An extended galaxy redshift survey – II. Virial constraints on $\Omega_0$

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Summary. We obtain new constraints on the cosmological density parameter, $\Omega_0^*$, from a cosmic virial theorem (Peebles) analysis of the 264 galaxies in the recently completed redshift survey of Metcalfe et al. Using methods similar to Bean et al. we obtain a value of $(600 \pm 140)$ km s$^{-1}$ for the line-of-sight rms relative peculiar velocity, $\langle W^2 \rangle^{1/2}$, at projected separations $\sigma < 1 \ h^{-1}$ Mpc ($H_0 = 100 \ h \ \text{km s}^{-1} \text{Mpc}^{-1}$) and although this is larger than the $(190 \pm 90)$ km s$^{-1}$ found in the redshift survey of Peterson et al., we show that the difference is not inconsistent with the range of values obtained from other redshift surveys. Comparing estimates of the two- and three-point correlation function amplitudes we find reasonable statistical agreement with the previous results. Based on these two redshift surveys and the survey of Parker et al. we find that our best estimate of $\langle W^2 \rangle^{1/2}$ at these scales is $(290 \pm 70)$ km s$^{-1}$. For the overall small-scale two- and three-point correlation functions we find $r_0 = (4.5 \pm 0.3) \ h^{-1}$ Mpc ($\gamma = 1.8$) and $Q = 0.58 \pm 0.05$, respectively. Thus, with the assumption that galaxies are good tracers of the mass distribution we obtain as our overall estimate of the mean mass density, $\Omega_0 = 0.18 \pm 0.09$ at scales $< 1 \ h^{-1}$ Mpc which is in good agreement with the value found by Davis & Peebles for the CfA Survey. Thus, in the extended survey the $\Omega_0 = 1$ model is rejected, although only at the same level of significance as found by Bean et al. due to a more realistic error estimate on $\langle W^2 \rangle^{1/2}$ from the larger combined sample.

1 Introduction

Measurements of the galaxy correlation functions and relative peculiar velocities at small scales place strong constraints on the value of the density parameter, $\Omega_0$. Using the virial theorem methods of Peebles (1976), previous studies (e.g. Bean et al. 1983; Davis & Peebles 1983; hereafter B83 and DP83, respectively) obtained values in the range 0.1–0.2 on the $\Omega_0^*$ is the ratio of the mean density to the critical density in a Friedmann world model.
assumption that the observed forms of the two- and three-point galaxy correlation functions, \( \xi = (r_0/r)^{-7} \) and \( \xi \propto Q^{-2} \), respectively, were representative of the mass distribution. As a direct consequence of this, 'biasing' (e.g. Kaiser 1984) was required to be introduced into models with \( \Omega_0 = 1 \) so that galaxies are assumed to be more clustered than the mass on these scales. Because of such fundamental implications, it is important to extend these 'cosmic virial analyses' to larger volumes of space. Here we check the previous result of B83 in two new redshift surveys which triple the volume sampled by the original survey (Peterson et al. 1986, hereafter the Durham/AAT Survey).

The basic survey material used here is the catalogue of 264 galaxy redshifts and apparent magnitudes prepared by Metcalfe et al. (1989, Paper I, hereafter the Durham/SAAO Survey). The nine fields of this survey are of similar construction to the Durham/AAT Survey and are limited to a similar apparent magnitude of \( b_J = 16.8 \) mag. However, in this case, only one in three of the galaxies were selected from the prepared magnitude lists for subsequent redshift determination. The principal aim of this larger volume survey was to place stronger constraints on the redshift correlation function, \( \xi(s) \), on scales from \( 2 \) to \( 100 \, h^{-1} \) Mpc (see Shanks et al. 1989, Paper III). However, as described in Section 2, the errors on our redshifts are accurate enough to provide more information on field galaxy peculiar velocities. Further, after a careful analysis of the significance of our results derived from field-to-field fluctuations and from simulations (see Section 3), we have found that the estimates of the relative peculiar velocity dispersion and small-scale correlation functions using the new survey are as well defined as the previous estimates of B83. We have, therefore, been able to use the new survey in an application of the cosmic virial theorem (Peebles 1976) to test the constraints on the density parameter.

In our overall analysis, we have also included the \( \sim 100 \) galaxy redshift survey of Parker et al. (1986) who sampled a single UK Schmidt (UKST) field to a magnitude limit similar to our own. As their instrumental set-up and reduction techniques were identical to those used to prepare the Durham/SAAO Survey, the results derived from this sample are easily combined with our own. Together with the previous Durham/AAT Survey we now have a dataset of \( \sim 700 \) galaxies, spread over 15 fields, on which to base our final conclusions.

Therefore, in Section 2 we describe, in more detail, the new catalogues mentioned above. In Section 3 we outline our methods for estimating correlation functions and peculiar velocities and for constructing the simulated catalogues used to test the significance of the results. In Section 4 we present our correlation and peculiar velocity results for the Durham/SAAO Survey. We also, in this section, compare these results with other surveys, including Parker et al. In Section 5 we derive, via the cosmic virial theorem, estimates of \( \Omega_0 \) for the Durham/SAAO Survey. Further, by combining the results from all three surveys, we obtain a global estimate of \( \Omega_0 \). Finally, in Section 6 we present our conclusions.

2 The data

2.1 THE DURHAM/SAAO SURVEY

This survey (Paper I) is basically an extension of the previous Durham/AAT Survey of Peterson et al. (1986). \( 3.75 \times 3.75 \) areas were chosen from nine separate UKST fields and surveyed to an apparent magnitude limit of \( b_J = 16.8 \) mag (the field centre coordinates and magnitude limits are given in Table 1). The galaxy magnitudes were obtained from the COSMOS automated measurements of UKST J photographic plates (\( J = \) Kodak IIIa-J plus Schott GG395 filter) and calibrated using CCD galaxy sequences. The CCD photometry

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Table 1. The Durham/SAAO Survey fields.

<table>
<thead>
<tr>
<th>Field</th>
<th>RA (h m)</th>
<th>Dec (° ' '')</th>
<th>w (°)</th>
<th>h (°)</th>
<th>m (m)</th>
<th>N_r</th>
<th>N_g</th>
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<tr>
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<td>-10 01</td>
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<td>3.75</td>
<td>16.68</td>
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<td>+00 23</td>
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<td>3.75</td>
<td>17.07</td>
<td>88</td>
<td>26</td>
</tr>
<tr>
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<td>14 40.8</td>
<td>-15 03</td>
<td>3.75</td>
<td>3.75</td>
<td>16.81</td>
<td>58</td>
<td>15</td>
</tr>
<tr>
<td>GNZ</td>
<td>15 19.4</td>
<td>+02 14</td>
<td>3.75</td>
<td>3.75</td>
<td>16.89</td>
<td>88</td>
<td>27</td>
</tr>
<tr>
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<td>-25 09</td>
<td>3.73</td>
<td>3.73</td>
<td>16.79</td>
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<td>3.73</td>
<td>17.00</td>
<td>65</td>
<td>21</td>
</tr>
</tbody>
</table>

The field centre coordinates (equinox 1950.0) are given in columns (1) and (2). The width and height of each field are given in columns (3) and (4), respectively. The number of galaxies brighter than the apparent magnitude limit (column (5)) is indicated in column (6). Column (7) gives the number of one-in-three galaxies with well-determined redshifts in each field (these galaxies have codes 0, 1, 2 and 5 in Paper I, table 5).

showed that the COSMOS/UKST galaxy magnitudes were linear in the range 14 mag ≤ b_r ≤ 17 mag. Using the SAAO 1.9-m telescope with the ITS spectograph and reticon (RPCS) detector, spectra were obtained for every third galaxy down the ordered apparent magnitude list for each field. Although the spectroscopic resolution (~6 Å) was lower than that used for the Durham/AAT Survey, velocities accurate to ~120 km s^{-1} rms were obtained using standard cross-correlation techniques (Tonry & Davis 1979). This error estimate was obtained from considerations of both internal and external redshift comparisons (see Paper I for full details of these data). The numbers of galaxies in each field with well-determined redshifts are shown in column 7 of Table 1. These galaxies provide the basis for the analysis in this paper.

The field centres were chosen, essentially, at random. Four fields have galactic latitudes b^II ≥ +40° and five have b^II ≤ −40°. As discussed in Paper I, two of the nine fields (GNX, GSG) were subsequently discovered to have rich clusters associated with them. The one-third sampling rate was decided upon to allow us to obtain, in the allotted observing time, a well-determined large-scale (≥10 h^{-1} Mpc) two-point correlation function whilst not compromising too much our correlation analyses at smaller scales. The survey was carried out in six one-week observing runs over the period 1983 September to 1984 October.

2.2 THE PARKER et al. SURVEY

This one-field survey of area 5°35 x 5°35 and centred on the UKST field at 00^h 00^m, −35° 00' contains 104 galaxy redshifts and apparent magnitudes to a limit of b_r = 16.5 mag. The number-magnitude counts indicate that this sample is only complete to b_r = 16.25 mag and this was the limit adopted in the analysis. Both photometry and spectroscopy were carried out using very similar methods to those described above. Velocities are claimed to be accurate to ~75 km s^{-1} rms. The field was, essentially, chosen at random for previous slitless 'redshift surveys' using UKST objective prism plates. The luminosity function for this sample has been derived by Hale-Sutton (in preparation) and is found to be consistent with that from the Durham surveys.
3 Methods for estimating basic quantities

3.1 THE TWO- AND THREE-POINT CORRELATION FUNCTIONS

At small scales (\( \lesssim 10 \ h^{-1} \ \text{Mpc} \)) the true spatial clustering distribution can be distorted in redshift space by relative peculiar velocities between galaxy pairs and redshift measurement errors. To quantify this smearing of the distribution along the line-of-sight we determine the redshift two-point correlation function, \( \xi_v \), for our sample in a similar manner to that described in Peebles (1979, hereafter P79) and B83. The estimation of \( \xi_v \) at small scales and its symmetry about the line-of-sight suggests we adopt coordinates \( (\sigma, \pi) \) for the perpendicular and parallel positions of one galaxy relative to another with respect to this direction. These are approximately

\[
\sigma = \theta v_1 / H_0 \quad \text{and} \quad \pi = (v_1 - v_2) / H_0,
\]

where \( v_1 \) and \( v_2 \) are the redshift velocities of two galaxies separated by an angle \( \theta \) on the sky.

The mean number of galaxies expected within a thin annulus of area \( \delta A_{\|}(\sigma) \) and thickness \( \delta \pi \) at coordinates \( (\sigma, \pi) \) from an arbitrary galaxy, \( i \), is

\[
\langle N(\sigma, \pi) \rangle = n(z) [1 + \xi_v(\sigma, \pi)] \delta \pi \delta A_{\|}(\sigma), \quad (3.2)
\]

As galaxies in the sample have been selected by apparent magnitude, the homogeneous number density, \( n \), depends on the redshift, \( z = v/c \), of the annulus. The area of the annuli that do not fall completely in the field volumes is found using the method of P79. To obtain an estimate of \( \xi_v(\sigma, \pi) \), \( n(z) \delta \pi \delta A_{\|}(\sigma) \) and the observed count \( N(\sigma, \pi) \) are determined for each galaxy, \( i \), in the sample and accumulated into the sums \( DR(\sigma, \pi) \) and \( DD(\sigma, \pi) \), respectively. It follows then that

\[
\xi_v(\sigma, \pi) = \frac{\langle DD(\sigma, \pi) \rangle}{\langle DR(\sigma, \pi) \rangle} - 1. \quad (3.3)
\]

We obtain an estimate of \( n(z) \) from

\[
n(z) = \int_{-\infty}^{M_{\text{max}}(z)} \Phi(M) \, dM, \quad (3.4)
\]

where \( \Phi(M) \, dM \) is the galaxy luminosity function. The upper limit \( M_{\text{max}}(z) \) on the absolute magnitude arises from requiring that each galaxy in the sample has an apparent magnitude, \( m \), brighter than the limit, \( m_{\text{lim}} \). Thus,

\[
M_{\text{max}}(z) = m_{\text{lim}} - 5 \log_{10} (cz/H_0) - 25 - kz. \quad (3.5)
\]

We adopt a mean value of \( k = 3.0 \) for the \( k \)-correction term. The slight variation in the apparent magnitude limit of the fields (Table 1) is allowed for by suitable changes in \( n(z) \). The luminosity function has been determined for the Durham/SAAO sample by Hale-Sutton et al. (Paper IV, in preparation). A Schechter (1976) form with

\[
M^* = -19.9 + 5 \log_{10} h \quad \text{and} \quad \alpha = -1.0 \quad (3.6)
\]

is found to represent the data reasonably well. Using this fit we then normalize \( n(z) \) to the number of galaxies within the velocity limits of the sample. The limits chosen for the Durham/SAAO Survey are

\[
1000 \leq \nu \leq 30\,000 \ \text{km s}^{-1}. \quad (3.7)
\]
Fig. 1 shows the observed velocity frequency distribution for this survey together with the predicted distribution using $n(z)$. Although there seems to be a slight under-density at $v \sim 15,000 \, \text{km s}^{-1}$, the model does provide a reasonable fit to the data.

The probability of finding triplets of galaxies in excess of that found for a random distribution is measured by the three-point correlation function. We use this statistic in Section 5 to provide an estimate of the mass distribution around pairs of galaxies. It has been implied from an analysis of two-dimensional catalogues that the reduced part of this function, $\xi(r_1, r_2, r_3)$, is reasonably well approximated by

$$\xi(r_1, r_2, r_3) = Q[\xi(r_1) \xi(r_2) + \xi(r_2) \xi(r_3) + \xi(r_1) \xi(r_3)]$$

(3.8)

(see, for example, Groth & Peebles 1977) with $\xi(r)$ being the two-point correlation function. In an attempt to minimize the effects of relative peculiar velocities (cf. Section 4.1.1) we estimate the amplitude, $Q$, by integrating the two- and reduced three-point functions over $\pi$ to a maximum $\pi_{\text{cut}}$. The method used here is identical to that described in B83. For the Durham/SAAO Survey we adopt a $\pi_{\text{cut}}$ of 5 $h^{-1}$ Mpc and a $n_{\text{R}}/n_{\text{G}}$ of 160 (see their notation) to reduce the noise in the mean counts $\langle DR \rangle$, $\langle DDR \rangle$ and $\langle DRR \rangle$.

3.2 SIMULATED CATALOGUES FOR TESTING THE SIGNIFICANCE OF THE RESULTS

In order to determine the relative significance of the results obtained from each of the Durham/SAAO, Durham/AAT and Parker et al. surveys, we applied our methods of analysis to catalogues of static Monte Carlo simulations based on the hierarchical clustering model. The prescription for such simulations has been fully described by Soneira & Peebles (1978)
and briefly summarized by B83. Here we outline our methods, presenting, in particular, the parameters for our simulations, which were chosen to match the results from our analysis of the surveys (Section 4).

The 20 simulations of the Durham/SAAO and Durham/AAT surveys were constructed so that interfield correlations could be investigated (see Paper III). This was achieved by giving the 14 fields in each simulation the same area and relative positions on the 'sky' as those in the real surveys. For the Parker et al. survey a separate set of 20 simulations was prepared and details can be found in Hale-Sutton (in preparation). The construction method for the Durham surveys was as follows. Clump centres were placed at random within the volumes defined by \( v < 30,000 \text{ km s}^{-1} \). At each site a nine-level hierarchy of subclumps was formed according to the method of B83. To match the power law and subsequent break in the observed two-point correlation function, the radius of the \( i \)th level was chosen to be

\[
(6 h^{-1} \text{ Mpc})/1.8^{i-1}. \tag{3.9}
\]

By randomly choosing 38 of the 512 points in the final level as galaxies, a correlation length, \( r_0 \), of \( \sim 5.6 h^{-1} \text{ Mpc} \) (\( \gamma = 1.8 \)) was obtained for the small-scale (\( \leq 6 h^{-1} \text{ Mpc} \)) two-point function. In the case of the three-point function the form approximately agrees with that of equation (3.8) with \( Q \sim 0.5 \) (Peebles 1980). The galaxies were then assigned an absolute magnitude in the range \( -21.5 \leq M \leq -17.5 \) drawn at random from a Schechter (1976) function with parameters \( M^* = -20.0 \) and \( \alpha = -1.0 \) (\( H_0 = 100 \)). Galaxies in the clump were then rejected if their apparent magnitude (including a 3\( z \) k-correction term, see equation 3.5) exceeded the mean Durham field limit of \( m_{\text{lim}} = 16.8 \text{ mag} \). Each remaining galaxy was assigned a peculiar velocity drawn at random from the \( \exp(-C|W|^{3/2}) \) distribution of Esfathiohi & Eastwood (1981). The dispersion of this distribution was adjusted to give an rms relative velocity of \( \sim 350 \text{ km s}^{-1} \). The simulation was stopped when the number in the catalogue equalled the total number of galaxies that are observed to be brighter than the mean Durham field limit. Finally, to match the sampling in the Durham/SAAO Survey we ranked the simulated SAAO galaxies in apparent magnitude and chose every third object for subsequent analysis.

We used the same analysis as we have applied to the actual data to find the ensemble means and adjusted rms deviations for these catalogues. It was apparent from this study that our third sampling procedure for the Durham/SAAO fields does not introduce any noticeable systematic bias in the quantities we derive. The rms deviations are used throughout this paper as the principal estimates of error. We have checked the validity of these estimates by comparing them with the equivalent errors deduced from field-to-field fluctuations. For example, we compare, in Table 2(a), the estimates of the uncertainty for the correlation length, \( r_0 \) (with \( \gamma = 1.8 \)), as derived from the simulated catalogues and from field-to-field fluctuations in the Durham/SAAO and AAT surveys (these latter estimates from the Durham/SAAO survey were obtained by combining the dataset into three groups of three fields to decrease the noise of having fewer pairs). The comparisons clearly show that our simulations seem to be providing a reasonable model of the fluctuations expected for our data. The only disagreement found was for the line-of-sight rms relative peculiar velocity, \( \langle W^2 \rangle^{1/2} \) (see Section 4.1.3). Table 2(b) again lists these quantities for the Durham/SAAO and AAT surveys and it can be seen that the field-to-field errors are about a factor of 2 larger than for the simulations. This inadequacy of the model may, for example, be evidence that individual clusters, at these scales, are more anisotropic than this hierarchical model would seem to suggest. When we quote errors on \( \langle W^2 \rangle^{1/2} \) we shall therefore use the simulated errors normalized upwards by a factor of 2 to produce overall agreement with the field-to-field errors, which we consider to be more realistic (the field-to-field errors cannot be used directly because in individual \( \sigma \) bins they can be too noisy).
To combine quantities estimated from different surveys we weight the estimates according to the inverse square of their simulated errors. For the error on the new estimate we assume that the surveys are statistically independent and combine the simulated errors accordingly. We prefer this method of approach over the direct, pair weighted, estimates because it takes into full account the significance of the undersampled fields.

4 Results from $\xi_*(\sigma, \pi)$ and the three-point correlation function

4.1 Estimates of $r_0$ and $\langle W^2 \rangle^{1/2}$ from $\xi_*(\sigma, \pi)$

We estimated the redshift two-point correlation function, $\xi_*(\sigma, \pi)$, for the Durham/SAAO sample according to the prescription outlined in Section 3.1. To model this statistic we continue to use the methods of B83 and P79 as these allow for a fairly comprehensive description of the small-scale clustering and dynamics. In Section 4.1.1 we look at $\xi_*(\sigma, \pi)$ in ‘semi-projection’ as this provides a value for the clustering length, $r_0$, of the underlying spatial correlation function, $\xi(r)$, that is reasonably independent of the effects of relative peculiar velocities. Following this, in Section 4.1.2, we determine the line-of-sight rms relative peculiar velocity, $\langle W^2 \rangle^{1/2}$, between galaxy pairs by looking at the characteristic width of $\xi_*(\sigma, \pi)$ in the $\pi$ direction. This method does not assume a value for $r_0$ nor a form for the distribution of peculiar velocities. Finally, in Section 4.1.3, $\xi_*(\sigma, \pi)$ is modelled directly using a distribution function and the data is fitted using $r_0$ and $\langle W^2 \rangle^{1/2}$ as free parameters.

4.1.1 $r_0$ from $\xi_*(\sigma, \pi)$ in ‘semi-projection’

The spatial function, $\xi(r)$, can be estimated from a projection of $\xi_*$ in the $\pi$ direction;

$$\omega_*(\sigma) = \int_{0}^{\pi_{\text{max}}} \xi_*(\sigma, \pi) d\pi.$$  \hfill (4.1)

Table 2. Comparison of simulated and field-to-field error

<table>
<thead>
<tr>
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<th>D/SAAO</th>
<th>D/AAT</th>
</tr>
</thead>
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<tr>
<td>$\sigma$ (\text{Mpc})</td>
<td>sim</td>
<td>$\pm$-\text{f-f}</td>
</tr>
<tr>
<td>0.25</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>0.75</td>
<td>0.8</td>
<td>1.0</td>
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<td>1.50</td>
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</tr>
<tr>
<td>3.00</td>
<td>1.0</td>
<td>0.9</td>
</tr>
</tbody>
</table>

(b) Error on the small scale peculiar velocity $\langle W^2 \rangle^{1/2}$.

<table>
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<tr>
<th></th>
<th>D/SAAO</th>
<th>D/AAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma &lt; 1.0$ (\text{Mpc})</td>
<td>sim</td>
<td>$\pm$-\text{f-f}</td>
</tr>
<tr>
<td>68</td>
<td>117</td>
<td>46</td>
</tr>
</tbody>
</table>

Error estimates for the correlation length, $r_0$ (in units of $h^{-1}$ Mpc), are presented for various projected separations, $\sigma$, in Table 2(a). The small-scale errors for the peculiar velocity, $\langle W^2 \rangle^{1/2}$ (in units of km s$^{-1}$), are given in Table 2(b). In both tables columns (1) and (2) are, respectively, the simulated and field-to-field errors for the Durham/SAAO Survey. Likewise, columns (3) and (4) are, respectively, the simulated and field-to-field errors for the Durham/AAT Survey.
Here, we truncate the integral to reduce the noise in $\omega_\nu(\sigma)$ arising from uncorrelated pairs at larger $\pi$, hence our use of the term ‘semi-projection’. Thus, provided $\pi_{\text{cut}} \gg (W^2)^{1/2}/H_0$ then

$$\omega_\nu(\sigma) = \int_0^{\pi_{\text{cut}}} \xi[\sigma^2 + \pi^2]^{1/2} d\pi. \quad (4.2)$$

For the spatial two-point function we adopt the power-law form of $\xi(r) = (r_0/r)^\gamma$ (Peebles 1974). It then follows that equation (4.2) becomes

$$\omega_\nu(\sigma) = A(\gamma, \pi_{\text{cut}}/\sigma) r_0^{(1-\gamma)}, \quad (4.3)$$

where the factor $A$ depends on $\gamma$ and on the ratio $\pi_{\text{cut}}/\sigma$. If we assume, as did B83, that $\pi_{\text{cut}} \gg \sigma$ then $A$ converges to

$$\frac{\Gamma(1/2)\Gamma(\gamma-1)/2}{2\Gamma(\gamma/2)} \quad (4.4)$$

provided $\gamma > 1$.

In Fig. 2 we present the Durham/SAAO results for $\omega_\nu(\sigma)$ using a $\pi_{\text{cut}}$ of 20 $h^{-1}$ Mpc. Following previous analyses, which have been based on taking the data to be reasonably consistent with a power-law form for $\xi$ with $\gamma = 1.8$, we have used the above model in a weighted least squares fit and find

$$r_0 = (5.3 \pm 0.5) \, h^{-1} \text{ Mpc}. \quad (4.5)$$

For comparison with the data this value of $r_0$ has been used in the model line in the figure. Although it is somewhat larger than the $r_0 = (4.1 \pm 0.6) \, h^{-1} \text{ Mpc}$ deduced for the Durham/

![Graph showing estimated values of $\omega_\nu(\sigma)$ against $\sigma$ (h^{-1} \text{ Mpc})$].

**Figure 2.** Estimates of the ‘semi-projected’ function $\omega_\nu(\sigma)$ for the Durham/SAAO Survey with $\pi_{\text{cut}} = 20 \, h^{-1} \text{ Mpc}$. The solid line shows the model $\omega_\nu(\sigma) = 1.84(r_0)^\gamma \sigma^{-0.8}$ (equation 4.3) with $r_0 = 5.3 \, h^{-1} \text{ Mpc}$. © Royal Astronomical Society • Provided by the NASA Astrophysics Data System
AAT Survey, we see from the errors in these estimates that the difference is not very significant.

The assumption, however, that the observed $\xi(r)$ is a continuous power law out to arbitrary large scales may lead to a poor estimate of the value of $r_0$ at $r \leq 10~h^{-1} \text{Mpc}$ (Bean 1983). The discrepancy arises from an uncertain large scale $\xi$ contributing significantly to $\omega_\varsigma(\sigma)$. Evidence for this is seen when applying this method of analysis to the simulations; here we find that the assumption of a power law to separations greater than the break scale at $\sim 6~h^{-1} \text{Mpc}$ causes the $r_0$ at separations $\lesssim 6~h^{-1} \text{Mpc}$ to be underestimated (Hale-Sutton, in preparation). To avoid the possible effects of non-power-law behaviour at larger scales, we have reduced $\pi_{\text{cut}}$ in the data to $10~h^{-1} \text{Mpc}$ and assumed that $\xi$ is a $-1.8$ power law out to at least this smaller separation. To estimate $r_0$ we have divided the new $\omega_\varsigma(\sigma)$ by $A(1.8, 10.0/\sigma)$ (using the non-asymptotic value in equation (4.3), thus allowing a least squares fit of the data with a $-0.8$ power law in the $\log[\omega_\varsigma(\sigma)/A] - \log \sigma$ plane. With this smaller $\pi_{\text{cut}}$ we find that $r_0$ increases to $5.6~h^{-1} \text{Mpc}$ for the Durham/SAAO sample and $5.0~h^{-1} \text{Mpc}$ for the Durham/AAT. As we shall see in Section 4.1.3 these values are more consistent with those determined from modelling $\xi_\varsigma(\sigma, \pi)$.

### 4.1.2 $\langle W^2 \rangle^{1/2}$ from the width of $\xi_\varsigma(\sigma, \pi)$

The mean square peculiar velocity between galaxy pairs can be determined from a second moment of $\xi_\varsigma(\sigma, \pi)$, namely

$$\langle W^2 \rangle^{1/2} \approx \frac{H_0^2}{\int_0^{\pi_{\text{cut}}} \pi^2 \xi(\sigma, \pi) d\pi} \int_0^{\pi_{\text{cut}}} \pi^2 \xi[[\sigma^2 + \pi^2]^{1/2}] d\pi$$

provided $\pi_{\text{cut}} > \langle W^2 \rangle^{1/2}/H_0$. To model the second term on the right-hand side of equation (4.6), which is the contribution to the width of $\xi_\varsigma(\sigma, \pi)$ coming from $\xi(r)$, we assume the usual power-law form for $\xi$ with $\gamma = 1.8$. As noted by B83, estimates of $\langle W^2 \rangle^{1/2}$ derived from this method are very sensitive to systematic differences between the observed and model $\xi$ at large $\pi$. However, provided that $\langle W^2 \rangle^{1/2}/H_0$ is still smaller than the separation at which $\xi$ departs from a pure power law then one expects equation (4.6) to yield reasonably stable values as $\pi_{\text{cut}}$.

<table>
<thead>
<tr>
<th>$\sigma$ (h^{-1} Mpc)</th>
<th>$\langle W^2 \rangle^{1/2}$ (km s^{-1})</th>
<th>$\langle W^2 \rangle^{1/2}$ (km s^{-1})</th>
<th>$\pi_{\text{cut}}$ (h^{-1} Mpc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 590</td>
<td>560+210</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>0.75 510</td>
<td>430+400</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>1.50 510</td>
<td>400+450</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3.00 400</td>
<td>110 -</td>
<td>9.5</td>
<td></td>
</tr>
</tbody>
</table>

Estimates of the line-of-sight rms peculiar velocity, $\langle W^2 \rangle^{1/2}$, from a second moment of $\xi_\varsigma(\sigma, \pi)$ are given at various projected separations, $\sigma$. In column (1), $\langle W^2 \rangle^{1/2}$ is the value derived from equation (4.6) when the second term on the right-hand side is neglected. Column (2) is the estimate of $\langle W^2 \rangle^{1/2}$ and its error. Column (3) is the mean $\pi_{\text{cut}}$ at which $\langle W^2 \rangle^{1/2}$ and $\langle W^2 \rangle^{1/2}$ were obtained.
is increased from smaller separations. It is these stable values that we have taken as the estimates of $\langle W^2 \rangle^{1/2}$.

In Table 3 we list our estimates of $\langle W^2 \rangle^{1/2}$ derived from the Durham/SAAO Survey. The column headed $\langle W^2 \rangle^{1/2}$ refers to the value of $\langle W^2 \rangle^{1/2}$ when the second term on the right-hand side of equation (4.6) is neglected, corresponding to the assumption that the galaxies at these scales are not moving with the Hubble flow. The similarity between the $\langle W^2 \rangle^{1/2}$ and $\langle W^2 \rangle^{1/2}$ estimates at separations $\sigma < 2.0 \ Mpc$ suggest that in this region peculiar velocities are tending to dominate the expansion. According to the errors these apparently reasonably well-determined values of $\langle W^2 \rangle^{1/2}$ lie in the range 400–560 km s$^{-1}$ and this is higher than the value of 200 km s$^{-1}$ observed for the Durham/AAT Survey (B83). However, because of the possible systematic errors in estimating $\langle W^2 \rangle^{1/2}$ using this method, we defer discussion of this difference until after we have again estimated this quantity in the following section using a method less prone to these errors.

### 4.1.3 $r_0$ and $\langle W^2 \rangle^{1/2}$ from direct models of $\xi_s(\sigma, \pi)$

Through the use of simulations, B83 showed that the more reliable and less biased estimate of $\langle W^2 \rangle^{1/2}$ came from modelling $\xi_s(\sigma, \pi)$ directly. We therefore take as our model

$$\xi_s(\sigma, \pi) = \int_{-\infty}^{\infty} \xi_s(\sigma^2 + (\pi - W/H_0)^2)^{1/2} dW,$$  \hspace{1cm} (4.7)

where it is assumed that line-of-sight relative peculiar velocities $W$ are drawn at random from a distribution $f(W, \langle W^2 \rangle^{1/2})$. For $f$ we assume the $\exp(-|W|^{1/2})$ form from the simulations of Efstathiou & Eastwood (1981, see also equation 15a of B83) and for $\xi_s$ we continue to use the power law with $\gamma = 1.8$. To estimate $r_0$ and $\langle W^2 \rangle^{1/2}$ we apply a two-parameter least-squares fit to the data in the $\xi_s(\sigma, \pi)$ versus $\pi$ plane with equal weighting for each point.

Our estimates of $\xi_s(\sigma, \pi)$ for the Durham/SAAO Survey (histograms) and the corresponding best models (solid lines) are shown at 4 separations in Fig. 3. The fits were truncated at $\pi_{cut} = 16 \ h^{-1} \ Mpc$ to include the possibly significant correlations seen out to large scales in the $\sigma < 1.0 \ h^{-1} \ Mpc$ figures. However, reducing $\pi_{cut}$ to 10 $h^{-1} \ Mpc$ does not significantly alter our results. Table 4(a) lists the values of $r_0$ and $\langle W^2 \rangle^{1/2}$ used in the models.

Except for a slightly flatter $\xi_s$ than the model predicts at separations $\pi < 8 \ h^{-1} \ Mpc$ seen in Fig. 3(c), it is clear that the model provides a good representation of the data. The values of $\langle W^2 \rangle^{1/2} \sim 600 \ km \ s^{-1}$ at $\sigma < 2 \ h^{-1} \ Mpc$ in the table are in reasonable agreement with the value of 460 km s$^{-1}$ from the moments method (previous section), confirming the suggestion that our estimates are much in excess of the 200 km s$^{-1}$ observed for the Durham/AAT Survey. These values also appear to be reasonably consistent with $\langle W^2 \rangle^{1/2}$ being independent of scale. We discuss these results in more detail in Section 4.3.

As we saw in Section 4.1.1, the uncertain nature of $\xi$ at large scales can lead to poor estimates of the small scale $r_0$ from $\omega_s(\sigma)$. We now consider how well the $r_0$ values are determined from the methods in this section. Again we have applied the analysis described here to the simulations. In this case we find that we can successfully predict the $r_0$ characterizing the power law below the simulated break scale even though we use a large $\pi_{cut}$ of 13 $h^{-1} \ Mpc$ for the fits. This shows that this method is more sensitive to small scales. Looking now at the $r_0$ values we obtain from the Durham/SAAO Survey (Table 4a), we see that the mean value of 5.8 $h^{-1} \ Mpc$ is larger than the value of 5.3 $h^{-1} \ Mpc$ obtained from $\omega_s(\sigma)$ assuming a continuous power law to large scales, but comparable to the 5.6 $h^{-1} \ Mpc$ when $\pi_{cut}$ was reduced to 10 $h^{-1} \ Mpc$ in that estimate.
Following B83 and DP83, we have also considered whether it is reasonable to neglect infall in this analysis. The dotted curves in Fig. 3 and the values in Table 4(b) were obtained when their infall model was incorporated into equation (4.7). As with estimates of $\langle W^2 \rangle^{1/2}$ from the moments method (previous section) the similarity between the values in Table 4(a) and (b)

---

**Figure 3.** Estimates of $\xi_v(\sigma, \pi)$ for the Durham/SAAO Survey. The solid and dotted curves show the best fits to these data using models which neglect and incorporate infall, respectively. The corresponding values for the parameters $r_e$ and $\langle W^2 \rangle^{1/2}$ are given in Table 4(a) and (b).
at $\sigma < 2.0 \, h^{-1} \text{Mpc}$ indicate that peculiar velocities tend to dominate the motions at small scales.

4.2 THE AMPLITUDE, $Q$, OF THE REDUCED THREE-POINT CORRELATION FUNCTION

The amplitude, $Q$, of the reduced three-point function was derived according to the prescription outlined in Section 3.1. In Table 5 we present the results for the Durham/SAAO
Table 4. Durham/SAAO Survey estimates of \( r_0 \) and \( \left\langle W^2 \right\rangle^{1/2} \) from model fits to \( \xi_r(\sigma, \pi) \).

(a) Without infall.  

<table>
<thead>
<tr>
<th>( \sigma ) (h(^{-1}) Mpc)</th>
<th>( \left\langle W^2 \right\rangle^{1/2} ) (km s(^{-1}))</th>
<th>( r_0 ) (h(^{-1}) Mpc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>680 ± 150</td>
<td>5.8 ± 0.7</td>
</tr>
<tr>
<td>0.75</td>
<td>560 ± 220</td>
<td>5.7 ± 0.8</td>
</tr>
<tr>
<td>1.50</td>
<td>620 ± 450</td>
<td>7.3 ± 0.7</td>
</tr>
<tr>
<td>3.00</td>
<td>70 ± 560</td>
<td>4.5 ± 1.0</td>
</tr>
</tbody>
</table>

(b) with infall.  

<table>
<thead>
<tr>
<th>( \sigma ) (h(^{-1}) Mpc)</th>
<th>( \left\langle W^2 \right\rangle^{1/2} ) (km s(^{-1}))</th>
<th>( r_0 ) (h(^{-1}) Mpc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>710</td>
<td>5.8</td>
</tr>
<tr>
<td>0.75</td>
<td>650</td>
<td>5.7</td>
</tr>
<tr>
<td>1.50</td>
<td>700</td>
<td>7.1</td>
</tr>
<tr>
<td>3.00</td>
<td>320</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Shown here are estimates of the correlation length, \( r_0 \), and line-of-sight rms peculiar velocity, \( \left\langle W^2 \right\rangle^{1/2} \), obtained from the best fit models to the \( \xi_r(\sigma, \pi) \) histograms in Fig. 3. In (a) the model for \( \xi_r(\sigma, \pi) \) is as given in equation (4.7), whereas in (b) this model was modified to incorporate infall.

Table 5. The amplitude, \( Q \), determined from the Durham/SAAO Survey.

<table>
<thead>
<tr>
<th>( \sigma ) (h(^{-1}) Mpc)</th>
<th>( u )</th>
<th>( 0 &lt; v &lt; 0.5 )</th>
<th>( 0.5 &lt; v &lt; 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1.11</td>
<td>0.44</td>
<td>0.5 &lt; v &lt; 1.0</td>
</tr>
<tr>
<td>0.75</td>
<td>1.14</td>
<td>0.47</td>
<td>0.5 &lt; v &lt; 1.0</td>
</tr>
<tr>
<td>1.50</td>
<td>0.24</td>
<td>0.39</td>
<td>0.5 &lt; v &lt; 1.0</td>
</tr>
<tr>
<td>3.00</td>
<td>0.09</td>
<td>-0.29</td>
<td>-0.07</td>
</tr>
</tbody>
</table>

The amplitude, \( Q \), of the reduced three-point function is shown at various values of the projected shape parameters \( \sigma \), \( u \), and \( v \). To make the significance of the results more uniform the smaller sized triangles have been combined into a larger bin in the parameter \( u \). The mean of the values in the table is

\[
Q = 0.48 \pm 0.1, \tag{4.8}
\]

where the uncertainty quoted is representative of the fluctuations in the various estimates.

4.3 COMPARISON WITH THE RESULTS FROM OTHER SURVEYS

In this section we compare the results we have derived for the Durham/SAAO Survey with those obtained from other magnitude-limited galaxy redshift surveys. Particular attention is given to the Durham/AAT and Parker et al. samples which we have analysed in an identical fashion to that described above. The results we present for \( r_0 \) and \( \left\langle W^2 \right\rangle^{1/2} \) from the Durham/AAT Survey are very similar to those of B83, but here the errors were obtained in a consistent manner from our simulations. The results and uncertainties for the Parker et al. survey are discussed in more detail by Hale-Sutton (in preparation). In this section we obtain average values for all three surveys and compare them directly with the results derived by DP83 for the larger volume, but smaller depth, CfA Survey (Huchra et al. 1983). Other comparisons are made with the results obtained by P79 for the Kirshner, Oemler & Schechter sample (1978, the
KOS Survey) and with the estimates for \( r_0 \) and \( Q \) from the two-dimensional Zwicky (Zwicky et al. 1961–68) and Lick (Shane & Wirtanen 1967) catalogues by Groth & Peebles (1977).

In Table 6(a) the unbracketed values of \( r_0 \) with \( \gamma = 1.8 \) for the Durham/SAAO, Durham/AAT and Parker et al. surveys have been obtained from fits to \( \xi_v(\sigma, \pi) \) neglecting streaming motion (Section 4.1.3). The same values are shown in Fig. 4 together with the \( r_0 \) values estimated from \( \xi_v(\sigma, \pi) \) for the CfA and KOS surveys [here \( \xi_v(\sigma, \pi) \) was integrated to \( \pi = 10 \) h\(^{-1}\) Mpc and \( \gamma \) was again constrained to be 1.8]. First, we note that there is some scatter in these small-scale values of \( r_0 \). Considering the range \( \sigma < 1 \) h\(^{-1}\) Mpc (which is the most important scale for virial analyses) we obtain an average of \( r_0 = (5.8 \pm 0.6) \) h\(^{-1}\) Mpc for the Durham/SAAO sample as compared with \( r_0 = (4.5 \pm 0.4) \) h\(^{-1}\) Mpc for the Durham/AAT. Parker et al.’s survey, in contrast, has an \( r_0 \) of \( (2.7 \pm 0.7) \) h\(^{-1}\) Mpc at this scale. Given the errors in these estimates, this scatter around our overall value of \( r_0 = (4.5 \pm 0.3) \) h\(^{-1}\) Mpc for these three surveys is reasonable. The good statistical agreement of this estimate with the \( r_0 = 5.2 \) h\(^{-1}\) Mpc (\( \gamma = 1.8 \)) for the CfA Survey (DP83) shows that all these results seem to be converging to around the value of \( 4.7 \) h\(^{-1}\) Mpc (\( \gamma = 1.77 \)) determined from the two sky catalogues and shown as a dashed line in Fig. 4.

We see from larger values of \( \sigma \), however, that there is some indication for an increasing \( r_0 \) with separation which would seem to imply \( \xi_v(\sigma) \) has a less rapidly decreasing slope at small scales than modelled here. We have to be cautious in interpreting this as such as we have limited our analysis to separations \( \pi < 10 \) h\(^{-1}\) Mpc where streaming motions may have an effect. However, the rise in \( r_0 \) is still possible there, even when an infall model is taken into account (Table 6a, values in brackets). Although the evidence for the rise in amplitude from the \( \xi_v(\sigma, \pi) \) data is still marginal, further evidence for the presence of a ‘shoulder’ feature in \( \xi_v(\pi) \) is seen in the redshift correlation function, \( \xi_v(s) \), for these surveys as discussed in Paper III.

Table 6(b) and Fig. 5 enable us to compare estimates of the rms relative peculiar velocity \( \langle W^2 \rangle^{1/2} \). Except in the case of the CfA Survey, where a different model for the distribution of peculiar velocities was used, the methods are as described in Section 4.1.3. The values of \( \langle W^2 \rangle^{1/2} \) which were obtained using a streaming motion model are indicated as open symbols in the figure and bracketed numbers in the table. As we are comparing between catalogues with

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Table 6. Estimates of \( r_0 \) and \( \langle W^2 \rangle^{1/2} \) from the Durham/SAAO, Durham/AAT and Parker et al. surveys.

(a) \( r_0 \) (h\(^{-1}\) Mpc)  
\begin{tabular}{cccc}
\hline
\hline
\( \sigma \) (h\(^{-1}\) Mpc) & D/SAAO & D/AAT & Parker & Overall \\
\hline
0.25 & 5.8 (5.8) \pm 0.7 & 4.5 (4.3) \pm 0.4 & 2.9\pm0.8 & 4.6 (4.8) \pm 0.3 \\
0.75 & 5.7 (5.7) \pm 0.8 & 4.5 (4.3) \pm 0.6 & 2.2\pm0.9 & 4.3 (4.8) \pm 0.4 \\
1.50 & 7.3 (7.1) \pm 0.7 & 5.6 (5.3) \pm 0.7 & 3.6\pm1.3 & 6.0 (6.1) \pm 0.5 \\
3.00 & 4.5 (3.4) \pm 1.0 & 6.9 (6.1) \pm 1.2 & - & 5.5 (4.6) \pm 0.8 \\
\hline
\end{tabular}

(b) \( \langle W^2 \rangle^{1/2} \) (km s\(^{-1}\))  
\begin{tabular}{cccc}
\hline
\hline
\( \sigma \) (h\(^{-1}\) Mpc) & D/SAAO & D/AAT & Parker & Overall \\
\hline
0.25 & 660 (690) \pm 150 & 220 (230) \pm 120 & 0\pm250 & 330 (400) \pm 90 \\
0.75 & 530 (630) \pm 220 & 170 (220) \pm 130 & 0\pm320 & 230 (320) \pm 100 \\
1.50 & 600 (680) \pm 450 & 190 (280) \pm 230 & 170\pm10 & 260 (360) \pm 190 \\
3.00 & 0 (270) \pm 560 & 0 (190) \pm 540 & - & 0 (230) \pm 390 \\
\hline
\end{tabular}

Estimates of \( r_0 \) and \( \langle W^2 \rangle^{1/2} \) from model fits to \( \xi_v(\sigma, \pi) \) are given at small projected separations, \( \sigma \). The unbracketed values with errors and the bracketed values are, respectively, for models that exclude and include infall. The combined estimates for all three surveys are given in the columns headed ‘Overall’. The infall values in this case are just for the Durham surveys.
Figure 4. Small-scale estimates of the correlation length, $r_0$ (in units of h$^{-1}$ Mpc), obtained from $\xi_0(\sigma, \pi)$ analyses in several redshift samples. The filled circles with error bars represent our combined estimate for the Durham/SAAO, Durham/AAT and Parker et al. surveys. The dashed line, with $r_0 = 4.7$ h$^{-1}$ Mpc, is the Groth & Peebles (1977) result for the Lick and Zwicky sky catalogues.

Figure 5. Small-scale estimates of the line-of-sight rms peculiar velocity, $\langle W^2 \rangle^{1/2}$, obtained from model fits to $\xi_0(\sigma, \pi)$. Filled and open symbols are for models which neglect and include infall, respectively. The combined estimates for the Durham/SAAO, Durham/AAT and Parker et al. surveys are shown as circles (the infall estimates omit the Parker et al. survey).
different redshift measurement errors, all values have been approximately corrected according to
\[ \left\langle W^2 \right\rangle_{corr} = \left[ \left\langle W^2 \right\rangle_{obs} - 2 \delta(V_z)^2 \right]^{1/2}, \]  
(4.9)
where \( \delta(V_z) \) is the quoted uncertainty for a single galaxy. The values used here are given in Table 7.

We see from Table 6(b) that the \( \sigma < 2 \) h\(^{-1}\) Mpc values obtained by neglecting streaming motion in the Durham/SAAO Survey appear to be larger than previously obtained for similar estimates in the Durham/AAT Survey. Again comparing the better determined estimates at separations \( \sigma < 1 \) h\(^{-1}\) Mpc we find that the mean value of \( \left\langle W^2 \right\rangle^{1/2} = (600 \pm 140) \) km s\(^{-1}\) for the Durham/SAAO differs from the value of \( \left\langle W^2 \right\rangle^{1/2} = (190 \pm 90) \) km s\(^{-1}\) for the Durham/AAT by about 2.5 s.d. As a result of this, it might be conceived that the higher peculiar velocity in the Durham/SAAO Survey was due to larger redshift measurement errors in this sample; however, we conclude from the study in Paper I that \( \delta(V_z) \approx 120 \) km s\(^{-1}\) is likely to be an upper limit for this sample. The Parker et al. survey, on the other hand, indicates little or no evidence for peculiar velocities at these small separations which again shows that we have obtained a wide range of values for \( \left\langle W^2 \right\rangle^{1/2} \) using a consistent method of analysis applied to all three samples.

The large range of \( \left\langle W^2 \right\rangle^{1/2} \) values we have obtained from fits to \( \xi_{\sigma, \pi} \) in the Durham/SAAO, Durham/AAT and Parker et al. surveys is, we believe, a result of sampling fluctuations in these surveys. Indeed, other authors have found a similar diversity in estimates of \( \left\langle W^2 \right\rangle^{1/2} \): for example, P79 reported a value of \( \left\langle W^2 \right\rangle^{1/2} \approx 500 \) km s\(^{-1}\) for the KOS Survey. As we discussed in Section 3.2, these statistical fluctuations are reflected in the field-to-field errors in the Durham/SAAO and Durham/AAT surveys but not in the hierarchical simulations. Here, we have attempted to incorporate such variations in our error estimates whereas previously B83 relied heavily on the simulations. However, in spite of these observations we find that our overall estimates of \( \left\langle W^2 \right\rangle^{1/2} \) do agree closely with the CfA Survey (see Fig. 5) and this may indicate that such surveys are reasonably fair samples of the Universe.

Our mean estimate of \( \left\langle W^2 \right\rangle^{1/2} = (290 \pm 70) \) km s\(^{-1}\) at \( \sigma < 1 \) h\(^{-1}\) Mpc for the combined Durham/SAAO, Durham/AAT and Parker et al. sample is drawn as a dashed line in Fig. 5. As this figure shows it is clearly not possible, with the present data, to say whether our results show any tendency to rise with increasing separation according to \( \left\langle W^2 \right\rangle^{1/2} \propto \sigma^{0.1} \) (\( \gamma = 1.8 \), solid line Fig. 5) as in the prediction of the cosmic virial theorem (Peebles 1976). However, as seen in the CfA Survey estimates (this diagram and fig. 6, DP83) any increase depends critically on the somewhat uncertain streaming motion model.

Finally, we consider estimates of the three-point amplitude, \( Q \). The value of \( Q = 0.48 \pm 0.1 \) we obtain for the Durham/SAAO Survey is in good agreement with the 0.60 \pm 0.06 obtained for the Durham/AAT. Further, Peebles (1981) derived \( Q = 0.68 \pm 0.05 \) for the compilation of

### Table 7. Redshift measurement errors for the different surveys.

<table>
<thead>
<tr>
<th>Survey</th>
<th>( \delta(V_z) ) (km s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>D/SAAO</td>
<td>120</td>
</tr>
<tr>
<td>D/AAT</td>
<td>50</td>
</tr>
<tr>
<td>Parker</td>
<td>75</td>
</tr>
<tr>
<td>CfA</td>
<td>50</td>
</tr>
</tbody>
</table>

The adopted mean redshift error for the four surveys discussed in the text.
redshifts by Rood (1982). However, these values are slightly at odds with the value of $Q = 1.3 \pm 0.3$ for the KOS Survey (B83) and $1.3 \pm 0.2$ for the two sky catalogues (Groth & Peebles 1977). As the three-point function is sensitive to regions of dense clustering, B83 argued that their lower value of $Q$ was as a result of undersampling such areas. However, we feel that this statement is no longer justified as the new Durham Survey, with its larger volume coverage, gives a similar result.

5 Application of the cosmic virial theorem

We now use the statistical virial theorem of Peebles (1976) to obtain estimates for the mean mass density of the Universe. The method is based on the assumption that the small-scale galaxy distribution is sufficiently clumpy and non-evolving that its internal dynamical pressure can be supported by the self-gravity of its mass. Thus, provided galaxies trace the mass distribution, the dynamics of a randomly chosen pair can be determined from the observed two- and three-point correlation functions. Modelling $\xi(r)$ as $(r_0/r)^{2\gamma}$ and $\zeta(r_1, r_2, r_3)$ as in equation (3.8), the theorem reduces, for an isotropic velocity dispersion, to

$$\langle V^2(r) \rangle = C_\gamma H_0^2 \Omega_0 r_0^2 r^{2\gamma}$$  \hspace{1cm} (5.1)

(Peebles 1976, equation 29), where $\langle V^2(r) \rangle$ is the mean square relative velocity in the radial direction, $\Omega_0$ is the density parameter and $C_\gamma$ is a constant depending only on $\gamma$.

In the detailed application to the Durham/SAAO Survey we adopt the same models as used by B83. In particular they included a cut-off parameter, $\epsilon$, in the potential of the mass distribution to account of the finite size of a galaxy. Using, as they did, the solid line of their fig. 7 we find that at a separation $\sigma = 0.5 \ Mpc$

$$\langle W^2(\sigma) \rangle^{1/2}(\Omega_0 Q)^{-1/2} = 833 \ km \ s^{-1},$$  \hspace{1cm} (5.2)

where $\langle W^2(\sigma) \rangle^{1/2}$ is the usual rms relative velocity along the line-of-sight. This model has $r_0 = 4.1 \ Mpc$ and $\epsilon = 10 \ h^{-1} \ kpc$. To derive estimates of $\Omega_0$ for our values of $r_0$ we scale equation (5.2) according to equation (5.1).

At this point we discuss what estimates of $r_0$ and $\langle W^2 \rangle^{1/2}$ would be most suited for use here. The assumptions inherent in the virial theorem imply that we should confine ourselves to the smallest scales. We consider that the methods of Section 4.1.3 provide us with an accurate determination of the required parameters especially if we choose $\sigma < 1 \ Mpc$, as this then reduces the problems of streaming motions and an uncertain $\xi$ at larger scales. Thus we find for the Durham/SAAO Survey

$$r_0 = (5.8 \pm 0.6) \ Mpc$$  \hspace{1cm} (5.3)

and

$$\langle W^2 \rangle^{1/2} = (600 \pm 140) \ km \ s^{-1}$$  \hspace{1cm} (5.4)

(see Section 4.3). Thus using equations (5.3), (5.4) and the estimate of $Q$ in (4.8) we find

$$\Omega_0 = 0.6 \pm 0.2.$$  \hspace{1cm} (5.5)

The error we have quoted here assumes that $r_0$, $\langle W^2 \rangle^{1/2}$ and $Q$ are independent. This value is larger than the $\Omega_0 = 0.1 \pm 0.1$ for the Durham/AAT Survey and this is directly as a result of the much higher peculiar velocities in the new sample. Indeed, the wide variation in estimates of $\langle W^2 \rangle^{1/2}$ seen in Section 4.3 carries through directly to our estimates of $\Omega_0$ and thus the conclusions that we derived above about such fluctuations being statistical will also apply here.
Combining the values from the Durham/SAAO, AAT and Parker et al. surveys we find

\[ r_0 = (4.5 \pm 0.3) \, h^{-1} \, \text{Mpc} \]  \hspace{1cm} (5.6)

and

\[ \langle W^2 \rangle^{1/2} = (290 \pm 70) \, \text{km s}^{-1}, \]  \hspace{1cm} (5.7)

which together with the mean value of \( Q = 0.58 \pm 0.05 \) for the Durham surveys implies

\[ \Omega_0 = 0.18 \pm 0.09. \]  \hspace{1cm} (5.8)

Using the CfA values (see Section 4.3) in equation (5.2) with \( Q = 0.6 \) we find \( \Omega_0 = 0.14 \), which again shows that the estimates from our different samples are, as a whole, in good agreement with the result from the CfA Survey.

6 Conclusions

Thus our conclusions are as follows:

(i) In the new Durham/SAAO Survey we have obtained a line-of-sight rms peculiar velocity difference of \( (600 \pm 140) \, \text{km s}^{-1} \) for galaxy pairs with projected separations \( \sigma < 1 \, h^{-1} \, \text{Mpc} \). This is somewhat higher than the previous estimate of \( (190 \pm 90) \, \text{km s}^{-1} \) found in the Durham/AAT Survey but we believe that the difference is probably consistent with sampling errors. From these two surveys and from the Parker et al. sample we find an overall value of

\[ \langle W^2 \rangle^{1/2} = (290 \pm 70) \, \text{km s}^{-1} \]  \hspace{1cm} (6.1)

and we note that this is in excellent agreement with the result from the CfA Survey (DP83).

(ii) In the Durham/SAAO Survey the two-point correlation function clustering scale length, \( r_0 \), is found to be \( (5.8 \pm 0.6) \, h^{-1} \, \text{Mpc} \) at \( \sigma < 1 \, h^{-1} \, \text{Mpc} \), in reasonable statistical agreement with the value of \( (4.5 \pm 0.4) \, h^{-1} \, \text{Mpc} \) in the Durham/AAT Survey. The overall value from the three surveys is

\[ r_0 = (4.5 \pm 0.3) \, h^{-1} \, \text{Mpc} \]  \hspace{1cm} (6.2)

in satisfactory agreement with the value of \( 5.2 \, h^{-1} \, \text{Mpc} \) from the CfA and \( 4.7 \, h^{-1} \, \text{Mpc} \) from the 2D Lick and Zwicky catalogues (Groth & Peebles 1977).

(iii) The amplitude of the three-point correlation function, \( Q \), is found, from the Durham/SAAO Survey, to be \( 0.48 \pm 0.1 \) and this is in good agreement with the value found in the previous survey by B83. Our overall value is thus

\[ Q = 0.58 \pm 0.05. \]  \hspace{1cm} (6.3)

(iv) On the assumption that galaxies trace the mass, the application of the cosmic virial theorem in the Durham/SAAO Survey gives \( \Omega_0 = 0.6 \pm 0.2 \) as compared with \( 0.1 \pm 0.1 \) from the Durham/AAT Survey. Our best estimate of \( \Omega_0 \) from the overall estimates of the velocity dispersions and correlation function amplitudes in the three surveys is

\[ \Omega_0 = 0.18 \pm 0.09. \]  \hspace{1cm} (6.4)

Thus, we confirm the result of B83 and DP83 that the \( \Omega_0 = 1 \) models are excluded unless galaxies are more clustered than the mass. However, despite the larger sample the significance of the rejection is about the same as before due to the increased error on the peculiar velocity estimates suggested by the new surveys.

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