The effect of the disc on the rotation curves of spiral galaxies

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Summary. We discuss the role of the galactic disc in the interpretation of the circular velocities of spiral galaxies at large galactocentric radii. The fractional amount of mass in the disc can be shown to be an increasing function of the luminosity. As a result, the shape of the rotation curve near the edge of the optical disc is expected to vary systematically with luminosity. Using a simple disc/halo model we find that in bright, disc-dominated galaxies the rotation curve should drop by a few tens of km s$^{-1}$ immediately outside the disc, even in the presence of an extended dark halo. Only in faint, halo-dominated galaxies, do we expect the rotation curve to remain flat or even to rise beyond the optical radius. We find evidence for this behaviour in several published rotation curves.

In the case of the Milky Way we show that if the disc dominates the gravitational potential in the inner regions, then the low mass estimated by Little & Tremaine from the dynamics of the outer satellites is consistent with a dark halo that extends beyond the region sampled. This and other available data are well fit by a model in which the luminous component of the Galaxy contributes $\sim 70$ per cent of the mass at 12 kpc from the centre and the mass at large radii increases in proportion to the radius.

1 Introduction

The rotation curves of spiral galaxies at large galactocentric radii are the best means to probe the mass distribution in galactic haloes (cf. Sancisi & van Albada 1987, for a recent review). Unfortunately, only a handful of rotation curves extend far enough beyond the disc to provide unambiguous information on the total mass and extent of galactic haloes. An important datum is the recent estimate by Little & Tremaine (1987) of a mass $M = 2.4^{+1.4}_{-1.0} \times 10^{11} M_{\odot}$ for our own Galaxy out to $\sim 8$ disc radii, obtained from an analysis of the dynamics of 10 satellites. Little & Tremaine concluded that an isothermal dark halo around our Galaxy would need to be truncated at $r \sim 50$ kpc in order to account for their low mass estimate. Similarly low masses at 3–5 disc radii and correspondingly small haloes have been inferred for a few other spirals from radio observations of small samples of satellites (Erickson, Gottesman & Hunter 1987).
In this paper we examine the contribution of galactic discs to the shape and amplitude of the rotation curves in the outer parts of spirals. Our main conclusion is that in bright galaxies like our own the disc is dynamically important and gives rise to a small but measurable drop in the rotation curve outside the optical radius. In fainter galaxies the disc is less important and the rotation curve is a true reflection of the halo-mass distribution, even in the inner regions. These results have a direct bearing on Little & Tremaine's conclusion concerning the extent of our Galaxy's halo. These authors assumed that the rotation curve is due exclusively to the halo and has the value measured near the Sun at all radii. By contrast, we find that if the contribution of the disc is subtracted, the asymptotic circular velocity of the halo is considerably smaller than its value at the Sun and, as a result, a small mass estimate is not inconsistent with an extended halo.

It now seems clear that the presence of nearly flat rotation curves in the inner parts of spiral galaxies involves a finely balanced interplay between the luminous material in the disc and the spheroidal dark halo (Rubin et al. 1985; van Albada et al. 1985; Persic & Salucci 1986). The relative contributions of these components can be estimated from the rotation curve and the light profile of the disc by a method of mass decomposition. From such an analysis applied to a sample of 43 Sb-Sc spirals, Persic & Salucci (1988) estimated the ratio, $\beta$, of visible to total mass within the optical radius. We define the latter as $R_0 = 3 \alpha^{-1}$, where $\alpha^{-1}$ denotes the radial disc scale length. [This radius coincides, on average, with $R_{25}$, the radius of the 25 B-mag arcsec$^{-2}$ contour, Freeman (1970); van der Kruit (1987); hereafter we will assume $R_0 = R_{25}$.] Persic & Salucci find

$$
\beta = \frac{V_c^2(R_0)}{V_a^2(R_0) + V_t^2(R_0)} \approx \frac{M_d(R_0)}{M_h(R_0) + M_d(R_0)} \\
= 0.7\left[1 - 2^{1/2}(1 + \langle d \log V_c / d \log r \rangle)^{1/2}/0.3\right]^{1.6} \\
\approx b(L_\odot/10^{10}h^{-2}L_\odot)^x,
$$

(1)

where $V_c(r)$, $M_d(r)$, $V_a(r)$ and $M_h(r)$ are the circular velocity and mass of the visible and halo components, respectively, $V_c$ is the total circular velocity and $h$ is Hubble's constant in units of 100 km s$^{-1}$ Mpc$^{-1}$. The square brackets here denote an average over the region $(0.2-1)R_0$. This relation was found by Persic & Salucci for galaxies with $L \geq 2.5 \times 10^8 h^{-2}L_\odot$, assuming that $d \log V_h / d \log r \big|_{R_0}$ is constant. For this sample they obtained $b = 0.25 \pm 0.05$ and $x = 0.56 \pm 0.1$. A more recent determination from a larger sample (Persic & Salucci, in preparation) gives $b = 0.57 \pm 0.1$ and $x = 0.25 \pm 0.05$; we shall adopt these values here. Thus, galaxies with flat rotation curves in the region of the optical disc or with $L \sim 2.3 h^{-2} \times 10^{10} L_\odot$ have $\beta \sim 0.7$. Similarly large values of $\beta$ are inferred from attempts to fit the rotation curves in the inner regions of spirals using a model with a 'maximum disc' and a spheroidal component (van Albada et al. 1985; Kent 1986).

For our own Galaxy the value of $\beta$ is uncertain because the shape of the inner rotation curve, the scalelength of the disc and the contribution of the stellar halo are all relatively uncertain (Kulkarni, Blitz & Heiles 1982; van der Kruit 1987). Hereafter we take $R_0 = 12$ kpc for our galaxy, but our results are insensitive to reasonable changes in this parameter. Detailed models of galactic structure imply values ranging from $\beta \sim 0.5$ (Bahcall, Schmidt & Soneira 1983) to $\beta \sim 0.75$ (Caldwell & Ostriker, 1981). In any case, a significant fraction of the measured circular velocity in our Galaxy is due to the disc.

Equation (1) provides the main motivation for this work. It implies that the dynamics of bright spirals are dominated by the disc in the inner regions and we might therefore expect an observable feature in the transition between this and the halo-dominated regime. As the luminosity decreases, so does the relative importance of the disc, and any transitional features may be expected to be small. In Section 2 we propose a simple mass model which enables us to
calculate this effect and we compare our results with published rotation curves. In Section 3 we apply our model to estimates of the mass of our Galaxy and of other similar spirals and reconsider the issue of whether Little & Tremaine’s (1987) estimate requires a truncated halo. In Section 4 we summarize our conclusions.

2 A simple model

As a first approximation we consider a model galaxy consisting of an exponential thin disc and a spherical dark halo. We model the halo as a pseudoisothermal sphere with

$$\rho_h = \frac{\rho_c}{1 + (r/a)^2},$$

where $a$ denotes the core radius. Outside the optical disc, for $r > R_0$, the mass contained within radius $r$ is given by

$$M(r) = G^{-1} r V_c^2(r) = \beta f(r) M(R_0) + (1 - \beta) M(R_0) \frac{r}{R_0} \frac{[1 - (a/r)\tan^{-1}(r/a)]}{[1 - (a/R_0)\tan^{-1}(R_0/a)]},$$

where $\beta$, the ratio of luminous to total mass, is defined in equation (1). The two terms in the right-hand side correspond to the contributions of the disc and of the halo, respectively. The function $f(r)$ is obtained by solving Poisson’s equation for an exponential thin disc and is given in terms of the modified Bessel functions, $I_n, K_n$, by (cf. Freeman 1970).

$$f(r) = \left( \frac{r}{R_0} \right)^3 \frac{[I_0 K_0 - I_1 K_1]_{r/2}}{[I_0 K_0 - I_1 K_1]_{R_0/2}}.$$

In addition to an overall normalization, $M(R_0)$, our model has two free parameters, $a$ and $\beta$, which specify the relative contributions of the disc and halo components. For their sample of 43 galaxies, Persic & Salucci (1988) found that the quantity $d\log V_h/d\log r \big|_{R_0}$ ranges between $\sim 0.7$ and $\sim 0.9$. For our model this translates into a range for $a$ of $\sim 0.25 R_0$ to $\sim 0.75 R_0$.

Equation (2) has often been used successfully to model dynamical data for spirals. In some cases, however, it leads to models which are not particularly good fits to the data in the inner regions of the galaxy (Lake & Feinswog 1988; Kent 1986; Begeman 1988). Such inaccuracies are not important for our analysis which is concerned exclusively with the behaviour of the rotation curve at $r > R_0$.

The fractional mass in the disc, $\beta$, determines the details of the transition between the disc-dominated and the halo-dominated regimes. From equation (1) we see that this parameter varies from $\sim 0.2$ for galaxies with $L_B \sim 2.5 h^{-2} \times 10^{8} L_\odot$ to $\sim 0.8$ for galaxies with $L_B \sim 4 h^{-2} \times 10^{10} L_\odot$. This, in turn, implies that the shape of the rotation curve outside $R_0$ must vary along the luminosity sequence. In a faint galaxy, the visible matter plays an insignificant role and the rotation curve will rise smoothly to the asymptotic halo value. In a bright galaxy, on the other hand, there is a region with $r < R_0$ where both the disc and the halo contribute significantly to the circular velocity. Such galaxies often have flat rotation curves in this region and a fine tuning of the two contributions is required since they have markedly different radial behaviours, $d\log V_h/d\log r = -[M_*(r)/M_0(r)] d\log V_*/d\log r$. Whenever the disc remains dominant near $R_0$, this fine tuning will break down because the circular velocity of the disc declines sharply but the contribution of the halo cannot increase fast enough (for any value of $a$) to maintain a flat rotation curve. In this case the rotation curve will drop for a while before rising
again to the asymptotic halo value. We can quantify these expectations with the use of equation (3).

For any value of the luminosity we obtain the corresponding value of $\beta$ from equation (1). We then use this in equation (3) to calculate the quantity $\delta V_c = [V_c(2R_0) - V_c(R_0)]/V_c(R_0)$ for any given value of $a$. In Fig. 1 we plot the result as a function of luminosity for values of the core radius corresponding to the limits of the range found by Persic & Salucci. [Note that we only show the model predictions for the range of $L_B$ for which equation (1) was obtained.] In all cases the ‘jump’ in circular velocity beyond $R_0$ varies monotonically with $L_B$. This jump is positive for sufficiently faint galaxies, implying a rising rotation curve, and is negative for bright galaxies, implying an actual drop in circular velocity beyond $R_0$. In either case the rotation curve will eventually become flat as the asymptotic value of the halo is reached. Note that, whereas the value of $L_B$ for which $\delta V_c = 0$ depends sensitively on the assumed value of $a$, the prediction of a drop in circular velocity for bright galaxies is almost independent of the core radius.

In order to compare our model predictions with observations, we searched the literature for galaxies with rotation curves measured at least out to $2 R_0$ with quoted errors less than 6 km s$^{-1}$. These are listed in Table 1 and our measured values of $\delta V_c$ are compared with the corresponding model predictions in Fig. 1. The data show the same general trend as the model predictions and in all cases fall within the region defined by the extreme values of the core radius inferred from Persic & Salucci’s sample. This agreement is particularly encouraging since it refers to a second-order prediction of the model. The case of NGC 6503 is especially interesting. This is one of the best-studied galaxies and has a rotation curve which is remarkably flat between $0.8 \leq r/R_0 \leq 4$ and thus provides some of the best evidence in favour of dark haloes around spirals (Begeman 1988). Its maximum circular velocity is $\sim 120$ km s$^{-1}$ and its luminosity is $\sim 2.7 h^{-2} \times 10^9 L_\odot$ so, from equation (1), $\beta \sim 0.4$. Thus according to our model this galaxy should be halo dominated at the optical radius and its flat rotation curve is consistent with equation (3) provided $a \sim 0.4 R_0$.

As a further test of our model we examined published rotation curves for galaxies with large values of $V_c$. For these our model predicts a relatively large drop in circular velocity at $R_0$, so

![Figure 1](https://academic.oup.com/mnras/article-abstract/237/1/247/1068536/7996032)

**Figure 1.** Relative change in circular velocity between two and one optical radii as a function of luminosity for galaxies with accurate rotation curves. The symbols represent the data from Table 1 with $\pm 1 \sigma$ error bars. The solid lines show the predictions of our model with core radius $a = 0.25 R_0$ (lower curve) and $a = 0.75 R_0$ (upper curve).
extremely accurate rotation curves are not required. In particular we selected all galaxies that we were able to find in the literature with rotation curves measured beyond $\sim 1.2 R_0$ and maximum circular velocities greater than 200 km s$^{-1}$. The difference between $V_c(R_0)$ and $V_c$ at the last measured point in each galaxy is given in Table 2. Without exception, $V_c(r)$ shows a significant drop beyond $R_0$. The size of the drop depends on the point at which it is measured, but can be as large as $\sim 60$ km s$^{-1}$ in the most extreme cases. (The HI discs of some of these galaxies exhibit warps or other asymmetries, but in all cases the authors of each study have applied a correction before deriving the final rotation curve.)

We now apply our model to galaxies similar to our own. More specifically, we consider galaxies with a flat rotation curve for $r < R_0$ of amplitude $V_c \sim 220$ km s$^{-1}$ [and thus $M(R_0) \sim 1.4 \times 10^{11} M_\odot$]. From the Tully–Fisher relation this value of $V_c$ corresponds to $L \sim 2.5 h^{-2} \times 10^{10} L_\odot$ for an Sbc galaxy (Rubin et al. 1985) and, from equation (1), to $\beta \sim 0.7$. Solving Poisson’s equation for our model with $\beta = 0.7$ and $dV_c/dr|_{R_0} = 0$ gives $a = 0.5 R_0$, a value near the middle of the range found for Persic & Salucci’s sample. Our predicted rotation curve with these parameter values and normalization is shown as a solid line in Fig. 2. There is a drop of $\sim 30$ km s$^{-1}$ between 1 and 3 $R_0$ and by $r \gtrsim R_0$ the rotation curve is again nearly flat and with asymptotic value of $V_c \sim 180$ km s$^{-1}$. Because of uncertainties in the slope of the galactic rotation curve at $R_0$ as well as uncertainties in our model, we feel that our model parameters for the galaxy are uncertain at about the 10 per cent level. We argue that this is a crude, but roughly correct model for the rotation curve in the outer parts of our Galaxy. Indeed, the galactic models of Bahcall et al. (1983) and Caldwell & Ostriker (1981) predict a decrease in $V_c$ of $\sim (10-30)$ km s$^{-1}$ in the region $1 \leq r/R_0 \less than 3$.

Since there are no direct measurements of rotation for our own Galaxy much beyond $R_0$, we can attempt to test our model of the Galaxy’s outer rotation curve by comparing with other galaxies of similar Hubble type and $V_c(R_0)$. For this purpose, we selected the subset of galaxies in Table 2 which have 200 km s$^{-1} < V_c(R_0) < 240$ km s$^{-1}$ and plot, in Fig. 2, the circular velocity at the last point measured. Only for three galaxies do the data extend beyond 3 $R_0$ and for these we plot the entire published rotation curve. In order to facilitate the inter-comparison of these data, rather than plotting $V_c(r)$ for each galaxy, we plot $220[V_c(r)/V_c(R_0)]$ km s$^{-1}$. Note that the model curve in Fig. 1 is not a fit to the data points, since having specified the shape of the rotation curve for $r \lesssim R_0$ and having assumed the empirical relation (1), our model contains no free parameters. It is remarkable that a relatively simple model, based on very few assumptions, can account so well for the general behaviour of the rotation curves in the outer parts of galaxies similar to our own.

### Table 1. Data for galaxies with accurate rotation curves.

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>$V_c(R_0)$ (km s$^{-1}$)</th>
<th>$\delta V_c^*$</th>
<th>Error</th>
<th>$L_B$ $10^9 h^{-2} L_\odot$</th>
<th>Ref.†</th>
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<td>0.039</td>
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<td>0.012</td>
<td>11.5</td>
<td>1</td>
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<tr>
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<td>0.023</td>
<td>8.6</td>
<td>1</td>
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<td>0.012</td>
<td>5.1</td>
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<td>N5033</td>
<td>220</td>
<td>−0.1</td>
<td>0.022</td>
<td>10.4</td>
<td>1</td>
</tr>
<tr>
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<td>0.025</td>
<td>41.9</td>
<td>1</td>
</tr>
<tr>
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<td>117</td>
<td>0</td>
<td>0.017</td>
<td>2.7</td>
<td>1</td>
</tr>
<tr>
<td>N7331</td>
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<td>−0.078</td>
<td>0.019</td>
<td>30.4</td>
<td>1</td>
</tr>
<tr>
<td>U2259</td>
<td>90</td>
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<td>0.02</td>
<td>0.6</td>
<td>2</td>
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</table>

* $\delta V_c$ is the fractional decrease in circular velocity between 2 $R_0$ and $R_0$.
† References: (1) Begeman (1988), (2) Carignan et al. (1988).

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Table 2. Data for galaxies with $V_c(R_o) > 200$ km s$^{-1}$.

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>$V_c(R_o)$ (km s$^{-1}$)</th>
<th>$r_{max}/R_o$</th>
<th>$\Delta V_c$ (km s$^{-1}$)</th>
<th>Error (km s$^{-1}$)</th>
<th>Ref.‡</th>
</tr>
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<td>N224</td>
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<td>$-25$</td>
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<td>2</td>
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<tr>
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<td>1</td>
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<tr>
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<td>$-50$</td>
<td>5</td>
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<td>1</td>
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<td>2.6</td>
<td>$-20$</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

*Sources for the photometry are listed in Persic & Salucci (1987) or given in the references in column 6, except for NGC 224 for which we have used data from Kent (1987).
†$\Delta V_c$ is the decrease in circular velocity between $R_o$ and the last point, at radius $r_{max}$, where the rotation curve has been measured. $\Delta V_c = V_c(r_{max}) - V_c(R_o)$.
§The rotation curve for N2841 has been measured out to 5.5 $R_o$, but it rises slightly after 3.5 $R_o$; $r_{max}$ here is taken as the point at which $V_c$ is a minimum.

![Graph showing $V_c(r/R_o)$ vs. $r/R_o$.]

Figure 2. Values of the circular velocity at the last measured point for the galaxies with 200 km s$^{-1} < V_c(R_o) < 240$ km s$^{-1}$ listed in Table 2 (filled circles). The ordinate gives $V'_c = 220[V_c(r)/V_c(R_o)]$ km s$^{-1}$. From left to right the data correspond to NGC 5907, 5383, 5033, 5371, 4013, 3031, 4017, 5055 and 2903. Full rotation curves for the latter three are shown as dotted lines. The solid line is the rotation curve predicted by the mass model of equations (2) and (3) with $\beta = 0.7$ and $a = 0.5 R_o$. For reference we plot a horizontal dashed line at $V'_c = 220$ km s$^{-1}$.

3 The large-scale mass distribution in the Galaxy

The discussion in the preceding section shows that our model for the rotation curve of the outer Galaxy is in good agreement with data on galaxies similar to our own. Although the rotation curve of the Milky Way is not known much beyond the solar neighbourhood, there have been attempts to estimate the mass at large radii using different probes.

The most recent and, in principle, the most reliable determination is Little & Tremaine's (1987) analysis of a sample of 10 satellites (6 dwarf ellipticals and 4 globular clusters), with
galactocentric distances in the range \( \sim (50-140) \) kpc and with good distance and radial velocity determinations. Assuming an isotropic velocity distribution for these satellites, they obtained a 95 per cent confidence limit, \( M \leq 5 \times 10^{11} M_\odot \) for the mass within \( \sim 100 \) kpc. Little & Tremaine attempted to infer the physical extent of the Galaxy's dark halo by comparing their mass estimate with the mass inferred from extrapolation of a flat rotation curve with the amplitude measured near the solar neighbourhood. Since the rotation curve already implies a mass \( \sim 1.4 \times 10^{11} M_\odot \) within \( \sim 12 \) kpc, the radius of the optical disc, they concluded that an isothermal dark halo would need to be truncated at \( r \sim 50 \) kpc in order to account for the low mass estimated further out.

Our discussion in Section 2 shows that the extent of the galactic halo cannot be inferred from a single mass measurement at large radius unless we have some knowledge of the disc/halo mass ratio and a model of the halo-mass distribution. For the parameters we found appropriate to our Galaxy, the asymptotic circular velocity of the halo is 180 km s\(^{-1}\) (with an uncertainty of \( \sim 10 \) per cent), not the value of 220 km s\(^{-1}\) assumed by Little & Tremaine. Since, at the effective radius of the satellite sample, the circular velocity in our model is close to its asymptotic value, the inferred cut-off radius, \( r_n \approx V_c^2(\infty) \). For the mass corresponding to Little & Tremaine's 95 per cent confidence limit we find \( r_n = 75 \pm 15 \) kpc, which is comparable to the effective sample size. Thus, although Little & Tremaine's mass estimate is significantly smaller than would be expected on the basis of our model, it is large enough that the hypothesis that the halo extends beyond the region of the satellite sample cannot be rejected.

The kinematics of the Magellanic stream have also been used to estimate the mass of the Galaxy (Lin & Lynden-Bell 1982). Assuming that the velocities of the stream are due only to gravity and assuming a particular model for the gravitational potential, Lin and Lynden-Bell estimated a mass of \( 1 \times 10^{12} M_\odot \) at a distance of 70 kpc. This result is difficult to interpret in the present context because their assumed model is very different from ours. We note however, that this mass estimate is significantly different, by about a factor 4 from that of Little and Tremaine although it refers to a similar radius.

Other mass determinations of the mass of our Galaxy and of other similar spirals using different probes include the following:

(i) Globular clusters. The dynamics of the galactic globular cluster system imply a rotation curve which is approximately flat or slightly rising in the region \( r \sim (2-30) \) kpc. However, the rotation curve is not well constrained beyond \( \sim 15 \) kpc by this method. The most plausible models for the orbital eccentricity of the clusters give a value of \( 210 \pm 30 \) km s\(^{-1}\) for the circular velocity at the position of the Sun (Frenk & White 1980, 1982; Lynden-Bell & Frenk 1981).

(ii) M31. M31 is approaching the Galaxy with a velocity of \( \sim 300 \) km s\(^{-1}\) relative to the Local Standard of Rest. If this motion is interpreted as the first approach over the Hubble time, the mass of the system is inferred to be several times \( 10^{12} M_\odot \) (Kahn & Woltjer 1959; Gunn 1974). A careful analysis of the kinematics of the Local Group gives a mass of \( \sim (3-7) \times 10^{12} M_\odot \) (Einasto & Lynden-Bell 1982), where the range of values reflects the uncertainties in the value of the transverse velocity and in the age of the system. From Table 2 the circular velocity of M31 at large radius is 225 km s\(^{-1}\), so, in our model, its mass at any large distance from the centre is 1.6 times the mass of our galaxy at the same distance from its centre. As a crude approximation, we divide the mass of the Local Group between both galaxies at their halfway point in this proportion.

(iii) Binary galaxies. From a statistical study of \( \sim 150 \) binary galaxies, White \textit{et al.} (1983) found that the data are best fit by a mass-to-light ratio which increases linearly with radius. Using the normalization factor found by Schweizer (1987) for late type spirals, \( M(S R_0) \sim 5 M(R_0) \) we find that \( \log M/M_\odot = 12 \) at \( \log r/kpc = 2.3 \) and \( \log M/M_\odot = 12.3 \) at \( \log r/kpc = 2.6 \). The
uncertainties in these results are difficult to assess since they stem primarily from uncertainties in the orbital eccentricity and from biases due to selection effects.

(iii) Satellite galaxies. In about half a dozen galaxies and detection of a few satellites has allowed a rough estimate of the mass out to \( \sim 50-100 \) kpc (Erickson et al. 1987). The best case is NGC 3992, an SBB galaxy with an optical rotation curve similar to that of the Galaxy. At \( r \sim 60 \) kpc, the estimated mass is \((2.3 \pm 1) \times 10^{11} M_\odot\), where the range of values reflects the uncertainties in the satellite orbits and in its distance.

In Fig. 3 we plot the mass estimates discussed above as a function of galactocentric distance and compare them with the mass model of equation (3) (with \( \beta = 0.7 \) and \( a = 0.5 R_0 \)) which is shown as a solid line. For comparison we also show, as a dotted line, the mass of a pure isothermal halo with no disc (\( \beta = 0, a = 0 \)). It is clear that the mass detected at large radii can be reconciled with the circular velocity in the inner regions only if the contribution of the disc to the circular velocity is taken into account. Our best model for the outer Galaxy gives an acceptable fit to the data over two orders of magnitude in distance and one in mass, although Little & Tremaine's estimate remains discrepant at about the \( \sim 2\sigma \) level.

4 Discussion and conclusions

We have proposed a simple model to estimate the effect of an exponential disc on the outer rotation curves of spiral galaxies. This model is motivated by Persic & Salucci's (1988) finding that the relative contribution of the visible mass to the total mass in spirals depends on luminosity. As a result, discs have rather different effects depending on the luminosity (or circular velocity) of the galaxy in which they occur. The dynamics of faint galaxies (with \( V_c \leq 100 \) km s\(^{-1}\)) are dominated by the dark halo, even inside the optical radius. Their rotation curves therefore rise at first until they attain the value appropriate to their halos. Bright galaxies (with \( V_c \geq 200 \) km s\(^{-1}\)), on the other hand, are dominated by the disc in the inner regions where their rotation curves tend to be nearly flat. If the disc is sufficiently massive, the halo is unable to compensate for the rapid decrease in the disc's circular velocity beyond the optical radius and this results in a limited drop in circular velocity just outside this radius;

![Figure 3](https://example.com/figure3.png)

**Figure 3.** The mass of the Galaxy estimated from the dynamics of globular clusters (filled triangle), the dynamics of satellites (filled circle), the Magellanic Stream (open circle) and the timing argument for the Local Group (filled square). Mass of NGC 3992 (open square) and of late type spirals in binary systems (open triangles). Error bars are crude \( \pm 1\sigma \) errors. The solid line shows the prediction of our model with \( \beta = 0.7 \) and \( a = 0.5 R_0 \); the dotted line corresponds to an isothermal halo with no disc contribution.
thereafter the rotation curve may once again become flat as the dark halo becomes the dominant source of gravity. For a galaxy like our own we predict a decrease in circular velocity of $\sim 30 \text{ km s}^{-1}$ within $\sim 30$ optical radii. In galaxies of intermediate luminosity (with $V_c \sim 130 \text{ km s}^{-1}$), the disc and the halo just balance one another to produce a nearly flat rotation curve extending beyond the optical radius.

We have found evidence in published rotation curves for an effect of the sign and approximately the magnitude predicted by our simple model. In particular, all published HI rotation curves of spirals with $V_c > 200 \text{ km s}^{-1}$ show a drop just outside the optical radius. Previous authors have remarked on this feature. For example, Sancisi & van Albada (1987), referring to the decline in their measured rotation curves state: ‘Possibly these features are pointing at some significant property of the mass distribution, and perhaps are related to the overall distribution of luminous matter as they occur over the region of the disc or close to its edge.’

Unfortunately, most measured HI rotation curves do not extend far enough to probe the halo-mass distribution in detail. However, Little & Tremaine’s recent determination of the mass of our Galaxy within $r \sim 100 \text{ kpc}$ using the dynamics of satellites, provides a unique data point in a region not previously sampled in any comparable galaxy. These authors interpreted their relatively low-mass estimate as signalling the truncation of the galactic halo because they assumed that the rotation curve measured near the position of the Sun remains flat out to the satellite’s region. When the contribution of the disc to the inner rotation curve is subtracted, their mass estimate, although still significantly smaller than expected in our model, no longer requires the halo to be truncated. Our best model for the outer Galaxy consists of an exponential disc which contributes $\sim 70$ per cent of the total mass at the optical radius and a pseudoisothermal dark halo with an asymptotic circular velocity of $\sim 180 \text{ km s}^{-1}$ with a likely uncertainty of $\sim 10$ per cent in both these numbers. This model provides an acceptable fit to other mass determinations of our Galaxy, including the estimate from the timing argument for the Local Group, and to mass estimates of galaxies similar to the Milky Way.

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