Tides, torques and the timing argument

Somak Raychaudhury and D. Lynden-Bell Institute of Astronomy, Madingley Road, Cambridge CB3 0HA

Accepted 1989 February 28. Received 1989 February 27; in original form 1988 September 23

Summary. The quadrupolar component of the gravitational field due to neighbouring galaxies at the barycentre of the Local Group is estimated. The tidal effect consists of an almost equal compression and extension along two principal axes, and of almost no effect along the third. The positive axis of the quadrupole component is aligned with the dipole component for galaxies within 10 Mpc of the Local Group. The nature of the binary orbit of the Andromeda galaxy and the Milky Way in the presence of this tidal field is investigated, and possible orbits are suggested. Though the tidal effects are appreciably strong, they hardly affect the mass of the Local Group derived from timing arguments. The direction along which Andromeda seems to have emerged from the Big Bang indicates that it was responsible for the orbit of the Magellanic Clouds about the Galaxy. The proper motion of Andromeda is predicted.

1 Introduction

Galaxies and clusters of galaxies are thought to have formed from the gravitational instability of small primordial density perturbations. In this scenario, the angular momentum of galaxies and their clusters is not primeval, but is rather a result of the tidal interaction between proto-objects. One would imagine that the material destined to end up in a single galaxy would have a very irregular shape, and consequently, as Hoyle (1949) pointed out, galaxies would have acquired angular momentum through gravitational interaction with the matter associated with neighbouring proto-systems. Tidal stresses are induced in an extended body (proto-galaxy) of mass $M_1$ due to the presence of a nearby mass $M_2$ a distance $D$ away from it, and the body experiences a torque about its centre of mass. The angular momentum thus induced in the proto-galaxy is of the order of

$$J \sim (\epsilon k^2) \frac{GM_1 M_2}{D^3} \tau,$$

where $k$ is its maximum radius of gyration, $\tau$ is the time for which the torque acts and $\epsilon$ is a factor depending upon its shape.
Hoyle, and later Sciama (1955), applied this to the problem of galaxy formation in a steady-state universe. With $\tau$ as the collapse time-scale of a proto-galaxy and a value of $M_2/D^3 \sim 10^{-28}$ g cm$^{-3}$ typical of clusters of galaxies, they obtained values of $J$ of the same order of magnitude as found in large galaxies. This, however, would fail to explain why the angular momentum of field galaxies would be of the same order of magnitude as that of cluster galaxies.

The problem is not as simple in the case of an expanding Friedmann universe, in which context this formalism was rediscovered by Peebles (1969). He assumed that a proto-galaxy picks up most of its angular momentum through tidal interactions before it reaches its state of maximum expansion, and his estimate of the acquired angular momentum, when applied to our Galaxy, fell short of Schmidt model estimates by almost an order of magnitude. Peebles' method has been criticized in the following years on the grounds that his value might have been an overestimate. In any case, a detailed analytic description has proved elusive, even in the linear regime.

There has been a surge of interest in recent years in studying the mechanism of the transfer of angular momentum between evolving proto-galaxies in the context of different cosmologies, in particular that of dark-matter-dominated models. Barnes & Efstatiou (1987), for instance, find in their $N$-body experiments that the specific angular momentum of a dark halo bears no simple relation with that of the galaxy forming inside it, and that the dimensionless spin parameter $\Lambda = J|E|^{1/2} M^{-5/2} G^{-1}$ has a broad distribution with $\Lambda_{med} \sim 0.05$. The same value has been arrived at analytically by Heavens & Peacock (1988), who also demonstrate that for most realistic power spectra, the angular momentum of a collapsing proto-galaxy is independent of the peak of the spectrum.

Thuan & Gott (1977) conducted a statistical study of the evolution of angular momentum through tidal interaction with neighbours, and found that the distribution of ellipticities and sizes for elliptical galaxies predicted by this method is consistent with observation, but for spirals the agreement is less spectacular. For the Milky Way ('the Galaxy' hereafter) they used a maximum expansion radius $R_p = 100$ kpc (Eggen, Lynden-Bell & Sandage 1962) instead of the value Peebles had used (20 kpc), and found that their tidal interaction picture gave a value for the acquired angular momentum that is very close to the standard Schmidt model value for the Galaxy.

Gott & Thuan (1978) later applied this model to the Local Group of galaxies, and found solutions for the relative orbit of the Galaxy and the Andromeda galaxy ('M31' hereafter) under the tidal influence of each other. They found two solutions which implied that the binary orbit had swung through either 37° or 143° in space due to their mutual tidal torque, of which they favoured the second solution (143°). In this analysis, they assumed that the net angular momentum of the Local Group is constant, since there is no interaction with nearby galaxies.

That the Local Group is isolated as far as the tidal influence of nearby galaxies is concerned is a questionable assumption. In view of this, in the next section we set out to estimate the value of the quadrupolar component of the field at the barycentre of the Local Group due to external galaxies. In the following section (Section 3), we point out certain problems that arise from the Kahn & Woltjer (1959) timing argument and from the Gott & Thuan analysis of the orbit of M31, and examine in Section 4 whether such problems are affected by the tidal torques evaluated in Section 2.

2 An estimate of the tidal influence on the Local Group

About 98 per cent of the luminous matter in the Local Group rests in M31 and the Galaxy, and its satellites. We can therefore safely assume that most of the mass of the Local Group lies in these two groups as well, unless (i) there are other equally massive nearby galaxies hidden
behind the plane of the Galaxy, (ii) the Maffei objects are close enough (≤ 1.5 Mpc) to be dynamically important, and/or (iii) the density of the intergalactic matter is appreciably high. Since we have no direct evidence for any of these possibilities, we assume that the barycentre of the Local Group lies on the line joining the Galaxy and M31.

The net gravitational potential at the barycentre of the Local Group (LG) due to all other neighbouring galaxies is given by

$$\psi = \sum_{\alpha \in LG} \frac{GM_\alpha}{|\mathbf{r} - \mathbf{r}_\alpha|},$$

where \( \mathbf{r}_\alpha, \mathbf{r} \) indicate the position vectors of the galaxy \( \alpha \) and the centre of mass of the Local Group respectively. The torque on the Local Group due to this field is

$$\mathbf{\Gamma}_i = \sum_{j,k} \varepsilon_{ijk} \mathbf{q}_{jk},$$

where

$$\mathbf{Q}_{ab} = \frac{\partial^2 \psi}{\partial r_a \partial r_b},$$

and \( \mathbf{q}_{ab} \) is the quadrupole moment of the Local Group. If the ratio of the masses of M31 and the Galaxy is \( \mu \), then

$$\mathbf{q}_{ab} = (3\mathbf{r}_a \mathbf{r}_b - \delta_{ab}) \frac{\mu}{(1 + \mu)^2} \mathbf{r}_A \mathcal{M},$$

where \( \mathcal{M} \) is the combined mass of the system, \( \mathbf{r}_A \) is the position vector of M31 with respect to the Galaxy, and \( \mathbf{i} \) is the corresponding unit vector.

The value of the quadrupolar component (3b) of the interaction due to external galaxies at the barycentre \( \mathbf{r} \) of the Local Group is estimated according to the expression

$$\mathbf{Q}_{ij} \bigg|_{\text{barycentre of LG}} = \sum_{\alpha \in LG} \frac{GM_\alpha}{|\mathbf{r} - \mathbf{r}_\alpha|^3} \left( \frac{3(\mathbf{r} - \mathbf{r}_\alpha)_{(i} (\mathbf{r} - \mathbf{r}_\alpha)_{j)} - \delta_{ij}}{|\mathbf{r} - \mathbf{r}_\alpha|^2} \right)$$

from a selected sample of galaxies as described in Section 2.3. The dimensions of \( \mathbf{Q}_{ij} \) are \([G\rho]_b\) or \([T^{-2}]\).

2.1 UNITS

The units of distance and velocity used in this paper are 1 Mpc and 100 km s\(^{-1}\), respectively, and \( G = 1 \). In this system, the unit of mass is \( 2.32 \times 10^{12} M_\odot \) and that of time is \( 0.98 \times 10^{10} \) yr. The units of angular momentum and density are respectively \( 1.43 \times 10^{77} \) g cm\(^2\) s\(^{-1}\) and \( 1.57 \times 10^{-28} \) g cm\(^{-3}\).

Mass-to-light ratios are calculated in terms of the corresponding ratio for the sun \( M/L_{B,\odot} \). Luminosities are estimated from blue magnitudes, the reference magnitude being \( M_{B,\odot} = 5.48 \).

2.2 THE BARYCENTRE OF THE LOCAL GROUP

The Local Group of galaxies essentially consists of two subgroups, each dominated by a massive spiral surrounded by a number of dwarf galaxies. We can thus treat its dynamics as a
two-body problem, with the two subgroups representing the two mass-points. It is therefore justified to assume that the barycentre of the Local Group lies on the line joining the Galaxy and M31, at a point determined by the ratio of the masses of the two major galaxies \( \mu = M_A/M_G \).

At a distance modulus of 24.2 ± 0.08 (692 ± 25 kpc, Feast & Walker 1987), the absolute blue magnitude of M31 would be \( M_B = -21.14 ± 0.08 \), using the Burstein & Heiles (1976) extinction value of \( A_B^0 = 0.32 \), and an internal extinction of \( A_B^1 = 1.03 \) (Kraan-Korteweg 1986). The corresponding number for the Galaxy is poorly determined, but taking \( M_B = -20.2 ± 0.2 \) (de Vaucouleurs 1983), the ratio of blue luminosity is 2.4 ± 0.6. The range quoted does not take into account the errors in \( A_B \), and the value of \( A_B^1 \) for M31 is quite uncertain, since it is highly inclined to the line of sight (\( i = 77° \)). Even if we use a conservative estimate of \( A_B^1 = 0.7 \) (Walterbos 1986), \( M_B = -20.8 ± 0.2 \), and the ratio is 1.7 ± 0.5.

From the Tully–Fisher (TF) relation, assuming \( M/L \) to be constant in a certain band, one can infer \( \mu = [(\mu)]/[(\mu)]_G \), where the adopted circular velocities are 260 km s\(^{-1}\) (Brinks & Burton 1984) and 220 km s\(^{-1}\), respectively, for M31 and the Galaxy, and \( n \) is obtained from the slope of the TF curve. Calibrations of the TF relation in the blue (e.g. Huchtmeier & Richter 1988) show that \( n = 3 \) whereas calibrations in the infrared \( H \) band (Aaronson, Huchra & Mould 1979) indicate \( n = 4 \), which correspond to \( \mu = 1.7 \) and \( \mu = 2.0 \), respectively.

To obtain the mass of the visible disc of the two galaxies, one generally assumes an exponential form \( \sigma_0 \exp(-r/r_s) \), where \( \sigma_0 \) is the central surface density in visible stars, and \( r_s \) is the scale-length, the mass of such a disc being \( 2\pi\sigma_0 r_s^2 \). Assuming that \( \sigma_0 \) is roughly constant in the centre of spirals (van der Kruit 1987), and that \( r_s = 6.4 ± 0.3 \) kpc for M31 (Walterbos 1986) and 4.5 ± 1.0 kpc for the Galaxy (Freeman 1987), one finds that the mass-ratio of the discs of the two galaxies is \( \mu_{\text{disc}} = 2.0 ± 0.5 \). In this paper, we use \( \mu = 2 \).

2.3 The Sample

The sample of galaxies used in this section has been drawn from Kraan-Korteweg’s catalogue of nearby galaxies (KK86, 1986). The velocities of these galaxies with respect to the Local Group were calculated from their heliocentric redshifts, assuming the motion of the Sun to be 300 km s\(^{-1}\) towards \( l = 90° \), \( b = 0° \). Distances to these galaxies were obtained from their velocities in terms of the distance to the Virgo cluster using a Virgocentric flow model as described in KK86, with an infall velocity of 220 km s\(^{-1}\). For galaxies having three solutions for the distance parameter by virtue of the ambiguity of the non-linear model in highly perturbed regions, the lowest value was taken except for galaxies clearly marked as members of Virgo. The adopted distance modulus of the Virgo cluster in this paper is 31.6, corresponding to 20.9 Mpc (Kraan-Korteweg, Cameron & Tammann 1988), and \( H_0 = 56 \) km s\(^{-1}\) Mpc\(^{-1}\).

All galaxies within 12 Mpc having velocities ≤ 1000 km s\(^{-1}\) with respect to the Local Group were chosen to constitute the sample, excluding those marked in the catalogue as members of the Local Group or of the Virgo cluster. Of the 218 galaxies left after the primary screening, all have published redshifts and all but two have apparent blue magnitudes. Corrected blue magnitudes \( m_{B,0}^i \) were obtained from the observed values \( m_B \) by correcting for internal absorption as given in KK86 and for galactic extinction as prescribed by Burstein & Heiles (BH, 1976).

Since the KK86 distances come from a model based on a dipole flow of all nearby galaxies towards Virgo, which is itself quite controversial in the light of emerging evidence for other attractors in the local neighbourhood (e.g. Faber & Burstein 1988), independent distance estimators are sought for the brighter galaxies. Table 1 lists eight galaxies that contribute the most to the evaluation of \( Q_{6i} \) in order of importance; it also lists other galaxies for which
Table 1. Distances to important nearby galaxies.

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>type</th>
<th>(1, b)</th>
<th>$m_0^{i}$</th>
<th>$D_{KK}$</th>
<th>$D_{FW}$</th>
<th>$D_{San}$</th>
<th>$D_{HR}$</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>I 342</td>
<td>SBcd</td>
<td>138,+11</td>
<td>5.8</td>
<td>5.9</td>
<td>6.3:</td>
<td>4.7:</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>M 81</td>
<td>Sb</td>
<td>142,+41</td>
<td>6.9</td>
<td>2.5</td>
<td>4.2:</td>
<td>5.8:</td>
<td>3.5</td>
<td>59</td>
</tr>
<tr>
<td>N 55</td>
<td>Sc</td>
<td>333,-76</td>
<td>7.4</td>
<td>1.9</td>
<td>1.7:</td>
<td>2.5:</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>N 253</td>
<td>Sc</td>
<td>98,-88</td>
<td>7.3</td>
<td>4.6</td>
<td>3.2:</td>
<td>3.0:</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>Cent A</td>
<td>S0p</td>
<td>310,+19</td>
<td>6.4</td>
<td>5.2</td>
<td>1.7</td>
<td>2.2</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>N 300</td>
<td>Sc</td>
<td>299,-79</td>
<td>8.3</td>
<td>2.1</td>
<td>1.6</td>
<td>6.9:</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>N 6946</td>
<td>Sc</td>
<td>96,+11</td>
<td>7.8</td>
<td>7.1</td>
<td>3.2</td>
<td>3.0:</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>N 247</td>
<td>Sc</td>
<td>114,-84</td>
<td>8.8</td>
<td>3.6</td>
<td>3.2</td>
<td>3.0:</td>
<td>73</td>
<td></td>
</tr>
</tbody>
</table>

M 101 Sc 102,+60 7.9 7.3 7.1 6.9 7.6: 18
N 2403 Sc 151,+29 8.3 6.5 2.9 2.6 3.5 55
M 83 Sc 315,+32 8.1 5.2  1.7 6.9: 25
N 3109 Sm 262,+23 9.8 2.5 1.7 1.6 2.5 86
Sex A Ibm 246,+40 11.6 2.1 1.3 1.7 2.2 35
Sex B Ibm 233,+44 11.6 2.3 1.3 1.7 2.6 45


Figures in bold indicate the values adopted in this paper.

Colons are appended to values believed to be more uncertain than the rest in the same column. In column 7, these are objects to which distances have been assigned based on assumed association, and in column 8 these refer to objects of low inclination.

Independent distances are available. Huchmeier & Richter (1988, HR) have published blue Tully–Fisher distances for almost all spirals and irregulars in KK86. In our final data set, we adopt their distances for the ones in the above list having a moderate inclination to the line-of-sight: namely NGC 55, 247 and 253 and M81. The late spirals NGC 247 and 253, and NGC 55 and 300 seem to be two pairs of galaxies bound together in the Sculptor group, which is in agreement with the HR distances. Feast & Walker (1987, FW) have compiled Cepheid distances to several nearby galaxies and treated them on a consistent basis: we accept their distances for NGC 2403, 3109, 5457 (M101) and the Sextans objects. The KK86 distances to IC 342, NGC 6946 and NGC 5236 (M83) were retained, since these are low-inclination galaxies, and the HR distances would not be more accurate. The adopted distances for all these galaxies are listed in Table 1.

The sample of 218 galaxies external to the Local Group and the 29 Local Group members are shown in Fig. 1 as a projection on to the galactic plane. A well-defined central plane, namely the Coma–Sculptor cloud (Tully & Fisher 1987, Nearby Galaxies Atlas) is easily seen. Galaxies belonging to the Leo cloud appear to the left of the Coma–Sculptor cloud; these include galaxies like M65, NGC 3521, etc. The spur seen emerging from the plane towards the right almost parallel to the horizontal axis lies almost totally within 7–9 Mpc: this is the M101 group, consisting of galaxies such as M51, M101, etc. The corresponding spur emerging towards the left of the plane is absent in the Nearby Galaxies Atlas; it points straight at the Virgo cluster ($x = 1.3$, $y = -5.4$, but outside the sphere that is plotted), and is probably an effect of the inclusion of a few Virgo members in the sample. Within this sample, the 136 galaxies lying within 7 Mpc of the Sun define a flat plane with 1σ half-thickness of 0.6 Mpc (Fig. 1).
2.4 Mass-to-Light Ratios

The masses $M_i$ of the galaxies can be replaced by a function of their apparent blue magnitudes $m_{B,i}$; if, as a starting point, one assumes that mass follows light. Schweizer (1987), in her study of binary galaxies in the southern sky, found that the median $M/L_V$ for late spirals and ellipticals is $21 \pm 5$ and $39 \pm 9$ solar units respectively ($H_0 = 50$). Most of the important nearby galaxies are late spirals (see Table 1), and since the value of $\langle B-V \rangle$ for spirals and that of the Sun ($= 0.65$) are similar, we have adopted a constant $\langle M/L_B \rangle = 20$ for all galaxies in the sample.

This value of $\langle M/L_B \rangle$, however, is quite small when one compares this with the value one obtains for the Local Group from simple ‘timing’ arguments (Section 3.1). Indeed, for $t_0 = 1.3 \times 10^{10}$ yr, $M/L_B = 65$ from the values of $L_B$ quoted in Section 2.2. One might therefore believe that the average $\langle M/L_B \rangle$ of the galaxies responsible for the tidal field is actually higher, in which case all one needs to do is to scale the eigenvalues of $Q_{kl}$ in proportion: the eigenvectors would be unchanged.

2.5 Evaluation of $Q_{kl}$

Since $Q_{kl} \propto r^{-3}$ (whereas $L_B \propto r^{-2}$), only closer and brighter galaxies are expected to be important. Therefore, the above sample is adequate for the evaluation of $Q_{kl}$. Again, since
apparent magnitudes are used, the values adopted for distance no longer play an important role ($Q_{kl} \propto r^{-1}$ for the same $m_p$). Of the 218 galaxies found eligible, two were excluded since blue magnitudes are not known for them. Four galaxies (Maffei I and II, Circinus and ESO 223-G09) were found to be in the region $|b| \leq 10^\circ$, and were also left out, since for these galaxies the values quoted for galactic extinction in the literature are very uncertain. The remaining 212 galaxies were used in the evaluation of $Q_{kl}$ as shown in Table 2(i). We would use this value for $Q_{kl}$ for the rest of the paper.

The $3 \times 3$ matrix $Q_{kl}$ is traceless and symmetric. If the sample consisted of only one object of mass $M$ a distance $R$ Mpc away from the barycentre of the Local Group, then the eigenvalues would have been of the form $(-a, -a, 2a)$, where $a = GM/R^3$, and the eigenvector corresponding to the positive eigenvalue would have pointed in the direction of the object from the barycentre. For a distribution of galaxies lying in a plane, the eigenvector corresponding to the most negative eigenvalue would be perpendicular to the plane. This is indeed the case with the eigenvectors listed in Table 2: the negative eigenvector in Table 2(i) is towards ($l = 43^\circ$, $b = 5^\circ$), only $6^\circ$ away from the direction perpendicular to the plane defined by the non-Local Group galaxies within 7 Mpc ($l = 45^\circ$, $b = -1^\circ$), reflecting the flatness of the Coma–Scout cloud. However, the eigenvalues show a curious form: the positive eigenvalue is almost equal in magnitude to the most negative one, the third being an order of magnitude lower – corresponding to an almost equal tidal extension and compression along two axes and virtually no effect along the third (Fig. 2). The same pattern can be seen in the evaluation of the quadrupolar component of the velocity field in the Local Supercluster (Lilje, Yahil & Jones 1986).

The directions of the axes of $Q_{kl}$ are shown on a plot (Fig. 3) of all galaxies $a$ belonging to the sample, where each galaxy is plotted as a circle whose area is proportional to $GM_a/|r_a - r|^3$.

### Table 2. Values for $Q_{kl}$ evaluated at the barycentre of the Local Group.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Eigenvalues*</th>
<th>Direction of Eigenvectors†</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Sample defined in §2.3 excluding galaxies within $</td>
<td>b</td>
<td>&lt; 10^\circ$</td>
</tr>
<tr>
<td>212 objects</td>
<td>0.069</td>
<td>$139 \pm 1$</td>
</tr>
<tr>
<td>(ii) As above including galaxies within $</td>
<td>b</td>
<td>&lt; 10^\circ$</td>
</tr>
<tr>
<td>216 objects</td>
<td>-0.006</td>
<td>309</td>
</tr>
<tr>
<td>(iii) The entire KK86 catalogue excluding galaxies within $</td>
<td>b</td>
<td>&lt; 10^\circ$</td>
</tr>
<tr>
<td>2483 objects</td>
<td>-0.002</td>
<td>310</td>
</tr>
<tr>
<td>(iv) As in (i), but after correction (cf. Appendix)</td>
<td>-0.064</td>
<td>$43$</td>
</tr>
<tr>
<td>upto n=5, 212 objects</td>
<td>-0.009</td>
<td>298</td>
</tr>
</tbody>
</table>

All data of position, magnitude and redshift taken from Kraan-Korteweg (1986). Distances are from the same source except for those in Table 1. Galaxies with unknown redshift or blue magnitude have been excluded in all cases.

*Unit of eigenvalues: $(G \times 2.32 \times 10^{12} M_\odot) (1 \text{ Mpc})^{-3}$. Errors on these values are discussed in Section 3.5. These values assume $M/L_B = 20$ for all galaxies responsible for $Q_{kl}$ and the distance modulus to Virgo = 31.6.

†The directions $(l \pm 180^\circ, -b)$ are equivalent. Errors as estimated in Section 3.5.
its contribution to the value of $Q_{4b}$. It clearly reveals the prominent characters dominating the tidal field, with the positive axis pointing almost towards the M81 group and the IC 342 group, the slight offset being due to the South Polar Group in the opposite part of the sky. The zero axis points towards Centaurus and the negative axis lies predictably in the middle of the void in the northern supergalactic hemisphere.

For comparison, we also evaluate the dipole vector

$$\mathbf{P} = \sum_{a \in \text{LG}} \frac{GM_a}{|\mathbf{r}_a - \mathbf{r}|^3} (\mathbf{r}_a - \mathbf{r})$$

from the same sample, where $\mathbf{r}$ is the position vector of the barycentre of the Local Group. Figs. 2(a) and (b) show the build-up of $\mathbf{P}$ and $Q_{4b}$ in distance bins of 2 Mpc from the barycentre of the Local Group. If a single galaxy or group of galaxies had been dominant in the values of both $\mathbf{P}$ and $Q_{4b}$, it would have been parallel to the positive axis of $Q_{4b}$. It is interesting to note from Figs 2 and 3 that at the 6 Mpc bin, where $Q_{4b}$ has started to converge, the direction of the dipole vector comes very close to the positive axis of the quadrupole tensor. Beyond this, $Q_{4b}$ converges, but $\mathbf{P}$ does not, since the predominant contribution to the dipole vector arises from galaxies further away than is covered by this sample (Lahav, Rowan-Robinson & Lynden-Bell 1988).

It can be seen that though individual galaxies such as IC 342, M81, Cen A, NGC 55, 253, 300 and 6946 contribute significantly towards the magnitude of the eigenvalues of $Q_{4b}$, the direction of its axes depend on the large-scale distribution of galaxies in the Coma–Sculptor cloud. It is also seen that the contribution of galaxies beyond $\sim 7$ Mpc is small (Fig. 2) and that the final values obtained are remarkably stable against the inclusion or exclusion of individual objects, within the anticipated errors. The exercise was repeated, this time including the four low-latitude galaxies we earlier excluded. In spite of using the highest estimated values of $A_B$
for three of these galaxies (Maffei I and II: Spinrad et al. 1973; Circinus: Freeman et al. 1977),
the result hardly changed [Table 2(ii)]. The inclusion of the rest of the KK86 catalogue shows
no appreciable effect [Table 2(iii)], even though it includes the whole of the Virgo subcatalogue
of 740 galaxies in the 10° around M87, of which 159 are flagged as ‘certain’ members.

In the remaining part of this paper, we shall refer to the eigenvectors corresponding to the
most negative, the median and the positive eigenvalues as the ‘negative’, the ‘zero’ and the
‘positive’ axes respectively.

2.6 UNCERTAINTIES AND ERRORS

It is difficult to estimate the formal errors on the values of $Q_{ab}$ quoted in Table 2, since these
involve the errors in the distances of the galaxies belonging to the sample. However, one can
make a rough estimate from Table 2(i), (ii) and (iii). Among the eigenvectors, the negative axis

![Figure 3](https://example.com/figure3)

**Figure 3.** The sample of galaxies used to evaluate $Q_{ab}$ shown in equal area projection. The numbers on the
circumference indicate Galactic longitude, and the tick marks indicate intervals of 10° in latitude. Each galaxy is
plotted as a circle whose area is proportional to $GM/r^3$, its contribution to $Q_{ab}$. Squares represent galaxies lying
in the region $|b| < 10°$, their area proportional to $L_{zh}/r$. Important galaxies are labelled. The positions $P_6$ indicate
the direction of the dipole vector $P$ due to all galaxies within $n$ Mpc. The positions of $Q^+$, $Q^0$, $Q^-$ indicate the
directions of the positive, zero and negative axes of $Q_{ab}$ for the whole sample.
seems to be determined to within a degree. This is not surprising, since \( Q_{\mu} \) is mostly dominated by galaxies lying in the well-defined plane shown in Fig. 1, and this direction is normal to the plane. The other two eigenvectors lie within this plane, which in turn is almost perpendicular to the galactic plane. Therefore, for these two directions, the value of \( l \) should again be fairly accurate (\( \sim 1^\circ \)), as seen in Table 2. The values of \( b \) for these two directions are probably known to within \( \sim 5^\circ \).

From Fig. 2, the eigenvalues of \( Q_{\mu} \) can be seen to have converged to within 10 per cent, which indicates the size of the error on them. However, the distances are evaluated as a fraction of the distance to the Virgo cluster, and so the eigenvalues are inversely proportional to \( D_{\text{Virgo}} \), and directly proportional to the adopted value of the average \( \langle M/L_B \rangle \) for the galaxies in the sample.

The largest source of error in \( Q_{\mu} \) is expected to arise due to the incompleteness of the sample, since in measuring the quadrupole component of a gravitational field, voids in the distribution of galaxies can be as important as clusters. KK86 claims to be complete all over the sky to 13.4 mag. The sample within 12 Mpc is reasonably complete to this brightness limit, except close to the plane of the Galaxy. There, too, very few galaxies have been left out since, as Fig. 1 shows, nearby galaxies lie in a very well defined plane which crosses the galactic plane almost at right angles.
For the sake of comparison, we estimate the contribution of galaxies in the zone of avoidance to the value of $Q_M$ by adopting a process of extrapolation, as elaborated in the Appendix. One assumes that one is looking at the sky through a medium of varying absorption (the ‘mask’), and then, constructing a model of the absorbing medium, attempts to infer the actual distribution of galaxies on the sky from the observed distribution [Table 2(iv)]. However, even this fails to produce any appreciable difference. One must point out that this scheme is not valid if a galaxy as massive as the Galaxy or M31 conspires to lurk behind the obscuring clouds of low galactic latitudes within a distance of 1–3 Mpc, as Maffei I was once thought to be.

The dependence of $Q_M$ on the parameters of the Virgocentric flow model used is not significant. The most sensitive parameter is the value of the infall velocity towards Virgo, and as KK86 demonstrates, changing that to 440 km s$^{-1}$ does not change the distances radically, and since apparent magnitudes are used, $Q_M$ varies only as $r^{-1}$. The notion of a pure dipole velocity field directed towards the Virgo cluster may itself be questionable (Faber & Burstein 1988), and the uncertainty in the distances of the few important nearby galaxies would then dominate the errors.

A major caveat in applying the tidal approximation (3) is that the external galaxies would be much closer to the Local Group galaxies in the distant past, and at an epoch when a few of our more massive neighbours are at least as close to one of the two-body system as the other member, terms of order higher than the quadrupole may be important. On tracing the orbits of the more important nearby galaxies back into the past, one realizes that at times earlier than $\sim 6$ billion years ago, M81 might have been as close to M31 as the Galaxy, but even then, the tidal field would probably have been weaker than the inverse-square force between the two galaxies. Also, the errors in the distances are quite large, and such extrapolation of orbits into the past is not reliable enough to yield conclusive results.

3 The mass of the Local Group

We assume that the mass of the Local Group is dominated by the two big spirals, M31 and the Galaxy. The mutual gravity of the two galaxies has forced each other to turn around after expanding to a maximum separation, and M31 is approaching the Galaxy at a speed of $(r_A)_0 = 124$ km s$^{-1}$, if the circular velocity of the Sun is $v_\odot = 220$ km s$^{-1}$. Even if $r_A$ is strictly radial, for $(r_A)_0 = 0.7$ Mpc, the combined mass of the two galaxies $\mathcal{M} = M_A + M_G \geq 1.2 \times 10^{12} M_\odot$. Any transverse velocity would only increase the mass required.

3.1 The Timing Argument of Kahn & Woltjer: Other Estimates of $\mathcal{M}$

Kahn & Woltjer (1959) first used the above argument to postulate that in order to be dynamically stable, the mass of the Local Group must be considerably higher than its luminous mass, since the two-body orbit is far from being 'just bound'. If $r_A$ be the distance of separation between the two galaxies on a rectilinear orbit, then

$$r_A = \alpha (1 - \cos \eta)$$

$$\dot{r}_A = \left( \frac{G \mathcal{M}}{\alpha} \right)^{1/2} \frac{\sin \eta}{1 - \cos \eta}$$

$$t = \left( \frac{\alpha^3}{G \mathcal{M}} \right)^{1/2} (\eta - \sin \eta),$$

(6)
where the maximum value achieved by $r_A$ is $2\alpha$ ($r_A = 0, \eta = 0$ when $t = 0$). Therefore for any acceptable age of the Universe, the two galaxies cannot have expanded to much more than 1 Mpc, given the above values of $\langle r_A \rangle_0$ and $\langle v_A \rangle_0$. Indeed, if $t_0 = 1.3 \times 10^{10}$ yr, then $M/L_B \approx 65$ for the Local Group. This indicates that the mass of the Galaxy $M_G$ should be at least $1 \times 10^{12} M_\odot$, which is clearly in direct conflict with mass estimates from other methods, as summarized in Table 3.

To estimate the amount of dark matter beyond the luminous disc, one needs to study the dynamics of dwarf satellite galaxies and distant globular clusters as point-mass probes of the gravitational field away from the centre of the Galaxy. The two most recent studies employing this method seem to agree on the value of $M_G$. Such methods normally assume an isotropic distribution of velocities of the satellites in a point-mass potential, such that the mean value of the eccentricity of the orbits $\langle e^2 \rangle = \frac{1}{3}$. Using improved measurements of velocities and distances of the Magellanic Clouds, Draco, Carina, Ursa Minor, Sculptor, Fornax and Leo I and II, Lynden-Bell, Cannon & Godwin (1983) found $M_G = 2.6 \pm 0.8 \times 10^{11} M_\odot$ out to 220 kpc from the centre. Little & Tremaine (1987) employed a Bayesian method to obtain $M_G = 2.4^{+1.3}_{-0.7} \times 10^{11} M_\odot$ from distant globular clusters and satellite galaxies (excluding Leo I and II), and $M_G \lesssim 5.2 \times 10^{11} M_\odot$ at the 95 per cent confidence limit. These estimates would be lower if the orbits were radial. If the potential were allowed to be of the form $r^{-\beta}$, where $\beta < 1$ (as it might be in the presence of an extended dark halo), then as shown by Kulesa (reported in Lynden-Bell 1987), a mass of the Galaxy of $10^{12} M_\odot$ is possible to the 90 per cent confidence limit ($\beta = 0.6$).

<table>
<thead>
<tr>
<th>Table 3. Estimates of the mass of the Local Group.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_G$ ($10^{11} M_\odot$)</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>Luminosity of Galaxy, $M/L_B = 10$</td>
</tr>
<tr>
<td>Minimum mass to be bound</td>
</tr>
<tr>
<td>Timing:</td>
</tr>
<tr>
<td>$t_0 = 10$ Gyr ($\Rightarrow 2\alpha = 0.9$ Mpc)</td>
</tr>
<tr>
<td>$t_0 = 13$ Gyr ($\Rightarrow 2\alpha = 1.0$ Mpc)</td>
</tr>
<tr>
<td>$t_0 = 20$ Gyr ($\Rightarrow 2\alpha = 1.1$ Mpc)</td>
</tr>
<tr>
<td>Satellites and globular clusters:</td>
</tr>
<tr>
<td>isotropic velocities, potential $\propto r^{-1}$:</td>
</tr>
<tr>
<td>50 to 220 kpc</td>
</tr>
<tr>
<td>50 to 140 kpc</td>
</tr>
<tr>
<td>potential $\propto r^{-0.8}$</td>
</tr>
<tr>
<td>Magellanic Stream</td>
</tr>
<tr>
<td>Deviation from Hubble Flow:</td>
</tr>
<tr>
<td>$\Omega_0 = 0, H_0 = 55$</td>
</tr>
<tr>
<td>$\Omega_0 = 1, H_0 = 55$</td>
</tr>
</tbody>
</table>


© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
The only truly high estimate of $M_\odot$ that is obtained from such methods is from the Magellanic Stream, based on the rather high radial velocity $v_H = -401$ km s$^{-1}$ ($v_G = -200$ km s$^{-1}$) obtained at the tip of the Stream near the SGP by Wannier & Wrixon (1972). The mass $M_\odot$ derived from this is very sensitive to the value of the eccentricity of the orbit adopted, since $M_\odot \propto (1 + e)e^{-2}$. From the simulation of the Magellanic Stream by Lin & Lynden-Bell (1982), $e = 0.6$, and $M_\odot = 2 \times 10^{12} M_\odot$.

Sandage (1986) evaluated the deceleration of the Hubble expansion in the local field due to the presence of the Local Group from the available velocity–distance data for very local galaxies, and concluded that for $\Omega_0 = 0$, the best fit mass of the Local Group is $M = 0.4 \times 10^{12} M_\odot$. Having noted the ‘traditional mass discrepancy’ between these values and those obtained from the timing argument, he agreed in a later paper (1987) that $H_0 = 55$, $\Omega_0 = 1$, $\tau_0 = 1.2 \times 10^{10}$ yr, and $M = 3.5 \times 10^{12} M_\odot$ is an equally possible ‘compromise solution’ from his analysis. However, most of the distances he uses are subject to rather large errors: IC 5152, Pegasus and Leo A have very uncertain distances; Sextans A and B may be substantially closer (Sandage: 1.7 Mpc, Feast & Walker: 1.2 Mpc); and IC 1613 should not feature in the solution, since it is probably a satellite of M31.

In view of this disagreement in the estimated mass of the Local Group, one might question the assumptions in the Kahn–Woltjer method. Any transverse velocities have been ignored in the analysis, but substantial transverse velocities would only increase the discrepancy. We have also assumed that the mass of the Local Group lies essentially in the two large spirals, whereas these two galaxies might be moving about in a background medium of cold dark matter, which in turn may have a significant mass. This would certainly reduce the required mass of the Local Group by at least a factor of 2 for the same value of $(r_A)_{\parallel}$ and $(r_A)_{\perp}$, but if the Local Group were essentially an extended potential well as a whole, one would not expect to see minor galaxies strongly clumped around M31 and the Galaxy. Also, one can obtain an upper limit to the density of this background from the analyses involving satellite galaxies as described above. On the other hand, Section 2 has shown us that the tidal influence of neighbouring galaxies on the Local Group cannot be neglected. We will address this issue in Section 4.

3.2 Spins and Swings

Gott & Thuan (1978) attempted to find the direction of the transverse motion of M31 in the sky with respect to the Galaxy by looking at the relative orientation of the spins of the two galaxies. They assumed that the angular momentum of the Local Group consists of three dominant constituents: the spin of the Galaxy $(J_G)$ and of M31 $(J_A)$, and the orbital angular momentum of M31 about the Galaxy $(L_A)$. Assuming that the Local Group is tidally isolated,

$$J_G + J_A + L_A = 0.$$  

(7)

Therefore, M31 must have swung to its present place in the sky from a position $(r_A)_n$ such that $(r_A)_n$ is parallel to $J_G \times J_A$. This means that M31 had started off on its orbit at $l = 152^\circ$, $b = 0^\circ$, or at $l = 332^\circ$, $b = 0^\circ$, implying a swing in space of $37^\circ$ or $143^\circ$ respectively for the orbit of M31 with respect to the Galaxy due to their mutual tidal torque. Gott & Thuan had to favour the second solution ($143^\circ$) because it turned out to be consistent with the assumption that external torques are absent, in which case $L_A$ must be antiparallel to the net spin angular momentum of the two galaxies.

In spite of this being a rather ingenious argument for finding the direction of $(r_A)_n$, one can question the validity of the following simplifying assumptions:

(i) Gott & Thuan assumed that the Local Group is tidally isolated from the neighbouring galaxies. We have demonstrated in Section 2 that the tidal torque on the Local Group is
appreciable at the present epoch, which implies that they were certainly stronger in the past, when the neighbouring galaxies responsible for the torque were closer to us than they are now.

(ii) They assumed an almost straight-line orbit for the two galaxies throughout their analysis, whereas most of the transverse motion would have come during the early stages of the orbit.

(iii) They do not consider the angular momentum due to the spins and swings of the other galaxies in the Local Group to be significant in (7). A glance at Table 4 tells us that the net angular momentum of the Magellanic orbit around the Galaxy is larger than the spin of the Galaxy, which may well be true even for the orbital motion of M33 around M31.

The tidal torques on the Local Group will always cause the relative orbit to swing in a direction so as to be aligned with the Coma–Sculptor Plane (Fig. 1), and within the plane, with the positive axis of $Q_{kb}$. Since we know the principal directions of $Q_{kb}$ we can easily find the direction from which M31 had to start on its orbit in order to get to its present position. In Fig. 4, the direction of the arrow at any point on the sky shows the direction of $Q_0 \cdot \mathbf{r}$, i.e. that of the tidal force on the orbit at that position. The present position of M31 is marked on the plot, along with the two Gott–Thuan solutions. One can easily see that if the orbit started off anywhere near the Gott–Thuan positions, M31 cannot have reached where it is now relative to the Galaxy, provided that $Q_0 \cdot \mathbf{r}$ is sufficiently greater than the mutual torque between the two galaxies.

Table 4. Angular momentum of Local Group members.

<table>
<thead>
<tr>
<th>Spin angular momentum of</th>
<th>$J$</th>
<th>$h$</th>
<th>(1, b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>the Galaxy $M/L_B \approx 2$</td>
<td>0.0003</td>
<td>0.02</td>
<td>0, −90</td>
</tr>
<tr>
<td>M 31 $M/L_B \approx 2$</td>
<td>0.0011</td>
<td>0.03</td>
<td>241, −30</td>
</tr>
</tbody>
</table>

Orbital angular momentum of

| LMC about Galaxy | 0.0008 | 0.18 | 185, 3 |
| M 33 about M 31 | 0.0003 | 0.09 | 190, 42* |
| Galaxy and M 31 about each other $M/L_B \approx 65$ | 0.011 | 0.26 | 69, 57 |

Values used in this table:

- Galaxy: $v_{\text{circ}} = 220 \text{ kms}^{-1}$, $r_* = 4.5 \text{ kpc}$
- M 31: $v_{\text{circ}} = 260 \text{ kms}^{-1}$, $r_* = 6.4 \text{ kpc}$
- LMC: $v_T = 373 \text{ kms}^{-1}$, $d = 49 \text{ kpc}$
- M33: $v_T = -46 \text{ kms}^{-1}$, $d = 760 \text{ kpc}$

*MThis assumes that the radial velocity of M33 towards the Galaxy is the same as its transverse velocity in its orbit around M31.*
In Section 4 we shall improve the situation by getting rid of assumptions (i) and (ii). To have a closer look at the relation between the spins and swings of the different major galaxies of the Local Group, we make rough estimates of the following in Table 4:

(a) An estimate of the spin angular momenta $J_G$, $J_A$ of the Galaxy and of M31. Assuming that the dark massive halo of a galaxy does not rotate appreciably, the fast-rotating disc of a spiral is responsible for most of the spin angular momentum $J_s$. We estimate the spin angular momentum of the Galaxy and of M31 using an exponential disc model of the form $\sigma_0 \exp(-r/r_s)$, where $\sigma_0$ is the central surface density in stars, and $r_s$ is the scale-length, for which $J_s = 4\pi \sigma_0 r_s^2 v_{\text{circ}}$ and $h_s = 2r_s v_{\text{circ}}$. For the Galaxy, $\sigma = 46 \pm 9 M_\odot$ pc$^{-2}$ (Kuijken 1988) in the solar neighbourhood ($r = 8.5$ kpc), and $r_s = 4.5$ kpc. For M31, we use the same value for $\sigma_0$ as for the Galaxy, with $r_s = 6.4$ kpc, as in Section 2.2.

(b) We estimate the value of the orbital angular momentum of LMC about the Galaxy, using a mass of $1 \times 10^{10} M_\odot$ (assuming the the LMC does not have a dark halo). We also use $v_1 = 373$ km s$^{-1}$ (Lin & Lynden-Bell 1982), which assumes that the orbit of the Magellanic Clouds follows the Magellanic Stream.

(c) We make a very rough estimate of the orbital angular momentum of M33 about M31. Here, we assume its radial velocity towards the Galaxy to be its transverse velocity with respect to M31 ($v_1 = -46$ km s$^{-1}$). We place it at a distance modulus of 24.4 (Feast 1988) from the Sun, and estimate its mass from its light, using the same $M/L_\beta$ obtained from the exponential disc model of M31.

Looking at Table 4, we find that contrary to the assumption (7), the orbital angular momentum of the LMC (and of the Magellanic Stream) about the Galaxy is larger than the spin angular momentum of the Galaxy. This, of course, uses the exponential disc mass of the Galaxy ($M_G = 3.9 \times 10^{10} M_\odot$), where we assume that the dark halo does not rotate appreciably, and therefore does not contribute to $J_G$. Similarly, the value of the orbital angular momentum of M33 about M31 is probably a lower limit, and we can see that it is easily comparable with the other values.
4 Dynamics in the presence of a tidal field

The direction to M31 ($l = 121^\circ$, $b = -22^\circ$), is $21^\circ$ away from the ‘zero’ axis ($l = 129^\circ$, $b = -41^\circ$) of $Q_{kl}$ as estimated in Section 2, and $81^\circ$, $72^\circ$ away from the negative and positive axes respectively. It is also found in the sky not very far ($9^\circ$) from the plane containing the ‘zero’ axis and the ‘positive’ axis ($l = 139^\circ$, $b = 49^\circ$).

In this section we set out to investigate whether the problem with the timing argument is alleviated to any extent by considering the tidal influence of external galaxies. The dynamics of the Galaxy and M31 can be represented as a two-body system moving under their mutual gravitational attraction in the presence of the tidal field due to galaxies not belonging to the Local Group,

$$\ddot{r}_A(t) + \frac{GM}{|r_A(t)|^3} r_A(t) = F(t),$$

where

$$F_k(t) = \sum_i r_i(t) Q_{ki}(t)$$

(8a)

(8b)

to first order, since the tidal field is weak compared to the inverse-squared term. Since the system is being spun up, its angular momentum is not conserved, and

$$\dot{h} = h_A \times F,$$

(9)

where $h = r_A \times \dot{r}_A$ is the angular momentum per unit mass.

To assume a time dependence for $Q_{kh}$ one integrates the Friedmann equation, which, in the case when $\Omega_0 = 1$, gives

$$a^2 = \frac{8\pi G \rho_0}{3} \times \frac{a_0^3}{a},$$

(10)

where $a_0$ is the present value of the scale factor $a(t)$. If galaxies had come into existence very early in the history of the Universe, the potential energy of the system would vary as $a^{-1}$, and $Q_{kl} \propto a^{-3}$. However, if perturbations which would ultimately form galaxies were to grow linearly for most of the age of the Universe, then the potential energy would be constant, implying $Q_{kl} \propto a^{-2}$. In reality, the evolution of the Local Universe has probably been through both phases, and the average value of the exponent $n$ in

$$Q_{kl} \propto a^{-n}$$

(11)

is somewhere between 2 and 3. We will consider the two extreme cases separately in the next two sections.

If $Q_{kl} \propto a^{-n}$, then it follows from (10) that $Q_{kl} \propto t^{-(2/3)n}$. Since the Universe outside the Local Group expands uniformly, we assume in the following analysis that $Q_{kl}$ evolves with $t$ only in magnitude, and that its eigenvectors remain fixed in space with respect to an inertial frame.

Scaling the variables in (8) as $r_A = a_1(t) r(t)$, where $a_1^3 = t^2$ and $dt = f(t) \, dt$, the equation reduces to

$$f^2 t^2 r'' + \left(t^2 f + \frac{4}{3} f^2 \right) r' - \frac{2}{9} r = -\frac{GM}{r^3} r + (Q_{kl} \cdot r)^{2-(2/3)n}$$

(12)
where primes refer to derivatives with respect to $\tau$, and $Q_0$ is the value of $Q_d$ at $t = 1$. In the units we use in this paper, this is equal to $\sim 10^{10}$ yr, which roughly corresponds to the present epoch ($t_0$). Otherwise, $Q_0 = t_0^{2/3}n(Q_d)_t = r^{-1}$. Furthermore, choosing $f = 1/r$ such that $\tau = \ln t$, one has

$$r'' + \frac{1}{3} r' - \frac{2}{9} r = -\frac{GM}{r^3} + \frac{Q_0 \cdot r}{r^2} t^{2 - [2/3]n}.$$  \hspace{1cm} (13)

In the above analysis, we have assumed $a(t) \propto r^{2/3}$ which holds for $\Omega_0 = 1$. This value, however, does not refer to the global value of $\Omega_0$ for the Universe, but to its local value, and this assumption is probably valid if one considers the value of $\Omega_0$ in the Coma–Sculptor Cloud, even at the present epoch. We shall retain this assumption in the remaining part of this paper.

4.1 THE ‘NON-LINEAR’ CASE ($n = 3$)

First, we consider the case where $n = 3$, i.e. where matter in the Universe has aggregated into galaxies at a very early phase of the Universe. This is a rather unphysical situation, but is relatively simple mathematically, and is worth exploring as a limiting case. This makes (13) independent of $t$ in explicit terms, but has the rather undesirable limitation that the $Q_0 \cdot r$ term remains very dominant even when $t \to 0$.

For the orbit (13), as one approaches $t \to 0$, everything should freeze regularly into the initial singularity as measured in the frame moving with the microwave background. Peculiar velocities in the comoving frame are only generated later due to gravitational attraction among pregalactic matter and, later still, among galaxies. As an initial condition, therefore, one expects the peculiar velocities in this frame to vanish as $t \to 0$. To show this from energy considerations, (13) is rewritten as

$$\frac{d\epsilon_0}{d\tau} = \frac{\epsilon_0}{dt} \left[ \frac{1}{2} r'' - \frac{1}{9} r'^2 - \frac{GM}{r} - \frac{1}{2} r \cdot Q_0 \cdot r \right] = -\frac{1}{3} r'^2 \leq 0$$ \hspace{1cm} (14)

where the scaled ‘energy’ $\epsilon_0$ describes the ‘energy’ of the motion relative to the comoving frame. As $\tau \to -\infty$ ($t \to 0$), in order to avoid $\epsilon_0 \to +\infty$, $r'$ must tend to zero. Since in this limit $r'' = 0$ as well, it is apparent from (13) that $r$ must be parallel to $Q_0 \cdot r$, and thus the orbit must emerge along one of the principal axes of $Q_0$.

Near $t = 0$, both $a(t)$ and $r_A$ are proportional to $t^{2/3}$, which implies that along the principal axis of emergence $r_1$, the magnitude of $r$ would be related to the eigenvalue of $Q_0$ along that axis as $r_1 = GM(Q_1 + 2/9)^{-1}$. Now suppose that the orbit emerges from $\tau \to -\infty$ along a principal axis $r_1$. To investigate the stability of the orbit along this axis, we perturb it by an amount $\delta r_1$, such that

$$\delta r_1'' + \frac{1}{3} \delta r_1' - 3 \left( Q_1 + \frac{2}{9} \right) \delta r_1 = 0,$$ \hspace{1cm} (15)

the solutions to such equations being of the form $\delta r_1 = e^{\lambda t}$. In the present case we cannot accept negative values of $\lambda$ since the values of $\delta r_1$ have to approach zero as $\tau \to -\infty$ to be consistent with the initial conditions adopted above. One of the solutions of (15), $\lambda_1 = (25/36 + 3 Q_1)^{1/2} - 1/6$, is positive unless $Q_1 < -2/9$. We have shown in Section 2 that the eigenvalues $Q_1$ are much smaller than this limit in magnitude. Therefore, the orbit (13) is unstable to perturbations along its axis of emergence, which has to be one of the eigenvectors.
of $Q_0$. If the orbit is perturbed, the component of a perturbation along the axis of emergence would grow as $t^4$. It can be shown in a similar fashion that the components perpendicular to the axis of emergence have no destabilizing effect if the orbit emerges along the axis with the largest eigenvalue. For an orbit starting off on any of the two other axes, an off-axis perturbation grows as a power-law function of time but with a smaller $\lambda = [1/36 + (Q_1 - Q_1)]^{1/2} - 1/6$, where the subscript ‘1’ refers to the axis of emergence. In this case, the orbit has to swing around, mainly towards the axis $i$ for which $Q_i$ is the largest, but with a milder $t$-dependence.

### 4.2 The ‘Linear’ Case ($n = 2$)

At first sight, that the orbit should emerge from the Big Bang along one of the principal axes of $Q_{ij}$ seems to be rather a surprising result, since the distribution of matter at large is expected to be independent of the structure of the cloud that fragmented into M31 and the Galaxy. At this stage, James Binney pointed out to us that Fig. 2 shows that most of $Q_{ij}$ results from galaxies within 6 Mpc of us, and that on this scale large-scale perturbations in the Universe would have probably evolved linearly for most of its age (Binney & Silk 1979).

This corresponds to the case $n = 2$ in (11). It is interesting to note that if $n \neq 3$, (13) is not independent of $t$ and the relative importance of the $GMr^{-2}$ term and the $Q_0 \cdot r$ term varies at different epochs. Moreover, if $n < 3$, the $Q_0 \cdot r$ term goes to zero as $t$ approaches zero, which means that at early epochs, as expected, the $GMr^{-2}$ term dominates over the $Q_0 \cdot r$ term. In this case, the orbit can no longer be expected to emerge along a principal axis of $Q_0$.

Equation (13) reduces, in this case, to

$$r'' + \frac{1}{3} r' - \frac{2}{9} r + \frac{GM}{r^3} r - (Q_0 \cdot r) t^{2/3} = 0. \tag{16}$$

This equation is solved numerically as a two-point boundary-value problem, using shooting techniques. The orbit (16) is started at $t = 0$ without the $Q_0 \cdot r$ term, and is allowed to evolve to $|r_A| = 0.1$ Mpc where the time-dependent perturbation is switched on. The values of $t_0$ and $M$, and of the initial position of M31 on the sky $(l_n, b_n)$ are kept as free parameters, which define the set of initial boundary conditions $r_n, v_n, r_0$, at $t = t_0$. The values of $(v_A(F), r_0)$ and $r_0$ at the present time $(t = t_0)$, are fixed from observation, keeping the transverse component of $(v_A)_{t=0}$ free.

One must at this point list the different parameters in the problem:

(i) We have assumed $\Omega_0 = 1$ for the very local universe, which justifies the use of $a(t) \propto t^{2/3}$.
(ii) We have not made any assumptions about the global value of $\Omega_0$ yet. This determines the value of $t_0$, the present age of the Universe. If we choose $\Omega_0 = 1$, then $t_0 = 13$ Gyr for $H_0 = 50$. We shall at present use this as our ‘preferred value’, and later show the effect of varying it.
(iii) We must choose a value for the average $\langle M/L_B \rangle$ of the galaxies responsible for $Q_{ij}$. In estimating $Q_{ij}$ in Table 2, we used $\langle M/L_B \rangle = 20$ (Section 2.4). We vary this value (thus varying the strength of the tide) to find possible orbits of M31 with respect to us, subject to the above boundary conditions at $t_0 = 13$ Gyr. The results obtained for different values of $\langle M/L_B \rangle$ are tabulated in Table 5.

We choose as our preferred value the case where we impose the condition that the $M/L_B$ of the Local Group ($L_B = 6.4 \times 10^{10} L_\odot$) has to be the same as the average $\langle M/L_B \rangle$ for the external galaxies that cause the tidal field. This leads to the solution in bold letters in Table 5, yielding $\langle M/L_B \rangle = 66$: this orbit is plotted in Fig. 5. We note that this does not affect the mass $M$ obtained from the timing argument (Table 5). In fact, for strong tides, a slightly larger value is required.
Table 5. Possible orbits of Andromeda.

<table>
<thead>
<tr>
<th>$t_0$ (Gyr)</th>
<th>$n$</th>
<th>$(M/L_B)$</th>
<th>$GM_{10^{12}M_\odot}$</th>
<th>$l_i$</th>
<th>$b_i$</th>
<th>$(v_T)_0$ (kms$^{-1}$)</th>
<th>*</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>2</td>
<td>0</td>
<td>4.2</td>
<td>121.2</td>
<td>-21.6</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>20</td>
<td>4.2</td>
<td>117.2</td>
<td>-25.5</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>4.2</td>
<td>20</td>
<td>108.2</td>
<td>-27.8</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>66</td>
<td>4.3</td>
<td>66</td>
<td>101.0</td>
<td>-27.0</td>
<td>38</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>4.3</td>
<td>80</td>
<td>93.4</td>
<td>-25.1</td>
<td>51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>20</td>
<td>3.1</td>
<td>107.1</td>
<td>-27.7</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>3.2</td>
<td>30</td>
<td>93.9</td>
<td>-25.1</td>
<td>53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>3.4</td>
<td>39</td>
<td>81.3</td>
<td>-20.2</td>
<td>62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>2.5</td>
<td>20</td>
<td>4.2</td>
<td>111.3</td>
<td>-27.5</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>4.3</td>
<td>40</td>
<td>91.7</td>
<td>-24.5</td>
<td>44</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

*These two orbits are plotted in Fig. 5.

Figure 5. 'Orbit 1' on this plot is the 'preferred orbit' of M31 relative to the Galaxy, in which M31 swings from $(101^\circ, -27^\circ)$ to its present position (open circle). Arrows mean the same as in Fig. 4. 'Orbit 2' represents the $n = 2.5$ orbit indicated in Table 5.

The mass of the Local Group that is required is relatively lower if $t_0$ is bigger, i.e. if the orbit gets a longer time to attain its present value of $(v_A, \dot{r}_A)_0 = -124$ km s$^{-1}$. This can occur if the value of the global $\Omega_0$ of the Universe is lower than unity. On comparison with Table 3, one notes that here too, the mass $M$ required is almost the same as that required for an unperturbed Kepler orbit. In this case, one fails to produce the present values of $r_A, v_A$ for
\( \langle M/L_B \rangle \geq 39 \), since to produce the present values for the orbit parameters, one requires a much higher velocity than can be obtained from a bound Kepler orbit at \( t = t_r \).

All the orbits calculated above use \( Q \propto a^{-2} \). We remarked above that the value of the exponent \( n \) may have changed from \( n = 2 \) early in the age of the Universe gradually to \( n = 3 \) as galaxies have formed, and so we repeat the exercise by substituting \( n = 2.5 \) in (13). We see that, as above, since the tide in this case is much stronger in the past, one fails to produce the present values of \( r_A, v_A \) for \( \langle M/L_B \rangle \geq 45 \) (Table 5).

4.3 The Magellanic Orbit

The Magellanic Stream is a long and narrow trail of high-velocity neutral hydrogen clouds stretching across the sky for more than 100° from the common envelope of the Magellanic Clouds to about (90°, −35°) via the SGP in a great circle. The Stream includes dwarf elliptical galaxies like Ursa Minor and Draco, and some of the high-velocity clouds in the northern galactic hemisphere are probably connected with it as well (for a plot see Lynden-Bell 1976). The orbit of the Stream lies in a plane that is perpendicular to the galactic plane, and can be determined to within a few degrees. Lin & Lynden-Bell (1982) determined the orbit of the barycentre of the Magellanic Clouds from the orientation of the Stream at the position of the Clouds, and found the direction of its angular momentum \( \mathbf{L}_{\text{MS}} \) to be towards \( l = 185°, b = 3° \).

From Table 5, it appears that in most realistic cases, the initial position of M31 is within \( \sim 5° \) of the orbit of the Magellanic Clouds about the Galaxy. Table 5 shows that the angular momentum of the Galaxy–Magellanic Stream system is dominated by \( \mathbf{L}_{\text{MS}} \). When M31 had been close enough to cause a significant tide, that tide was therefore so oriented as to generate orbital angular momentum of the Magellanic Clouds within \( \sim 5° \) of the observed orbital plane.

In this picture, the total spin of the Local Group

\[ \mathbf{J}_A + \mathbf{J}_G + \mathbf{L}_{\text{MS}} + \mathbf{L}_{\text{M33}} + \mathbf{L}_A = \mathbf{J}, \tag{17} \]

where the net angular momentum \( \mathbf{J} \) comes from the driving torque \( \mathbf{\Gamma} \) due to external galaxies, where \( \mathbf{\Gamma} = -\mathcal{M} (\mathbf{r}_A \times \mathbf{Q} \mathbf{r}_A) \). From this, we can go on step further and try to say something about the orbit of M33. We know the directions of \( \mathbf{J}_G, \mathbf{J}_A \) and \( \mathbf{L}_{\text{MS}} \) to within a couple of degrees, but do not know their relative magnitudes very well. Furthermore, since in our estimate in Section 4.2 we have used \( \mathbf{L}_A = \mathbf{J} \), we cannot estimate \( \mathbf{L}_{\text{M33}} \) from (17). We can, however, remark that \( \langle \mathbf{J}_A + \mathbf{L}_{\text{M33}} \rangle \cdot \langle \mathbf{r}_A \rangle = 0 \), if we believe that the Galaxy was responsible for the orbital angular momentum of M33 about M31, in the same way as M31 caused the orbit of the Magellanic Clouds.

4.4 Angular Momentum and the Local Group

For the unperturbed case, i.e. when \( \mathbf{F} = 0 \), the solution to (13) is the rectilinear Kepler orbit (6). The angular momentum in this case, of course, is \( \mathbf{h} = 0 \) and the energy of the system is conserved:

\[ E_0 = \frac{1}{2} r_0^2 \frac{GM}{r_0} = -\frac{1}{2} \frac{GM}{a} \]

In the presence of \( \mathbf{F} \), however, the orbit gains energy from the torque-inducing tidal field, and the angular momentum has a non-zero finite value at any instant of time. From (8b), (9) and (11), therefore,

\[ \frac{d}{dt} \mathbf{h}(t) = \frac{\mathbf{r}_A(t) \times \mathbf{Q}_0 \cdot \mathbf{r}_A(t)}{a^3(t)} \tag{18} \]

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
With $n = 2$ as is adopted in this paper, $h$ starts growing as $h \sim t$. In the later stages of this orbit, its Kepler nature dominates, and $r_A$ becomes progressively slower than the Hubble flow, reaches a stage of maximum expansion and then turns around. The value of $h$ begins to converge only when $r_A(t)$ ceases to increase faster than $t$ to the power of $(n/3 - 1/2)$, i.e. $t^{1/6}$ in this case.

From our ‘preferred’ orbit (Table 5), $v_T = 38$ km s$^{-1}$ at the present epoch, and so the present value of $|h| = 0.26$. Therefore, the net orbital angular momentum $|J| = 0.48$. Fig. 6 shows the growth of $h$ with time for this case, along with the angle of swing of M31 on the sky as a function of time.

One would like to compare this value with those of the spin angular momentum of galaxies within the Local Group (Table 4). We find that the values of $J$, for both galaxies are two orders of magnitude lower in magnitude than the value of $|J|$ quoted above. This is partly due to the fact that the exponential disc models of the Galaxy and of M31 give their disc masses as $3.9 \times 10^{10} M_\odot$ ($M/L_B = 2.1$) and $7.8 \times 10^{10} M_\odot$ ($M/L_B = 1.7$), respectively, whereas the masses of the two galaxies used above yields $1.4 \times 10^{12} M_\odot$ ($M/L_B = 75$) and $2.8 \times 10^{12} M_\odot$ ($M/L_B = 61$) respectively. If, as a way of comparison, one uses the exponential disc masses, the value of $|J|$ still turns out to be an order of magnitude higher than the spin angular momenta of individual objects (Table 4).

The value of the dimensionless spin parameter

$$\Lambda = \frac{h |E|^{1/2}}{G \mathcal{M}^{3/2}} = \frac{\mu^{1/2}}{\mu + 1} \frac{h}{G \mathcal{M}} \left[ \frac{G \mathcal{M}}{r_0} \frac{1}{2} (v^2 + v^2) \right]^{1/2} = 0.09,$$

where $\mu = M_A/M_G$ as before. This is consistent with the values obtained by Barnes & Efstatthiou (1987) for small groups of particles from numerical simulations.

Another interesting parameter is

$$\chi = \frac{E_B}{|E_p|} = \frac{h^2}{2 G \mathcal{M} r}$$

which is the ratio of the centrifugal energy to the energy due to the gravitational force between the two galaxies. For our ‘preferred’ orbit, $\chi_0 = 0.027$. Assuming that the orbit has ceased to

![Figure 6](https://academic.oup.com/mnras/article-abstract/240/2/195/992074/16 December 2018)

**Figure 6.** The growth of the specific angular momentum of the two-body orbit of M31 and the Galaxy for the ‘preferred orbit’ (Table 5). The dashed line shows the angle of swing of the binary orbit from its initial direction, as a function of time.

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
gain angular momentum any further, only $|E_p|$ increases in the later stages of the orbit, and $\chi = 1$ when the centre of M31 is at a distance $r_A = 19 \text{ kpc}$ from the centre of the Galaxy. The radial component of the motion of M31 towards the centre of the Galaxy would turn around just before reaching this point (indeed, if the orbit were parabolic, then the pericentre would have been at $\chi = 1$). This leaves us with the rather discomforting conclusion that most of the inner parts of the Galaxy, including the Sun, might not be saved from M31 after all, and the possibility of a merger in another four billion years cannot be ruled out.

5 Conclusions

In order to estimate the tidal torque exerted on the Local Group by external galaxies, we have evaluated the quadrupolar component of the gravitational field at the barycentre of the Local Group, due to galaxies within 12 Mpc of the Sun. To do this, a well-defined sample of nearby galaxies has been compiled, for which the eigenvalues and eigenvectors of $Q_{ij}$ are evaluated. It is shown that the chosen sample is reasonably complete, and the inclusion of galaxies lying further away or in the zone of avoidance does not introduce any significant correction. The quadrupole tensor consists of an almost equal tidal extension and compression along two of its axes, and of virtually no effect along the third. The directions of the eigenvectors are related to the distribution of galaxies belonging to the Coma–Sculptor cloud, which lie in a well-defined plane (Fig. 1). The ‘negative’ axis is perpendicular to this plane, and is thus determined to within a degree. The other two axes are probably known to within $\sim 5^\circ$, and the ‘positive’ axis is almost aligned with the dipole moment of the field due to galaxies within 8–10 Mpc.

The dynamical behaviour of the binary orbit of the Galaxy and M31 in the presence of this tidal field due to the galaxies outside the Local Group is investigated. Our analysis is based on the following assumptions: (i) that the tidal force is weak compared to the gravitational pull between the two galaxies; (ii) that the principal axes of the quadrupole remain fixed with time, due to the uniform and isotropic expansion of the Universe, and (iii) that in the immediate vicinity of the Local Group, the local value of $\Omega_0 = 1$.

The principal conclusions are as follows:

(i) The orbital motion of M31 with respect to the Galaxy is due to the tidal torque exerted on the two galaxies by galaxies lying outside the Local Group.

(ii) The inclusion of this external torque in the dynamics of the orbit does not affect the value of $M_{LG}$ obtained from the timing argument.

(iii) Imposing the condition that the $M/L_B$ of the Local Group is the same as the average $\langle M/L_B \rangle$ of neighbouring galaxies, it turns out that the mass of the Local Group is $4.3 \times 10^{12} M_\odot$, and that M31 had emerged from the Big Bang along $l = 101^\circ, b = -27^\circ$.

(iv) The orbital motion of the Magellanic Clouds (and possibly the Magellanic Stream) around the Galaxy is due to M31, which exerted a substantial torque on the Clouds in the initial phase of its orbit.

6 Prediction

The proper motion of M31 is $0.25 \text{ km s}^{-1} \text{ kpc}^{-1}$ ($5.3 \times 10^{-5} \text{ arcsec yr}^{-1}$) towards $(-0.733, -0.595, -0.334)$ in galactic coordinates. This is well below the limits of observation at the present moment. Its components on the sky are $\mu_\phi = -1.9 \times 10^{-5} \text{ arcsec yr}^{-1}$ towards the galactic north, and $\mu_\ell = 5.0 \times 10^{-5} \text{ arcsec yr}^{-1}$ in the direction of the increase of $l$. 

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
Acknowledgments

We are grateful to the referee, James Binney, whose very constructive criticism of an earlier version of this paper has helped to improve it considerably. We also wish to thank Renée Kraan-Korteweg for supplying us with updated tape/printed versions of her catalogue of nearby galaxies, and Ofer Lahav for useful discussions. SR is supported by an Isaac Newton Studentship.

References


© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
Appendix: Correction for extinction in the Galaxy and for galaxies hidden behind its plane

Consider a function $Q(\theta, \phi)$ of a distribution of mass-points, each point being weighted by the mass associated with it. Expanding in terms of spherical harmonics,

$$Q(\theta, \phi) = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} (-1)^m [a_{lm} \cos m\phi + b_{lm} \sin m\phi] P_l^m(\cos \theta)$$  \hspace{1cm} (A1)

where, ideally, $n \to \infty$. In the galactic system of coordinates, $\theta = 90^\circ - b^\circ$, $\phi = l^\circ$.

When one observes this distribution through an absorbing medium (the 'mask'), $W(\theta, \phi)$, the observed multipole coefficients are given by

$$\hat{a}_l(\theta) = \frac{2l+1}{4\pi} \int_0^{2\pi} Q(\theta, \phi) W(\theta, \phi) P_l(\cos \theta) d\Omega$$

$$\hat{b}_l(\theta) = \frac{2l+1}{4\pi} \int_0^{2\pi} Q(\theta, \phi) W(\theta, \phi) P_l^m(\cos \theta) \cos m\phi d\Omega$$

$$\hat{b}_l(\theta) = \frac{(2l+1)(l-m)!}{2\pi(l+m)!} (-1)^m \int_0^{2\pi} Q(\theta, \phi) W(\theta, \phi) P_l^m(\cos \theta) \sin m\phi d\Omega.$$  \hspace{1cm} (A2)

In the context of this paper, we are concerned with the quadrupole terms of this distribution. The observed coefficients can be obtained from the positions $r_j$ of the galaxies in the sample and their masses $M_j$. As in Section 3, $M_j$ is a function of the apparent magnitude $m_j$ (corrected for internal absorption), the value of the extinction in the Galaxy $A_E$, and the adopted distance $r_j$, together with an assumed $M/L_B$ ($=20$, the same as in Section 3).

The 'mask' through which we are looking at the sky here consists of an opaque strip for the region $-10^\circ \leq b \leq 10^\circ$ and a medium of absorption as estimated by Burstein & Heiles (BH, 1976) elsewhere. Since the evaluation of the integrals (A2) using the discrete BH values $A^\text{BH}$ proves difficult, we have used a continuous approximation of the BH values as given by Fisher & Tully (FT, 1981),

$$A^\text{FT} = -0.149 + 6.41 \times 10^{-4} N_h$$

where

$$N_h = 323 \sec \theta$$

$$= 323 \sec \theta + [3.8\theta - 89.0 \cos(\phi - 140^\circ) - 237.0]$$

when $\theta > 100^\circ$

and $\theta$, $\phi$ are in degrees. Since we use the FT values in place of the BH mask which we assume to be correct,

$$\Delta A_{E,j} = A^\text{BH} - A^\text{FT}$$

is used in the evaluation of the observed coefficients.

Finally, to obtain the coefficients of the actual distribution, one uses equations (A1) and (A2). In practice, the summation over $l$ is carried out up to $n=5$, and no significant contribution is seen beyond $n=3$. 

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System