The mass distribution in the galactic disc – III.
The local volume mass density

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Summary. Evidence for missing mass associated with the galactic disc is usually
derived from solutions of the combined Poisson and Boltzmann equations
which describe the velocity dispersion and density data for samples of F dwarfs
and K giants near the Sun. Previous analyses have deduced that perhaps 50 per
cent of the local volume mass density is not identified with known stars or with
the interstellar medium. We show here that some of the data on which such
conclusions were based are internally inconsistent. Similarly the velocity
models previously applied to analyse the remaining data are not consistent with
the observed age-velocity dispersion relations near the Sun, and with stellar
evolutionary tracks. Reanalysis of the available F dwarf and K giant data with
more plausible models provides no evidence for any missing matter near the
Sun in addition to the identified \( \rho_{o, id} = 0.10 M_\odot \text{ pc}^{-3} \).

1 Introduction
If one knew both the total surface mass density of the galactic disc and the volume mass density
in the plane, then one could readily deduce the scaleheight (equivalently, the vertical velocity
dispersion in the plane) of any unidentified contribution to the local volume mass density. This
might in turn help elucidate its nature. For example, if any missing mass in the solar
neighbourhood volume mass density (the Oort limit) were found to have a contribution to the
surface mass density consistent with a scaleheight of 50–100 pc, one might reasonably explain
the local ‘missing’ mass as an underestimate of the mass density in the local interstellar medium
(ISM). Similarly, if the deduced scaleheight were similar to that of the gravitationally dominant
old stellar population (\( \sim 300 \) pc), then either or both of very low-mass stars (brown dwarfs)
and very low-luminosity degenerates (cool white dwarfs) would be a plausible identification.

In Paper I of this series (Kuijken & Gilmore 1989a) we developed a new method for
deducing the total surface mass density of the galactic disc, and in Paper II (Kuijken & Gilmore
1989b) we applied it to deduce a surface mass density of \( 46 \pm 9 M_\odot \text{ pc}^{-2} \). Using the galactic
vertical force law \( K_z(z) \) deduced in the same paper, we showed that the surface mass density
accounted for by identified stars in the solar neighbourhood integrated through the \( K_z \) force
law \( \sim 35 \pm 5 M_\odot \) pc\(^{-2}\). The directly observed column density of the ISM, whose spatial distribution and contribution to the local volume mass density is, however, poorly known, is \( \sim 13 \pm 3 M_\odot \) pc\(^{-2}\). Thus an amount of \(-2 \pm 12 M_\odot\) pc\(^{-2}\) remains unidentified in the integral surface mass density (Paper II).

By contrast, recent determinations of the volume mass density near the Sun – the Oort limit – deduce that 50 per cent or more of the local volume mass density remains unidentified near the Sun (Bahcall 1984b,c). Comparison of this result with that of Papers I and II referred to above clearly requires that the unidentified contribution to the local volume mass density must have a vertical scaleheight characterizing its distribution no greater than that of the bulk of the ISM. If one accepts that there is some form of age-velocity dispersion relationship followed by stars in the disc, regardless of its physical cause, then old stars are thereby excluded as candidates for the local missing mass. Thus brown dwarfs and old white dwarfs cannot contribute significantly to the mass of the disc.

This important conclusion depends sensitively on the amount of unidentified mass in the local volume density. If the local volume mass density were significantly lower than the \( \sim 0.18 M_\odot\) pc\(^{-3}\) deduced by Bahcall (1984b,c) then a correspondingly larger scaleheight would be allowed. Similarly, the range of candidates for this missing mass would also widen. Hence in this paper we reconsider the determination of the volume mass density near the Sun, with particular emphasis on possible systematic effects in the analyses of local F star and K giant data. Two alternative methods to those generally applied for measuring \( \rho_0\) are noted in Section 5.

## 2 Determination of the local volume mass density, \( \rho_0\)

Measurement of the gravitational potential perpendicular to the galactic disc is based on the collisionless Boltzmann equation, and is discussed in detail by Bahcall (1984a) and in Paper I. In practice, the moment equation including the velocity dispersion perpendicular to the galactic plane – the vertical Jeans’ equation – is of most interest (cf. Paper II). This equation reads

\[
- K_z = \frac{1}{v} \frac{\partial}{\partial z} \left( v \sigma_z^2 \right) + \frac{1}{vR} \frac{\partial}{\partial R} \left( v R \sigma_R^2 \right),
\]

where \(K_z\) is the force component perpendicular to the disc plane \(z = 0\), and \(R\) is the cylindrical radius from the axis of symmetry of the disc. \(v(z)\) refers to the volume density of a tracer population from whose kinematics we deduce the potential, and \(\sigma_z^2 = \langle v^2 \rangle\) is its velocity dispersion tensor. The second term on the RHS (containing \(\sigma_R^2\)) is discussed at length in Paper I, and utilized in the analysis of Paper II. For data restricted to be very near the galactic plane it contributes only a few per cent to \(K_z\), and may be ignored safely (cf. Bahcall 1984c). A further assumption which has been made in previous analyses (and shown to be consistent with available data) is that the available tracer population is isothermal, or a sum of a few such isothermals. In this case (cf. Paper I), equation (1) reduces to

\[
K_z = \sigma_z^2 \frac{\partial}{\partial z} \ln v(z).
\]

The important feature of this equation is that it depends on the gradient of the spatial density distribution of the tracer population. Thus considerable care is required to define the density law appropriately.
Derivation of the local volume mass density $\rho_0$ is possible using the Poisson equation, which relates the volume mass density $\rho$ to the gravitational potential $\psi$,
\[ \nabla^2 \psi = -4\pi G \rho. \tag{3} \]

The Jeans’ equation (1) may be combined with the Poisson equation (3) to yield, for a tracer population near $z = 0$ in a disc with a flat rotation curve,
\[ 4\pi G \rho_0 = \frac{\partial}{\partial z} \left[ \frac{1}{v} \frac{\partial}{\partial z} (v \sigma_z^2) \right]. \tag{4} \]

This combined Poisson–Boltzmann equation can be solved self-consistently to derive the local volume mass density $\rho_0$ from local velocity dispersion and density law data. (One must also consider the contribution of the dark halo required to generate a flat rotation curve to the local volume density. In practice this contribution is small – typically $\lesssim 0.1 \rho_0$ – simplifying the analysis and interpretation.) Full details are provided by Bahcall (1984a,b,c), whose analysis technique we adopt in this paper.

An important feature of equation (4) is that it depends on the second derivative of the space density data, and hence involves a factor of the square of the distance scale. Since the space densities are themselves derived from observations, one must therefore be very careful of the possibility of systematic errors. (The effects of random errors are discussed further below.) In order to test the reliability of the derived value of the local volume mass density, we repeat the analysis of Bahcall (1984b,c), utilizing his analysis technique and applying it to the same datasets analysed by him. We investigate here the sensitivity of the resulting value of $\rho_0$ to the way in which the data are binned before analysis, to the adopted set of isothermal fits to the local velocity data, and to systematic uncertainties in the density law.

3 Determinations of $\rho_0$ Based on F Stars

3.1 Analysis of the Data

A suitable tracer stellar population for the determination of the local volume mass density $\rho_0$ must be numerous and of precisely determinable absolute magnitude. The former requirement is necessary to ensure useful statistical significance in the result, the second since the derived value depends on the square of the distance scale, through equation (4). The surface mass density by comparison is linearly proportional to the distance scale. A potentially suitable tracer population which has been utilized extensively is the population of F stars.

An extensive study of these stars was reported by Hill, Hilditch & Barnes (1979, henceforth HHB); subsequent observations are reported in Adamson et al. (1988), and used by them to deduce a local volume mass density of $\rho_0 = 0.14 M_\odot \text{pc}^{-3}$. However, as pointed out by Bahcall (1984b), the analysis of HHB was not dynamically self-consistent, since the density of each of the mass components they used to model the solar neighbourhood was taken to be that of an isolated, self-gravitating population, rather than one which responds to the total gravitational field.

The analysis of HHB was superseded by the work of Bahcall (1984a,b), who devised a self-consistent solution technique to equations (2) and (4). This is a substantial improvement on earlier work, and is adopted here. As Bahcall emphasized, a major source of uncertainty in such an analysis is due to the possibility that there is unidentified mass in the galactic disc, whose distribution is a priori unknown. A wide range of self-consistent models is then possible. These are parameterized conveniently by a single number $P$, which is the volume mass density...
at $z=0$ of dark matter expressed as a fraction of the mass density of identified material. The vertical distribution of the dark matter is taken to be the same as that of some or all component(s) of the identified mass.

Bahcall (1984b) showed that the sum of the two F star samples of HHB (F5 and F8 stars) was consistent (to within 10 per cent) with having a single isothermal distribution, and so combined them into a single sample. He fitted a wide range of self-consistent potentials to these data, and adopted a preferred model in which $\rho_0 = 0.185 \pm 0.02 M_\odot \, \text{pc}^{-3}$. This is derived from the HHB data over the $z$-range 0–200 pc, for a model in which the unobserved matter is distributed proportionately to all components of the identified matter.

To provide an estimate of the uncertainty in the value of $\rho_0$ due to the manner of binning the data, we investigate the fits allowed by the F5 and F8 samples separately. Each has been shown to be consistent (to within 10 per cent) with an isothermal velocity distribution, with the same velocity dispersion applying to each sample.

Any isothermal population must obey the Jeans' equation (2). In particular, since all stars move in the same potential, any two relaxed populations with the same velocity dispersion must follow the same relative density law. As the subsamples of F5 and F8 stars which make up the F star sample appear to have identical kinematics (both are isothermal at a velocity dispersion of $\sim 11 \, \text{km s}^{-1}$, see Adamson et al. 1988), they must follow the same density law $\nu(z)/\nu(0)$. However, as Fig. 1 shows, this is not the case. The density decrease of F5 stars is significantly steeper than that of the F8 stars, to the extent that (relative to the ratio at $z=0$) there are 45 per cent fewer F5 stars than F8 stars at 200 pc. To illustrate the effect of this difference on determinations of $\rho_0$ we also show in Fig. 1 model fits to the F5 and the F8 samples separately, using the same technique as was used in Bahcall (1984b). The resulting values of the local volume mass density are $\rho_0 = 0.11 M_\odot \, \text{pc}^{-3}$ (F8 sample) and $\rho_0 = 0.29 M_\odot \, \text{pc}^{-3}$ (F5 sample). For comparison, the value derived from a fit to the combined (F5+F8) sample is $\rho_0 = 0.18 M_\odot \, \text{pc}^{-3}$, and the identified mass density is $\rho_{\text{obs}} = 0.10 M_\odot \, \text{pc}^{-3}$.

Thus, analysis of the F8 star sample of HHB using the technique of Bahcall provides evidence for no 'missing' mass near the Sun. Analysis of the F5 star sample from the same

![Figure 1](https://example.com/figure1.png)

**Figure 1.** The Hill et al. (1979) F star samples. The difference between the supposedly identical density profiles of the F5 and the F8 samples is evident. The curves show separate Bahcall model fits to the F5 and the F8 subsets of the data defined by HHB. Only the averaged sample (solid points) was analysed by Bahcall (1984b). The models shown are defined by: $P=0.2$, $\rho_0 = 0.11 M_\odot \, \text{pc}^{-3}$ (solid line); $P=2$, $\rho_0 = 0.29 M_\odot \, \text{pc}^{-3}$ (dashed line), with $P$ as defined in the text.
source using the same technique provides a quite discordant result. Which, if either, of these results ought to have greater weight, and what is the origin of the discrepancy? We briefly discuss each of the more obvious possibilities in turn.

3.2 Sampling Errors in the Density Laws

Gilden & Bahcall (1985, henceforth GB) investigated the effect of sampling errors due to Poisson noise in the observational dataset from which the density law and the velocity dispersion were derived. They found that this caused a formal random error on $\rho_0$ of 15 per cent. However, a similar analysis by Crézé, Robin & Bienaymé (1989, hereafter CRB) concludes that random errors in the F star density law are in fact much larger than this. CRB deduce that the available F star data are consistent with a larger range of models, including ones without any missing matter. Both GB and CRB generated simulated star catalogues by sampling stars from theoretical distributions, and then treated these simulated data in the same way as the observed F stars. The main difference is that GB do not allow for observational errors in their simulated catalogues, whereas CRB fold in distance errors and analyse the $(m, \log \pi)$-table for each of their simulations. The latter group claim that this substantially increases the uncertainty when one is working with small numbers of stars, and in fact biases results towards too high a value for $\rho_0$.

3.3 The F Stars Are Not Dynamically Mixed

Early F stars are young compared to the vertical oscillation time-scale of stellar orbits. This means that they will not make many oscillations up and down through the disc before ceasing to look like F stars. Consequently, any correlations in their kinematics at birth (e.g. stellar associations, starbursts,...) will still be present now. This effect is clearly seen in the $(U, V)$-diagrams for A stars (Eggen 1965), and in those for F stars younger than NGC 752 (age $\sim 10^9$ yr) (Eggen 1969). The F5 sample of HBB, which contains a larger fraction of these young stars, will be particularly affected in this way. Nevertheless, assuming a constant star formation rate, c. 15–20 per cent of F8 stars are younger than NGC 752. In fact, Adamson et al. (1988) note that there appears to be structure in the velocity distribution of F stars near the plane, and that the distribution function they fit at greater height does not fit for $z < 100$ pc (see Fig. 2 and the discussion below). Unfortunately, it is at low $z$ that the weight of a determination of $\rho_0$ lies. A priori, we would expect the early type F stars to be less well-mixed than the later ones. Consequently the F8 density distribution may be a better tracer of the potential of the Galaxy, and the low value of $\rho_0$ derived from it should be given greatest weight.

3.4 The F Stars Are Not Isothermal

If we wish to use the F stars to estimate $\rho_0$, then we must know their velocity distribution at various $z$-heights. The analyses of Bahcall and of Adamson et al. conclude that the F stars are well-modelled as a single isothermal population. This is a critical assumption as far as a derivation of $\rho_0$ is concerned. Considering the evidence for a continuously rising velocity dispersion of F stars with age (Twarog 1980; Carlberg et al. 1985; Knude, Schnedler, Nielsen & Winther 1987), one would expect that any sample on which no age limits are set picks up a range of different components of different velocity dispersion. Observational studies of the rate of dynamical heating based on the nearby stars (Fuchs & Wielen 1987) show that a 'cold' population will heat up quite quickly, but that the rate of heating decreases with increasing velocity dispersion, as the stars then move too fast to have strong encounters. However, the
early F stars are still in the regime of strong heating, so that if the local studies are correct, the F stars should not form a single isothermal population. The young, low-dispersion populations will be concentrated towards the plane, and hence be an important contributor to the volume density there. They can in fact lead to a substantially reduced velocity dispersion near the plane relative to that at large \( z \)-distances (for a fuller discussion see Fuchs & Wielen 1987). In the Adamson et al. (1988) velocity data, the distribution of the F stars with \( z < 100 \) pc is not at all well-fitted by the same Gaussian which fits the higher \( z \) data. To illustrate this, Fig. 2 shows the single-Gaussian fit for \( z > 100 \) pc together with an alternative double-Gaussian fit containing stars in a 6 km s\(^{-1}\) component as well as the 11.5 km s\(^{-1}\) population we see at higher \( z \). The effects of a low-dispersion component which contaminates an otherwise isothermal population near the plane will be discussed further below in relation to the K giant samples.

3.5 ONE OR OTHER OF THE DENSITY LAWS IS IN ERROR

Any error in the distance scale will affect the density laws. If we assume that the difference between the HHB density laws is solely due to an error in the distance scale, then we would need to change the distances of the F5 stars relative to those of the F8 stars by a factor of \( c \cdot 1.7 \) (corresponding to a change in the absolute magnitude of \( \sim -1.2 \) mag) to bring the two densities into line. For comparison, the \( uvby\beta \) calibration of absolute magnitudes (Crawford 1975) has a formal relative error of 0.30 mag for a single star. Since this was tied into the parallax stars using 43 stars, the formal error in the mean is only 0.04 mag.

Could such a large difference as required by the data arise? F stars are near the turnoff of the bulk of the Population I stars, and hence we expect a lot of subgiants in any sample. The effect of such a contamination is poorly determined. The density laws adopted in these analyses were derived from Upgren's (1963) objective prism survey towards the north galactic pole. However, his spectra were not sufficiently detailed to distinguish dwarf stars from subgiants. We may estimate the possible effect of this indirectly. The Gliese catalogue (Gliese 1969) contains 125 F5-G2 dwarfs, and six subgiants in the same spectral range. Assuming subgiants are 1 mag brighter than dwarfs, this produces a contamination of 20 per cent in any
magnitude-limited catalogue. A *uvbyβ* study of stars in the AGK3 catalogue (Knude et al. 1987) yields an even greater subgiant fraction: among F stars brighter than $V = 10.05$ there are as many stars which are 'evolved' (i.e. 0.4–1.4 mag above the ZAMS) as there are within 0.2 mag of the ZAMS. This applies equally to 'early' (F1–F4) and 'middle' (F5–F7) F stars. The effect of subgiant contamination has been ignored in the density law calculations, which assume that all stars are main sequence stars with a dispersion in absolute magnitude of 0.5 mag. Since there will soon be a new extensive dataset of intermediate band photometry of F stars, there is little point in a detailed (and necessarily uncertain) modelling of the counts as they exist at present.

However, the likely direction and size of the change to the estimate of $\rho_0$ this will cause can be estimated crudely, as follows. The Malmquist correction (Malmquist 1936):

$$\Delta(M) = \frac{a}{\sigma_m} \frac{d \log A}{dm}$$

is large for stars close to the plane, where the number counts $A(m)$ have a slope of $\sim 0.6$, so knowing the dispersion in absolute magnitude is crucial. If the F stars are indeed contaminated by subgiants, then their average absolute magnitudes should be decreased (perhaps by $\sim 0.3$ mag), while the dispersion should be increased to $\sim 0.7$ mag. These would combine to give a decrease of the average magnitude-limited absolute magnitude of $\sim 0.6$ mag, which is equivalent to a distance scaling factor of 1.3. The estimates for $\rho_0$ would come down by the square of this factor, and in fact then be consistent with the best estimates for the density of observed material.

We emphasize that the above arguments are merely estimates of the possible size of effects that have not been taken fully into account in the analyses published to date. They do not constitute a full reanalysis, which would require an (as yet unavailable) new density law based on accurate photometric parallaxes. However, it is clear that large uncertainties in the value of $\rho_0$ as derived from F stars still remain, with the derived values ranging from $0.11 M_\odot$ pc$^{-3}$ to $0.29 M_\odot$ pc$^{-3}$, depending on which subset of the data is modelled.

4 Determinations of $\rho_0$ based on K giants

Ever since the first studies of $K_i$ (Kapteyn 1922; Oort 1932), most work has involved the use of K giants as tracers. These stars are intrinsically bright, have strong spectral lines, and are easy to identify on objective prism plates, all of which make them convenient to work with. However, the astrophysics of these stars is such as to require considerable care when using them as distance indicators.

The location of the red giant branch in the H–R diagram is a strong function of stellar age and metallicity, as well as having a very steep colour-magnitude relation. Obtaining precise absolute magnitudes of K giants is therefore difficult, as both systematic errors (due to metallicity or age differences) and random errors (due to photometric errors) can lead to large errors in the derived absolute magnitudes. Nevertheless, by using appropriate photometric indices, such as those of the DDO system, good results can be obtained. A project to do just this is currently under way (Flynn 1987). Until that is completed, the only available density laws for K giants are ones derived from objective prism surveys (Ungren 1962; Hill 1960; McNeil 1986), for which detailed luminosity classifications are difficult, and precise abundances unavailable. Thus the possible problems just mentioned are acute, and the density laws should be used with care. Assigning a constant value to the absolute magnitude of all K giants is also dangerous, since the known correlations between stellar age (hence, through dynamical heating, velocity dispersion and vertical scaleheight) and metallicity will lead to a z-
gradient in the average absolute magnitudes of K giants. Ignoring this possibility will lead to a systematic distance-dependent error in the assigned absolute magnitudes, and hence will skew the density law of K giants and confuse the $K_s$ analysis. Nevertheless, no allowance for a vertical age/abundance gradient has been made in investigations to date.

Using new values for the absolute magnitudes for K giants, Bahcall (1984c) rescaled the Hill and Ugpren density laws.* Models of the same type as used in conjunction with the F star samples were then fitted to the $z < 600$ pc space density data, assuming that over this distance range the K giants are adequately described by a single isothermal population of dispersion 20.3 km s$^{-1}$ [Hartkopf & Yoss 1982 (HY)]. The fits to the density laws $\nu(z)/\nu(0)$ were unweighted, and assume that the density at $z = 0$ is known with infinite precision – since the densities are derived from counts of a constant surface area on the sky, however, the volume near the plane sampled by such a survey is necessarily small, and hence the counting statistics poor. We have therefore repeated the fits to the K giant data as rescaled by Bahcall using the same models as discussed by him, but with a weighting of each data point inversely proportional to its sampling error, and with a free zero point normalization of the density law at $z = 0$. Both of these changes have the effect of lowering the potential of the best-fitting model (see Fig. 3) quite substantially (and of improving the fit). Rather than requiring 1.5 times as much dark matter as identified, after these changes the proportional amount of dark matter required is reduced to only 0.6.

The fact that K giants are astrophysically such a mixed bag of old and young stars means that we expect their kinematics to be a mixture of that observed for young and old stars. It would therefore be surprising if they comprised a single isothermal population. The evidence for isothermality cited in Bahcall (1984c) is the result of HY that the velocity dispersion is constant (at the typical ‘old disc’ value of $\sim 20$ km s$^{-1}$) below $z = 600$ pc; however, the sample analysed in fact contains only 10 stars below $z = 200$ pc on Bahcall’s distance scale (which is a factor of 1.8 larger than that used by HY). The velocity dispersion of these 10 stars is

*Flynn (1987) performed the same analysis on the recent results of McNeil’s (1986) objective prism survey, and came to the same conclusions as Bahcall.
21 ± 4.7 km s⁻¹, which can hide considerable non-isothermality. In his analysis of a larger sample of stars near both galactic poles, Hill (1960) obtained the distribution of z-velocities (from Wilson 1953) for 259 K giants near the plane (Fig. 4). This distribution is inconsistent with a Gaussian of dispersion 20.3 km s⁻¹, but can be fitted adequately with a two-component distribution containing similar numbers of stars in Gaussians of dispersions 11 km s⁻¹ and 20 km s⁻¹. This suggests that there is an extra component, important at low z, in the K giant population. This would indeed be expected for the younger stars among the giants, such as the ones with F star progenitors, which would have a velocity dispersion of c. 11 km s⁻¹. By the time we reach a z-height of 200 pc, this population is very depleted relative to the 20 km s⁻¹ one, so that the HY sampling does not see it. Repeating the Bahcall fits with this extra component to the K giant tracer population then gives an excellent fit to the P = 0 model (see Fig. 5). That is, if one analyses published K giant data in the way devised by Bahcall, but also

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adopts a set of two isothermal components to fit to the local velocity data (to allow for the existence of young stars), rather than the single old-disc isothermal favoured by Bahcall (1984c), and weights the density data by its $\sqrt{N}$ uncertainty, then the K giant data provide evidence for no unidentified matter in the solar neighbourhood.

Because of the caveats mentioned earlier as regards the suitability of K giants for this investigation, this result should be viewed with the necessary reserve. Nevertheless, it does illustrate the kind of problems and uncertainties that can arise when attempting to measure $\rho_0$ from data that are not of the highest precision. In particular, the kinematics very near to the plane (within a scaleheight of the youngest stars that might be in the sample) are of crucial importance. Until considerably improved space density and velocity dispersion data are available, one cannot unambiguously prove the existence of any dark matter in a disc-like distribution with a significant volume density in the solar neighbourhood.

5 Other methods of measuring $\rho_0$

5.1 Low-latitude proper motion data

The Jeans’ equation (1) combined with the Poisson equation (3) yields equation (4) above. The right-hand side of this equation has dimensions of $[V^2/L]^2$. Thus, if we can derive the vertical velocities from proper motions, the distance scale factors out, and the problem of requiring accurate distances to each star in the sample is alleviated: velocities turn into proper motions, and $z$-heights into galactic latitudes. To measure $\sigma^2_z$ from proper motions, we need to observe stars near the galactic plane, which may cause serious problems of confusion if we use faint stars. Therefore this would probably be done better with brighter stars near the turnoff of the disc population, in spite of all the caveats regarding their dynamical suitability discussed in Section 3. In order to know where we are measuring the Oort limit, we need some idea of the distances to these stars. For example, we would not want to use giant stars half-way to the centre of the Galaxy without being aware of it. This again suggests the use of turnoff stars, which have a fairly narrow range of absolute magnitudes, and hence magnitude-limited samples of such stars are well-confined in space.

To see how this method might work, consider the following toy model. Assume that the vertical density profile of tracer stars is Gaussian

$$v(z) = v_0 e^{-z^2/2\sigma^2_z}$$

near the plane, and that the population is isothermal

$$f_\nu(z, v_z) = v(z) \frac{1}{\sqrt{2\pi}\sigma^2_z} e^{-v_z^2/2\sigma^2_z}.$$  

Then from the Jeans’ equation (2) we see that

$$-K_z = \frac{\sigma^2_z}{\sigma^2_v} z,$$

and hence the volume density $\rho_0$ is constant with $z$-height while this is a reasonable approximation. For the purpose of this illustrative discussion, we will ignore any radial density gradients, or effects due to the Sun’s vertical motion, and assume that we are dealing with a plane-parallel galaxy. Assuming further that we are looking at small galactic latitudes $b$, we have $z \approx rb$ and $v_z \approx r\mu_z$ for a star at distance $r$. Then the proper motion distribution $f_\mu$ along a
line-of-sight at galactic latitude $b$ is (dropping the $z$-suffix on the vertical proper motion $\mu_z$)

$$f_\mu(b, \mu) \, db \, d\mu = \int dr \, f_c(rb, r\mu) \, r db \, r d\mu$$

$$= f_\mu(b, \mu) = \int \frac{r^2 \, dr}{\sqrt{2\pi} \sigma_{zz}^2} \, v_0 \exp \left[ -r^2 \left( \frac{b^2}{2\sigma_v^2} + \frac{\mu^2}{2\sigma_{zz}^2} \right) \right].$$

If we sample stars to infinity (see below), so that we can integrate from $r = 0 \to \infty$, then

$$f_\mu(b, \mu) = C \left( \frac{b^2 + \mu^2}{\sigma_v^2 + \sigma_{zz}^2} \right)^{-3/2}.$$

$C$ is an arbitrary normalization constant. From equation (4),

$$-K_z = z \frac{\sigma_{zz}^2}{\sigma_v} = 4\pi G \rho_0 z,$$

and hence

$$f_\mu(b, \mu) = C [(4\pi G \rho_0) b^2 + \mu^2]^{-3/2}$$

is the distribution of proper motions $\mu$ in the $z$-direction at latitude $b$ for a sample of stars with space and velocity distributions given by equations (5) and (6). In fact, any superposition of isothermal populations will give the same shape distribution, so that the isothermality condition we used to derive equation (12) was redundant.

In principle, then, by matching the proper motion distribution of a sample of stars to the form given in equation (12), we can derive $\rho_0$. If $\mu$ is measured in units of 0.01 arcsec yr $^{-1}$, and latitude $b$ in 10°, then $\rho_0$ is in units of $1.43 \, M_\odot$ pc $^{-3}$ – about a factor of 10 larger than the value we expect to find. Thus accurate measurements are necessary. This is illustrated in Fig. 6,

**Figure 6.** The proper motion distributions calculated from equation (12), for $\rho_0 = 0.143 M_\odot$ pc $^{-3}$. The galactic latitudes for the models are listed from top to bottom.
where the proper motion distributions for various galactic latitudes are plotted. These distributions are not convolved with any observational error functions; clearly such errors must not be worse than 0.01 arcsec yr$^{-1}$ for this method to work.

These models were derived by integrating the proper motion distribution to a distance of infinity. Clearly this is a gross idealization: especially at low latitudes many stars will be sampled at distances where the approximate density model of equation (5) is inapplicable. However, at higher latitudes this is not so much of a problem, as such lines-of-sight quickly leave the disc, and hence do not meet many distant stars. As long as we include all the stars within a few multiples of the distance

$$\left( \frac{b^2}{2\sigma_v^2} + \frac{\mu^2}{2\sigma_{\mu}^2} \right)^{-1/2},$$

where the integrant of equation (9) reaches a maximum, the model calculation above may be reasonable (although the form of $K_z$ is no longer appropriate for stars more than a few hundred pc out of the plane). For $\sigma_v = 200$ pc, $\sigma_{\mu} = 20$ km s$^{-1}$, this yields $\mu > 0.004$ arcsec yr$^{-1}$, $b > 15^\circ$. At such latitudes projection effects in the velocities begin to be significant, so that the models would have to be made more sophisticated than those presented here; nevertheless with development this method might prove a fruitful new addition to the armoury of $K_z$-hunters.

5.2 Flattening of Star Clusters

$N$-body simulations of open clusters show that star clusters respond to the tidal field of the Galaxy by flattening in the $z$-direction (Wielen 1967; Aarseth 1973; Terlevich 1987), and this effect is indeed seen in some open clusters (Hyades: Oort 1979; Pleiades: van Leeuwen 1983; NGC 3532: Terlevich 1987). A film of one such simulation (E. Terlevich & S. J. Aarseth, private communication) shows that this flattening in fact takes place strikingly quickly. An initially spherical cluster flattens to its equilibrium axis ratio after only $\sim 26$ Myr, just a few crossing times. Since this does not happen for completely isolated clusters, which evolve spherically symmetrically, the degree of flattening should be related to the strength of the tidal field.

$$\frac{\partial K_{\mu}}{\partial z}, \frac{\partial K_{\mu}}{\partial r},$$

which is directly proportional to $\rho_0$. This has not been investigated in any great detail, since such simulations are expensive. To date, most $N$-body simulations have used the same tidal field (based on the Oort (1960) value of $\rho_0 = 0.15 M_\odot$ pc$^{-3}$) and are agreed on the size of the resultant flattening, about 2:1. Clearly what is needed is a systematic investigation of the effect of the tidal term in the potential, for comparison with the observed shapes of clusters. Unfortunately, a variety of observational and evolutionary effects may act to confuse the appearance of an open cluster. Contamination of the field by background and foreground stars, patchy extinction in the background or across the cluster, a 'halo' of recently escaped stars (Terlevich 1987), projection effects which may hide the flattening (Oort 1979), dynamical heating from passing clouds (this is not thought to be a very significant process, but somewhere between the total disruption that would be caused by a passing giant molecular cloud and a quiet encounter with a 'standard' cloud there must be a regime of dynamically significant non-disruptive encounters with clouds), vertical oscillation through the galactic plane producing a varying tidal field, ... are all complicating factors. Nevertheless, the possibility of directly mea-
suring the tidal surface of a galactic (or disc globular) cluster from a detailed study of its outer regions warrants further study. Similar considerations apply to the vertical distribution of wide binary stars.

6 Conclusions

Determination of the local volume mass density near the Sun – the Oort limit – typically shows that perhaps 50 per cent of the mass measured dynamically remains unidentified. Measurement of the surface mass density of the galactic disc near the Sun shows no significant difference between the disc mass measured dynamically and that identified. Thus, either the unidentified mass in the Oort limit has a very small scaleheight – in effect it must be distributed like the cold interstellar medium – or systematic errors remain in the determination of the Oort limit.

The primary sensitivity of determinations of the local volume mass density \( \rho_0 \) to uncertain data lies in the modelling of the stellar velocity distribution near the galactic plane, and in the determination of the stellar density distribution with distance from this plane. Both F dwarf and K giant tracer samples have been analysed to determine \( \rho_0 \), and each leads to \( \rho_0 \approx 0.20 \, M_\odot \, \text{pc}^{-3} \) (Bahcall 1984c). (The identified mass provides \( \rho_{0,\text{obs}} = 0.10 \, M_\odot \, \text{pc}^{-3} \).) However, potentially large systematic problems with the data remain.

The F star sample analysed is the sum of two sub-samples (F5 and F8, Hill et al. 1979), with no evidence for a difference between their velocity distributions (Adamson et al. 1988). For steady-state stellar populations, the same kinematics in the same gravitational potential must lead to the same spatial density distribution, but for the F5 and F8 samples this is not the case. One or both of the data or the assumptions underlying the modelling of the F star kinematics is thus clearly in error.

The amplitude of the resulting uncertainty can be found by deducing \( \rho_0 \) from each of the three F star samples, F5, F5 + F8 and F8. The resulting values of \( \rho_0 \) are 0.29, 0.185 and 0.11 \( M_\odot \, \text{pc}^{-3} \), respectively. Thus one may deduce that there is twice as much mass missing as observed in the local volume density, just as much missing as observed, or no missing mass at all, depending on which sample of stars one chooses to analyse. Clearly, the available F star data are not capable of providing any evidence either for or against the concept of missing mass near the Sun.

The sample of K giants which has been analysed previously has been shown to have a velocity distribution which is consistent with a single isothermal, with a velocity dispersion of \( c. \, 20 \, \text{km s}^{-1} \) (Bahcall 1984c). Thus, unlike the F stars, in this model the K giants consist entirely of old disc stars, with neither young disc nor thick disc star representatives. Since stars of a wide range of masses become K giants, this model is inherently implausible. Velocity data from Hill (1960) have been used here to provide an improved model. The K giant density law is also uncertain, as the relevant colour–magnitude relation is a strong function of age and metallicity, both of which appear to be correlated with distance from the galactic plane. Higher precision data are necessary to derive a reliable density profile.

A further complication follows from a feature of previous analyses, which assigns high weight to the density profile near the plane, where the number of stars counted is smallest. Reanalysis of published data including weighting of the density data by their Poisson noise and using the more detailed fit to the velocity data mentioned above leads to a value of \( \rho_0 = 0.10 \, M_\odot \, \text{pc}^{-3} \). The previously derived value from the same data was \( \rho_0 = 0.21 \, M_\odot \, \text{pc}^{-3} \).

We conclude that available determinations of the volume mass density near the Sun – the Oort limit – remain limited by systematic and random difficulties with the available data. One may deduce a local unexplained mass density which is up to a factor of 2 larger than that mass density which is identified with stars and the interstellar medium near the sun from some
samples of (young) F stars. Other samples of (older) F stars and of K giants, when analysed using velocity distributions consistent with the structure of the local galactic disc, provide no evidence for any unexplained mass near the Sun. Determinations of the integral surface mass density of the galactic disc near the Sun (Kuijken & Gilmore 1989b) also shows evidence for no missing mass in the galactic disc. In brief, available data are either internally inconsistent, or provide no robust evidence for the existence of any missing mass associated with the galactic disc.

References