructed for these models.

2 Dynamo theory

There is now a strong consensus that rotating, convecting, electrically conducting fluid can drive an effective dynamo. The properties of the directly observed magnetic fields of the Earth, Sun and a few other lower main-sequence stars, together with those of the inferred fields of a much larger number of cool dwarfs, are all consistent in general terms with the presence of dynamo action. The plethora of theoretical dynamo models produced in the last 20 yr give firm support in broad terms, although a detailed explanation of, for example, the solar cycle, is still lacking (see, for example, Moss 1986a). Thus the ability of a region such as the convective core of a CP star to generate a large-scale magnetic field by a dynamo process, given suitable statistical properties of the turbulence and values of the dynamo number or similar parameters, will not be questioned here. The problem to be examined is whether such a mechanism can explain the field observed at the stellar surface.

The first question that must be addressed is whether a core-generated field can manifest itself at the stellar surface within a time of order $5 \times 10^6$ yr (the age of the magnetic CP stars in the OB1 association). Clearly, classical diffusion is much too slow—with a mean temperature of $10^7$ K and conductivity $\sigma \sim 10^7 T^{3/2}$ cgs, the slowest decaying mode has a time-scale of order of the age of the Galaxy. Schussler & Pähler (1978) discuss a rather more rapid ‘forced’
diffusion (whose physical mechanism is not altogether clear), but characteristic time-scales are
still of order $3 \times 10^6$ yr or more – much too slow to be relevant to any but the oldest and least-
massee magnetic CP stars. Clearly some more rapid mechanism of field transport is required.
If field is to penetrate the stellar surface within $5 \times 10^6$ yr, a rise speed of $\sim 10^{-3}$ cm s$^{-1}$ is
required. An immediate suggestion is magnetic buoyancy (e.g. Krause 1983; Dolginov 1986).
In a region stable against convection, however, buoyancy of individual flux tubes is a slow
(‘Eddington–Sweet’) process, see, for example, Parker (1979). He gives the estimate

$$u = k \frac{F}{B^2} \frac{B}{8 \pi p} \left( \frac{\lambda}{R_T} \right)^2$$

(2.1)

for the rise velocity of a flux tube of radius $R_T$. [$F$ is the radiative flux, $\lambda$ the temperature scale
height, $p$ the ambient pressure and $k=0(1)$.] Substituting values appropriate to a fractional
radius 0.2 in a $2.5 M_\odot$ zero-age main sequence model (e.g. Schwarzschild 1958) into (2.1) gives
a rise time of

$$t_{\text{rise}} \sim 3 \times 10^{12} \frac{R_T}{B_6^2} \frac{B_6^2}{\lambda} \text{yr},$$

(2.2)

where $B_6$ is the field strength in units of $10^6$ G. Typically, deep in the interior $\lambda \sim R_\ast$, the stellar
radius. This demonstrates that the factor $(R_T/\lambda)^2$ must be small if $t_{\text{rise}} \leq 5 \times 10^6$ yr but, even if
$R_T \sim 10^{-2} R_\ast$, $t_{\text{rise}} \sim 3 \times 10^8 B_6^{-2}$, so a field strength of at least $10^7$ G is needed.

This leads to the question of the maximum strength of a field generated by dynamo action in
a convective core. A non-linear dynamo will be limited by dynamical effects, possibly to
approximate equipartition with the kinetic energy of the convection. [There is a small amount
of numerical evidence that this may overestimate the field strength, e.g. Gilman & Miller (1981);
Gilman (1983)]. Turbulent convection is, of course, poorly understood even in the absence of
dynamically significant magnetic fields, but the simple analysis given in section 7 of
Schwarzschild (1958) can be used to provide an estimate of the equipartition field. Equations
(7.5) and (7.7) of Schwarzschild lead to the estimate

$$v_c^3 \approx \frac{F\bar{r}GM_\ast}{r^2p}$$

(2.3)

for the speed $v_c$ of the convective elements, where $\bar{r}$ is the mean displacement of a fluid
element. Again, inserting values appropriate to a $2.5 M_\odot$ ZAMS star, with $\bar{r}$ equal to the core
radius $\sim 0.2 R_\ast$ (an upper limit), (2.3) gives:

$$B_{\text{eq}} \sim 4\pi \rho \nu v_c \sim 10^5 \text{G},$$

(2.4)

This figure is insensitive to the stellar mass assumed. Another possible limitation on the field
strength is given by equating the Coriolis and Lorentz forces,

$$\frac{|B_x B_y|}{4\pi D} \sim \rho v_c \Omega,$$

(2.5)

where $D$ is a typical length scale. With a naïve model of the dynamical back-reaction of the
Lorentz forces on the small-scale fluid motions, this mechanism is only likely to be important
in a turbulent core if the cut-off given by (2.4) has not already operated, i.e. if (2.5) gives a
smaller limit than (2.4). With plausible numbers,
\[ |B_{eq}|/|B_p| = 0(10^{-1})(B_p/B_p)^{1/2}. \]

(For an \( \alpha^2 \) dynamo \(|B_p/B_p| \sim 1\), whereas for an \( \alpha\omega \) dynamo \(|B_p/B_p| \) may considerably exceed unity.) Other possible limiting mechanisms, which may operate before (2.4) is effective, include suppression of differential rotation and flux-loss due to buoyancy of toroidal flux tubes. Thus (2.4) may be a generous upper limit, and it appears unlikely that relation (2.2) can yield \( t_{rise} \leq 5 \times 10^6 \) yr unless \( (R_p/\lambda) \) is very small indeed. This would lead to a picture of the field escaping from the vicinity of the core in the form of fine ‘strands’ (flux ‘threads’ rather than ‘ropes’), that manage to maintain their identity through much of their ascent through the stellar interior, but which nevertheless combine to give a surface field with the general large-scale order inferred from the observations. It is difficult to understand how \( v_c \) could be completely unaffected by a magnetic field of approximately equipartition strength, but in the ‘worst case’ when the effects of the field on the small-scale motion are ignored completely, (2.5) would give

\[ |B_p| \lesssim 3 \times 10^5(B_p/B_p)^{1/2} G \]

using values for a middle main-sequence star with a period of about 5 days. This estimate, whilst larger than \( B_{eq} \) (equation 2.4), would still require \( R_p/\lambda \) to be very small for plausible values of \(|B_p/B_p|\).

In a steady state, if the core field is connected to the surface field then the signed surface flux cannot exceed the signed core flux. A fractional core radius of 0.2, together with the limit (2.4), then implies a surface field of no more than about \( 4 \times 10^3 \) G, and so a maximum measured effective field of similar magnitude. The figures are similar whether the core field is thought to be predominantly poloidal, or predominantly toroidal but then twisted so as to give the observed poloidal field. If any field lines are close within the radiative envelope (as seems plausible) these estimates will be reduced. If, alternatively, the surface flux is part of a loop broken free from the core then comparable, but in some circumstances perhaps slightly larger, estimates can be derived. (Note that in this case there would have to be an enhanced envelope resistivity to allow sufficiently rapid reconnection so that the loop can detach itself from the core field.) These estimates can be compared with the observed effective fields significantly in excess of \( 10^3 \) G, which are seen in a number of CP stars. Clearly, they do not prove that it is impossible for a dynamo in the core to produce surface fields as large as those observed, but it does appear that the assumptions have to be ‘stretched’ to their limits in order to do so. The very large fields, in excess of \( 10^4 \) G, seen in exceptional objects pose a major problem.

There remains the possibility of some other, more rapid, form of instability which would allow the core field to reach the surface within the required time. A core dynamo initially immersed in a more-or-less field-free radiative envelope will initially generate a configuration in which the field strength decreases quite rapidly outwards. A unidirectional horizontal field with this property is likely to be unstable (e.g. Hughes & Cattaneo 1987), but the topology of the fields at the outside of a core dynamo may well be more complex. Further, the non-linear development of the instability is unknown.

A further complication is that any upward fluid motion near the core surface will be affected by the developing molecular weight gradient as the star evolves from its zero-age configuration (e.g. Mestel 1965). For stars of mass typical of that of the magnetic CP stars, the mass of the convective core decreases as the star evolves from the ZAMS, leaving behind a zone with negative molecular weight gradient. This can stabilize the region immediately exterior to the core against the rise of flux tubes. If the horizontal scale of the upward motions is \( d_h \) (corresponding roughly to \( R_p \)), then for a given \( \mu \)-gradient the downward (stabilizing) velocity induced by a disturbance is proportional to \( d_h^{-3} \), i.e. motions of smaller tubes are more strongly resisted. Typical estimates of the inhibitory velocities (for \( R_p/d_h = 10 \)) at age \( 5 \times 10^6 \) or \( 10^7 \) yr.
are of order $|v_\mu| \sim 10^{-3} \, \text{cm s}^{-1}$. Of course the $\mu$-gradient only develops after a finite amount of time, and even if $\mu$-gradients when established were totally effective at preventing the rise of flux tubes, there might still be a 'window' near age zero when field could rise freely. This period would be shorter for more massive stars. Convective overshoot may also complicate matters.

Leaving these possibilities aside, only rise mechanisms with velocities greater than $|v_\mu|$ are likely to be effective at transporting flux away from the core. This would appear to rule out thermally driven buoyancy as a mechanism after a modest amount of hydrogen burning has occurred, unless the dynamics of the convection succeed in pushing the field beyond the shell of varying chemical composition. (It is possible that continued stirring could substantially reduce the composition gradient outside of the core, but at the expense of increasing the radial extent of the inhomogeneous zone.) The substantive uncertainty in the above discussion is the possibility of instabilities driving the fields outwards at rates which are much faster than those from strictly thermal mechanisms. It is not enough for the linear analysis of the instability to give such a speed – it must be maintained as the field moves through a substantial fraction of the envelope. Some such mechanism seems to be essential to bring a substantial fraction of the core flux to the surface within the required $5 \times 10^6 \, \text{yr}$.

There are few attempts in the literature to construct models of an entire star consistent with the core dynamo theory, in contrast to the large amount of effort that has been put into investigating properties of linear kinematic dynamos in spheres and spherical shells. In hindsight, given the uncertainties discussed above, perhaps this lack is well founded! Krause (1971, 1983) claims, from kinematic theory, that modes with an axis of symmetry either parallel or perpendicular to the rotation axis will be excited preferentially and, in the linear theory at least, the mode with fastest growth rate will dominate. However, more recent work on simple nonlinear dynamos (Krause & Meinel 1988; Brandenburg et al. 1989) supports the idea that the relative sizes of the kinematic growth rates are often irrelevant and that, in some instances, the dominant mode will be that with the smallest marginal dynamo number.

Krause argues the case for the 'symmetric rotator'; in order to explain the effective fields of the observed CP stars, which are conventionally modelled by the oblique displaced dipole, it is then necessary to invoke a non-uniform distribution of line-forming elements on the stellar surface, such that the conventional analysis misinterprets a configuration with an equatorially symmetric field as an oblique configuration (e.g. Oetken 1977, 1979; see also criticism in Borra & Landstreet 1978). No truly self-consistent models have yet been produced – to be convincing, detailed analysis and modelling along the lines pioneered by Landstreet (1988) is needed. Non-linear effects might even allow simultaneous excitation of modes with an axis of symmetry parallel and perpendicular to the rotation axis, but no steady spherical dynamo models with two modes of different parity-types simultaneously present have yet been discovered (e.g. Brandenburg et al. 1989).

The ideas of Krause and his colleagues, referenced above, have assumed implicitly that the surface field of a core dynamo model is a simple extension of the field in the core. If a sufficiently efficient (in the sense discussed in this section) mechanism is present to transport the core field to the surface within the time available, then, provided the general symmetry of the field with respect to the magnetic and rotational equators is preserved by this transport process, the above particular problems concerning the interpretation of the observed magnetic variations that are associated with the perpendicular rotator remain.

3 Fossil theory

This theory has been discussed at length in previous papers in this series, so only brief comments will be made here. The wide range of possible initial conditions for the magnetic
field is simultaneously a strength and a weakness of the theory. It is convenient, if not essential, to have the total magnetic flux present in a star as a free parameter, in order to explain the lack of correlation between the measured fields and other stellar properties. However, this means that it is difficult to use the properties of observed fields to construct incisive tests of the theory.

Most modelling has assumed that the flux distribution within the star when it arrives on the zero-age main sequence is fairly homogeneous. This is not obviously true, especially if there is a phase of Hayashi turbulence. (Whether or not stars of mass appropriate to the magnetic CP stars experience such turbulence still seems to be uncertain.) In this case any field surviving is likely to be concentrated into ropes in the stellar interior which, if only 'ordinary' diffusion acts, will not have time to diffuse into a more homogeneous distribution in the time between the cessation of the turbulence and the onset of hydrogen burning. Alternatively, a field of intermittent spatial structure might also be generated if a 'turbulent dynamo' operates during pre-main-sequence evolution (e.g. Schussler 1975). This leaves the possibility that the observed surface field configurations might be able to yield information on the ZAMS configurations throughout the star, and thus on the field origin. For example, even a moderate degree of inhomogeneity in the initial field configuration might produce surface fields which are inconsistent with the observed $B_0$ (and to a more limited extent $B_1$) variations during a cycle. Preliminary calculations for slightly inhomogeneous but axisymmetric interior field models were presented in Moss (1987b), and the question is addressed at length, for non-axisymmetric 'rope-like' fields, in Sections 4 and 5 of this paper.

Existing self-consistent fossil theory calculations (e.g. Moss 1986b, 1987a) are broadly consistent with almost all the observational evidence. Given the freedom available when choosing initial flux distribution and the comparatively coarse-grained field observations (averages over the stellar disc, mostly of $B_0$, i.e. the mean longitudinal component only), this agreement cannot yet be regarded as totally convincing. Hints at deviations from axisymmetry of the field, which have emerged in a few of the observations, and the development of non-axisymmetric numerical models, possibly point to one way of testing the theory more rigorously. A larger and more precise body of observational evidence relating to the field geometry (including, e.g., the distribution of the values of the inclination, $\beta$, between the rotation and magnetic axes as a function of stellar age) will, however, need extensive and careful observations.

The one piece of observational evidence which might be used to argue against the fossil field/oblique rotator theory is the recent discovery by Thompson & Landstreet (1985) that the CP star HD37776 has effective field variations of 'double wave' type, whose most obvious interpretation is in terms of dipolar and quadrupolar components, with the quadrupolar substantially larger than the dipolar. The variant of the fossil theory that takes the contemporary field to be the descendant of that in the ISM requires the global topology of the contemporary field to be that of a uniform or dipolar field. Of course if the observed field is generated by a contemporary dynamo or is a fossil from a Hayashi phase dynamo, this constraint need not apply. However, Moss (1987c) showed that, in principle, a double wave effective field variation can be produced by a field with topology compatible with an origin in the ISM, if the surface flux is concentrated into four 'spots' (two positive, two negative) with positive surface field in one hemisphere and negative field in the other, although a realistic configuration would necessarily have a less discontinuous structure. In reality, the anomalous field variations of HD37776 must present a challenge to any theory – fossil, dynamo or whatever. Why should this star display $B_0$ variations of a different form from any other? If a dynamo mechanism is responsible for the field, what special conditions apply that a predominantly (but not entirely) quadrupole mode is excited in this star but apparently in none other (at least, that we have so far observed)?
4 The flux rope model

Moss (1987a) describes a program which follows the generation of non-axisymmetric field components, with azimuthal dependence \(\sin(\cos 2\lambda)\), \(\sin(\cos 4\lambda)\), ..., from an initially axisymmetric field in the case of strictly perpendicular magnetic and rotation axes. The meridional circulation generated by the magneto-centrifugal perturbation is included self-consistently in the magneto-hydrodynamical equation. This program was modified as follows.

Consider an initial magnetic field configuration of the form

\[
B_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} - \frac{L_n(\Phi_n)}{r^2} \cos n\lambda
\]

\[= B_0 + B_n \text{ say,}
\]

\[
B_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r} + \frac{1}{r} \frac{\partial^2 \Phi_n}{\partial \theta^2} \cos n\lambda,
\]

\[
B_\lambda = -\frac{n}{r \sin \theta} \frac{\partial \Phi_n}{\partial r} \sin n\lambda,
\]

where

\[
\psi = \psi(r, \theta), \quad \Phi_n = \Phi_n(r, \theta),
\]

\[
L_n(\Phi_n) = \frac{\partial^2 \Phi_n}{\partial \theta^2} + \cot \theta \frac{\partial \Phi_n}{\partial \theta} - n^2 \frac{\Phi_n}{\sin^2 \theta},
\]

and \(n\) is a positive integer (cf. Chandrasekhar 1961). Choose \(\psi\) so that

\[
\frac{1}{\sin \theta} \frac{\partial \psi}{\partial \theta} \neq 0, \quad \theta_1 \leq \theta \leq \theta_2,
\]

\[
\frac{1}{\sin \theta} \frac{\partial \psi}{\partial \theta} = 0, \quad 0 < \theta < \theta_1 \text{ and } \theta_2 < \theta < \pi/2,
\]

where

\[
0 < \theta_1 < \theta_2 \leq \pi/2, \quad \text{and} \quad \frac{1}{\sin \theta} \frac{\partial \psi}{\partial \theta}
\]

has a single maximum in \((\theta_1, \theta_2)\); and

\[
L_n(\Phi_n) \approx \pm \frac{1}{\sin \theta} \frac{\partial \psi}{\partial \theta}.
\]

The resulting field, \(\mathbf{B} = B_0 + B_n\), say, consists approximately of \(n\) ‘ropes’, of approximate latitudinal extent \(\theta_2 - \theta_1\) and longitudinal half-width of about \(\pi/n\). It satisfies \(\nabla \cdot \mathbf{B} = 0\), but has not yet been constrained by the equations (hydrostatic equilibrium, the MHD equation with self-consistent meridional circulation, etc.), appropriate to a magnetic field immersed in a
radiative region of a star. The evolution of initial field structures of the types described above
for the cases \( n = 1 \) and \( n = 2 \) was investigated. The magneto-centrifugal circulation, in the form
\[
\nu = v_2(r) P_2(\cos \theta) + v_{1n}(r) P_n(\cos \theta) \cos n\lambda, \tag{4.3}
\]
(with corresponding expressions for the other components) is calculated self-consistently. In
practice, except close to the stellar surface and boundary of the convective core, the centrifugal
terms dominate the circulation when the rotation is sufficiently rapid for a mean magnetic
Reynolds number to be greater than about unity. For the models investigated this happens
when the parameter,
\[
\Omega^2 R^3 / GM = \lambda_\Omega \gtrsim 10^{-3},
\]
the critical value depending somewhat on the stellar mass and the field strength. (The zero-
order ZAMS model used here has mass \( 2.5M_\odot \) and luminosity \( 41L_\odot \).) Of course other
components of circulation will be generated, but the above representation includes all of the
dominant centrifugal terms in the relevant components of the MHD equation. The \( n = 1 \)
calculations were restricted to the case where the rotation axis \( \Omega \) coincided with the axis \( \theta = 0 \)
(with unit vector \( k \)) of the spherical polar coordinate system since, as shown in Moss (1987a),
the case with \( \Omega \cdot k = 0 \) generates field components with \( \sin / \cos 2\lambda, \sin / \cos 4\lambda \) ... dependence
from the interaction of the rotationally driven meridional circulation with the axisymmetric
field component. In the \( n = 2 \) case the rotation axis can be taken to be either parallel or
perpendicular to \( k \), as the \( \sin / \cos 2\lambda \) component generated by the field/circulation interaction
can now be included in the basic formulation.

In Section 5 calculations are described for a variety of \( n = 1 \) and \( n = 2 \) (‘one-rope’ and ‘two-
rope’) initial configurations. It was found that, starting with the initial fields described above,
the fields in the interiors of the first relaxed models (i.e. those derived from the above formulation
but satisfying all the relevant structural equations – hydrostatic equilibrium, Poisson’s
equation, energy equation, radiative transfer and the MHD equation – retain quite closely the
prescribed rope-like structure. In particular, the approximate relation (4.2) holds through
much of the interior. However, near the stellar surface there is a marked departure from this
relation, in the sense that
\[
\left| \frac{1}{\sin \theta} \frac{\partial \psi}{\partial \theta} \right| > |L_n(\Phi_n)|.
\]

The rope-like structure is diluted near the surface and the observable fields are much more
homogeneous in structure than those in the interior.

With hindsight, the following appears to be a plausible explanation of this effect. At the edge
of a discrete flux rope the field gradients and hence also the Lorentz force, \((\nabla \times B) \times B / 4\pi\), will
be large. In hydrostatic equilibrium the Lorentz force has to be balanced by perturbations to
the zero-order pressure gradient and, in particular, a more-or-less radial rope will induce
relatively large horizontal pressure gradients. Deep in the stellar interior, where the field
energy density is a small fraction of the thermal energy density, the star can easily adjust its
structure in this way. However, this is not possible in the low density, low pressure surface
regions where field and thermal energy densities may be nearly comparable. Thus the field
gradients must be substantially lower near the surface. This is in contrast to the situation in
cool stars, such as the Sun, which have convective outer layers where the dynamical interaction
of the turbulence with the flux tubes is important.

These surface field distributions can be illustrated by contour maps of equal values of \( B_r \)
and \( |B| \). In order to compare the calculated fields with those observed, effective \( (B_v) \) and
surface \((B)\) field variations are also plotted, using the program described in Moss (1987b). As pointed out above, strictly speaking when \(n = 1\) this formulation requires \(\Omega\) to be parallel to \(\theta = 0\). However, the only influence of the relative orientation of the rotation axis is via the effects of the approximately Eddington–Sweet circulation acting on the magnetic field in the MHD equation. Thus the field variations with \(n = 1\) and \(\Omega\) perpendicular to \(\theta = 0\) can be approximated if either \(|\Omega|\) is small enough for the global magnetic Reynolds number of the Eddington–Sweet circulation to be less than about unity or, if \(|\Omega|\) is larger than this value, for times small enough for the effects of advection of field by the circulation to be not yet apparent.

Although the one-rope configuration is simpler and in some ways more satisfactory to study, the two-rope model was also developed to give a fully self-consistent treatment of the case \(\Omega\) perpendicular to \(\theta = 0\) with \(\lambda_\Omega \geq 10^{-3}\), as well as to investigate the effects of a more complex internal structure on the observable field. (As discussed above, it is not possible to model the case \(\Omega \cdot \mathbf{k} = 0\) when \(n = 1\).) A summary of the parameters of the models discussed in detail is given in Table 1. Here and subsequently \(\lambda_\Omega = \hat{H}^2 R^4/(GM^2)\), where \(\hat{H}\) is of the order of the initial surface field strength.

5 Summary of the calculations

5.1 One-rope model

The general principles can be illustrated by a detailed discussion of a typical calculation, for parameters \(\lambda_\Omega = 10^{-3}\), \(\lambda_\Omega = 10^{-10}\) (model 1). For a globally distributed field most of whose initial flux passes through the stellar surface this value of \(\lambda_\Omega\) is just large enough that noticeable effects are caused by field advection within a few \(\times 10^8\) yr, whilst also allowing the field evolution to be followed at relatively low spatial resolution. (For these computations, usually 11 points were distributed uniformly in the range \(0 \leq \theta \leq \pi/2\).)

With larger \(\lambda_\Omega\), qualitatively similar changes will occur on shorter time-scales, whereas for smaller values of the field, advection is never important, however long the time-scale. The initial field is given by (4.1), and Fig. 1(a) sketches the field lines of the \(\lambda\)-independent field in a meridian plane for the first relaxed model (time 0 yr). In Fig. 2(a) \(B_{0r}\) and \(B_{1r}\), at \(\lambda = 0\), are plotted against colatitude \(\theta\) at fractional radii 0.5 and 1.0. These figures illustrate clearly that a thick rope-like structure of the type discussed above is present in the stellar interior, but is diluted at the surface. With ‘fine tuning’ it is possible to arrange that relation (4.2) is more closely satisfied in the interior, but this makes little or no difference to the following discussion. Contours of constant radial field component, \(B_r\), and constant field strength, \(|\mathbf{B}|\), at the surface are shown in Fig. 3(a). (When interpreting all these contour maps, it must be remembered that there is massive areal distortion in the region of the pole and that the entire upper boundary of the diagram maps on to the pole \(\theta = 0\).) The model was allowed to evolve for a few \(\times 10^8\) yr, and Fig. 3(b) shows the surface field contours at time \(3.5 \times 10^8\) yr. In Figs 1(b) and 2(b) the corresponding quantities to those plotted in Figs 1(a) and 2(a) are given at \(3.5 \times 10^8\) yr. These figures demonstrate the anticipated advection of the surface field towards the rotational

<table>
<thead>
<tr>
<th>Model</th>
<th>(\mathbf{\hat{\Omega}} \cdot \mathbf{k})</th>
<th>(n)</th>
<th>(\lambda_\Omega)</th>
<th>(\lambda_\Omega)</th>
<th>(\theta_1)</th>
<th>(\theta_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>(10^{-3})</td>
<td>(10^{-10})</td>
<td>0.2</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>(10^{-3})</td>
<td>(10^{-8})</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>(10^{-3})</td>
<td>(10^{-10})</td>
<td>0.0</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>2</td>
<td>(10^{-3})</td>
<td>(10^{-10})</td>
<td>0.0</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>2</td>
<td>(3 \times 10^{-3})</td>
<td>(10^{-10})</td>
<td>0.0</td>
<td>0.7</td>
</tr>
</tbody>
</table>
equator, in a similar manner to that seen for more globally distributed fields in previous studies. It is worth noting again that, whereas the interior field has largely retained its initially rope-like form, the surface field is even more uniformly distributed than at earlier times. A significant $B_2$ component is present by $2 \times 10^8$ yr (calculated in the kinematic approximation, see Section 4), and the dashed contours in Fig. 3(b) give the contours of equal $B$, when this component is omitted.

Variations of $B_s$ and $B_t$ with time for angle $i$ between rotation axis and lines-of-sight equal to 60° are given in Fig. 4(a) for time $3.5 \times 10^8$ yr. Although, as explained above, for $n = 1$ the program can only follow the field evolution wholly self-consistently when $\boldsymbol{\Omega} \cdot \mathbf{k} = 1$, the approximate variations for small time for $\boldsymbol{\Omega} \cdot \mathbf{k} = 0$ can be calculated. Thus Fig. 4(b) gives $B_s$ and $B_t$ curves for the age-zero model with $\boldsymbol{\Omega} \cdot \mathbf{k} = 0$.

Varying the parameters $\lambda_\Omega$ and $\lambda_H$ produces effects which are comparable to those found in previous studies with more homogeneous field distributions – for example increasing $\lambda_\Omega$ tends to increase the rate of field advection, whilst increasing $\lambda_H$ reduces these effects. A number of models were run, with results generally consistent with those just described, and so will not be discussed further here, with one exception. This case (model 2) has a more asymmetric initial field configuration whose initial field contours ($\lambda$-independent part) are sketched in Fig. 5(a) at
The magnetic fields of the CP stars

5.2 TWO-ROPE MODEL

The general nature of the results for $n=2$ is similar to that described in the previous section for $n=1$, except that now the case $\Omega \cdot \mathbf{k} = 0$ can be followed explicitly. Moss (1984, 1987a) demonstrated that for initially axisymmetric and fairly homogeneous fields, the nature of the field evolution for larger values of $\lambda_\Omega$ was fundamentally different for the cases $\Omega \cdot \mathbf{k} = 1, 0$ in that with parallel axes the surface field was advected towards the equator and eventually largely 'buried', whereas with perpendicular axes the axisymmetric component of surface flux was concentrated towards the initial magnetic poles but not buried. Advection of the surface field structures towards the magnetic poles/equator for $\Omega \cdot \mathbf{k} = 0/1$ is seen when $\lambda_\Omega \approx 10^{-3}$ for $\lambda_H = 10^{-10}$. In Fig. 6 $B_{0r}$ and $B_{1r}$ at $\lambda = 0$ are plotted against colatitude $\theta$ for time $t = 0$ for Model 3 with $\lambda_\Omega = 10^{-3}, \lambda_H = 10^{-10}, \Omega \cdot \mathbf{k} = 1$. Fig. 7(a) and (b) shows the field contours at the stellar surface at $t = 0$ and $t = 4 \times 10^8$ yr, respectively, and Fig. 8 shows the effective and surface field variations for $i = 60^\circ$ and $t = 0, 4 \times 10^8$ yr. Fig. 9 shows the surface field contours for
Figure 3. (a) Contours of equal $B_r$ and $|\mathbf{B}|$ at the stellar surface, Model 1, $t = 0$. (b) Contours of equal $B_r$ and $|\mathbf{B}|$ at the stellar surface, Model 1, at $3.5 \times 10^8$ yr. Dashed contours are when the $B_z$ component is omitted.

Figure 4. (a) Effective ($B_e$) and surface ($B_s$) variations at $3.5 \times 10^8$ yr, $i = 60^\circ$, Model 1. (b) $B_e$ and $B_s$ variations at $t = 0$, $i = 60^\circ$, for Model 1 assuming $\Omega \cdot \mathbf{k} = 0$.

Figure 5. (a) Field lines of the $\lambda$-independent field at $2.5 \times 10^7$ yr, for Model 2. (b) $B_e$ and $B_s$ variations at $2.5 \times 10^7$ yr, for Model 2.
Figure 6. $B_{ir}$ and $B_{2r}$ at $\lambda = 0$ as a function of $\theta$ at $t = 0$ at $x = 0.5$ (dashed curves) and $x = 1$ (full curves) for Model 3.

Figure 7. (a) Surface field contours at $t = 0$, Model 3. (b) Surface field contours at $t = 4 \times 10^8$ yr, Model 3.
642

D. Moss

Figure 8. (a) $B_e$ and $B_s$ variations, $i = 60^\circ$, $t = 0$, Model 3. (b) $B_e$ and $B_s$ variations, $i = 60^\circ$, $t = 4 \times 10^8$ yr, Model 3.

Figure 9. Field contours at the stellar surface, $t = 4 \times 10^8$ yr, Model 4.

Figure 10. $B_e$ and $B_s$ variations, $i = 60^\circ$, $t = 4 \times 10^8$ yr, Model 4.

Model 4 with $\mathbf{\Omega} \cdot \mathbf{k} = 0$ at time $4 \times 10^8$ yr [the contours at $t = 0$ are identical to those shown in Fig. 7(a)], and Fig. 10 gives the observable field variations for this model, again for $i = 60^\circ$. Unsurprisingly, the $B_s$ curves show a distinct double wave behaviour. In this case the advection of the field towards the poles reduces the asymmetry of the model. This can just be seen in Model 4 at $4 \times 10^8$ yr, but it is much more evident for $\lambda_{\Omega} = 3 \times 10^{-3}$ when, however, the resolution in the $\theta$-direction is breaking down (Model 5, see Fig. 11).

6 Conclusions

Sections 2 and 3 have reviewed the current status of the contemporary core-dynamo and fossil field theories for the origin of the fields of the magnetic CP stars. Whilst it is not at present

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
possible to decide conclusively between these two theories, a number of constraints have been identified. In particular, the dynamo theory has difficulty in transporting the fields observed in very young CP stars from the core to the surface in the time available, unless there is a mechanism operating on a time-scale much shorter than a thermal time-scale or, alternatively, the field rises in very fine strands which combine to give a coherent field near the surface. In either case there may be difficulties in generating surface fields which are as strong as those observed, provided that the core fields are limited by equipartition with the kinetic energy of the convection (and that the simple order-of-magnitude estimates are correct).

The surface fields of at least the more slowly rotating CP stars with relatively strong fields are deduced not to be concentrated into more or less discrete spots, but to be distributed over much of the stellar surface (Preston 1971). The computations discussed in Sections 4 and 5 (together with those of Moss 1987b) demonstrate that the surface field distributions resulting from a fossil origin will satisfy this constraint, even if the interior fields are quite inhomogeneous. However, it seems unlikely that internal field structures of two (or more) approximately equi-strength ropes, viewed from arbitrary orientations, would be consistent with the body of observational evidence, and so any such field configurations must be rare, if they exist at all.

Conversely, detailed knowledge of surface field distributions seems unlikely to yield definitive models of the internal field distributions in CP stars, and thus it is even more unlikely to throw light on the pre-main-sequence history of the fields. The only major exception to this statement is knowledge of the overall topology of the stellar field. With the exception of HD37776, all observed magnetic CP stars appear to have the topology of a uniform (or dipole) field. Moss (1987c) showed how, in principle, a field distribution with topology compatible with a fossil origin could produce the ‘double wave’ effective field variations observed in the subject. The calculations presented here re-emphasize the crudeness of the field approximations (radial field only, confined to four circular spots) of that paper; any more realistic simulation would have a more smoothly distributed surface field. The extreme non-axisymmetry of such a configuration is beyond the scope of the techniques discussed here, but it is very plausible that a ‘four-spot’ surface field of this basic structure, but with a somewhat more homogeneous surface field distribution, would be consistent with calculations of the sort described here, for a more general ‘two-rope’ internal structure.

In conclusion, this work is not intended to suggest that the standard displaced dipole/oblique rotator model for the fields of the CP stars is seriously deficient. Clearly it provides a good first approximation for the majority of magnetic CP star fields at the current resolution of
the observations. The point is that it need not be taken too literally. Departures from the
displaced dipole structure, including lack of axisymmetry, can arise quite naturally and a range
of interior field structures are consistent with the observed field variations.

**Acknowledgment**

DM is grateful to Leon Mestel for his comments on a preliminary version of this paper.

**References**

Dolginov, A. Z., 1986 In: Upper Main Sequence Stars with Anomalous Abundances, eds Cowley, C. R.,